Bayes' theorem

Problem:

A manufacture claims that its drug test shows positive for an athlete who uses steroids at 95% of the time.

Your friend on the football team has just tested positive. What is the probability that he uses steroids?

Bayes' theorem

E = positive test result

F = uses steroids

$P(E \mid F)$ = probability that the test is positive from an athlete who uses steriods

$P(F \mid E)$ = probability that an athlete uses steriods given that the test is positive

Bayes' theorem

Question:

What information do we need to calculate

$P(F \mid E)$ from a knowledge of $P(E \mid F)$?

Bayes' theorem

A manufacturer claims that its drug test will detect steroid use (that is, show positive for an athlete who uses steroids) 95% of the time.

Bayes' theorem

15% of all steroid-free individuals also test positive (the false positive rate). **Assume 10% of the football team members use steroids.**

Take

E = positive test result

F = uses steroids

Bayes' theorem

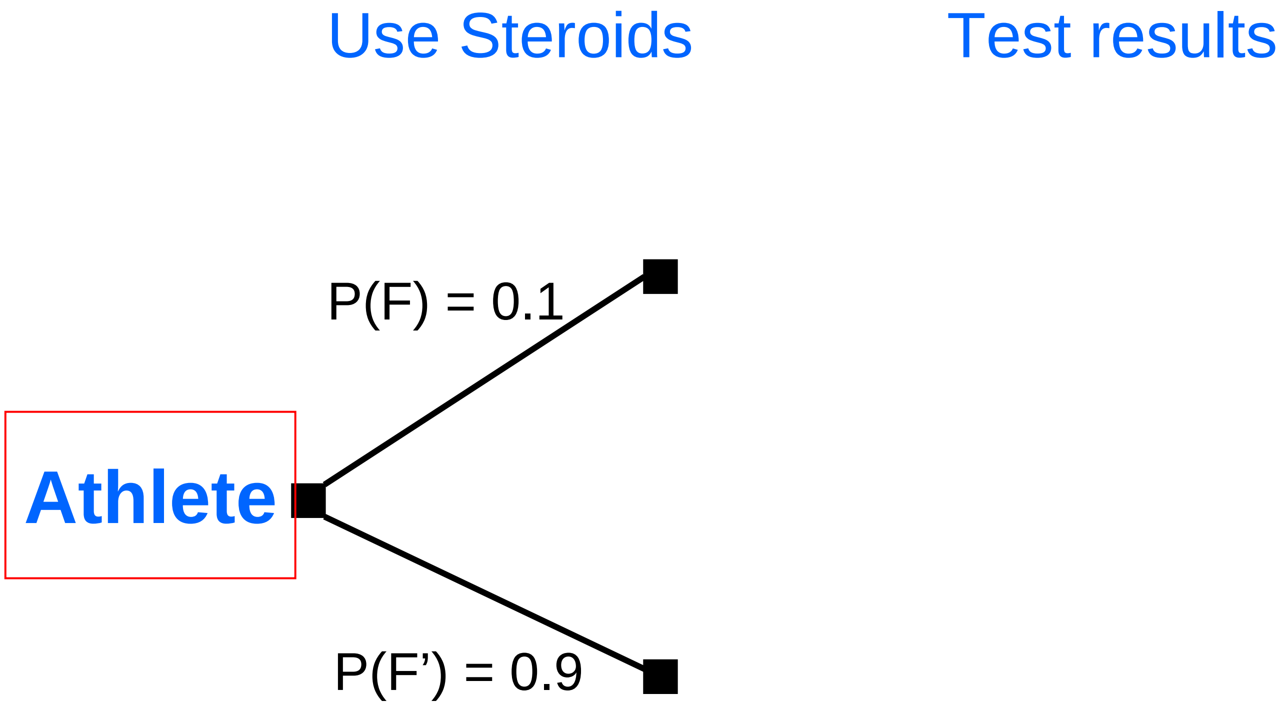
Based on the above information, please calculate

$$P(F) = 0.1 \qquad P(F') = 0.9$$

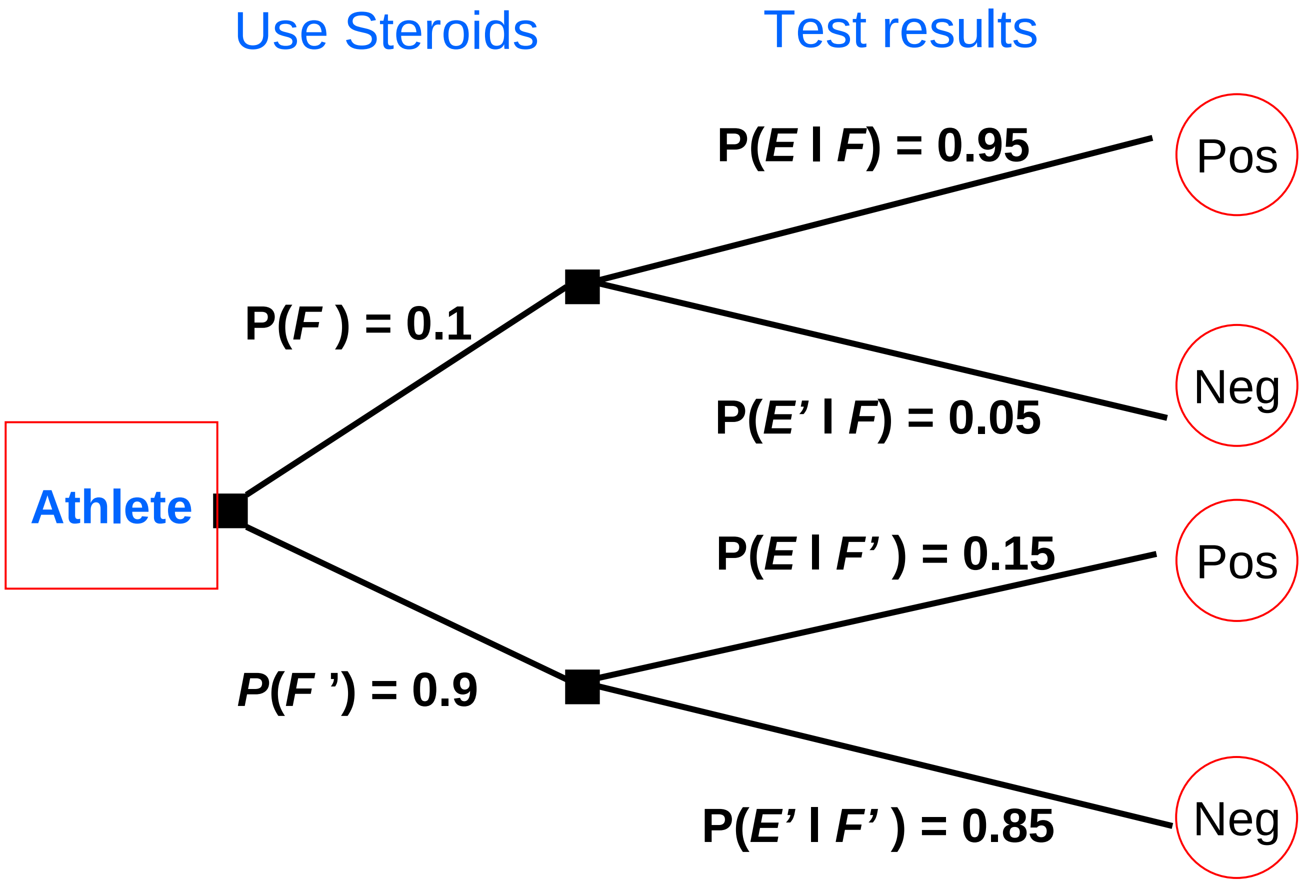
$$P(E|F) = 0.95 \qquad P(E'|F) = 0.05$$

$$P(E|F') = 0.15 \qquad P(E'|F') = 0.85$$

Tree Diagrams



Tree Diagrams



Bayes' theorem

E = positive test result

F = uses steroids

$$P(F \mid E) = \frac{P(E \mid F)P(F)}{\boxed{P(E)}}$$

$$= \frac{P(E \mid F)P(F)}{\boxed{P(E \mid F)P(F) + P(E \mid F')P(F')}}$$

In our example, we consider

$P(E|F) = 0.95 \qquad P(E|F') = 0.15 \qquad P(F) = 0.1 \qquad P(F') = 0.9$

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F')P(F')}$$

$$= \frac{(0.95)(0.1)}{(0.95)(0.1) + (0.15)(0.9)} \approx 0.4130$$

Confusion of the Inverse

Example*: *Diagnostic Testing

- Confuse *P* (Use steroids | Positive), with *P* (Positive | Use steroids): known as the *sensitivity* of the test

- Often forget to incorporate the *base rate* for a disease.