新高中數學課程學與教策略：統計的新重點

統計的學與教—「區間估計」

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2006年4月

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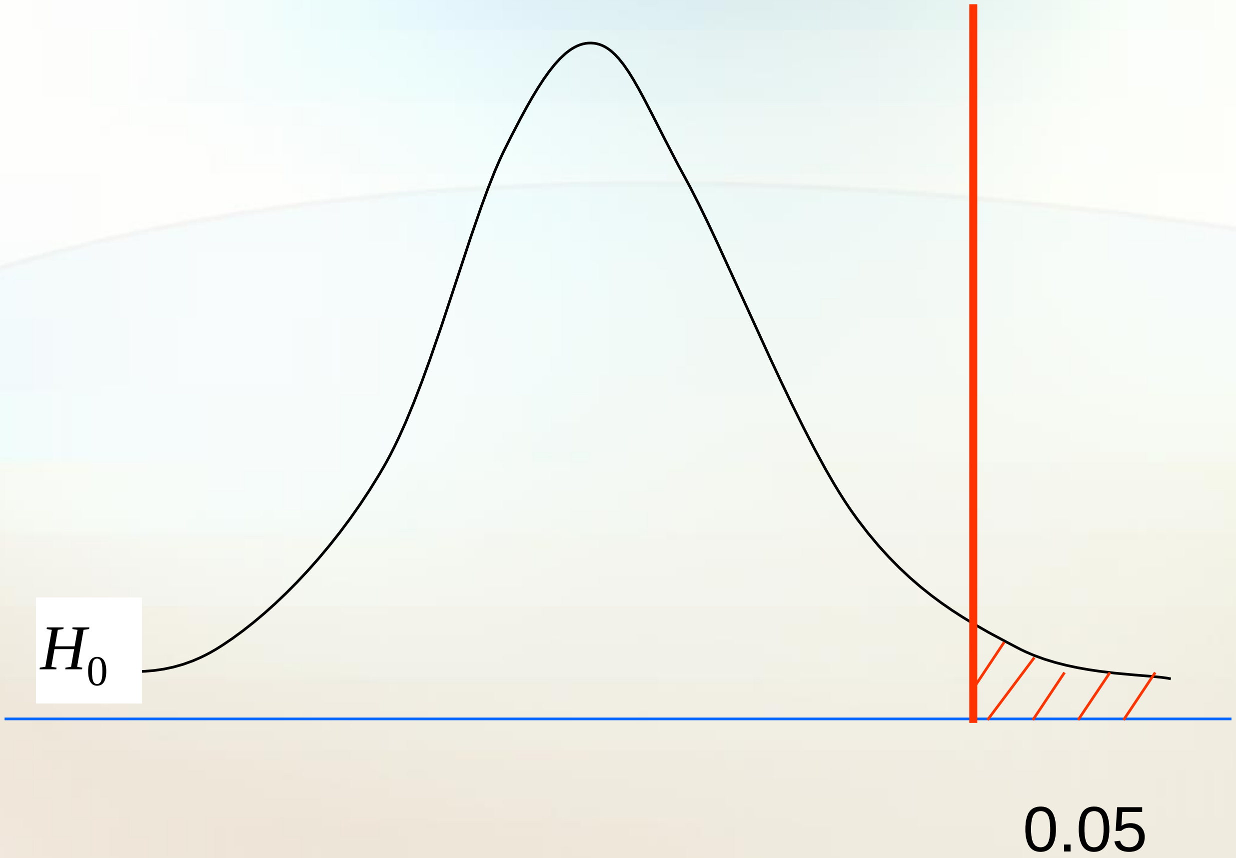
1.5假設檢驗的困境


H_0
0.05

1.5 假設檢驗的困境

- α、 顯著水平(significance level)、 第一類誤差 (Type I error)
- 統計上顯著性(Statistical significance) vs 實際上重要性(Practical significance)
- $P(H_0|D)$ vs $P(D|H_0)$
- 虛假設(Null hypothesis) *必然為假*
- H_0 拒收 (reject)與否，與 *n*的值之大小有關

1.5 假設檢驗的困境

- Fisher hypothesis testing vs Neyman-Pearson significance test
- 如果H_0真，則D出現的概率很低

  D真的出現了
  ____

  H_0為真的概率很低
- APA, Wilkinson & Task Force on Statistical Inference, 1997
- Cohen, 1994; Harlow, Mulaik, & Steiger (Eds.), 1997; Kline, 2005; Morrison, & Henkel (Eds.), 1970.
- *Mathematics Education Research Journal* 5(1) 1993

2.認識置信區間的意義

2.1 從樣本平均值推算總體平均值

- 點估計→區間估計
- 總體是正態分佈（方差為σ^2）

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

- 無論總體分佈如何（方差為σ^2），當n 足夠大時，根據中心極限定理，

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

探索與研究

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$P(-z_{\frac{\alpha}{2}} < z < z_{\frac{\alpha}{2}}) = 1 - \alpha$$

$$P(-z_{\frac{\alpha}{2}} < \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < z_{\frac{\alpha}{2}}) = 1 - \alpha$$

$$P(\overline{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}) = 1 - \alpha$$

$$(\overline{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \ , \ \overline{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$$

為 總體平均數(參數)的(1 – α)100%置信區間(CI)

當未知總體分佈的方差σ^2時，

用樣本方差S^2，作為σ^2的估計量

但無論總體分佈是否正態，$\dfrac{\overline{X}-\mu}{\dfrac{S}{\sqrt{n}}}$ 都不是正態分佈

若總體分佈是正態分佈

t 分佈

$$\left(\overline{x}-z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\ ,\ \overline{x}+z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right)$$

n 越大，t 分佈越接近正態分佈

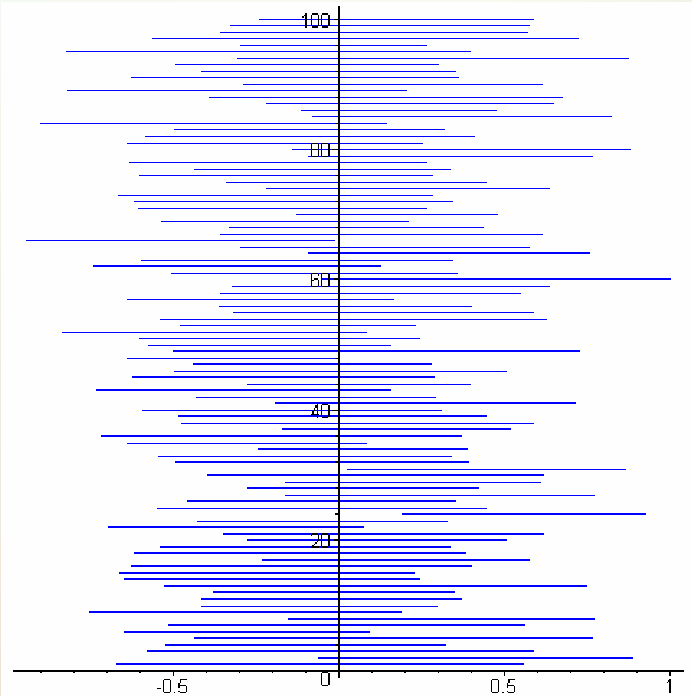
為 總體平均數(參數)的(1 – α)100%置信區間(CI)

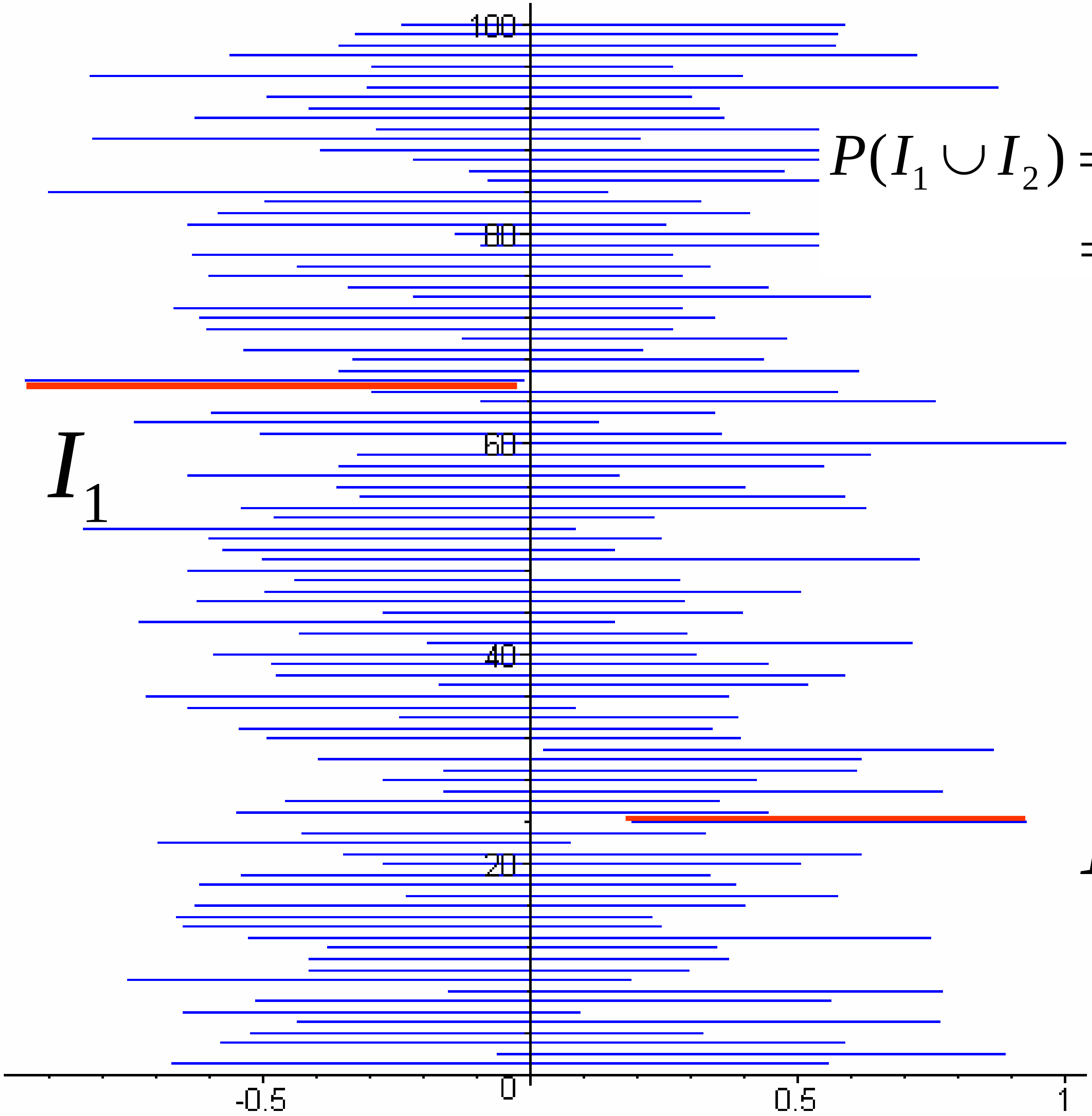
μ 落在區間 $\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} , \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right)$ 的概率為 $1-\alpha$？

參數（固定）

$$P\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right) = 1-\alpha$$

隨機





$$P(I_1 \cup I_2) = P(I_1) + P(I_2)$$
$$= 0.95 + 0.95 > 1$$

I_1

I_2

對置信區間的誤解

- '… perhaps the most obvious difficulty with confidence intervals lies in how we interpret what the confidence statement means' (Smithson, 2003, p.16)
- 大學生及研究者的誤解 (Fidler and Cumming, 2005; Cumming, Williams & Fidler, 2004)

2.2 確定總體比值(population proportion/ratio)的置信區間

- p 為二項分佈 的比值，樣本統計量 $\hat{p}=\frac{X}{n}$

- 根據中心極限定理，當 n 很大時， $\hat{p}$ 的分佈接近於正態分佈，且其平均值和方差分別為

$$\begin{aligned}\mu_{\hat{p}} &= \mathrm{E}(\hat{p}) \\ &= \mathrm{E}(\frac{X}{n}) \\ &= \frac{1}{n}\mathrm{E}(X) \\ &= \frac{np}{n} \\ &= p\end{aligned}$$

$$\begin{aligned}\sigma_{\hat{p}}^{\ 2} &= \sigma_{\frac{X}{n}}^{\ 2} \\ &= \frac{1}{n^2}\sigma_X^{\ 2} \\ &= \frac{1}{n^2}[np(1-p)] \\ &= \frac{p(1-p)}{n}\end{aligned}$$

$$P\left(-z_{\frac{\alpha}{2}} < \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} < z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

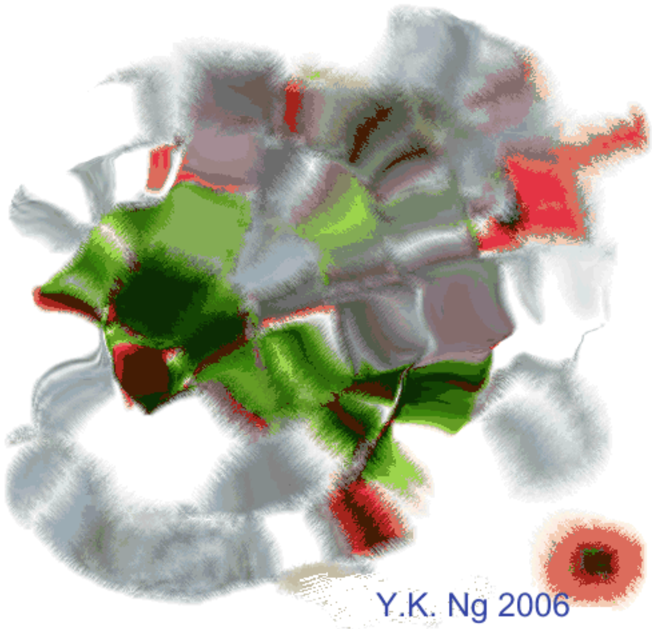
$$P\left(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right) = 1-\alpha$$

p 的 (1-α)100%　置信區間:

$$\left(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}} \quad , \quad \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right)$$

當 n 很大時，

$$\left(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad , \quad \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$



完

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