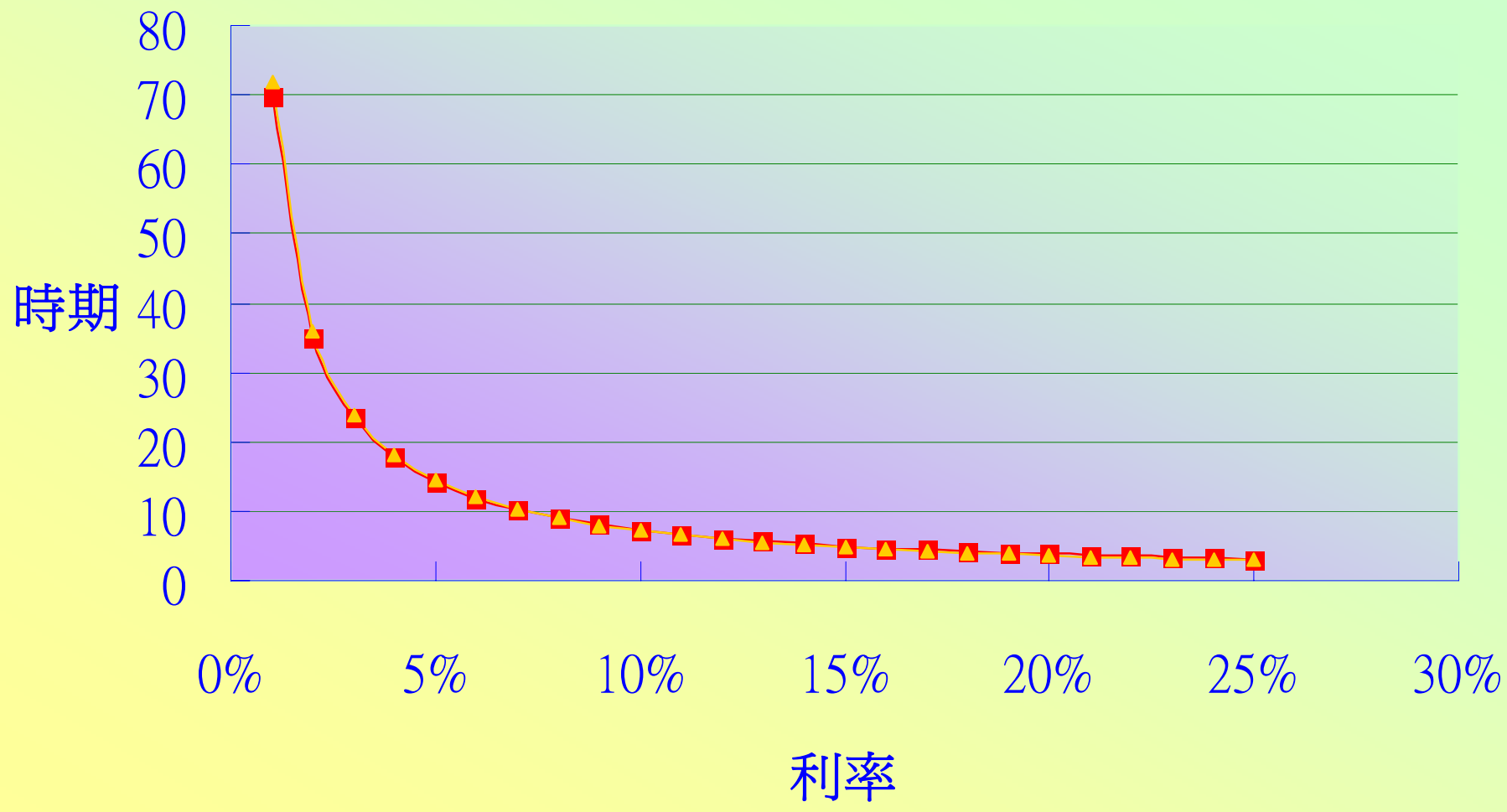


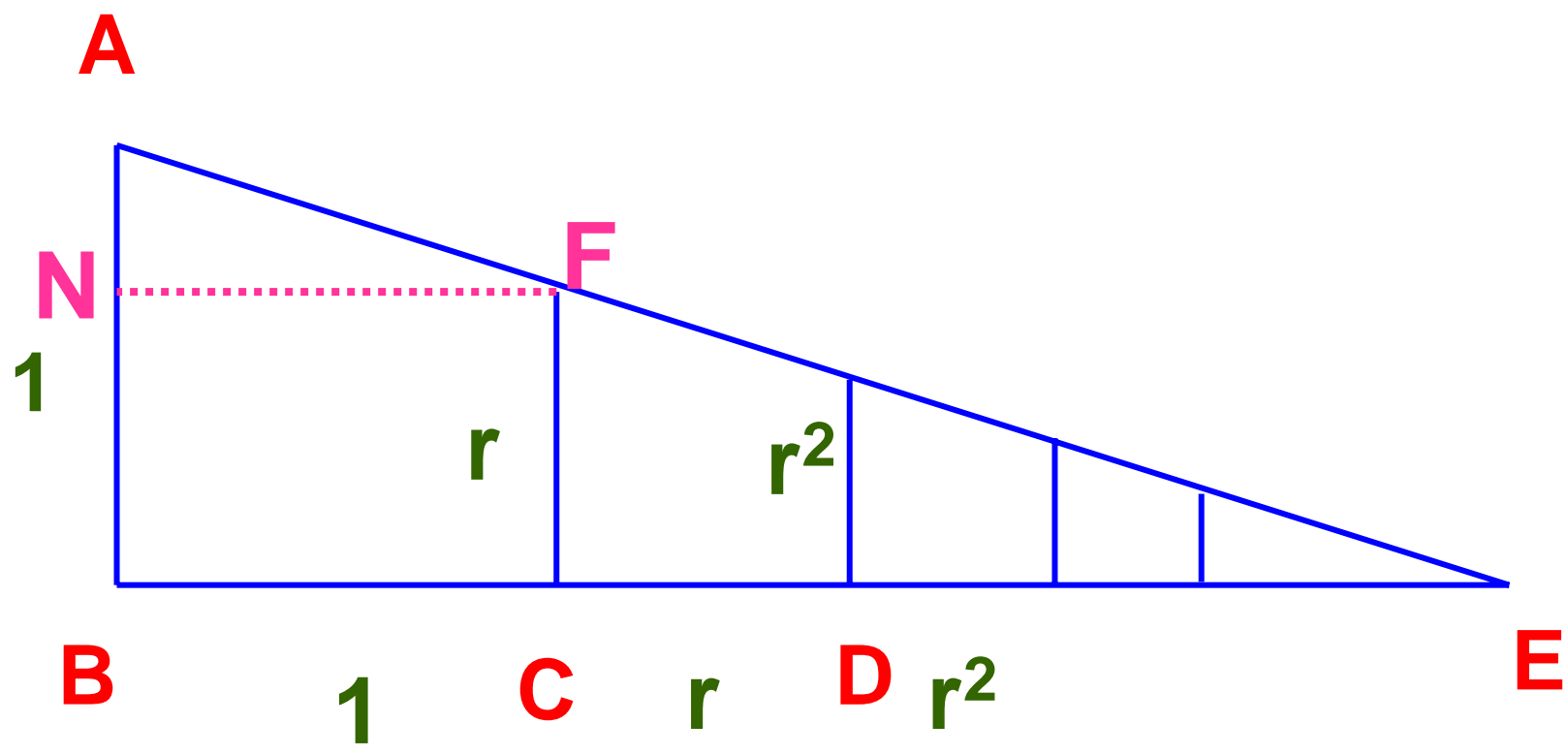
趣味數學

數與代數



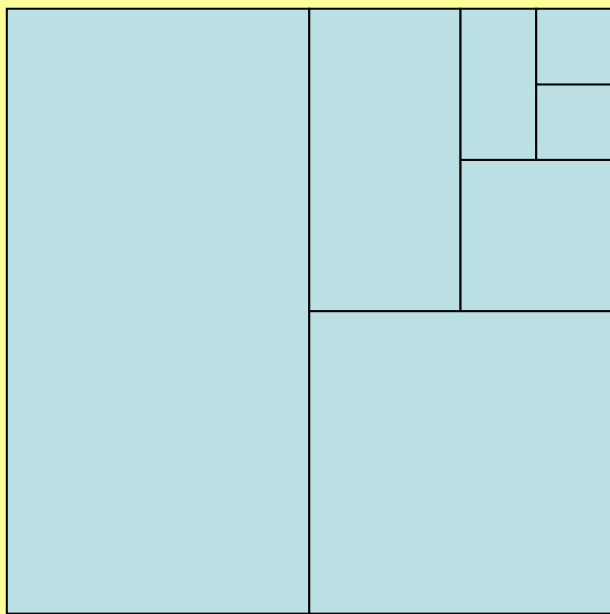
年利率	實際結果	72 法	年利率	實際結果	72 法
1	69.66	72	9	8.04	8
2	35.00	36	10	7.27	7.2
3	23.45	24	15	4.96	4.8
4	17.67	18	20	3.80	3.6
5	14.21	14.4	40	2.06	1.8
6	11.90	12	50	1.71	1.44
7	10.24	10.29	100	1.00	0.72
8	9.01	9			

當 $5\% < r < 15\%$ 時，72 法提供了一個很好的近似值。



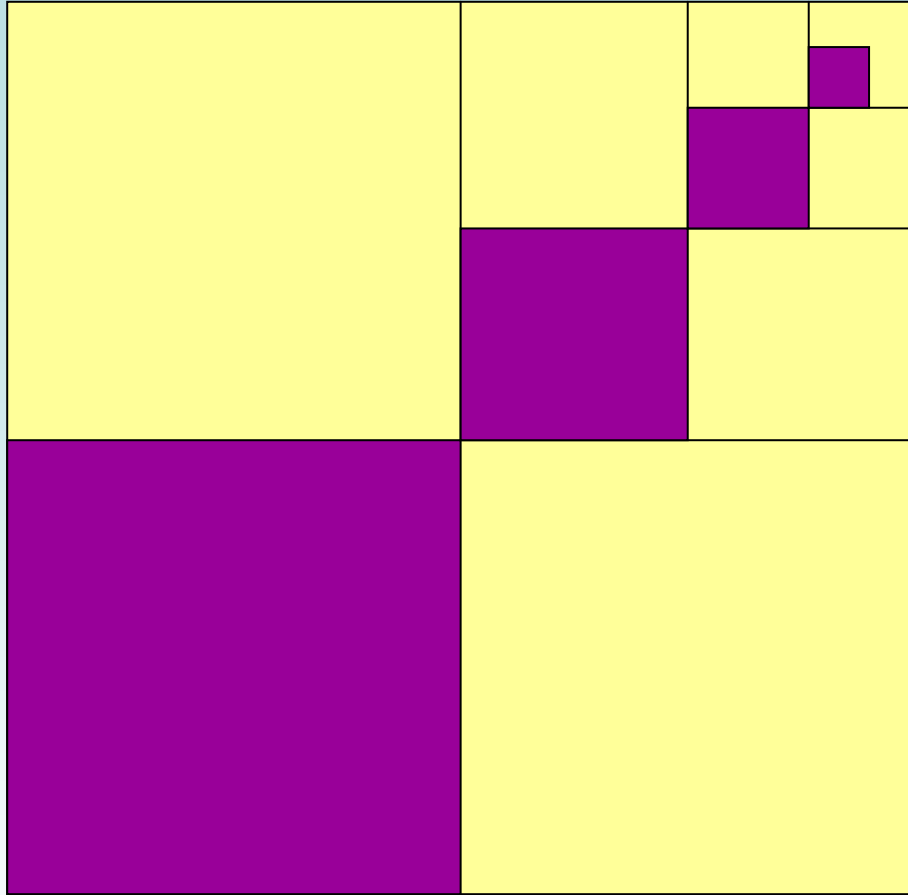
$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \dots$$

1



邊長一單位的正方形，其面積為一平方單位。

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots = 1$$



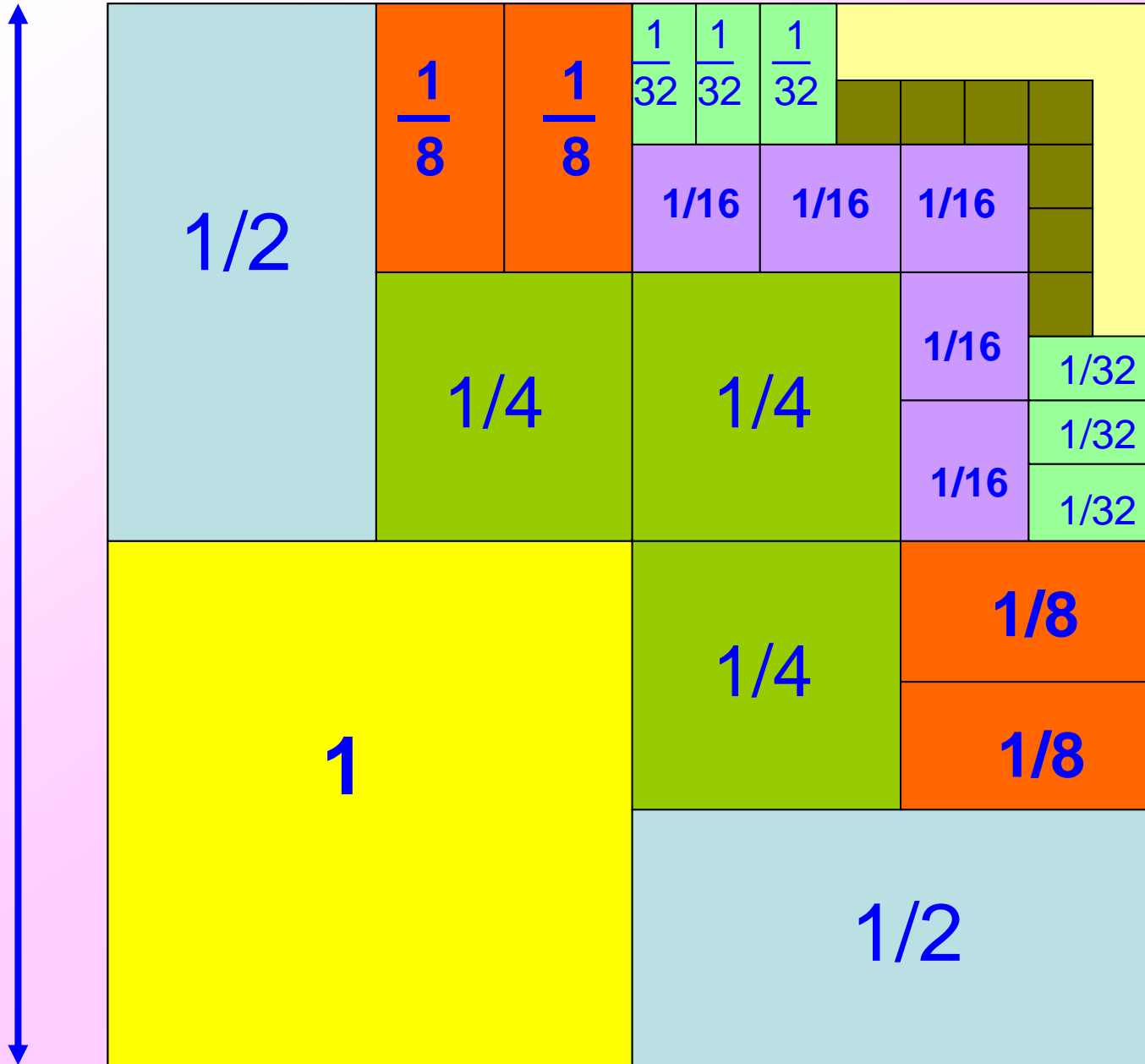
$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots = \frac{1}{3}$$

我們也可以用類似方法得到下列公式：

$$a + ar + ar^2 + \dots = \frac{a}{1-r}$$

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right) + \dots = 4$$

2

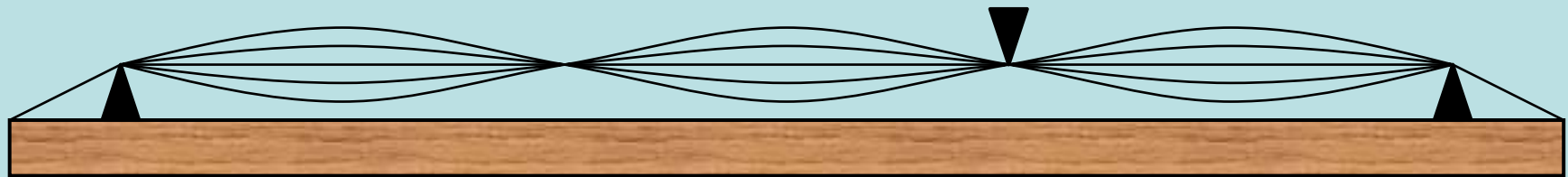


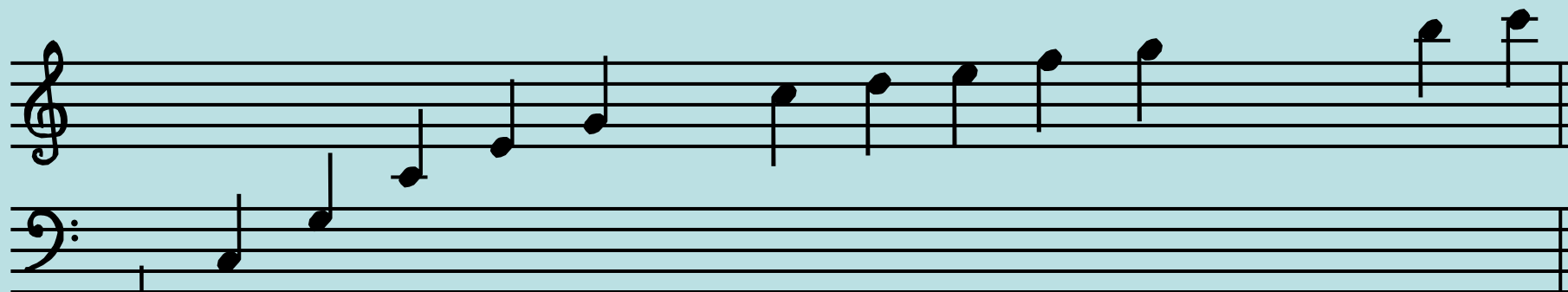
我們也可以用類似方法得到下列公式：

$$1 + 2r + 3r^2 + 4r^3 + \dots = \left(\frac{1}{1-r} \right)^2$$

物理、數學與音樂

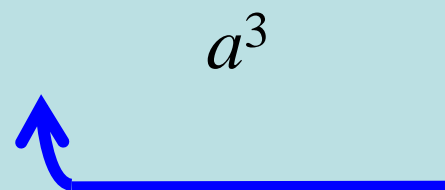
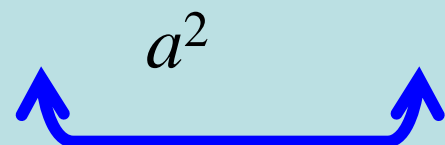
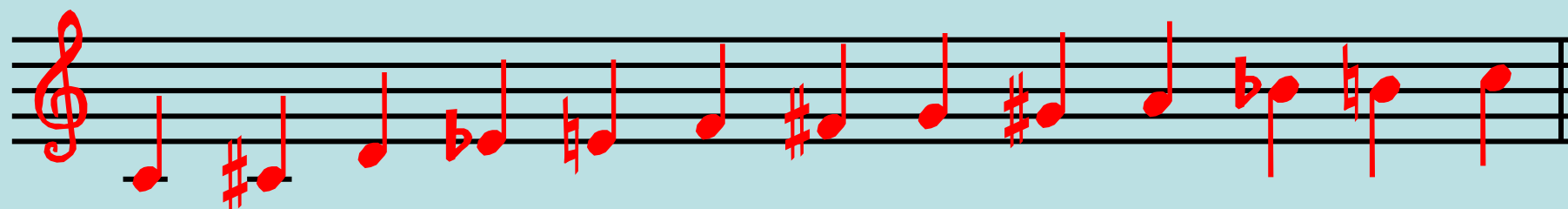
泛音 (Overtone)





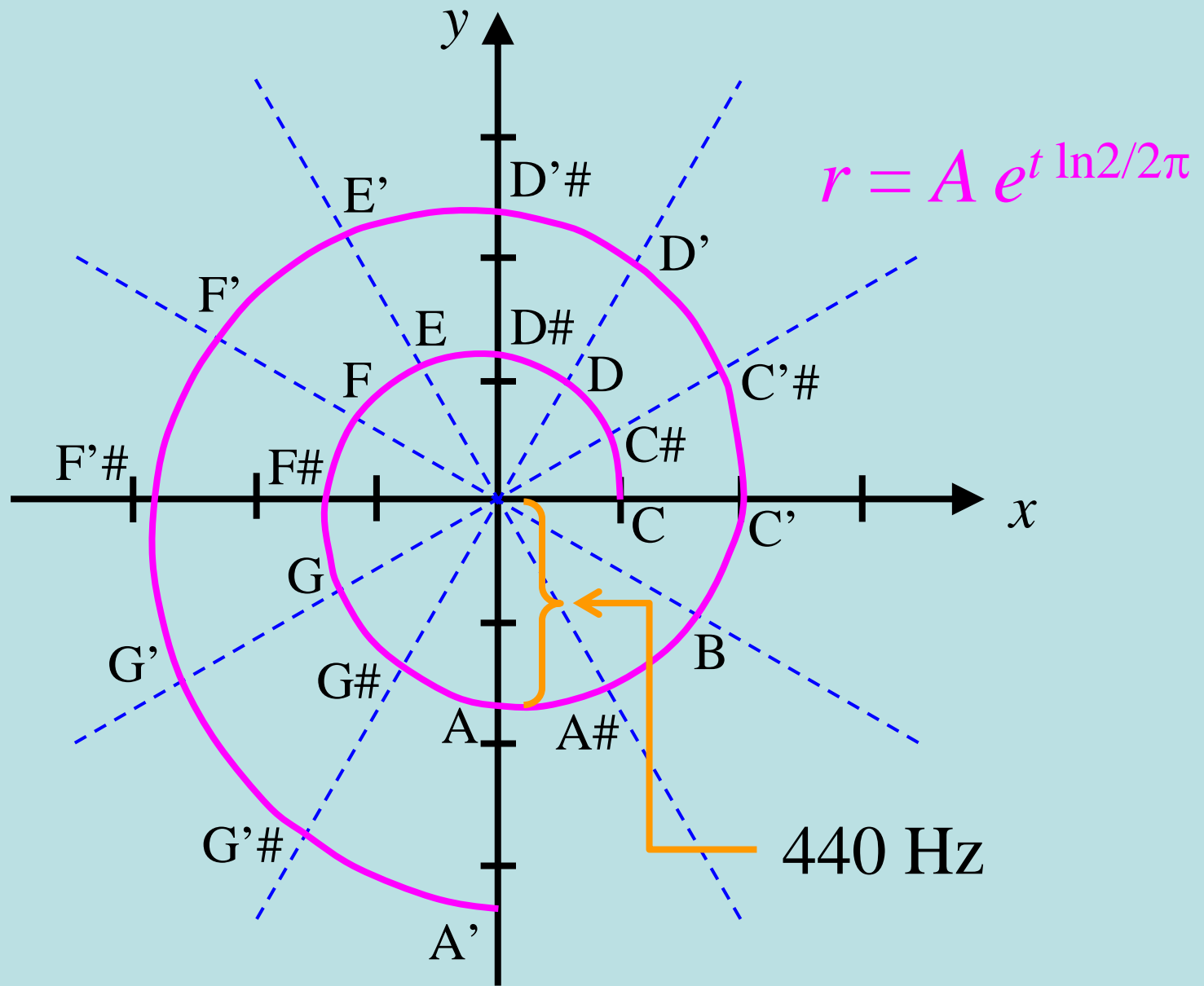
1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$
						$\frac{9}{8}$	$\frac{16}{15}$	$\frac{9}{8}$		$\frac{16}{15}$	$\frac{15}{16}$

注意： $\frac{9}{8} \times \frac{16}{15} = \frac{6}{5} = \frac{12}{10}$

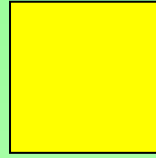


$$a^{12} = 2$$

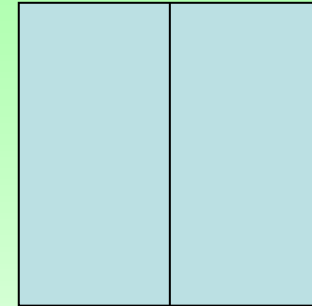
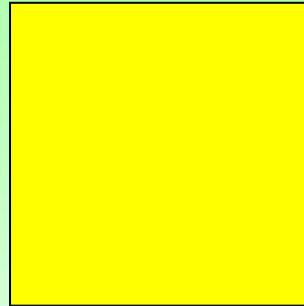
$$\therefore a = \sqrt[12]{2}$$



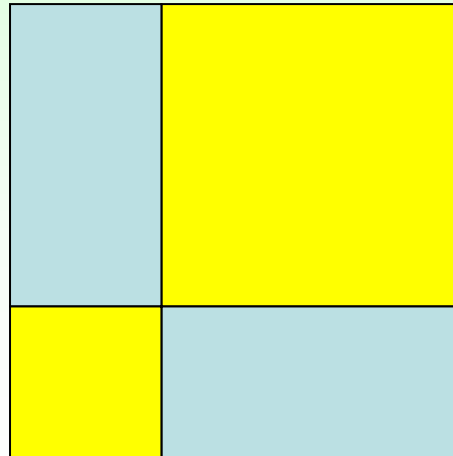
$$1^3 = 1 \times 1^2$$

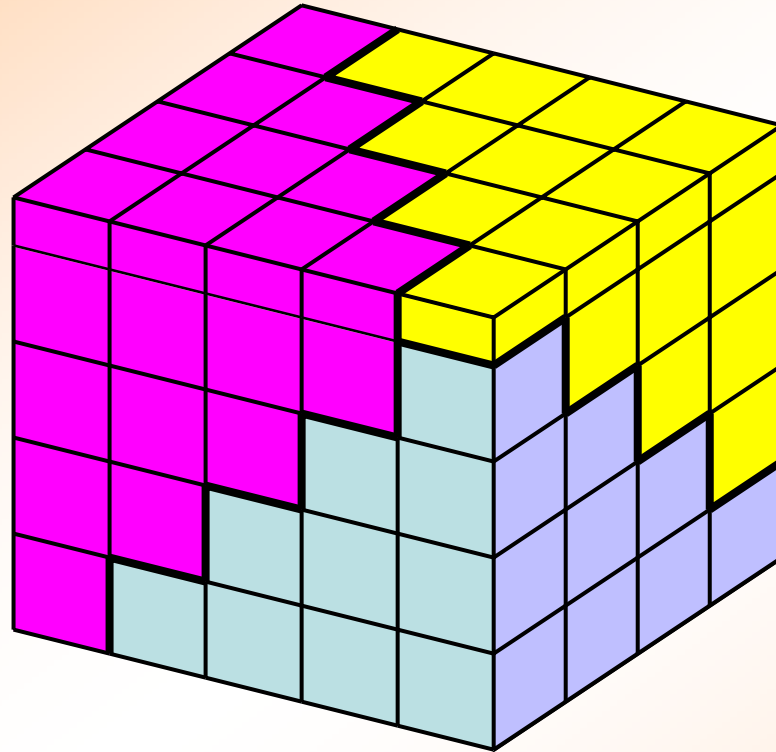


$$2^3 = 2 \times 2^2$$



$$1^3 + 2^3 = (1+2)^2$$



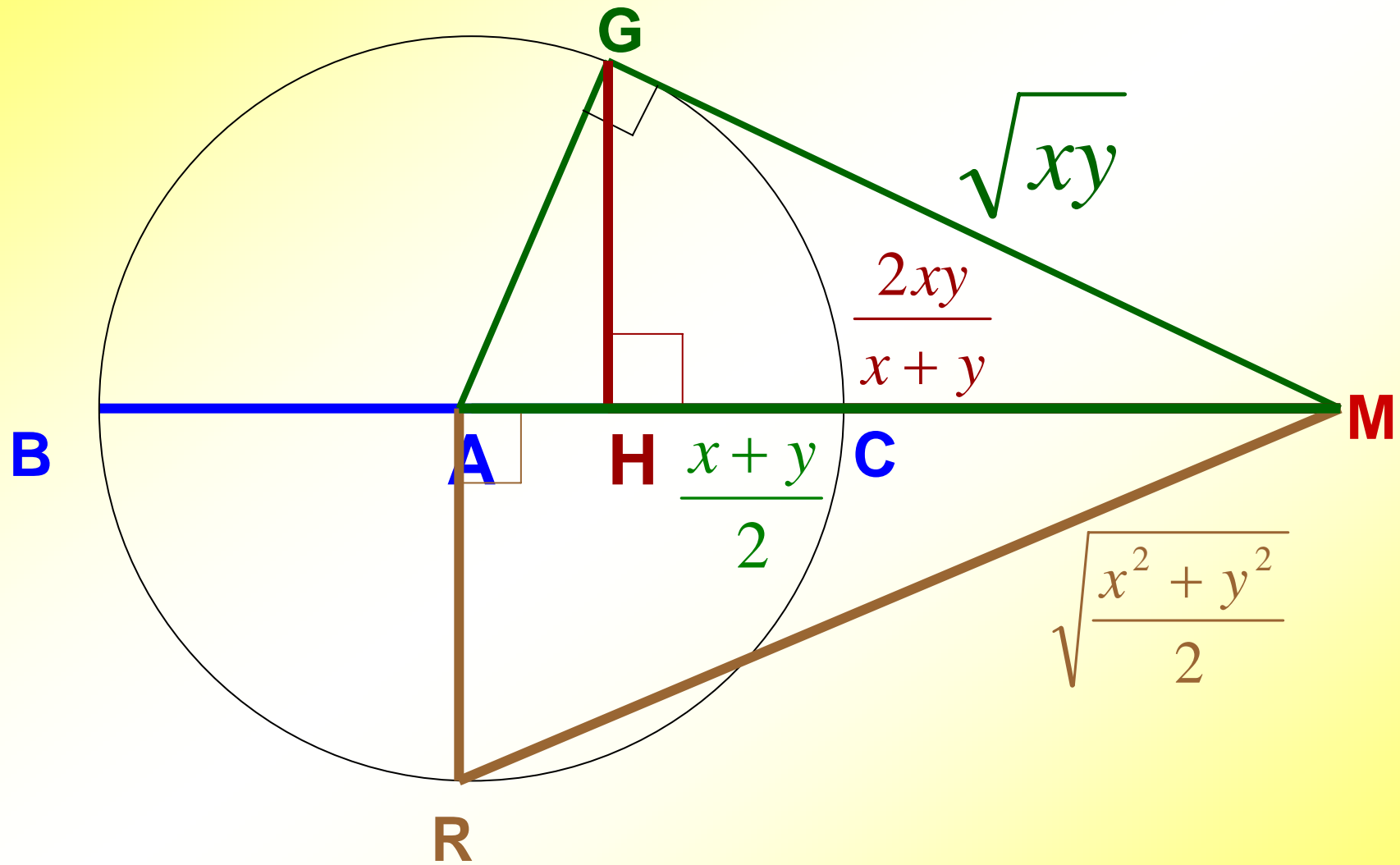


$$1^2 + 2^2 + \dots + n^2$$
$$= \frac{1}{6} n (n+1) (2n+1)$$

當 x 和 y 不是負數時:

$$\frac{x + y}{2} \geq \sqrt{xy}$$

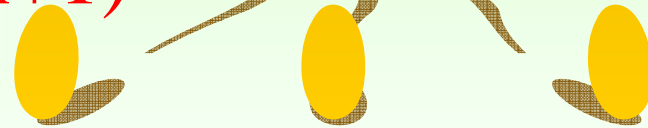
$BM = x, CM = y$



$$1+2+3+4+\dots+(n-1)+n+(n-1)+\dots+2+1 = n^2$$

$$1+3+\dots+(2n-1)+(2n+1)+(2n-1)+\dots+3+1$$

$$= n^2 + (n+1)^2$$



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- 張景中 好玩的數學