57th IMO HONG KONG 2016

2016 年國際數學奧林匹克通訊　IMO 2016 NEWSLETTER

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2016 年第五十七屆國際數學奧林匹克籌備委員會
Organising Committee of the 57th International Mathematical Olympiad 2016

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2016 年第五十七屆
國際數學奧林匹克籌備委員會

香港將於 2016 年 7 月舉辦第五十七屆國際數學奧林匹克（IMO），迎接來自超過 100 個國家的中學生數學精英。希望《數聞》可在我們邁向 2016 年 IMO 期間帶動同學和公眾對數學的興趣，更希望這種氣氛歷久不衰。

歡迎讀者向《數聞》投稿。文章須為原著，以中文或英文寫成（或兩種文本兼備），長度為一至四頁（就一種語言而言），並以電郵附件方式傳送至 info@imohkc.org.hk，或郵寄至 九龍油麻地 405 號九龍政府合署 4 樓 403 室 教育局數學教育組《數聞》編輯，標題為「Submission to IMOment」。

Hong Kong is proud to be organizing the brightest secondary school mathematics talents from over 100 countries at the 57th International Mathematical Olympiad (IMO) in July 2016. We hope that IMOment will promote interest in mathematics among students and the public in this period leading up to IMO 2016, and beyond.

Readers are welcome to submit articles on mathematics and/or Mathematical Olympiad to IMOment. Submissions should be original, one to four pages in length in either Chinese or English (or both), and should be sent by attachment to an email to info@imohkc.org.hk, or be mailed to Rm . 403, 4/F, Kowloon Government Offices, 405 Nathan Road, Kowloon, titled "Submission to IMOment."

Organising Committee of the 57th International Mathematical Olympiad 2016

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WWW.IMO2016.ORG

ALICE WONG: STRENGTH THROUGH CONFIDENCE
王詩雅：信心滿路途

盧安迪/ ANDY LOO

王詩雅畢業於拔萃女書院，曾代表香港參加 2012、2013、2014 年國際數學奧林匹克，共奪一銀兩銅，並於 2012 年中國女子數學奧林匹克勇奪滿分。她現就讀於美國麻省理工學院。

Alice Wong Sze-nga, a graduate of Diocesan Girls' School, represented Hong Kong at the International Mathematical Olympiad in 2012, 2013 and 2014, winning one silver and two bronze medals. She got a perfect score in the 2012 China Girls Mathematical Olympiad. She is now studying at the Massachusetts Institute of Technology in the USA.

盧 = 盧安迪　王 = 王詩雅

AL = Andy Loo　AW = Alice Wong

盧：詩雅，很久不見！不如我們談談你為甚麼喜歡數學？

AL: Long time no see, Alice! Let's talk about why do you like mathematics?

王：安迪，你好！我喜歡數學的優雅。一旦明白問題背後的原理，問題便迎刃而解，而這種通透的理解對學好數學是至關重要的。此外，不同數學範疇之間千絲萬縷的關聯也令我讚嘆。一條看似代數的題目，有時可以幾何方法解答。

AW: Hello Andy! I like the elegance of mathematics. Once you understand the principle behind the question everything becomes natural. The full understanding is crucial to excellence in this field. Moreover, the inter-connectedness of different areas of mathematics is particularly fascinating. A problem may appear as an algebraic one but can in fact be solved by geometric methods.

盧：原來如此！可否分享一下你參加數學奧林匹克的經歷？

AL: Oh I see! Tell us about your math Olympiad story.

王：我在小學四年級開始參加數學奧林匹克課程，自此便對奧數著迷。那些簡單而又巧妙的解題方法使我心往神馳。正是這種對數學的熱情，加上一點運氣，讓我嘗到成功的滋味。為了不斷提升自己，我向一個又一個挑戰進發。在這段路途上，固然有挫折和沮喪的時刻，但我沒有放棄。經過堅持不懈的努力，我終於踏上國際數學奧林匹克（IMO）的舞台。

AW: I started taking Mathematical Olympiad classes in fourth grade and became fascinated by this subject. The simple yet ingenious solutions were beyond my imagination. My enthusiasm in mathematics, along with some luck, earned me a taste of success. Fueled by the desire to achieve more, I challenged myself to more ambitious goals. During this journey, there were times of failure and discouragement. However, my persistence did not allow me to give up and eventually brought me to the International Mathematical Olympiad (IMO).

盧：作為一個女性奧數選手，有沒有特別的挑戰？你怎樣克服這些挑戰？

AL: Are there any particular challenges you have faced as a female math Olympian? How did you overcome them?

王：作為女生，我是奧數選手中的少數。不少人覺得男性比女性更擅長數學，但我並不接受這種性別定型。我相信兩性或有不同的思考模式，互補長短。當然，有人認為我不能在數學上取得成就。為了推翻這種誤解，我發奮圖強，最終取得IMO 的入場劵。我對我的成績感到自豪，我的成績打破了陳觀舊念。

AW: As a female Math Olympian, I often find myself among the minority in the Math Olympiad community. It is a common stereotype that males do better than females in mathematics. Personally I refuse to accept this. I believe females and males may have different thinking modes which can be complementary. Still it is no surprise that there were people who believed I wouldn't do great in mathematics. In order to prove them wrong, I studied hard and eventually earned a ticket to IMO. I am proud of what I have achieved, and it has disproved the ideological stereotype.

盧：你有沒有結識了很多其他國家的女性奧數選手？

AL: Did you meet many fellow female Math Olympians from other countries?

王：在不同的數學比賽中，我交到了不同國家的女性朋友。這種友誼彌足珍貴，因為把我們聯繫起來的不只是對數學的熱愛，更是向世界證明女性也可學好數學這個使命。更有趣的是，我跟一些在比賽中結交的女生現在成為了大學同學。這不但讓我們的友誼更進一步，也讓我意識到，抱著同一理想的女性可以團結合作，自信地立足世上。

AW: During different Math Olympiad contests, I have made friends with female competitors from other countries. Such friendship is particularly valuable, for what links us together is not only the passion for mathematics, but also the mission to prove to the world that the performance of females in mathematics can be as good as anybody else's. What surprises me more is to have become college classmates with these girls I met at contests. This not only helps us sustain our friendship, but also makes me realize girls with the same vision can indeed come together and face the world with confidence.

盧：如果要選你從奧數學到的最重要的東西，你會選什麼？

AL: What is the most important thing you have gained from Math Olympiad?

王：我從奧數學到堅持的重要性。追求卓越的道路往往佈滿崎嶇，更會不時絆倒。關鍵是我們能否重新站起，繼續前進。有時候我想了幾天都想不通一條題目。憑著堅持不懈的態度，我跟題目一直搏鬥下去。這種性格不單對數學比賽，甚至對生活的其他事情也很重要。

AW: The most important lesson I have learnt is the importance of persistence. The road to striving for excellence is not always smooth. One may be tripped over many times. What makes the difference is persistence would make one stand up and march forward again. Sometimes I can be stuck in a problem for a few days. Persistence grants me the motivation to continue working on it no matter what. This character applies not only to math contests but also general life matters.

「不要讓閒言閒語澆滅你心中的那團火。」

"Don't let the fire in your heart die just because of what others say."

盧：你將來有什麼志願？

AL: What are your future aspirations?

王：我希望將來運用我的知識，幫助少數群體──尤其是女性──發現自己的潛質，並明白到自己有能力戰勝刻板的定型，跟其他人一樣取得成功。

AW: In the future, I hope to apply my knowledge to help the minority, especially females, recognize what they are capable of and strive to eliminate inequality caused by unjust stereotypes.

盧：有不少對數學感興趣的女生，可能都視你為榜樣。你有什麼想跟她們說？

AL: Many female students interested in mathematics may look up to you as a role model. What would you like to say to them?

王：的確，我們是數學界中的少數。我們有時會被看淡，但最重要的是對自己有信心。不要讓閒言閒語澆滅你心中的那團火。

AW: Yes indeed, we are the minority. We may have been looked down at and discouraged by others, but the most important part is to have faith in ourselves. Don't let the fire in your heart die just because of what others say.

幾何雙胞胎：梅涅勞斯定理及塞瓦定理

GEOMTRIC TWINS: MENELAUS' THEOREM AND CEVA'S THEOREM

周智康/ JIMMY CHOW CHI-HONG
2011、2012 年國際數學奧林匹克香港代表隊隊員
Hong Kong team member for International Mathematical Olympiad 2011, 2012

以下是兩道平面幾何的定理：

In plane geometry, we have the following two theorems:

定理一：在 ΔABC 中，設 E、F 分別為線段 AC、AB 上的點，而 D 是 BC 向 C 延長線上的一點。若 D、E、F 共線（見圖一），則

$$\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$$

Theorem 1: In a ΔABC, let E, F be two points on the segments AC, AB respectively, and D be a point on the ray BC beyond C. If D, E and F are collinear (see Figure 1). Then

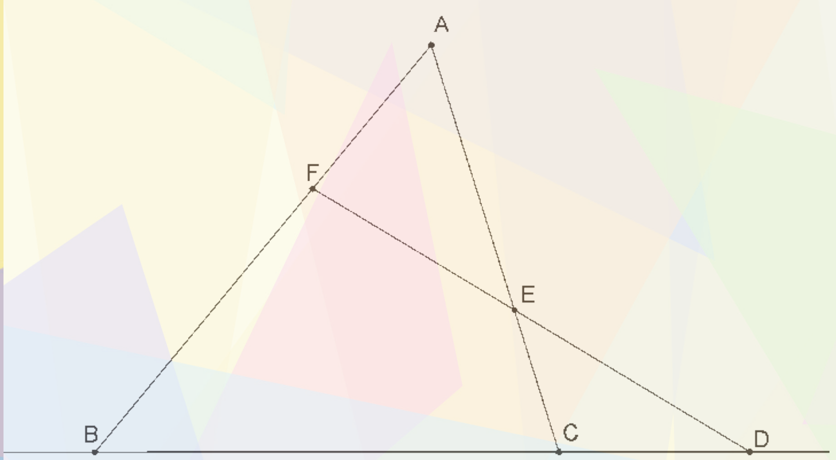
$$\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$$

定理二：在 ΔABC 中，設 D、E、F 分別為三邊 BC、AC、AB 上的一點，若 AD、BE、CF 共點（見圖二），則

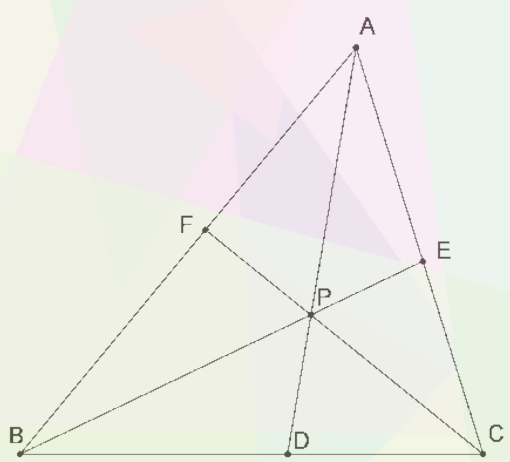
$$\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$$

Theorem 2: In a ΔABC, let D, E and F be three points on the segments BC, AC and AB respectively. If AD, BE and CF are concurrent (see Figure 2). Then

$$\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$$



圖一 Figure 1



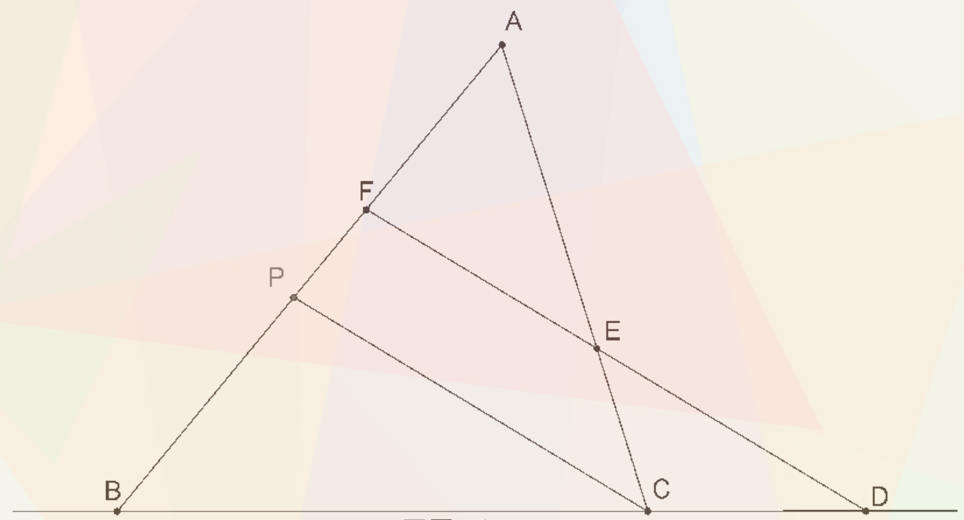
圖二 Figure 2

簡單來說，以上定理令涉及共線及共點的問題變得易於運算。它們分別名為**梅涅勞斯定理**及**塞瓦定理**。梅涅勞斯定理被普遍認為是源於希臘數學家亞歷山大里亞的梅內勞斯（70 年 - 140 年）。另一相似定理在 1500 年後才被證實，並以意大利數學家塞瓦（1647 年 - 1734 年）命名。現在讓我們看看如何證明這兩道定理。

Roughly speaking, these theorems convert problems related to collinearity and concurrency into ones where we can do calculations. They are respectively called **Menelaus' Theorem** and **Ceva's Theorem**. It is generally accepted that the first theorem is due to Greek mathematician Menelaus of Alexandria (70–140). The other similar theorem, which was proved 1500 years later, bears the name of Italian mathematician Giovanni Ceva (1647–1734). Now let us see how they are proved.

梅涅勞斯定理的證明：設 P 為 AB 上的一點，使得 CP 與 EF 平行（見圖三）。我們有 $\frac{AE}{CE}=\frac{AF}{FP}$ 及 $\frac{BD}{CD}=\frac{BF}{FP}$，然後從兩道算式中消去 FP，便得到 $\frac{CE\cdot AF}{AE}=\frac{CD\cdot BF}{BD}$，從而得到我們所需的方程 $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$。

Proof of Menelaus' Theorem: Let P be the point on AB such that CP is parallel to EF (see Figure 3). Then we have $\frac{AE}{CE}=\frac{AF}{FP}$ and $\frac{BD}{CD}=\frac{BF}{FP}$, and by eliminating FP from these two equations, we have $\frac{CE\cdot AF}{AE}=\frac{CD\cdot BF}{BD}$ or $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$ which is the desired equation.



圖三 Figure 3

塞瓦定理的證明：在圖二中，設 P 為 AD、BE、CF 的交點。（這些線段皆稱為塞瓦線段。）若 $[XYZ]$ 代表 ΔXYZ 的面積，我們便得出 $\frac{BD}{CD}=\frac{[ABD]}{[ACD]}=\frac{[PBD]}{[PCD]}=\frac{[ABD]-[PBD]}{[ACD]-[PCD]}=\frac{[ABP]}{[ACP]}$，類似地 $\frac{CE}{AE}=\frac{[BCP]}{[ABP]}$ 及 $\frac{AF}{BF}=\frac{[ACP]}{[BCP]}$，因此 $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=\frac{[ABP]}{[ACP]}\cdot\frac{[BCP]}{[ABP]}\cdot\frac{[ACP]}{[BCP]}=1$。

Proof of Ceva's Theorem: As in Figure 2, let P be the point where AD, BE and CF meet. (Any one of these segments is called a cevian.) If $[XYZ]$ denotes the area of a ΔXYZ, then we have $\frac{BD}{CD}=\frac{[ABD]}{[ACD]}=\frac{[PBD]}{[PCD]}=\frac{[ABD]-[PBD]}{[ACD]-[PCD]}=\frac{[ABP]}{[ACP]}$, and similarly $\frac{CE}{AE}=\frac{[BCP]}{[ABP]}$ and $\frac{AF}{BF}=\frac{[ACP]}{[BCP]}$, and hence $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=\frac{[ABP]}{[ACP]}\cdot\frac{[BCP]}{[ABP]}\cdot\frac{[ACP]}{[BCP]}=1$.

在探討一些有趣的例子前，我們須注意：兩道定理雖然有著一樣的算式，算式的意思卻並不相同。在梅涅勞斯定理中，我們假定 D 位於 ΔABC 之外；而在塞瓦定理中，D、E、F 三點均在三角形的邊上。

Before we proceed to some interesting examples, we remark that although the equations appearing in the above two theorems are the same, they do not mean the same thing: in Menelaus' Theorem, we require that D lies outside the ΔABC, while in Ceva's Theorem, all the three points D, E and F lie on its sides.

例一：試用梅涅勞斯定理證明塞瓦定理。

見圖二。運用梅涅勞斯定理於 ΔABD 及線 CPF，我們得出 $\frac{BC}{CD}\cdot\frac{DP}{AP}\cdot\frac{AF}{BF}=1$。另外，若運用同一定理於 ΔACD 及線 BPE，便得出 $\frac{BC}{BD}\cdot\frac{DP}{AP}\cdot\frac{AE}{CE}=1$。結合以上兩道算式及消去 AP、DP 和 BC 後，就會得出我們所需的方程。

Example 1: Prove Ceva's Theorem by Menelaus' Theorem.

See Figure 2 again. Applying Menelaus' Theorem to the ΔABD and the line CPF, we have $\frac{BC}{CD}\cdot\frac{DP}{AP}\cdot\frac{AF}{BF}=1$. On the other hand, if we apply the same theorem to the ΔACD and the line BPE, then we have $\frac{BC}{BD}\cdot\frac{DP}{AP}\cdot\frac{AE}{CE}=1$. The desired equation will be obtained after we combine these two equations and eliminate the terms AP, OP and BC.

例二：設 AD、BE、CF 為三條塞瓦線段，並設 AD 垂直於 BC。試證明 $\angle ADE = \angle ADF$。

解答方法是先加一條穿過 A 並與 BC 平行的直線，再考慮由 DE、DF 和這條線所組成的三角形（見圖四）。由於 AD 垂直於 BC，而 XY 亦然，故可見 $\angle ADE = \angle ADF$ 當且僅當 $AX = AY$。換言之，只要我們能證明 $AX = AY$，問題便迎刃而解。

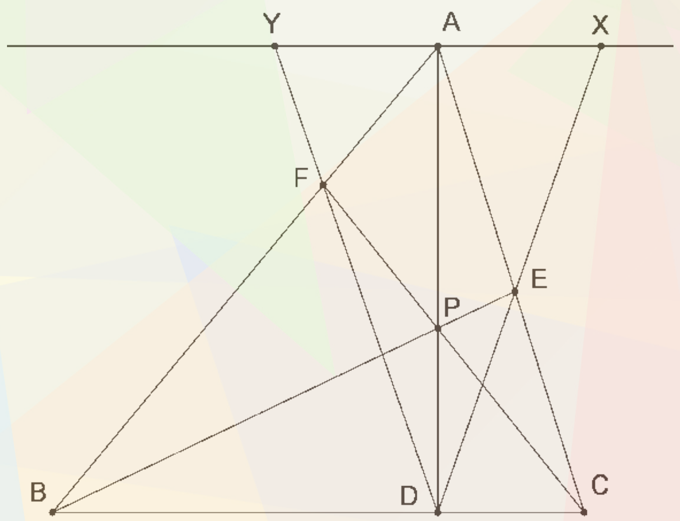
現在，我們用塞瓦定理便能得出 $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$。由於 $AX//CD$，因此 $\frac{AE}{CE}=\frac{AX}{CD}$。同理 $\frac{AF}{BF}=\frac{AY}{BD}$。於是 $AX=\frac{AE\cdot CD}{CE}=\frac{AF\cdot BD}{BF}=AY$。

Example 2: Let AD, BE and CF be three cevians with an additional assumption that AD is perpendicular to BC. Prove that $\angle ADE = \angle ADF$.

The idea is to draw the line through A parallel to BC and consider the triangle formed by ADE, ADF and this line (see Figure 4). Since AD is perpendicular to BC and so XY, we see that $\angle ADE = \angle ADF$ is equivalent to $AX = AY$. In other words, the problem will be solved if we can show $AX = AY$.

Now by Ceva's Theorem, we have $\frac{BD}{CD}\cdot\frac{CE}{AE}\cdot\frac{AF}{BF}=1$. Since $AX//CD$, it follows that $\frac{AE}{CE}=\frac{AX}{CD}$, and similarly $\frac{AF}{BF}=\frac{AY}{BD}$. Therefore,

$$AX=\frac{AE\cdot CD}{CE}=\frac{AF\cdot BD}{BF}=AY.$$



圖四 Figure 4

其實，梅涅勞斯定理和塞瓦定理的逆定理也成立，即是說，若這條優雅的等式成立，那麼就必然有共線或共點的性質。當證明一些美妙的數學定理，例如著名的帕斯卡六邊形定理及三角形各心的存在性時，這些逆定理往往能大派用場。以上定理及其逆定理是奧數中不可或缺的一部分，嘗試過這一期「挑戰園地」的第三題後，相信你也會有同感！

The converses of Menelaus' Theorem and Ceva's Theorem actually hold; that is, the collinearity or the concurrence follows if that elegant equality holds. These converses turn out to be particularly useful in proving some beautiful theorems such as the famous Pascal's hexagon theorem and the existence of various centers of a triangle. In Mathematical Olympiad these theorems and their converses have proved to be indispensable. You may agree with this after playing with problem 3 of the Challenge Corner in this issue!

我們比想像中更親近！

We Are Closer Than We Think!

盧安迪 / ANDY LOO

你或許聽說過「六度分隔」的理論。這個理論指，你可以通過一條不超過 6 人的「朋友鏈」，聯繫到世界上幾乎任何人（例如，你認識 A，A 認識 B，B 認識 C，C 認識 D，D 認識 E，E 認識 X，因此你聯繫到 X）。雖然這個現象跟直覺相悖，但如果我們明白指數增長的本質，它還是不太令我們感到意外的。

You may have heard of the theory of six degrees of separation, which suggests that you are connected with almost every person in the world via a chain of at most 6 friends (e.g. you are connected with X in the sense that you know A, A knows B, B knows C, C knows D, D knows E and E knows X). This phenomenon, though counter-intuitive, is hardly surprising if we understand the nature of exponential growth.

假設一般人有 m 個朋友。因為你的 m 個朋友每個都有 m 個朋友，所以你可通過不超過 2 人的「朋友鏈」聯繫到約 m^2 人。當然，你的朋友可能跟你的朋友的朋友重複，而你的不同朋友的朋友們也可能有所重複，但我們暫時可放下這些問題。

同樣道理，我們可通過不超過 3 人的「朋友鏈」聯繫到約 m^3人，通過不超過 4 人的「朋友鏈」聯繫到約 m^4人，如此類推。一般來說，我們可通過不超過 n 人的「朋友鏈」聯繫到約 m^n 人，而 m^n 是 n 的一個指數函數。（當 m^n 超出總人口數量時，上一段提到的重複會變得顯著。此外，世上亦可能有一些不跟任何人聯繫的隱士。）

指數函數的增長率是驚人的。傳說中，一名印度大臣送了一個精美的棋盤給國王。當被問到想要什麼獎賞時，大臣答道：「我想在棋盤的第一格上要一粒稻穀，而在此後每一格，請給我上一格的穀粒數目的兩倍。」國王高興地答應了。豈料第 64 格所需的穀粒數目是 $2^{63} \approx 9 \times 10^{18}$，已超過世界上所有稻穀的數目！

對於「朋友」的概念，我們可作不同變化。二十世紀有一名非常多產的數學家愛多仕（Paul Erdős），人們定義了「愛多仕數」：愛多仕自己的愛多仕數是 0，跟他合作過寫論文的人的愛多仕數是 1，跟愛多仕的合作者合作過的人的愛多仕數是 2（如果他們沒有更小的愛多仕數的話），如此類推。愛多仕數為 1 的超過 500 人，愛多仕數為 2 的超過 6000人，愛多仕數為 3 的超過 30000 人。這些數字並不完全符合我們預測的指數增長規律（試想想為什麼！），但增長率還是甚高的。

Supposing that a typical person has m friends, the number of people you can connect with via chains of at most 2 friends is roughly m^2, as each of your m friends in turn has m friends. Of course there may be overlaps between your friends as your friends' friends, as well as between the friends of different friends of yours, but we can neglect these complications for now.

By the same token, you can connect with roughly m^3 friends via chains of at most 3 friends, and m^4 via chains of at most 4 friends, and so forth. In general, the number of people you can connect with via a chain of at most n friends is m^n, which is an exponential function in n. (When mn exceeds the total population, the overlaps mentioned in the previous paragraph become more obvious. Also, there may be hermits who are not connected with anybody.)

The exponential function grows at a staggering rate. Legend has it that an Indian minister once gave the king a beautiful chessboard. When asked what reward he wanted, the minister answered, "I want one grain of rice on the first square of the chessboard, and on each subsequent square, please place double the number of grains of the preceding square." The king gladly agreed, only to find that the number of grains needed for the 64th square is $2^{63} \approx 9 \times 10^{18}$, more than the rice available in the whole world!

There are many variations to the concept of "friendship." Twentieth-century mathematician Paul Erdős was such a prolific collaborator that people defined the Erdős number: Erdős himself has an Erdős number of 0, his collaborators have an Erdős number of 1, those who have collaborated with Erdős's collaborators (and do not qualify for a smaller Erdős number) have an Erdős number of 2, and so forth. There are more than 500 people with Erdős number 1, more than 6000 people with Erdős number 2, and more than 30000 people with Erdős number 3. While these figures do not closely exhibit the exponential pattern we predicted (can you think of some possible explanations?), the growth rate is high nonetheless.

讀者亦可研究其他類似的現象。舉例說，如果一個人在我們一生中某時刻進入過我們的視線範圍，我們就算作跟他有聯繫，那麼一個人要通過多少步才能聯繫到世界上幾乎所有人呢？

現在，讓我們來考慮另一層面的聯繫：血緣關係。你有沒有想過，我們每個人都是自己的表親？

如果我們不是自己的表親，當我們沿家族圖追溯上去時，所有祖先都必須是不同的。我們有 2 名父母、4 名祖父母、8 名曾祖父母，如此類推，我們在第 n 代有 2^n 名祖先。向上推到第 30 代（即少於 1000年 前）我們便會有 $2^{30} \approx 10^9$ 個不同的祖先，但這已比當時的世界人口還要多！按抽屜原理，這 10^9 個祖先中必有重複。

那麼，我們可否證明世界上任何兩人 A、B 都是親戚呢？畢竟，如果他們的祖先全部不同，他們約 1000 年前便需有 $2 \times 2^{29} = 2^{30} \approx 10^9$名祖先，而這超出了當時的世界人口。可惜，這個證明是有問題的，因為每對「重複」的祖先，可能兩者都是 A 的祖先，或兩者都是 B 的祖先。上述的論說不能保證有 A 的某個祖先跟 B 的某個祖先重複。然而，它意味著任何兩個人都頗有機會在不久的過去有著共同祖先，人類的大家庭比我們想像中親近得多！

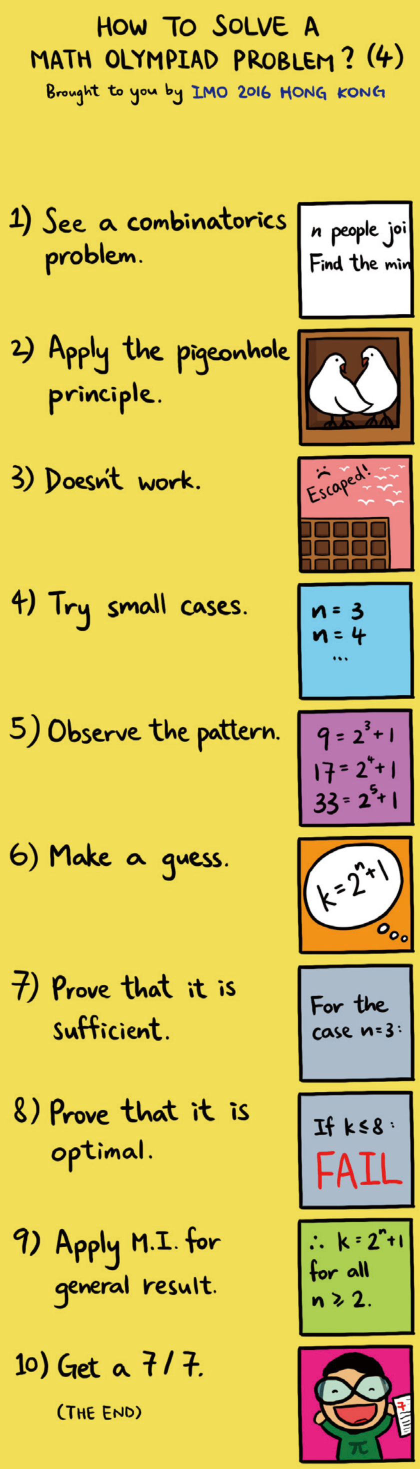
Readers can also investigate other phenomena of this type. For example, if a person is connected with everybody that has entered his field of vision some time in his life, how many steps of connection does it take for one to reach almost everybody in the world?

Now let's consider connection in another dimension: ancestry. Have you ever thought that every one of us is our own cousin?

If we weren't our own cousin, all our ancestors would have to be distinct as we move up along our family tree. We have 2 parents, 4 grandparents, 8 great-grandparents, and in general, 2^n ancestors in the nth generation. We would need to have $2^{30} \approx 10^9$ ancestors in the 30th generation (or less than 1000 years ago), but that exceeds the total world population at that time! By the pigeonhole principle, there must be repetitions.

Can we, then, prove that every two people in the world, A and B, are related? After all, if all their ancestors were different, there would be $2 \times 2^{29} = 2^{30} \approx 10^9$ distinct ancestors around 1000 years ago, more than the world population at that time! Unfortunately, this proof doesn't work because each pair of "overlapping" ancestors may just both be ancestors of A or ancestors of B. The above argument does not guarantee that some ancestor of A is the same person as some ancestor of B. What it does suggest, however, is that it is quite likely for two people to have a common ancestor from not too long ago, and that the human race is a much closer family than we might have imagined!

笑一笑 Laugh Out Loud



挑戰園地 Challenge Corner

第三期挑戰園地的解答及得獎名單，可見:
For the solutions and list of awardees of the Challenge Corner of the 1st issue, please see:
http://www.edb.gov.hk/tc/curriculum-development/kla/ma/IMO/IMOment.html

1. 在社交網站 Mathbook 上，朋友關係是相互的（即如果 A 是 B 的朋友，則 B 也是 A 的朋友）。是否可能有 5 名用戶，其中既不存在 3 人全部互為朋友（即這 3 人中每人都跟其餘兩人為朋友），也不存在 3 人全部互相不是朋友（即這 3人中沒有兩人是朋友）？證明你的答案。

On the social networking website Mathbook, friendship is mutual (i.e. if A is a friend of B, then B is also a friend of A). Is it possible to have 5 users such that no 3 of them are all friends with each other (i.e. each of these 3 users is friends with the other two) and no 3 of them are all strangers with each other (no two of these 3 users are friends)? Prove your answer.

2. 若 x 為正實數，求 $x+\frac{1}{x}$ 的最小值。

If x is a positive real number, find the minimum value of $x+\frac{1}{x}$.

3. 設 $ABCD$ 為平行四邊形，並設 P、Q、R、S 分別為四邊 AB、BC、CD、DA 上的點，使得 $PR//BC$ 及 $QS//AB$。若直線 AC、PQ、RS 沒有兩條平行，求證這三條直線交於一點。

Let $ABCD$ be a parallelogram and P, Q, R, and S be points on the sides AB, BC, CD, DA respectively such that $PR//BC$ and $QS//AB$. If no two of the lines AC, PQ and RS are parallel, prove that these three lines meet at one point.

4. 求以下數式的值：

Evaluate the following expression:

$$\frac{1}{1\times2\times4}+\frac{1}{2\times3\times5}+\cdots+\frac{1}{97\times98\times100}$$

歡迎香港中、小學生讀者電郵至 info@imohkc.org.hk 提交解答（包括證明），並於電郵中列明學生中英文姓名、學校中英文名稱及學生班級。每一名學生只可發送一份電郵。首 20名 答對最多題目的同學將獲贈紀念品，但每間學校最多有 3 名同學得獎。解答可以中文或英文提交。打字及掃描文件皆可接受。得獎者將於下一期公布。2016 年第五十七屆國際數學奧林匹克籌備委員會對本活動安排有最終決定權。如有疑問，可電郵至 info@imohkc.org.hk 查詢。

Hong Kong secondary and primary school student readers are welcome to submit solutions (with proofs) via email to info@imohkc.org.hk, specifying the student's name in Chinese and in English, the school's name in Chinese and in English, and the student's class in the email. Each student may send at most one email. Souvenirs will be awarded to the first 20 students solving the most questions on the condition that each school can have at most 3 awardees. Solutions can be submitted in Chinese or English. Both typed and scanned files are acceptable. The awardees will be announced in the next issue. The decision of the Organising Committee of the 57th International Mathematical Olympiad on any matter of this activity is final. Enquiries may be emailed to info@imohkc.org.hk.