Learning Objectives of Module 1 (Calculus and Statistics)

Notes:

1. *Learning units are grouped under three areas (“Foundation Knowledge”, “Calculus” and “Statistics”) and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the “Remarks” column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	Students are required to recognise the summation notation ($\sum$). The following content are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of the number e and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$	8	
	2.2 understand exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are required to solve problems including those related to compound interest, population growth and radioactive decay.
	2.4 transform $y = ka^x$ and $y = k\left[f(x)\right]^n$ to linear relations, where a , n and k are real numbers, $a > 0$, $a \neq 1$, $f(x) > 0$ and $f(x) \neq 1$		When experimental values of x and y are given, students are required to plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercepts.
	Subtotal in hours	11	

Learning Unit	Learning Objective	Time	Remarks
Calculus			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	5	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are **not** required).
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following algebraic functions are required: • polynomial functions • rational functions • power functions x^α • functions derived from the above ones through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$

Learning Unit	Learning Objective	Time	Remarks
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Students are required to recognise the notations: y', $f'(x)$ and $\dfrac{dy}{dx}$.
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Students are required to recognise the notations: $f'(x_0)$ and $\left.\dfrac{dy}{dx}\right\|_{x=x_0}$.
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	8	The rules include: • $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$ • $\dfrac{d}{dx}(uv) = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$ • $\dfrac{d}{dx}(\dfrac{u}{v}) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ • $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Learning Unit	Learning Objective	Time	Remarks
	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The formulae that students are required to use include: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ • $(\log_a x)' = \frac{1}{x \ln a}$ • $(a^x)' = a^x \ln a$ Implicit differentiation and logarithmic differentiation are **not** required.
5. Second derivative	5.1 recognise the concept of the second derivative of a function	2	Students are required to recognise the notations: y'' , $f''(x)$ and $\frac{d^2y}{dx^2}$. Third and higher order derivatives are **not** required.

Learning Unit	Learning Objective	Time	Remarks
	5.2 find the second derivative of an explicit function		Students are required to recognise the second derivative tests and concavity.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangent, rate of change, maximum and minimum	10	Local and global extrema are required.
7. Indefinite integration and its applications	7.1 recognise the concept of indefinite integration	10	Indefinite integration as the reverse process of differentiation should be introduced.
	7.2 understand the basic properties of indefinite integrals and basic integration formulae		Students are required to recognise the notation: $\int f(x)\,dx$. The properties include: • $\int kf(x)dx = k\int f(x)dx$ • $\int [f(x) \pm g(x)]dx$ $= \int f(x)dx \pm \int g(x)dx$ The formulae include: • $\int kdx = kx + C$

Learning Unit	Learning Objective	Time	Remarks
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions 7.4 use integration by substitution to find indefinite integrals 7.5 use indefinite integration to solve problems		• $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ • $\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ • $\int e^x dx = e^x + C$ Students are required to understand the meaning of the constant of integration *C*. Integration by parts is **not** required.
8. Definite integration and its applications	8.1 recognise the concept of definite integration	12	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. Students are required to recognise the notation: $\int_a^b f(x)\,dx$.

Learning Unit	Learning Objective	Time	Remarks
			The concept of dummy variables is required, for example, $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$.
	8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals		The Fundamental Theorem of Calculus that students are required to recognise is: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$. The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$

Learning Unit	Learning Objective	Time	Remarks
	8.3 find the definite integrals of algebraic functions and exponential functions 8.4 use integration by substitution to find definite integrals 8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems		$=\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves.
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required. Students are required to determine whether an estimate is an over-estimate or under-estimate by using the second derivative and concavity.
	Subtotal in hours	51	
Statistics			

Learning Unit	Learning Objective	Time	Remarks
10. Conditional probability and Bayes' theorem	10.1 understand the concept of conditional probability 10.2 use Bayes' theorem to solve simple problems	6	
11. Discrete random variables	11.1 recognise the concept of discrete random variables	1	
12. Probability distribution, expectation and variance	12.1 recognise the concept of discrete probability distribution and represent the distribution in the form of tables, graphs and mathematical formulae 12.2 recognise the concepts of expectation $E[X]$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems	7	The formulae that students are required to use include: • $E[X]=\sum xP(X=x)$ • $\mathrm{Var}(X)=E\left[(X-\mu)^2\right]$ • $E[g(X)]=\sum g(x)P(X=x)$ • $E[aX+b]=aE[X]+b$ • $\mathrm{Var}(X)=E\left[X^2\right]-(E[X])^2$ • $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$

Learning Unit	Learning Objective	Time	Remarks
13. The binomial distribution	13.1 recognise the concept and properties of the binomial distribution	5	The Bernoulli distribution should be introduced. The mean and variance of the binomial distribution are required (the proofs are **not** required).
	13.2 calculate probabilities involving the binomial distribution		Use of the binomial distribution table is **not** required.
14. The Poisson distribution	14.1 recognise the concept and properties of the Poisson distribution	5	The mean and variance of Poisson distribution are required (the proofs are **not** required).
	14.2 calculate probabilities involving the Poisson distribution		Use of the Poisson distribution table is **not** required.
15. Applications of the binomial and the Poisson distributions	15.1 use the binomial and the Poisson distributions to solve problems	5	
16. Basic definition	16.1 recognise the concepts of continuous random variables and	3	Derivations of the mean and variance of

Learning Unit	Learning Objective	Time	Remarks
and properties of the normal distribution	continuous probability distributions, with reference to the normal distribution		the normal distribution are **not** required. Students are required to recognise that the formulae in Learning Objective 12.3 are also applicable to continuous random variables.
	16.2 recognise the concept and properties of the normal distribution		The properties include: ● the curve is bell-shaped and symmetrical about the mean ● the mean, mode and median are all equal ● the flatness can be determined by the value of σ ● the area under the curve is 1
17. Standardisation of a normal variable and use of the standard normal table	17.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	
18. Applications of	18.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$	7	

Learning Unit	Learning Objective	Time	Remarks
the normal distribution	and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu,\sigma^2)$ 18.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu,\sigma^2)$ 18.3 use the normal distribution to solve problems		
19. Sampling distribution and point estimates	19.1 recognise the concepts of sample statistics and population parameters 19.2 recognise the sampling distribution of the sample mean $\overline{X}$ from a random sample of size n	9	Students are required to recognise: If the population mean is μ and the population size is N, then the population variance is $\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$. Students are required to recognise: • If the population mean is μ and the population variance is σ^2, then $E\left[\overline{X}\right] = \mu$ and $\mathrm{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$.

Learning Unit	Learning Objective	Time	Remarks
			• If $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ (the proof is **not** required).
	19.3 use the Central Limit Theorem to treat $\bar{X}$ as being normally distributed when the random sample size n is sufficiently large		
	19.4 recognise the concept of point estimates including the sample mean and sample variance		Students are required to recognise: If the sample mean is $\bar{x}$ and the sample size is n, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Students are required to recognise the concept of unbiased estimator.
20. Confidence interval for a population mean	20.1 recognise the concept of confidence interval 20.2 find the confidence interval for a population mean	6	Students are required to recognise: • A $100(1-\alpha)\%$ confidence interval for the mean μ of a normal population with known variance σ^2 is

Learning Unit	Learning Objective	Time	Remarks
			given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$. • When the sample size n is sufficiently large, a $100(1-\alpha)\%$ confidence interval for the mean μ of a population with unknown variance is given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$, where s is the sample standard deviation.
	Subtotal in hours	56	
Further Learning Unit			
21. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours

Learning Objectives of Module 2 (Algebra and Calculus)

Notes:

1. *Learning units are grouped under three areas (“Foundation Knowledge”, “Algebra” and “Calculus”) and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the “Remarks” column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Odd and even functions	1.1 recognise odd and even functions and their graphs	2	Students are required to recognise that the absolute value function is an example of even functions.
2. Mathematical induction	2.1 understand the principle of mathematical induction	3	The First Principle of Mathematical Induction is required. Students are required to prove propositions related to the summation of a finite sequence. Proving propositions involving inequalities is **not** required.

Learning Unit	Learning Objective	Time	Remarks
3. The binomial Theorem	3.1 expand binomials with positive integral indices using the binomial theorem	3	Proving the binomial theorem is required. Students are required to recognise the summation notation ($\sum$). The following content are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation
4. More about trigonometric functions	4.1 understand the concept of radian measure 4.2 understand the functions cosecant, secant and cotangent	13	 The formulae that students are required to use include: $1+\tan^2\theta=\sec^2\theta$ and $1+\cot^2\theta=\operatorname{cosec}^2\theta$ Simplifying trigonometric expressions by identities is required.

Learning Unit	Learning Objective	Time	Remarks
	4.3 understand compound angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine		The formulae include: • $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ • $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ • $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ • $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ • $2\sin A \sin B = \cos(A-B) - \cos(A+B)$ • $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ • $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ • $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ • $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
5. Introduction to	5.1 recognise the definitions and notations of the number e and the	2	Two approaches for the introduction to

Learning Unit	Learning Objective	Time	Remarks
the number e	natural logarithm		e can be considered: • $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ These definitions may be introduced in Learning Objective 6.1.
	Subtotal in hours	23	
Calculus			
6. Limits	6.1 understand the intuitive concept of the limit of a function 6.2 find the limit of a function	3	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are not required). The formulae that students are required to use include:

Learning Unit	Learning Objective	Time	Remarks
			• $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.
7. Differentiation	7.1 understand the concept of the derivative of a function	14	Students are required to find the derivatives of elementary functions, including C, x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x, $\ln x$ from first principles. Students are required to recognise the notations: y', $f'(x)$ and $\frac{dy}{dx}$. Testing differentiability of functions is **not** required.
	7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation		The rules include: • $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Learning Unit	Learning Objective	Time	Remarks
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		• $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$ • $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ The formulae that students are required to use include: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ The following algebraic functions are required:

Learning Unit	Learning Objective	Time	Remarks
			• polynomial functions • rational functions • power functions x^{α} • functions formed from the above functions through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$
	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Students are required to recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Students are required to recognise the second derivative tests and concavity. Third and higher order derivatives are **not** required.
8. Applications of differentiation	8.1 find the equations of tangents to a curve	14	
	8.2 find the maximum and minimum value of a function		Local and global extrema are required.

Learning Unit	Learning Objective	Time	Remarks
	8.3 sketch curves of polynomial functions and rational functions		The following points should be considered in curve sketching: • symmetry of the curve • limitations on the values of *x* and *y* • intercepts with the axes • maximum and minimum points • points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students are required to deduce the equation of the oblique asymptote to the curve of a rational function by division.
	8.4 solve the problems relating to rate of change, maximum and minimum		
9. Indefinite integration and its applications	9.1 recognise the concept of indefinite integration	16	Indefinite integration as the reverse process of differentiation should be introduced.
	9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals		The formulae include: • $\int k \ dx = kx + C$

Learning Unit	Learning Objective	Time	Remarks
			• $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ • $\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ • $\int e^x dx = e^x + C$ • $\int \sin x\, dx = -\cos x + C$ • $\int \cos x\, dx = \sin x + C$ • $\int \sec^2 x\, dx = \tan x + C$
	9.3 understand the applications of indefinite integrals in mathematical contexts		Applications of indefinite integrals in some fields such as geometry is required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2 - x^2}$, $\frac{1}{\sqrt{a^2 - x^2}}$ or $\frac{1}{x^2 + a^2}$		Students are required to recognise the notations: $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$, and their related principal values.

Learning Unit	Learning Objective	Time	Remarks
	9.6 use integration by parts to find indefinite integrals		Teachers may use $\int \ln x \, dx$ as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.
10. Definite integration	10.1 recognise the concept of definite integration	10	The definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The concept of dummy variables is required, for example, $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. Using definite integration to find the sum to infinity of a sequence is **not** required.
	10.2 understand the properties of definite integrals		The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$

Learning Unit	Learning Objective	Time	Remarks
			• $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ • $\int_{-a}^a f(x)\,dx = 0$ if $f(x)$ is an odd function • $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(x)$ is an even function
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions		The Fundamental Theorem of Calculus that students are required to recognise is: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$.
	10.4 use integration by substitution to find definite integrals		
	10.5 use integration by parts to find definite integrals		The use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time	Remarks
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure 11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	4	“Disc method” is required.
	Subtotal in hours	61	
Algebra			
12. Determinants	12.1 recognise the concept of determinants of order 2 and order 3	2	Students are required to recognise the notations: $\|A\|$ and $\det A$.
13. Matrices	13.1 understand the concept, operations and properties of matrices	10	The addition, scalar multiplication and multiplication of matrices are required. The properties include: • $A+B=B+A$ • $A+(B+C)=(A+B)+C$ • $(\lambda+\mu)A=\lambda A+\mu A$ • $\lambda(A+B)=\lambda A+\lambda B$

Learning Unit	Learning Objective	Time	Remarks
			• $A(BC) = (AB)C$ • $A(B+C) = AB + AC$ • $(A+B)C = AC + BC$ • $(\lambda A)(\mu B) = (\lambda\mu)AB$ • $\lvert AB\rvert = \lvert A\rvert\lvert B\rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The properties include: • the inverse of A is unique • $(A^{-1})^{-1} = A$ • $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$ • $(A^n)^{-1} = (A^{-1})^n$ • $(A^T)^{-1} = (A^{-1})^T$ • $\lvert A^{-1}\rvert = \lvert A\rvert^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.

Learning Unit	Learning Objective	Time	Remarks
14. Systems of linear equations	14.1 solve the systems of linear equations in two and three variables by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: A system of homogeneous linear equations has nontrivial solutions if and only if the coefficient matrix is singular
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students are required to recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underset{\sim}{a}$) ; and some notations for magnitude (including $\lvert a\rvert$ and $\lvert\vec{a}\rvert$).
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The properties include: • $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$ • $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$ • $\mathbf{a}+\mathbf{0}=\mathbf{a}$ • $0\,\mathbf{a}=\mathbf{0}$

Learning Unit	Learning Objective	Time	Remarks
			• $\lambda(\mu\mathbf{a}) = (\lambda\mu)\mathbf{a}$ • $(\lambda + \mu)\mathbf{a} = \lambda\mathbf{a} + \mu\mathbf{a}$ • $\lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b}$ • If $\alpha\mathbf{a} + \beta\mathbf{b} = \alpha_1\mathbf{a} + \beta_1\mathbf{b}$ (**a** and **b** are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$
	15.3 understand the representation of a vector in the rectangular coordinate system		The formulae that students are required to use include: • $\lvert\overrightarrow{OP}\rvert = \sqrt{x^2 + y^2 + z^2}$ in $\mathbb{R}^3$ • $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$ and $\cos\theta = \dfrac{x}{\sqrt{x^2 + y^2}}$ in $\mathbb{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not**

Learning Unit	Learning Objective	Time	Remarks
			required.
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors	5	The properties include: • $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$ • $\mathbf{a}\cdot(\lambda\mathbf{b})=\lambda(\mathbf{a}\cdot\mathbf{b})$ • $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ • $\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2\geq 0$ • $\mathbf{a}\cdot\mathbf{a}=0$ if and only if $\mathbf{a}=\mathbf{0}$ • $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\geq\lvert\mathbf{a}\cdot\mathbf{b}\rvert$ • $\lvert\mathbf{a}-\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-2(\mathbf{a}\cdot\mathbf{b})$
	16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbb{R}^3$		The properties include: • $\mathbf{a}\times\mathbf{a}=\mathbf{0}$ • $\mathbf{b}\times\mathbf{a}=-(\mathbf{a}\times\mathbf{b})$ • $(\mathbf{a}+\mathbf{b})\times\mathbf{c}=\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$ • $\mathbf{a}\times(\mathbf{b}+\mathbf{c})=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ • $(\lambda\mathbf{a})\times\mathbf{b}=\mathbf{a}\times(\lambda\mathbf{b})=\lambda(\mathbf{a}\times\mathbf{b})$

Learning Unit	Learning Objective	Time	Remarks
			• $\lvert\mathbf{a}\times\mathbf{b}\rvert^2 = \lvert\mathbf{a}\rvert^2\lvert\mathbf{b}\rvert^2 - (\mathbf{a}\cdot\mathbf{b})^2$
17. Applications of vectors	17.1 understand the applications of vectors	6	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector and the area of a triangle are required.
	Subtotal in hours	34	
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours

The Learning Units for Key Stage 4 (S4 – S6) Further Mathematics (Elective)			
Foundation Knowledge	Calculus	Statistics	Algebra
1. Odd and even functions 2. Mathematical induction 3. The binomial theorem 4. Exponential and logarithmic functions 5. More about trigonometric functions	6. Limits 7. Differentiation and its applications 8. Indefinite integration and its applications 9. Definite integration and its applications 10. The trapezoidal rule	11. Conditional probability and Bayes' theorem 12. Discrete random variables and probability distributions 13. The binomial, the geometric and the Poisson distributions, and their applications 14. The normal distribution and its applications 15. Sampling distribution and point estimates 16. Confidence interval for a population mean	17. Determinants 18. Matrices 19. Systems of linear equations 20. Vectors and their applications 21. Complex numbers
Further Learning Unit			
22. Inquiry and investigation			

Further Mathematics (Elective)

Learning Unit	Learning Objective	Time	Remarks
21. Complex numbers	21.1 understand the concepts and properties of the conjugate and modulus of a complex number	22	The properties include: • $z\overline{z} = \lvert z\rvert^2$ • $\overline{\overline{z}} = z$ • $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$ • $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ • $\overline{(\frac{z_1}{z_2})} = \frac{\overline{z_1}}{\overline{z_2}}$ • $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ • $\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ • $\lvert z_1 + z_2\rvert \le \lvert z_1\rvert + \lvert z_2\rvert$
	21.2 understand the polar form of a complex number		The real part $(\mathrm{Re}\, z)$, imaginary part $(\mathrm{Im}\, z)$, argument $(\arg z)$ and principal value of argument $(\mathrm{Arg}\, z)$ of a complex number z are required. $"r\mathrm{cis}\theta"$ as a short form of complex number

Learning Unit	Learning Objective	Time	Remarks
			$r(\cos\theta + i\sin\theta)$ should be introduced. Students are required to represent complex numbers on an Argand diagram. Students are required to convert any complex number z from standard form $x + yi$ to polar form $r(\sin\theta + i\cos\theta)$ and vice versa.
	21.3 perform multiplication and division of complex numbers in polar form		Students are required to understand: If $z_1 = r_1(\cos\theta_1 + \sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then $z_1z_2 = r_1r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$ and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$
	21.4 describe and sketch the locus of points satisfying given conditions on an Argand diagram		The conditions include: • $\lvert z - z_1\rvert = k$ • $\lvert z - z_1\rvert = \lvert z - z_2\rvert$ • $\arg(z - z_1) = \theta$

Learning Unit	Learning Objective	Time	Remarks
	21.5 understand de Moivre's theorem and its applications		• $\arg(\frac{z-z_1}{z-z_2}) = \frac{\pi}{2}$ or π Students are required to: • find z^n, where *n* is an integer • find the *n*th roots of *z* • understand the cube roots of unity: 1, ω, ω^2 and their properties $\omega^3 = 1$, $1+\omega+\omega^2 = 0$ • solve problems related to trigonometric identities
	Subtotal in hours	64	
Further Learning Unit			
22. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	10	

Grand total: 250 hours