2.1 Curriculum Aims of Mathematics Education KLA

The general aims of mathematics curriculum are as follows:

To enable students to cope confidently with the mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, so that each student is ready for lifelong learning, the mathematics curriculum aims at developing in students:

(a) the ability to think critically and creatively, to conceptualize, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines;

(b) the ability to communicate with others and express their views clearly and logically in mathematical language;

(c) the ability to manipulate numbers, symbols and other mathematical objects;

(d) number sense, symbol sense, spatial sense and a sense of measurement as well as the capability of appreciating structures and patterns;

(e) a positive attitude towards mathematics learning and the capability of appreciating the aesthetic nature and cultural aspect of mathematics.

2.2 The Curriculum Framework

The curriculum framework for Mathematics Education is the overall structure for organizing learning and teaching for the subject of Mathematics. The framework comprises a set of interlocking components including subject **knowledge and skills**, which are expressed in the form of learning targets and learning objectives under the Strands or Dimensions (see Section 2.2.1 below); **generic skills**; and **positive values and attitudes**.

The framework sets out what students should know, value and be able to do at various stages of schooling. It gives schools and teachers flexibility to adapt the central mathematics curriculum to meet their varied needs. In general, the framework of the mathematics curriculum can be represented diagrammatically as shown in Figure 2 on the next page.

Diagrammatic Representation of the Framework of the Mathematics Curriculum

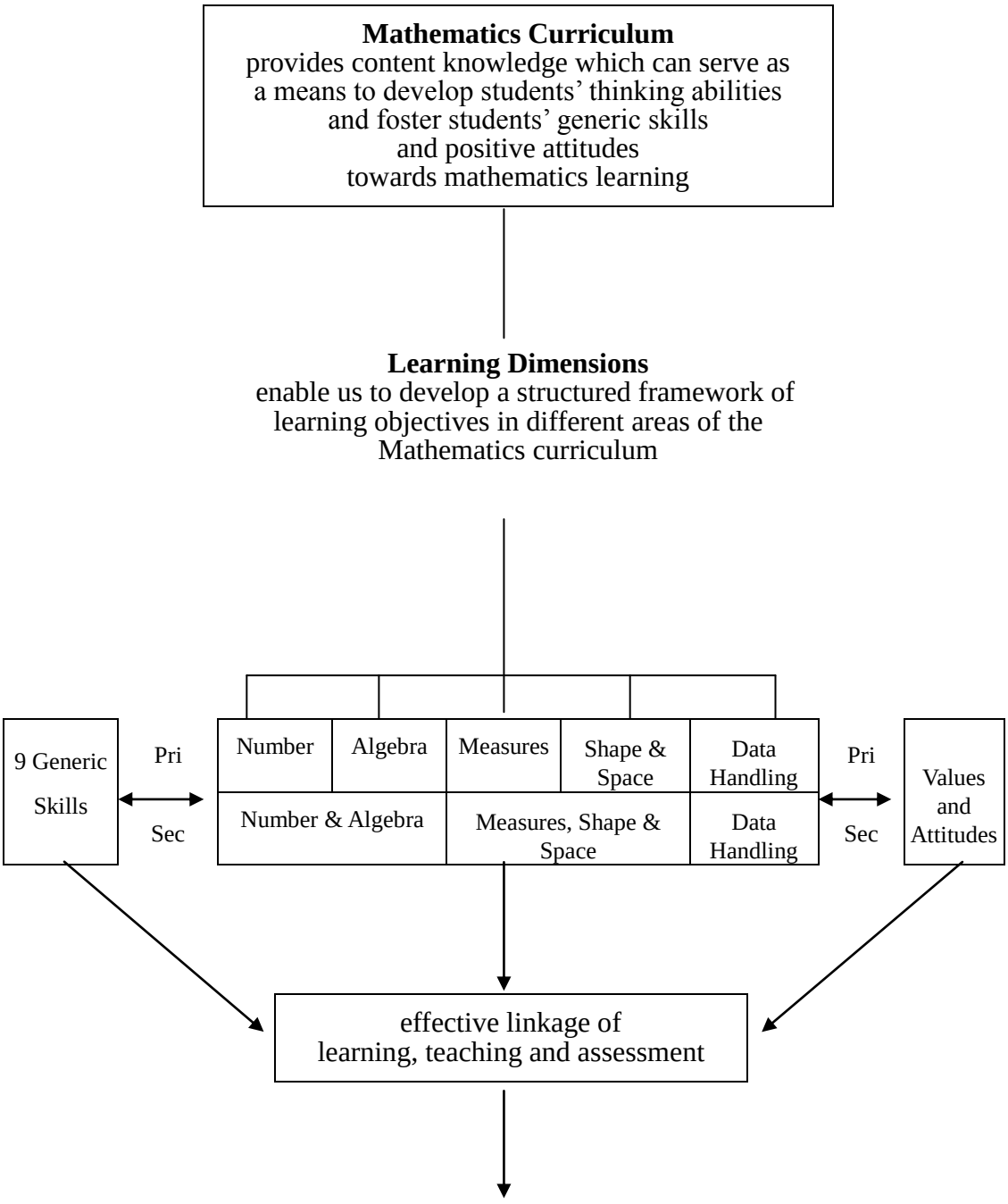

Figure 2

Table 1
Learning Targets of the Mathematics Curriculum

Knowledge and Skills

The Learning Targets for Key Stage 1 (P1-P3)				
Number and Algebra Dimensions		Measures, Shape and Space Dimensions		Data Handling Dimension
Number	Algebra	Shape & Space	Measures	Data Handling
• To understand and manipulate whole numbers • To understand simple fractions • To examine the reasonableness of results • To formulate and solve simple problems involving numbers	The ALGEBRA Dimension is not included at this key stage.	• To identify, describe and group lines, angles, 2-dimensional & 3-dimensional shapes • To recognize intuitively the elementary properties of 3-dimensional shapes • To recognize the properties of 2-dimensional shapes • To make 2-dimensional and 3-dimensional shapes from given information • To recognize and appreciate shapes • To identify the four directions	• To choose and use a variety of non-standard units to record results in basic measuring activities • To understand the need to use standard units of measurement • To select appropriate measuring tools and standard units of measurement • To integrate knowledge of Number, Measures, Shape & Space to solve simple problems in measurement	• To collect, compare and group discrete statistical data according to given criteria • To construct and interpret simple statistical graphs showing relations among data • To formulate and solve simple problems arising from collected data and constructed graphs

The Learning Targets for Key Stage 2 (P4-P6)				
Number and Algebra Dimensions		Measures, Shape and Space Dimensions		Data Handling Dimension
Number	Algebra	Shape & Space	Measures	Data Handling
• To understand whole numbers, fractions, decimals, percentages and the relations among them • To manipulate numbers and examine the reasonableness of results • To formulate and solve problems involving numbers	• To use symbols to represent unknown numbers • To communicate simple mathematical facts and relations using symbols • To formulate and solve simple problems and examine the results	• To understand the properties of 2-dimensional and 3-dimensional shapes • To group and make 2-dimensional and 3-dimensional shapes • To identify the eight compass points	• To choose and use a variety of non-standard and standard units to record results in various measuring activities • To select and justify appropriate measuring tools and standard units of measurement • To recognize the degree of accuracy and the approximate nature of measurement • To inquire and use simple measurement formulae • To integrate knowledge of Number, Measures, Shape & Space to formulate and solve simple problems in measurement	• To understand the criteria for organizing and grouping discrete statistical data • To apply simple arithmetic and appropriate scales in constructing and interpreting more complex statistical graphs • To show relationships among data using a variety of statistical and graphical representations • To recognize relations and patterns from graphs • To formulate and solve problems arising from collected data and constructed graphs

The Learning Targets for Key Stage 3 (S1-S3)		
Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension
• To experience rational and irrational numbers • To develop various strategies in using numbers to formulate and solve problems, and to examine results • To develop and refine strategies for estimating • To extend the use of algebraic symbols in communicating mathematical ideas • To explore and describe patterns of sequences of numbers using algebraic symbols • To interpret simple algebraic relations from numerical, symbolic and graphical perspectives • To manipulate algebraic expressions and relations; and apply the knowledge and skills to formulate and solve simple problems and to examine results • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To understand the nature of measurement and be aware of issues of precision and accuracy • To apply a variety of techniques, tools and formulas for measurements and solving measurement problems • To explore and visualize geometric properties of 2-dimensional and 3-dimensional objects intuitively • To use inductive reasoning, deductive reasoning and an analytic approach to study the properties of 2-dimensional rectilinear shapes • To formulate and write simple geometric proofs involving 2-dimensional rectilinear shapes with appropriate symbols, terminology and reasons • To inquire, describe and represent geometric knowledge in 2-dimensional figures using numeric and algebraic relations • To inquire into geometric knowledge in 2-dimensional space using trigonometric relations • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To understand the criteria for organizing discrete and continuous statistical data • To choose and construct appropriate statistical diagrams and graphs to represent given data and interpret them • To compute, interpret and select the appropriate measure to describe the central tendency of a set of data • To judge the appropriateness of the methods used in handling statistical data • To understand the notion of probability and handle simple probability problems by listing and drawing diagrams

Table 2
Overview of Learning Units

The Learning Units for Key Stage 1 (P1 – P3)

Unit				
Number	**Shape and Space**	**Measures**	**Data Handling**	**Algebra**
• Numbers to 10 • Numbers to 20 • Numbers to 100 • 3-digit numbers • 4-digit numbers • 5-digit numbers • Basic addition and subtraction (within 18) • Addition and subtraction (I) (addition within 2 places; subtraction within 2 places, excluding decomposition) • Addition and subtraction (II) (addition within 3 places; subtraction within 2 places) • Addition and subtraction (III) (subtraction within 3 places; mixed operations of addition & subtraction) • Addition and subtraction (IV) (within 4 places) • Basic multiplication (basic concept and computation) • Multiplication (I) (multiplier 1 digit and multiplicand 2 or 3 digits) • Basic division (basic concept and computation) • Division (I) (divisor 1 digit and dividend 2 or 3 digits) • Mixed operations (I) (addition, subtraction, multiplication and brackets) • Fractions (I) (basic concept, comparison)	• 3-D shapes (I) (prisms, pyramids and spheres) • 3-D shapes (II) (prisms, cylinders, pyramids and cones) • Straight lines and curves • 2-D shapes (polygons and circles) • Quadrilaterals (I) (rectangles, squares, trapeziums, rhombuses, etc.) • Quadrilaterals (II) (characteristics of parallelograms) • Triangles • Angles (I) (angles and right angles) • Angles (II) (acute and obtuse angles) • The four directions • Parallel and perpendicular	• Length and distance (I) (basic concept, direct comparison, improvised unit) • Length and distance (II) (centimetre) • Length and distance (III) (metre) • Length and distance (IV) (kilometre and millimetre) • Hong Kong money (I) (coins) • Hong Kong money (II) (bank-notes) • Time (I) (hour, year, month, day, week) • Time (II) (hour, minute, a.m., p.m., day, year) • Time (III) (second) • Time (IV) (the 24-hour time) • Weight (gram and kilogram) • Capacity (litre and millilitre)	• Pictograms (I) (1 picture represents 1 unit) • Block graphs (1 square represents 1 unit, average value)	

Units in the overview are not arranged in the order of teaching sequence.

The Learning Units for Key Stage 2 (P4 – P6)

Unit				
Number	**Shape and Space**	**Measures**	**Data Handling**	**Algebra**
• Large numbers (approximation) • Multiplication (II) (multiplier 2 digits and multiplicand 2 or 3 digits) • Division (II) (divisor 2 digits and dividend 2 or 3 digits, divisibility) • Acquaintance with modern calculating devices (calculators) • Multiples and factors • Common multiples and common factors • Mixed operations (II) (the four operations) • Fractions (II) (types, equivalent fractions, addition and subtraction of fractions with the same denominator) • Fractions (III) (addition and subtraction of fractions with different denominators) • Fractions (IV) (multiplication) • Fractions (V) (division) • Decimals (I) (basic concept) • Decimals (II) (addition and subtraction) • Decimals (III) (multiplication) • Decimals (IV) (division) • Decimals (V) (conversion between decimals and fractions, comparison of fractions) • Percentages (I) (basic concept, convert percentages into decimals or fractions and vice versa) • Percentages (II) (uses of percentages)	• Quadrilaterals (III) (characteristics of quadrilaterals) • Fitting and dissecting shapes • Symmetry • The eight compass points • 3-D shapes (III) (characteristics of prisms, pyramids and spheres) • 3-D shapes (IV) (vertices, edges, faces and sections) • Circles	• Perimeter (I) (irregular shapes, squares and rectangles) • Perimeter (II) (circumference) • Area (I) (square centimetre, square metre, squares, rectangles) • Area (II) (parallelograms, triangles, trapeziums and polygons) • Volume (I) (cubic centimetre, cubic metre, cuboids, cubes) • Volume (II) (capacity and volume) • Speed (metre per second, kilometre per hour)	• Pictograms (II) (1 picture represents 10 or 100 units) • Bar charts (I) (1 square represents 1, 2, 5 or 10 units, average value) • Bar charts (II) (compound bar charts, 1 square represents 50 or 100 units) • Bar charts (III) (frequency counts of 1000 or above) • Averages • Broken line graphs	• Elementary algebra (algebraic symbols) • Simple equations (I) (involving one step in finding solution) • Simple equations (II) (involving two steps in finding solution)

Units in the overview are not arranged in the order of teaching sequence.

The Learning Units for Key Stage 3 (S1 – S3)

Unit		
Number and Algebra	**Measures, Shape and Space**	**Data Handling**
• Directed Numbers and the Number Line • Numerical Estimation • Approximation and Errors • Rational and Irrational Numbers • Using Percentages • More about Percentages • Rate and Ratio • Formulating Problems with Algebraic Language • Manipulations of Simple Polynomials • Laws of Integral Indices • Factorization of Simple Polynomials • Linear Equations in One Unknown • Linear Equations in Two Unknowns • Identities • Formulas • Linear Inequalities in One Unknown	• Estimation in Measurement • Simple Idea of Areas and Volumes • More about Areas and Volumes • Introduction to Geometry • Transformation and Symmetry • Congruence and Similarity • Angles Related with Lines and Rectilinear Figures • More about 3-D Figures • Simple Introduction to Deductive Geometry • Pythagoras' Theorem • Quadrilaterals • Introduction to Coordinates • Coordinate Geometry of Straight Lines • Trigonometric Ratios and Using Trigonometry	• Introduction to Various Stages of Statistics • Construction and Interpretation of Simple Diagrams and Graphs • Measures of Central Tendency • Simple Idea of Probability

Units in the overview are not arranged in the order of teaching sequence.

Note: The learning units of Key Stages 1 and 2, and Key Stage 3 are extracted respectively from the two curriculum documents: *Mathematics Education Key Learning Area – Mathematics Curriculum Guide (P1 – P6) (2000)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5) (1999).*

2.2.1 Strands[8], Learning Targets and Objectives

Strands (or learning dimensions) are categories of mathematical knowledge and concepts for organizing the curriculum. Their main function is to organize mathematical contents for the purpose of developing knowledge, skill, values and attitudes as a holistic process. It can be seen from Figure 2 that the contents of the mathematics curriculum are organized into 5 learning dimensions at the primary level and 3 at the secondary.

The learning targets and learning units in the first three key learning stages are respectively shown in Tables 1 and 2.

2.2.2 Generic Skills

Generic skills (e.g. problem-solving, critical thinking, creativity and communication) can be seen as both process skills and learning outcomes in the Mathematics Education KLA. It should be noted that **generic skills are not something to be added on to the learning and teaching of mathematical concepts. They should be embedded within them. In fact, they serve as means to help acquire and master the mathematical knowledge and concepts.** An emphasis on the priority generic skills of communication, creativity and critical thinking in the context of mathematical activity will help to strengthen students' overall abilities to achieve the overall learning goals of the curriculum reform. Daily life applications and HOTS are emphasized in both the Primary Mathematics Curriculum (2000) and the Secondary Mathematics Curriculum (1999). The 9 generic skills and how they can be developed through the course of mathematics are illustrated as follows:

<Please turn to the next page.>

[8] The term "learning dimension" has been used in the Primary Mathematics Curriculum (2000) and Secondary Mathematics Curriculum (1999).

Collaboration Skills

Problem solving, planning and making decisions in a small group require collaboration skills, namely the skills of listening, appreciation, communication, negotiation, making compromises, asserting leadership, making judgement, as well as influencing and motivating others. Learners with these skills will be able to effectively engage in tasks and teamwork as well as working with others. Ultimately, learners will be able to form relationships that are mutually beneficial.

(The expected achievements of learners in this type of generic skills cannot be suitably classified according to Key Stages)

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Understanding working relationships Learners will learn to • clarify and accept various roles and responsibilities of individual members in a team and be willing to follow team rules • recognize that individuals as well as the team members have to take the consequences for their own actions	Learners 1. share responsibilities and understand the roles of individual members in doing mathematical group work like collecting data, measuring objects and presenting projects 2. understand and accept that members with different cultural backgrounds may have different interpretations of a mathematical problem (e.g. analyzing statistical data) 3. accept and follow the group decision in doing mathematical group work
Developing attitudes which contribute to good working relationships Learners will learn to • be open and responsive to others' ideas; appreciate, encourage and support the ideas and efforts of others • be active in discussing and posing questions to others, as well as in exchanging, asserting, defending and rethinking ideas • recognize and avoid stereotypes; withhold premature judgement until the facts are known • be willing to adjust their own behaviour to fit the dynamics of various groups and situations	Learners 1. discuss and exchange ideas openly with others in completing tasks and solving mathematical problems 2. exercise patience and listen to others in the discussion of mathematical problems (e.g. when sharing experience in the process of investigating number patterns or formulating proofs of geometric problems) 3. value the contributions of others in accomplishing mathematical tasks or solving mathematical problems together 4. appreciate different solutions to mathematical problems presented by others (e.g. using different approaches to prove mathematical theorems) 5. participate actively and pose questions in clarifying arguments in the solution of mathematical problems (e.g. discussing the strategies to be adopted in investigating practical statistical problems)
Achieving effective working relationships Learners will learn to • select a strategy and plan cooperatively to complete a task in a team • understand the strengths and weaknesses of members and build on the strengths to maximize the potential of the team • liaise, negotiate and compromise with others • reflect on and evaluate the strategy used by the group and make necessary adjustments	Learners 1. share experience in solving mathematical problems and select cooperatively a suitable strategy to solve a mathematical problem 2. clarify arguments objectively and rationally in solving mathematical problems (e.g. when examining the appropriateness of a particular strategy used to solve a mathematical problem) 3. liaise, negotiate and compromise with others in selecting a suitable strategy for solving a mathematical problem (e.g. use a synthetic or analytic approach in solving a geometrical problem)

Communication Skills

Communication is a dynamic and ongoing process in which two or more people interact in order to achieve a desired outcome or goal. In learning to communicate effectively, learners should learn to speak, listen, read and write effectively. They should learn to select the most appropriate means to convey a message in accordance with the purpose and context of the communication. They should use accurate and relevant information and organize it systematically and coherently for their audience. They should also evaluate the effectiveness of their communication and identify areas of improvement for action.

Descriptors of Expected Achievements Across the School Curriculum	**Exemplars of Implementation in Mathematics Education**
Key Stage One (Junior Primary) Learners will learn to • comprehend and act appropriately on spoken instructions • use clear and appropriate means of communication, both verbal and non-verbal, to express meaning and feelings • read and write simple texts	Learners 1. describe objects such as cubes and prisms orally with simple and appropriate mathematical terms (e.g. a cube has six faces) 2. interpret drawings, tables, graphs (e.g. pictograms) and symbols (e.g. $+$, $-$, $\times$, $\div$) 3. present findings with drawings and symbols 4. present data with tables and graphs (e.g. block graphs) 5. describe drawings and symbols in plain language (e.g. $2+3$ as 2 plus 3) 6. express simple daily-life problems in mathematical language (e.g. use symbols like \$2 $\times$3 and graphs like bar graphs)
Key Stage Two (Senior Primary) Learners will learn to • comprehend and respond to different types of texts • use spoken, written, graphic and other non-verbal means of expression to convey information and opinions, and to explain ideas • work and negotiate with others to develop ideas and achieve goals	Learners 1. interpret drawings, symbols (e.g. %), tables and graphs (e.g. broken line graphs) 2. describe and explain findings/results/data of mathematical tasks in both oral and written forms (e.g. the average score of a student's performance in a test, the favorite fruit) 3. present results of tasks with appropriate drawings and symbols 4. present data with tables, charts and graphs (e.g. broken line graphs, straight line graphs) 5. describe and analyze data 6. present solutions of problems logically (e.g. use of "=" properly) 7. express simple problems in mathematical language (e.g. the percentage of discount is 10%) 8. discuss with others in accomplishing tasks such as projects
Key Stage Three (Junior Secondary) Learners will learn to • understand, analyze, evaluate and respond to a range of different types of texts • use appropriate language and/or other forms of communication to present information and different points of view, and to express feelings • reflect and improve on the effectiveness of their own communication • work and negotiate with others to solve problems and accomplish tasks	Learners 1. interpret numeric, symbolic and graphical presentations 2. describe findings or explain conjectures in both oral and written forms using mathematical language (e.g. the two triangles are congruent) 3. choose appropriate statistical diagrams/graphs to present given data and use appropriate mathematical terminology or symbols in explaining ideas 4. formulate and write simple geometric proofs involving 2-D rectilinear shapes with appropriate symbols, terminology and reasons 5. interpret and respond appropriately to others' mathematical arguments in both oral and written forms 6. distinguish the difference between the language used in a mathematical context and daily life (e.g. rate, similar) 7. use mathematical language including graphs, figures and symbols to analyze and present possible solutions to a problem and discuss with others

Creativity

Creativity is an important but elusive concept. It has been defined in a variety of ways. Some people define it as an ability to produce original ideas and solve problems, others see it as a process, and yet others take it as certain personal qualities. In fact, creativity is a complex and multifaceted construct. Within the individual, creative behaviour is the result of a complex of cognitive skills/abilities, personality factors, motivation, strategies, and metacognitive skills. A person's creative performance may not correspond to his/her developmental stages.

Although the demanding process of teaching for creativity is hard to make routine, some principles apply in general. To develop students' creativity, we ask them to go beyond the given information, allow them time to think, strengthen their creative abilities, reward their creative efforts, value their creative attributes, teach them creative thinking techniques and the Creative Problem Solving model, and create a climate conducive to creativity[1]. These principles can be employed in all KLAs.

(The expected achievements of the learners in this type of generic skills cannot be suitably classified according to key learning stages)

Descriptors of Expected Achievements Across the School Curriculum	**Exemplars of Implementation in Mathematics Education**
Learners will learn to • strengthen creative abilities: fluency[2], flexibility[3], originality[4], elaboration[5], sensitivity to problems[6], problem defining[7], visualization[8], imagination, analogical thinking[9], analysis, synthesis, evaluation, transformation[10], intuition, logical thinking, etc. • develop creative attitudes and attributes: imagination, curiosity, self-confidence, independent judgement, persistence and commitment, tolerance for ambiguity, openness to new and unusual ideas/methods/approaches, deferment of judgement, adaptability, willingness to take sensible risks, etc. • Use and apply the Creative Problem Solving (CPS) Model and creative thinking techniques: brainstorming, 6W thinking technique, 6 hats method, attribute listing[11], idea checklists, synectics[12], mind mapping, etc.	Learners 1. create geometric patterns with different shapes and tell stories with given mathematical sentences 2. devise their own way/strategy when solving problems such as different solutions to a problem of plane geometry 3. adopt different approaches to a task or problem, such as proving a geometrical theorem using a synthetic or an analytical approach 4. pose related problems such as "Can triangles other than equilateral triangles be used in tessellation?" and "Will the same relationship $a^2 + b^2 = c^2$ in Pythagoras' Theorem still hold if the triangle is not right-angled?" 5. formulate hypotheses such as the value of a fraction decreases as the denominator increases if the numerator is kept constant 6. be imaginative in visualizing 3-D shapes 7. use and apply the technique of synectics to relate different given information, and utilize analogies to help analyze problems (e.g. deducing the formula of the volume of a cylinder from that of a prism)

Notes:

1. Climate conducive to creativity: Respecting the novel and unusual, providing challenges, appreciating individuality and openness, encouraging open discussion, absence of conflicts, allowing time for thinking, encouraging confidence and a willingness to take risks, appreciating and supporting new ideas, etc.
2. Fluency: The ability to produce many ideas in response to an open-ended problem, question or task.
3. Flexibility: The ability to take different approaches to a task or problem, to think of ideas in different categories, or to view a situation from several perspectives.
4. Originality: Uniqueness, nonconformity in thought and action.
5. Elaboration: The ability to add details to a given idea, such as to develop and embellish the idea.
6. Sensitivity to problems: The ability to identify problems, list out difficulties, detect missing information, and ask good questions.
7. Problem defining: The capability to 1) identify the "real" problem, 2) isolate the important aspects of a problem, 3) clarify and simplify a problem, 4) identify sub-problems, 5) propose alternative problem definitions, and 6) define a problem broadly.
8. Visualization: The ability to fantasize and imagine, "see" things in the "mind's eye" and mentally manipulate images and ideas.
9. Analogical thinking: The ability to borrow ideas from one context and use them in another; or the ability to borrow the solution to a problem and transfer it to another.
10. Transformation: The ability to adapt something to a new use, to "see" new meanings, implications, and applications, or to change an object or idea into another creatively.
11. Attribute listing: A creative thinking technique that involves listing out all the important characteristics of an item and suggesting possible changes or improvements in the various attributes.
12. Synectics: The joining together of apparently unrelated elements. This technique utilizes analogies and metaphors to help the thinker analyze problems and form different viewpoints.

Critical Thinking Skills

Critical Thinking is drawing out meaning from given data or statements. It is concerned with the accuracy of given statements. It aims at generating and evaluating arguments. Critical thinking is the questioning and enquiry we engage in to judge what to believe and what not to.

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • extract, classify and organize information from a source • identify and express main ideas, problems or central issues • understand straightforward cause-and-effect relationships • distinguish between obvious fact and opinion • recognize obvious stereotypes, assumptions, inconsistencies and contradictions • formulate questions, make predictions/estimations and hypotheses • draw simple but logical conclusions not contradictory to given evidence and data	Learners 1. sort objects using various criteria such as shapes and sizes 2. choose the right tools to measure objects such as using measuring tapes to measure the "circumference" of a head 3. reason inductively (e.g. when exploring the commutative property of addition) 4. check the reasonableness of the answer to a problem (e.g. the number of apples eaten by a boy per day might be too large to be realistic)
Key Stage Two (Senior Primary) Learners will learn to • understand and make deductions/ inferences from sources • cross reference other sources to determine the reliability of a source • understand the concepts of relevance and irrelevance • distinguish fact and opinion as well as source and evidence • question obvious bias, propaganda, omissions, and less obvious fallacies • formulate appropriate questions, make reasonable predictions and hypotheses • draw logical conclusions base on adequate data and evidence, and make predictions about consequences	Learners 1. categorize information using various criteria such as properties of quadrilaterals 2. choose appropriate methods and units to measure objects, such as using the method of displacement to measure the volumes of irregular objects 3. reason inductively (e.g. when exploring the formula for the area of a rectangle) 4. check the reasonableness of the solution to a problem (e.g. the steps for solving a problem might be unreasonably complicated)
Key Stage Three (Junior Secondary) Learners will learn to • compare different sources, note contrasts and similarities, and determine their reliability • distinguish fact, opinion and reasoned judgment • be aware that value orientations and ideologies would affect the perspective of a source • recognize and challenge stereotypes, inconsistencies, emotional factors, and propaganda • draw and test conclusions as well as hypotheses, identify reasonable alternatives and predict probable consequences	Learners 1. use inductive and deductive reasoning to study the properties of geometric shapes, such as when proving "the sum of the exterior angles of a convex polygon is 4 right angles" and "the base angles of an isosceles triangle are equal" 2. generalize observations in symbolic forms from concrete experiences (e.g. when generalizing the index laws from observing several examples in numbers) 3. judge whether the information given about a problem is relevant or not (e.g. extraneous data given in a geometrical problem) 4. examine the reasonableness of the solution to a problem and evaluate the strategy adopted (e.g. evaluate the effectiveness of using the graphical method to solve simple linear equations)

Information Technology Skills

IT skills include the ability to use IT to seek, absorb, analyze, manage and present information critically and intelligently. In addition, IT will motivate and empower our learners to learn at their own pace and help them develop habits of self-learning, which will benefit them for life.

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • operate computers in school • input Chinese characters with a handwriting recognition device • use multimedia resources to support learning with the help of teachers • communicate and handle information with IT tools in learning activities	Learners 1. use suitable software to investigate number patterns and properties of numbers (e.g. odd, even, ascending, descending) 2. use suitable software to create and explore geometric patterns (e.g. squares, rectangles, triangles)
Key Stage Two (Senior Primary) Learners will learn to • use a number of software packages for different purposes • input Chinese characters with devices and the aid of an input method • access information via computer networks and other media • process information using IT tools	Learners 1. use suitable software to investigate the properties of shapes, draw and create geometric patterns (e.g. draw squares, rectangles, triangles, circles and create geometric patterns with these figures) 2. use a spreadsheet to record data and create graphs for doing statistical projects (e.g. input data in a spreadsheet and present the data with line graphs, bar charts) 3. use the information obtained through Internet/Intranet in self-directed learning and when doing projects (e.g. symmetry)
Key Stage Three (Junior Secondary) Learners will learn to • use appropriate IT tools to facilitate learning • use IT tools and strategies for processing and presenting information • communicate with others via e-mail • verify and evaluate the accuracy and reliability of information	Learners 1. use scientific calculators/graphing calculators for various computational and exploratory activities (e.g. input data and create statistical graphs; draw straight lines and explore their relationship with slope) 2. use suitable software to explore the relations of numbers (e.g. number patterns), algebraic formula (e.g. formulae of area and volume) and graphical representations (e.g. pie charts and straight lines) 3. use suitable software to construct/explore appropriate statistical diagrams/graphs (e.g. bar charts, pie charts, line charts) to represent given data; to find simple statistical measures (e.g. mean, mode) and to explore the meaning of experimental probability (e.g. tossing coin simulation) 4. use geometry software packages to explore properties of 2-D rectilinear geometric figures dynamically (e.g. the relationship among the angles or sides of a parallelogram); to explore and visualize geometric properties of 2-D and 3-D figures intuitively (e.g. transformation and symmetry) 5. use the information obtained through Internet/Intranet in self-directed learning and when doing projects (e.g. statistical projects, projects on the development of mathematics in China, stories and achievements of mathematicians) 6. judge the appropriateness of using IT in solving mathematical problems (e.g. quicker to calculate $2\sin 30°$ mentally)

Numeracy Skills

Numeracy skills include the ability to perform basic computations, to use basic mathematical concepts in practical situations, to make reasonable estimates, to understand graphs, charts and numerical concepts in languages, to manage data, to handle money and do stock inventories.

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • perform basic computations • recognize and describe shape, position and direction • develop an intuitive knowledge of measurement and measuring units, and use appropriate tools for measurements e.g. ruler, thermometer • formulate and solve simple problems arising from collected data and constructed graphs • read and use simple quantitative information	Learners 1. describe the number of objects with natural numbers 2. perform properly basic computations involving whole numbers 3. use non-standard and standard measuring tools in comparing measures of different objects 4. recognize basic directions: east, south, west and north 5. recognize and describe 2D-shapes (e.g. triangles and quadrilaterals) and 3-D shapes (e.g. pyramids and prisms) 6. read simple statistical graphs and charts (e.g. block graphs and simple pictograms)
Key Stage Two (Senior Primary) Learners will learn to • perform numerical computations, calculate mentally and provide quick estimates of the accuracy of a calculation • understand intuitively the properties of shape, position and direction • extend measurement skills to concept areas such as volume • collect, process, present and evaluate quantitative information • use mathematical concepts to solve simple real-life problems	Learners 1. choose the correct forms of numbers in presenting information (e.g. using percentages to tell the discount obtained) 2. perform numerical computations to solve daily-life problems and evaluate their own work 3. apply formulae to find measures of simple 2-D shapes (e.g. the area of a triangle) 4. describe measures with appropriate units (e.g. using m^2 to measure the area of a courtyard while using cm^2 to measure the area of a sheet of paper) 5. organize simple data and interpret simple statistical graphs in various daily-life situations 6. recognize the 8 compass points, e.g north-east, south-west, etc. 7. use equations to solve simple problems
Key Stage Three (Junior Secondary) Learners will learn to • perform numerical manipulations and quick estimates of the accuracy of a calculation • understand properties of shape, position, direction and movement • apply formulae or choose the appropriate tools and strategies to find measures and note the approximate nature of measurement • use appropriate tools and strategies for collecting, processing and presenting quantitative information • estimate risks and chances through the use of elementary probability • solve real-life problems utilizing quantitative information	Learners 1. manipulate numbers, algebraic symbols, trigonometric relations, etc. to solve real-life problems 2. apply numerical estimation strategies and estimation strategies in measurement to various real-life situations (e.g. the number of significant figures) 3. apply formulae to find measures of 2-D and 3-D shapes (e.g. the area of a sector and the volume of a sphere) 4. apply spatial concepts in real-life situations (e.g. the angle of elevation and bearings) 5. apply simple ideas of data handling to understand statistical data and graphs in various sources from real-life experience 6. apply simple ideas of probability in various real-life situations (e.g. the fairness of games)

Problem Solving Skills

Problem solving involves using thinking skills to resolve a difficulty. It assembles facts about the problem and determines the best course of action.

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • develop ideas about the problem and identify sources of information and help • identify, under guidance, different ways of tackling the problem • choose and implement a solution plan, using support and advice given • follow the given step-by-step methods to check and describe the outcomes	Learners 1. use simple methods for solving problems (e.g. using addition to find the sum of money used in buying goods) 2. adopt various ways of solving problems (e.g. using drawing and manipulatives to do addition) 3. solve problems by choosing the correct given data (e.g. choosing the correct combination of coins for one exact bus fare)
Key Stage Two (Senior Primary) Learners will learn to • identify the problem and describe its main features • propose alternative courses of action for solving it • plan and try out the selected option, obtain support and make changes when needed • develop an appropriate method to measure the outcomes and examine the approach chosen	Learners 1. use different approaches to solving problems (e.g. finding the area of a shape by means of counting the number of squares, dissecting the shape into parts or using formulae) 2. make use of various tools in solving problems (e.g. measuring tapes and calculators) 3. choose the correct given data and relevant information to solve problems (e.g. identify the correct height and base in finding the area of a triangle) 4. use past experience to solve new problems, for example, by comparing the new problems with similar ones solved before (e.g. comparing the ways of finding the area of a right-angled triangle and that of a general one) 5. adopt various ways of solving problems such as using tables and formulae (e.g. formulae for perimeter, area, etc.)
Key Stage Three (Junior Secondary) Learners will learn to • explore the problem and identify the issue(s) at stake • suggest and compare the possible outcomes of each alternative course of action and justify the option selected • execute the planned strategy, monitor progress and revise the approach when necessary • evaluate against established criteria the quality of outcomes, and review the effectiveness of the solution process	Learners 1. understand a given problem; devise an appropriate plan for solving problems (e.g. geometrical problem) and justify or evaluate the solution presented 2. appreciate and pursue a better strategy for solving problems (e.g. by comparing different strategies for solving geometrical problems and comparing different data sets with means and medians) 3. judge and evaluate arguments of their own or others in presenting a solution to a mathematical problem (e.g. explain why "mode" is the best measure of the most popular size of shoes in a market) 4. formulate a mathematical solution when tackling a real-life problem (e.g. finding the area of a layout plan of a flat)

Self Management Skills

Self-management skills are essential for the building up of self-esteem and the accomplishment of goals. Learners who have mastered self-management skills understand their own feelings and preserve emotional stability. They are positive and proactive towards work. They set appropriate goals, make plans and initiate actions to achieve them. They manage time, money and other resources well. They are able to handle stress and tolerate ambiguities.

(The expected achievements of the learners in this type of generic skills cannot suitably be classified according to key learning stages)

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Learners will learn to 1. evaluate their own feelings, strengths, weaknesses, progress and objectives (self-assessment) 2. consider aspects of their performance, attitudes and behaviour in order to change or enhance future outcomes (self-reflection) 3. be confident in their own judgements, performance and capabilities (self-confidence) 4. make informed decisions and safe choices in reaching goals and carrying out tasks, develop good habits and maintain a healthy life style (self-discipline) 5. work under unfamiliar, stressful or adverse conditions, accept changes and new ideas and be able to handle diversity and tolerate ambiguity (adaptability) 6. make decisions and initiate actions on their own and draw satisfaction from their own effort (self-motivation) 7. keep promises and fulfill obligations (sense of responsibility) 8. control their own emotions and impulses and maintain emotional balance (emotional stability)	Learners 1. work neatly and tidily in accomplishing tasks (e.g. statistical projects) and doing mathematical problems (e.g. drawing geometrical figures with rulers and pencils) 2. appreciate/accept and evaluate others' opinions in accomplishing tasks (e.g. different ways of collecting data) and solving problems (e.g. different strategies for solving geometrical problems) 3. self-evaluate their own strengths and weaknesses in mathematics learning 4. should be responsible for their own work, for example, by checking the answers to problems 5. should be self-reliant, confident and willing to apply mathematics in solving problems independently 6. should be persistent in solving difficult/complicated problems 7. keep emotions under control when facing failure in solving problems 8. communicate their own feeling to others when facing failure in solving problems 9. set goals and priorities properly in mathematics learning

Study Skills

Study skills help to improve the effectiveness and efficiency of learning. They underpin the learning habits, abilities and attitudes that form the essential foundation for lifelong learning.

Descriptors of Expected Achievements Across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • identify the main points and ideas in different types of straightforward reading material • use different forms of writing to present main ideas clearly • collect information from given sources, organize them into predetermined categories and analyze them according to preset guidelines • understand the need to set up a study plan and follow a given plan to meet short-term targets	Learners 1. understand concepts and do not learn only by rote memorization (e.g. students have to understand the concept of multiplication and build up the multiplication tables themselves instead of just memorizing the multiplication tables; understand the concept of perimeters and use various ways to find the perimeters of figures instead of using formulae only) 2. present simple problems in mathematical languages including symbols (e.g. 1+2=3) and graphs (e.g. pictograms) 3. use simple objects to build up mathematical concepts (e.g. use $1cm^3$ cubes or an abacus to grasp the concept of addition, subtraction; and use a pin board to grasp the concept of the perimeter and area of a rectangle)
Key Stage Two (Senior Primary) Learners will learn to • identify main lines of reasoning, skim materials to gain general ideas of content and scan text to obtain main points • use different forms and styles of writing for different purposes and present main ideas coherently in a given form and style of writing • locate required information from a variety of sources, organize it into self-defined categories and assess it for completeness, accuracy and relevance • develop short-term and intermediate study plans to meet targets and purposes of study identified by themselves	Learners 1. try to understand concepts and not to learn only by rote memorization (e.g. students have to understand the concepts and formulae of areas and volumes instead of memorizing the formulae in solving problems) 2. use more aids to build up mathematical concepts (e.g. use a calculator to learn number patterns) 3. present problems in mathematical languages including symbols (e.g. simple algebraic equations) and graphs (e.g. bar charts) 4. use diagrams, pictures and charts to help understand mathematical concepts (e.g. use travel graphs to understand the idea of speed) 5. learn from mistakes made in solving mathematical problems in homework exercises, tests, etc.
Key Stage Three (Junior Secondary) Learners will learn to • identify accurately complex lines of reasoning and hidden ideas and distinguish facts from opinions • select an appropriate form and style of writing for a specific purpose and develop a writing strategy for organizing ideas and information clearly and coherently • define purposes of collecting information, critically investigate sources to distil relevant information and evaluate its quality and validity • review and revise study plans developed for short-term, intermediate and long-term targets to meet new demands and to improve study performance	Learners 1. understand and do not only memorize by rote the meanings of different forms of mathematical objects, concepts and principles 2. generalize observations to a higher level of abstraction for better memorization and for transferring strategies to solving a wider range of problems 3. learn from mistakes made in solving mathematical problems in homework exercises, tests, etc. 4. identify key similarities and differences from working among various types of mathematical problems 5. search and select information from various sources including libraries, reference books, Internet, etc. 6. use diagrams, pictures and charts to help understand mathematical concepts (e.g. use charts to present the hierarchy of the real number system)

2.2.3 Values and Attitudes

Besides knowledge and skills, the development of positive values and attitudes is also important in mathematics education. Values and attitudes such as responsibility, commitment and open-mindedness are necessary for developing goals in life and learning. Learning effectiveness can be enhanced by the inculcation of positive attitudes through appropriate learning and teaching strategies (such as giving positive feedback to student performance). These will in turn reinforce students' development of those values and attitudes as part of character formation. These values and attitudes permeate the mathematics curriculum in different learning dimensions and key stages of learning and have been incorporated into the learning objectives of the Primary Mathematics Curriculum (2000) and the Secondary Mathematics Curriculum (1999). The following learning objectives illustrate how the revised mathematics curricula relate to the development of positive values and attitudes and aim at facilitating the planning of relevant learning experiences in the Mathematics Education KLA. These learning objectives, however, are neither exhaustive nor do they imply that the related values and attitudes should progress in the order stated. In fact, they can be realized at all key stages to different extents.

- Develop **interest** in learning mathematics.
- Show **keenness** to participate in mathematical activities.
- Develop **sensitivity** towards the importance of mathematics in daily life.
- Show **confidence** in applying mathematical knowledge in daily life, clarifying one's argument and challenging others' statements.
- Share ideas and experience and work **co-operatively** with others in accomplishing mathematical tasks/activities and solving mathematical problems.
- Understand and take up one's **responsibilities** in group work.
- Be **open-minded** in doing group work, willing to listen to others in the discussion of mathematical problems, respect others' opinions, and value and appreciate others' contributions when doing mathematics together.
- **Think independently** in solving mathematical problems.
- Be **persistent** in solving mathematical problems.
- **Appreciate** the precise, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

Generic skills, values and attitudes are not developed in a vacuum. They are expected to be developed/fostered through the learning of mathematical knowledge in the content areas. It is desirable for teachers to help students cultivate them through planned learning activities. Figure 3 illustrates how they intertwine to form a reference grid.

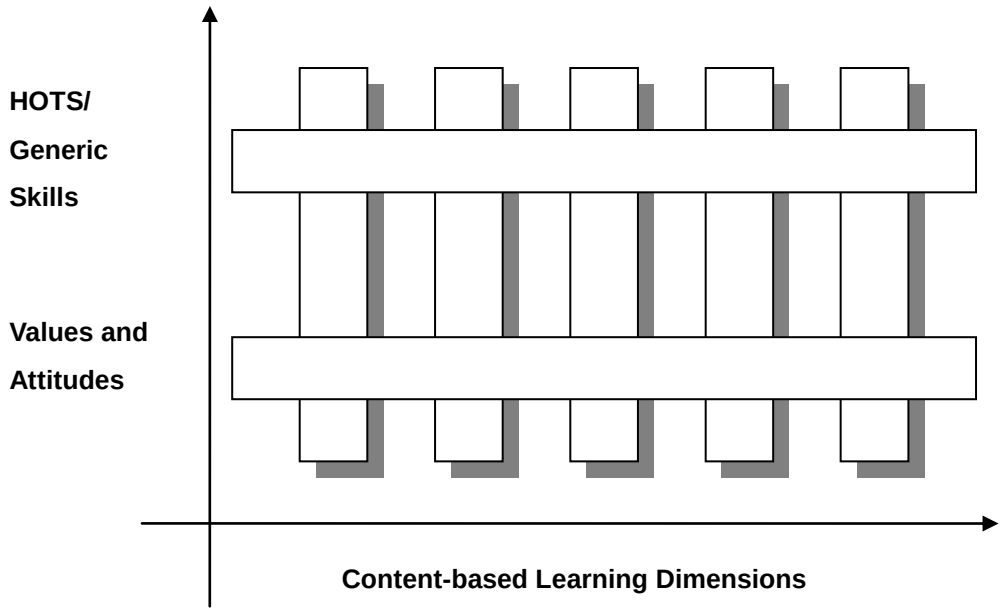

Figure 3

The exemplars at the end of this Guide also illustrate how the learning targets and generic skills can be linked. In particular, the linkage between exemplars 1 – 5 (exemplars 1 – 3 are for primary level and exemplars 4 – 5 for junior secondary level) and the main learning targets and generic skills is summarized in the following table.

Exemplar	Main learning targets	Main generic skills
1	"To understand … whole numbers" of the Number Dimension in Key Stage 1	• Critical thinking • Communication • Problem-solving
2	"To group and make … 3-dimensional shapes" in the Shape & Space Dimension in Key Stage 2	• Communication • Critical Thinking • Problem-solving

3	"To formulate and solve problems arising from collected data and constructed graphs" in the Data Handling Dimension in Key Stage 2	• Collaboration • Critical Thinking • Problem-solving • Creativity • Numeracy
4	"To interpret simple algebraic relations from numerical, symbolic and graphical perspectives" in the Number and Algebra Dimension in Key Stage 3	• Information Technology • Numeracy • Critical Thinking • Problem-solving
5	"To explore and visualize geometric properties of 2-dimensional and 3-dimensional objects intuitively" in the Measures, Shape and Space Dimension in Key Stage 3.	• Information Technology • Numeracy • Creativity • Problem-solving

Schools can develop their own school-based mathematics curriculum along the same line as the framework with reference to the detailed set of learning objectives of the Primary Mathematics Curriculum (2000) and the Secondary Mathematics Curriculum (1999).

2.3 Core and Extension

Upon the implementation of universal education in Hong Kong, a wider range of students gain access to mathematics than has been in the past. Therefore, the issue of student diversities has been considered in the development of the Primary Mathematics Curriculum (2000) and the Secondary Mathematics Curriculum (1999).

The Secondary Mathematics Curriculum (1999) has been divided into two parts, namely the Foundation Part and the Non-Foundation Part. The Foundation Part represents the topics that ALL students should strive to master. It is identified under the principles that:

(a) it is the essential part of the Curriculum stressing the basic concepts, knowledge, properties and simple applications in real life situations;

(b) it contains different components that constitute a coherent curriculum.

On the other hand, no such division is made in the Primary Mathematics Curriculum (2000) as most of the concepts are basic and fundamental at the primary level.

For more able students, teachers can adopt some enrichment topics (like *chance* and *number pattern* at the primary level and *properties of the centres of a triangle* and *rotational symmetries in regular polyhedra* at the secondary levels) at their discretion to extend students' horizon and exposure in mathematics.

2.4 Curriculum Organization

The principle of proposing different modes of curriculum planning is to build on the particular strengths of schools and teachers, and provide them with curriculum flexibility and diversity to meet different purposes of learning and teaching. At the basic education level, the curricula are planned by objectives and a "dimension approach" is adopted. Although the dimensions are content-based, HOTS, generic skills, values and attitudes should be incorporated into the content (see Figure 3 of Section 2.2.3) during teaching.

The learning units and modules for each dimension are developed in terms of key stages in both the primary and secondary curricula. Teachers are free to select the learning units and modules for each year level from those suggested in the key stage concerned, provided that all the units and modules selected are presented logically and consistently. Enrichment activities/topics are provided in the Primary Mathematics Curriculum (2000), while the Foundation Part, Non-Foundation Part and enrichment activities/topics are provided in the Secondary Mathematics Curriculum (1999) to suit the different abilities of students. Spare periods are also reserved at each key stage to provide teachers with curriculum space to rearrange or to adapt the content and depth of the teaching materials, use IT in mathematics learning and teaching and organize exploratory activities or projects. (Note: Teachers should not use these spare periods for unnecessary drilling.) Teachers have the flexibility to design their school-based mathematics curriculum to cater for the needs of their students. For example, at Key Stage 3 (see Figure 4), teachers are suggested to teach *Linear Equations in One Unknown* at the beginning. After that, they are free to choose the order of teaching on the following combinations of topics:

(a) *Linear Inequalities in One Unknown*
(b) *Formulas* followed by *Linear Equations in Two Unknowns*
(c) *Identities* followed by *Factorization of Simple Polynomials*

Teachers can teach according to the order (b), (a), (c) or (b), (c), (a). In addition, teachers are giving a free hand to teach the combinations of topics anytime at the key stage. All the topics mentioned form the prerequisites for learning the related topics at Key Stage 4.

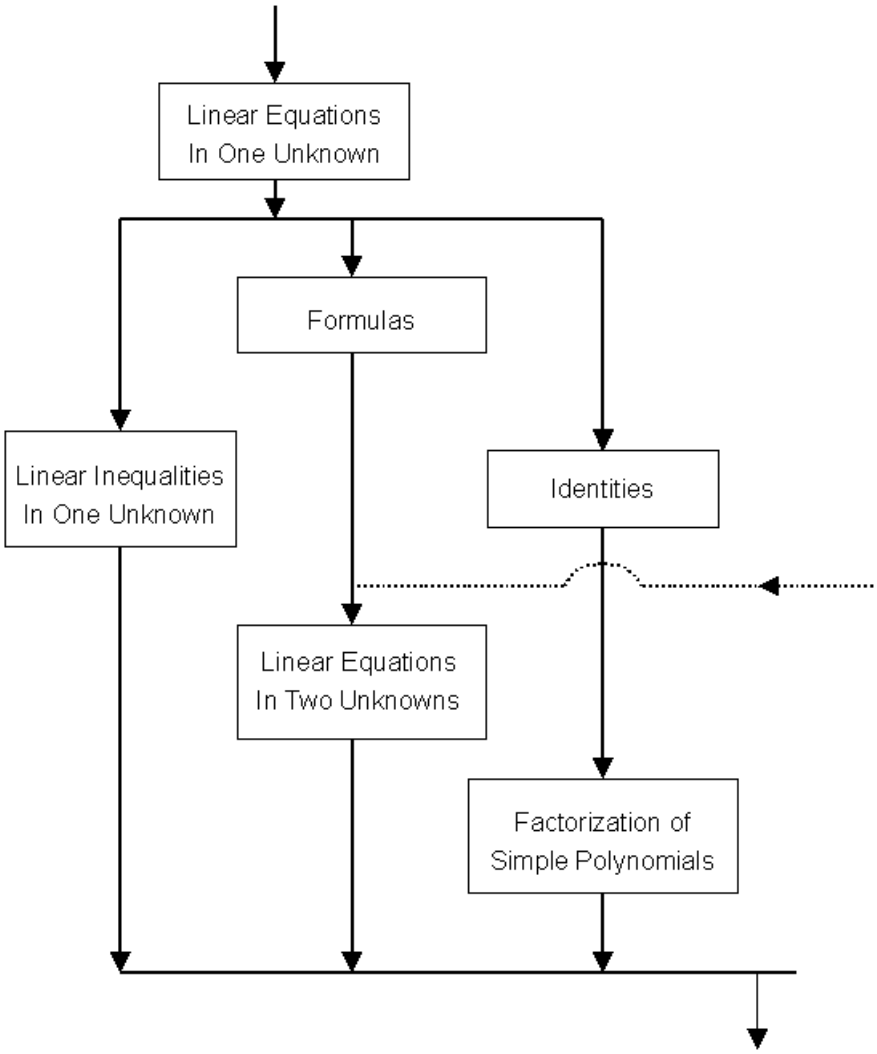

Figure 4

2.5 Transition between Kindergarten and Primary School, Primary and Secondary School

Students adapt to changes during the transition from Kindergarten to P1 and also from P6 to S1. To ease the transition, the following approach is expected in curriculum planning and classroom teaching:

- Children at the pre-primary level are expected to acquire pre-mathematics concepts through hands-on activities and experiences. Formal computation and calculations are not recommended. Emphases should be put on developing children's interest in mathematics learning. Learning activities could be integrated through thematic approach, but there should be flexibility in curriculum integration.

- Integrating children's experience and focusing on the application of mathematics in daily-life could be continued at the junior primary levels. Diversified learning activities should be organized to arouse students' interests and develop their mathematical abilities, in particular, their high-order thinking abilities. Exercise could be used for consolidation but the amount and quantity of assignments should be reasonable. There should be a due amount of drill-and-practice, but over-drilling is not recommended.

- At the upper primary levels, mathematical concepts are further strengthened by providing a wide range of materials and learning activities. Learning and teaching should be closely related to students' hands-on experiences. Apart from solving routine problems which involve mostly rote learning, more emphases should be put on exploratory activities which involve diversified thinking abilities. There should be a balance between exposition and exploration in the learning and teaching of mathematics.

- The teaching approach adopted at the upper primary levels could be continued at the junior secondary levels. Use of IT could be properly enhanced to illustrate mathematical concepts and to serve as a tool to develop conceptual understanding.

2.6 Interface with S4-5

As far as teaching is concerned, there is no much difference between S3 and S4. As regards to the curriculum, the Foundation Part is self-contained. Students learning only the topics of the Foundation Part at Key Stage 3 have sufficient background knowledge to study the mathematics curriculum at Key Stage 4. Nevertheless, it should be noted that students studying only the Foundation Part at Key Stage 3 may have difficulties in studying Additional Mathematics at Key Stage 4. Teachers should spend more time to help students master the concepts and skills which have been skipped (such as trigonometry with angles greater than 90 degrees).