## 4.1 Guiding Principles

In designing learning and teaching activities in the subject of Mathematics, the following principles should be noted:

- Our main concern is to help students *learn to learn* rather than merely delivering subject content to them.
- All students can learn, but they learn at different speeds.
- A learner-focused approach should be adopted.
- There should be a balance between what students are expected to learn in the mathematics curriculum prepared by the CDC Committee on Mathematics Education and aspects of school-based curriculum development based on the needs of the students in school.

The Primary Mathematics Curriculum (2000) and Secondary Mathematics Curriculum (1999) have been developed in line with these principles. Their main focus is not only on what mathematical topics should be learnt but also on how mathematics is learnt. The acquisition of HOTS and generic skills and the fostering of positive attitudes towards mathematics learning should be embedded within the learning of mathematical content. The teachers' role is to help students to learn how to learn mathematics.

There is flexibility in the mathematics curriculum of Hong Kong. For example, the Foundation & Non-Foundation Parts and enrichment activities in the Secondary Mathematics Curriculum (1999) provide teachers with a viable way to tailor the mathematics curriculum to the needs of students of different abilities.

A learner-focused approach is recommended. Due consideration should be taken of students' abilities, experience and interests when designing a teaching approach. Diversified learning/teaching activities including projects are encouraged to suit the different abilities and interests of students. Greater emphasis is placed on getting students to apply mathematical concepts as this provides students with the motivation for learning mathematics. The use of daily-life situations helps students to relate what they learn in the classroom to reality and to realize the need for mathematics.

Apart from the formal mathematics curriculum, mathematics-related extra-curricular activities also play an important role in mathematics learning. It is generally agreed that well chosen and organized mathematics-related activities help to promote students' interest in learning the subject. Examples include mathematical games/puzzles, mathematics

competitions/quizzes, mathematics workshops, projects, talks, plays, film shows, mathematics bulletins, newspaper cuttings and board displays, etc.

## 4.2 Approaches to Learning and Teaching

To address the needs of our students to face the challenges of the 21st Century and to help them develop the capability of learning to learn, mathematics learning and teaching should focus on the learning process and the fostering of thinking abilities, generic skills and positive attitude of learning mathematics. However, it should be noted that the teacher is always the key person in the classroom teaching. The liveliness and clear explanation of the teachers are students' main concerns and is the key element leading to a successful mathematics lesson. Teachers have the responsibility of delivering clear explanation, designing and conducting activities in lessons, creating a good environment and showing concern for students' progress.

### 4.2.1 The Four Key Tasks

The four key tasks, namely moral and civic education, reading to learn, project learning and information technology for interactive learning can help students to develop independent learning capabilities and hence help to realize the spirit of "learning to learn" in schools.

*Moral and Civic Education*

The development of positive attitudes and a sense of commitment provide the affective basis for students to learn more effectively. In mathematics, the moral and civic elements could be introduced in the following ways:

- Through tackling problems, students foster appropriate attitudes towards alternative opinions (as there may be no unique solution to a problem). Although some solutions may be more efficient than others, it is often a matter of taste.
- The inclusion of daily life examples in the mathematics classroom will enhance students' awareness of its relation to real life.
- By organizing project work or extra-curricular activities related to mathematics, students are provided with opportunities to develop an inquiring mind, to take up responsibilities, to learn to be cooperative and to acquire leadership and social skills.

*Reading to Learn*

To encourage students to read more mathematics reference books, providing them with magazines and journals might help to promote independent learning capabilities and achieve the learning targets of the school curriculum. Mathematics-related recreational books like the story of $\pi$, the stories of mathematicians, etc. can be considered. Students might present a brief report on the topics that have interested them.

*Project Learning*

Students learn best when they (a) are actively involved in learning; (b) are making discoveries, building on what they already know and looking for answers to questions that interest them; (c) are challenged to think for themselves; and (d) can pursue their own interests and discover new ones.

Working on projects is one effective method of learning which meets the above requirements. Project learning is a powerful learning and teaching strategy to promote self-directed, self-regulated and reflective learning. It can provide students with exciting, challenging and action-oriented activities and hence can enable students to connect knowledge, skills, values and attitudes when constructing their mathematical knowledge.

Teachers have a role to play in project learning. They need to provide guidance, support and advice to their students. If their students are becoming more independent in the learning process, teachers merely act as facilitators. Projects can be done by students individually or in groups depending on their nature. Collaboration and communication skills, however, will be more effectively developed by projects done in groups.

Projects should be selected to respond to students' interests and abilities. Investigations, surveys, problem-solving strategies, applications of mathematics, research studies, the history of mathematics, famous mathematics problems, biographies of mathematicians and book reviews are typical topics and types of projects. Exemplars 8 – 10 of the exemplars provided at the end of this Guide illustrate how projects can be adopted in mathematics learning and teaching. The depth of treatment of the projects can vary according to students' abilities. Similarly, different levels of guidance can be provided to cater for students' abilities, the focus and the difficulty and nature of the project. There should be continuous

support, monitoring and regular feedback from teachers. Project work could be assessed holistically by some pre-determined criteria (see Section 5.2).

*Information Technology for Interactive Learning*

The positive role that IT can play in mathematics learning has been emphasized in the Primary Mathematics Curriculum (2000) and Secondary Mathematics Curriculum (1999). The popularity of calculators, the availability of computers and other IT aids in the classrooms has had a considerable impact on the mathematics curriculum in terms of content and strategies for learning and teaching. There are many ways in which IT may be used in the mathematics classrooms. These include using computers for data analysis, as a simulation device, graphical presentation, symbolic manipulation and for observing patterns. The appropriate use of IT in the learning and teaching of mathematics has become one of the important emphases in the mathematics curriculum. Exemplars 4 – 5 of the exemplars at the end of this Guide show how IT can be utilized in mathematics learning and teaching. Nevertheless, teachers should pay special attention to the situation that extensive use of IT tools in mathematics learning may lead to the de-emphasis of skills. Care needs to be taken to ensure that the necessary knowledge and skills are learnt.

(School may refer to Booklet 3 of the ***Basic Education Guide – Building on Strengths (2002)*** for more ideas on four key tasks.)

### 4.2.2 Life-Wide Learning

The provision of life-wide learning opportunities enables students to learn from experience in real contexts. This sort of learning cannot always be provided in the classroom. The choice of life-wide learning activities should be both school-based and student-focused. There are many opportunities for students to learn mathematics through experience outside schools. Typical examples include visits to place such as supermarkets, Science Museum and Space Museum, public lectures, activities and learning packages jointly developed by tertiary institutions, professional bodies and/or the government. Some popular activities are:

| Activities | Specific Purposes | Organizing Bodies |
|---|---|---|
| Mathematics Competition for Hong Kong Primary Schools – a mathematics competition for primary students | The Competition aims to promote students' interest in studying mathematics and hence improve mathematics learning in primary schools. | Hong Kong Professional Teachers' Union |
| Po Leung Kuk Primary Mathematics World Contest<br>– an international event for primary students | The Contest aims to discover mathematically gifted primary students in all countries. It also aims to create an opportunity for the exchange of learning experiences among gifted students of all countries. | Po Leung Kuk (Training is provided to potential participants by Po Leung Kuk.) |
| National Primary Mathematics Olympiad (HK Competition) – a mathematics competition for primary students | The competition aims to promote and sustain primary students' interest in the study of mathematics. Through the Olympiad, students with special talent in mathematics are identified. | • Hong Kong Association of Science and Mathematics Education<br>• Mathematics Section of ED |
| Mathematics & Science Trail – a mathematics and science competition for secondary students | Mathematics & Science Trail aims to enhance students' problem-solving skills in mathematics and science through applications in real life situations in the physical environment. | • Hong Kong Association for Science and Mathematics Education<br>• Mathematics Section of ED<br>• Science Section of ED. |

| | | |
|---|---|---|
| The Hong Kong Mathematical High Achievers Selection Contest | The Contest aims to discover and challenge mathematical high achievers. | • Po Leung Kuk<br>• Hong Kong Association for Science and Mathematics Education |
| Statistical Project Competition for Secondary School Students – a statistics competition for secondary students | The Competition aims to promote students' interest in studying statistics and to encourage them to understand the community in a scientific manner through the use of statistics. | Hong Kong Statistical Society |
| Mathematics Project Competition for Secondary Schools | The Competition aims to promote students' interest in studying mathematics and provide students with an opportunity to develop their independent learning capabilities. | Mathematics Section of ED |
| HK Mathematics Olympiad (HKMO) – a mathematics competition for secondary students | The Olympiad aims to promote and sustain students' interest in the study of mathematics. | • Department of Mathematics of Hong Kong Institute of Education<br>• Mathematics Section of ED |
| Poster Design Competition for the Hong Kong Mathematics Olympiad (HKMO) | The Poster Design Competition aims at promoting the HKMO. | • Department of Mathematics of Hong Kong Institute of Education<br>• Mathematics Section of ED |

| | | |
|---|---|---|
| International Mathematical Olympiad – an international event for secondary students | The Olympiad aims to discover and challenge mathematically gifted young people from all countries who have not formally enrolled at any university. It also aims to foster friendly relations among gifted students of all countries. | • International Mathematical Olympiad HK Committee which is affiliated to the HK Mathematics Society<br>• Mathematics Section of ED<br>(Training is provided to potential participants by the Committee.) |

Details of some activities can be found in the booklet 《全方位學習活動簡介》 (pp.8-9) published by the Education Commission. School may also refer to Booklet 6 of the ***Basic Education Guide – Building on Strengths (2002)*** for more ideas on life-wide learning.

## 4.3 Catering for Student Diversities

Since each student has his/her own strengths and weaknesses, the student diversities become a key issue in mathematics learning and teaching. Three aspects in planning strategies to cater for student diversities, namely the central curriculum aspect, the school aspect and the classroom aspect, are suggested below.

### 4.3.1 Central Curriculum Aspect

In designing the central curriculum, the needs of students at both ends of the ability scale are equally important. Opportunities to learn should be maximized for all students. That is to say, attention should not be placed only on lower academic achievers. The needs of the more able students should also be catered for. Flexibility in the curriculum organization is provided in the mathematics curriculum of Hong Kong. Further details can be referred to Section 2.3 and 3.2 above.

### 4.3.2 School Aspect

The panel head, in collaboration with other panelists, should make a careful diagnosis of the students' general strengths and weaknesses in mathematics as well as their needs. Based on this information, schools are expected, with close reference to the mathematics curriculum, to draw up their school-based mathematics curriculum. Strategies in catering for student diversities at the school level include:

(a) Adopting organizational arrangements such as providing additional lessons to certain students and ability grouping strategies like streaming, split class, withdrawal and cross-level subject setting.

(b) Selecting the coverage of the learning units from the Non-Foundation Part of the Secondary Mathematics Curriculum (1999) or select the depth of treatment of topics in the Primary Mathematics Curriculum (2000) as the common core learning contents for all students.

(c) Arranging the learning contents in a logical sequence for each year level taking into consideration the cognitive development and the mathematical abilities and interests of students; the learning objectives of each learning unit; the inter-relation of learning units; the inter-relation of mathematical learning at different year levels.

For low ability students in schools, it is necessary to review the essential topics regularly. The spiral approach assists students to refresh their knowledge. However, it should be noted that though the spiral approach has its strong points, the teaching of too many topics in a single year and fragmentation of learning (i.e. breaking learning down into unconnected bits of knowledge or skills) should be avoided. In addition, measures like organizing bridging programmes can be introduced to ensure that students of different abilities can follow.

(d) Choosing an appropriate textbook and adapting or producing instructional materials. Schools may use different textbooks for different ability groups of students in the same year level, or use the same textbook but centrally produce different instructional materials to support students of similar ability in different classes.

(e) Designing a wide variety of informal and non-formal learning activities such as statistical projects, weekly questions posted in the mathematics bulletin boards, mathematics books reading scheme, poster design using transformation of shapes, mathematics camp, etc. Students with different inclinations and abilities may participate in different activities that suit their needs or interest.

High ability students should also be encouraged to participate in outside school activities like National Primary Mathematics Olympiad (HK Competition), Hong Kong Mathematics Olympiad, etc. On the other hand, outside-school activities like Mathematics Project Competition for Secondary Schools, Poster Design Competition for the Hong Kong Mathematics Olympiad (HKMO) are open to all secondary school students no matter whether their mathematical achievements are high or low.

(f) Formulating the assessment policies and the method of recording and reporting to provide feedback for learning and teaching. Schools may empower teachers to design their own assessment methods to suit the needs of individual classes such as allowing certain percentages, say 5% to 10%, of the mathematics scores in students' report cards to teachers' own discretion. Class teachers may design their own test papers, project works, daily marks, etc., which account for 5% – 10% marks.

Schools may also consider using different test papers for classes of different mathematical abilities. The advantage of using different test papers can provide flexibility for each class to assess what they have learnt and at their particular ability levels. Information provided would reflect the performance of students suitable to the ability criteria and would serve the purpose of assessment for learning.

Exemplars of arranging learning units and activities to cater for different learning abilities of students at S1 are given below. It shows how the learning units can be organized, having considered the depth of learning of each unit and the time allocation for the year and students' learning abilities. Similar arrangement can be made at other year levels.

**Exemplar on Arrangement of Learning Units at S1**
**for Schools with a Majority of Academically Lower Achievers**

| Year | Number & Algebra | Data Handling | Measures, Shape & Space | Sub-Total |
|---|---|---|---|---|
| S1 | Revision of arithmetic * (+10)<br>Directed numbers and the number line (12+3)<br>Numerical estimation (5+2)<br>Using percentages (17+3)<br>Formulating problems with algebraic language (14+4)<br><br>**48+22** | Introduction to various stages of statistics (5)<br>Revision of statistical graphs learnt in primary schools & simple projects (+5)<br><br>**5+5** | Introduction to Geometry(10+3)<br>Transformation and symmetry (6+2)<br>Congruence and similarity excluding construction (14–6+3)**<br>Angles related with lines and rectilinear figures excluding construction (18–3+2)<br>Estimation in Measurement (6)<br>Simple Idea of Areas and Volumes (15+3)<br><br>**69–9+13** | 153[@] |

**Exemplar on Arrangement of Learning Units at S1**
**for Schools with a Majority of High Ability Students**

| Year | Number & Algebra | Data Handling | Measures, Shape & Space | Sub-Total |
|---|---|---|---|---|
| S1 | Directed numbers and the number line (12)<br>Numerical estimation (5+2)<br>Formulating problems with algebraic language (14+1)<br>Laws of Integral Indices (10+3)<br>Manipulation of Simple Polynomials (10)<br>Linear Equations in One Unknown (7)<br><br>**58+6** | | Introduction to Geometry (10+3)<br>Transformation and symmetry (6)<br>Congruence and similarity (14+3)<br>Angles related with lines and rectilinear figures (18+4)<br>Estimation in Measurement (6)<br>Simple Idea of Areas and Volumes (15+1)<br>Introduction to Coordinates (9)<br><br>**78+11** | 153[@] |

Notes: * The items shaded are suggested learning contents to consolidate learning in primary school levels.

** Numbers in brackets such as (14-6+3) are interpreted as follows:

14: number of periods as described in the Secondary Mathematics Curriculum (1999)

-6: number of periods to be deducted because of not treating the topics in the Non-Foundation Part.

+3: additional number of periods for enrichment or consolidation activities

[@] The total number of periods allocated for each year level in Key Stage 3 is 160.

### 4.3.3 Classroom Aspect

No matter how the curriculum is designed or how students are organized in schools, it is important that the class teacher should be flexible enough to adjust his/her teaching plan to suit the needs of students. Below are some of the strategies for teachers in designing their classroom activities:

*Diagnosis of Students' Needs and Differences*

Teachers need to gather background information of students, including their interests, their strong and weak areas. Self-designed tests can be used, but teachers' own observation of students' performance in class and in written assignments is also a reliable basis for diagnosis.

*Variation on Level of Difficulties and Contents Covered*

Based on the above findings, teachers can plan relevant learning activities for each lesson. Teachers have to select, adapt or design materials to suit the range of abilities of their students. Too easy or too difficult tasks will not stimulate and sustain student's internal drive to learn. For less able students, tasks should be relatively simple and fundamental in nature as these activities can give students greater sense of satisfaction and hence greater confidence. On the other hand, for more able students, tasks assigned should be challenging enough to cultivate as well as to sustain their interest in mathematics learning.

Below is one example in the Key Stage 3 learning unit "Congruence and Similarity" in the Measures, Shape and Space Dimension of the Secondary Mathematics Curriculum (1999). For less able students, teachers can consider not to include the underlined learning objectives, which are not in the Foundation Part. For more able students, teachers can cover all learning objectives and select the enrichment topic, denoted with **.

- Recognize the properties for congruent and similar triangles
- Extend the ideas of transformation and symmetry to explore the conditions for congruent and similar triangles
- Recognize the minimal conditions in fixing a triangle
- Identify whether 2 triangles are congruent/similar with simple reasons
- Explore and justify the methods to construct angle bisectors, perpendicular bisectors and special angles by compasses and straight edges
- Appreciate the construction of lines and angles with minimal tools at hand
- **Explore some shapes in fractal geometry

*Variation in Questioning Techniques*

On the other hand, through providing students with different clues when asking questions, teachers can enable students to learn the same topic at the same year levels. In general, teachers can ask simple and straightforward questions to less able students and comparatively more challenging questions to more able ones. Even for the less able students, teachers can request them to modify their answers, to explain their strategies of solving the problems instead of giving the solutions right after they give wrong answers. Exemplar 7 of the given exemplars at the end of this Guide can be referred to for the details.

*Variation in Clues provided in Tasks*

Teachers can also provide students with the same task or exercise but with additional supports such as diagrams to aid comprehension and structuring long question for less able students. For the more able students, teachers may ask open-ended questions and provide fewer hints in the process of solving problems. Further, open-ended problems, such as Exemplars 5, 11 – 13 at the end of this Guide, can also be used to motivate students to solve the problems with strategies suited their abilities and concerns.

*Variation in Approaches in Introducing Concepts*

Teachers can introduce mathematical concepts with different approaches. Concrete examples may be used to illustrate the concepts for young students but can use symbolic language for more mature students. For example, the number of petals on flowers from our environment can be used to illustrate the pattern of Fibonacci Sequence for less able students, whereas the recursion idea of the Sequence can be introduced to more able students.

*Variation in Using Computer Packages*

Different levels of exercises or activities are always included in the educational software packages. Appropriate use of IT provides teachers with a way to cater for student diversities as it allows students with different abilities to learn at different paces. The facilities to record students' performance in these software packages also provide information for teachers to diagnose students' misconceptions or general weaknesses so that they can re-adjust the teaching pace and the teaching strategies. Teachers can also make use of the built-in functions in different software to design activities of different levels of difficulty.

*Variation in Peer Learning*

Besides whole-class teaching, teachers can also consider different grouping strategies to cater for the needs of different students. However, it should be note that the way the groups are formed, the suitability of the tasks designed for the groups, the durability of the grouping and the ongoing assessment of the group dynamics are ingredients for successful collaborative learning. Further, it is very important to build up the collaborative instead of competitive atmosphere that is found undesirable in effective students' learning.

Teachers may consider grouping students with similar learning abilities, different learning abilities, or in different group size for collaborative learning. However, care must be taken to avoid labeling effect on grouping students of same abilities especially for a long duration. Instead, heterogeneous groups may lead to both positive academic and remedial outcomes. When students of different abilities are grouped together, the high-ability students benefit from group interaction as much as the low- or average-ability students. Nevertheless, for maximum communication among members, the ability range in the groups should not be too wide. Groups of 3 to 4 students work quite well.

*Importance in Arousing Learning Motivation*

Motivation is probably one of the most important factors in affecting learning performance because a well-motivated learner is more determinative to achieve and to overcome a lot of learning difficulties. Motivation is not constant over time but may change according to the circumstance and disposition of the learner. Teachers must be aware of the possibilities of such changes and be flexible enough to adjust their strategies when necessary. It is crucial for teachers to plan learning activities with particular attention paid to initiate their students' motivation.

For more details on the catering of student diversities, the documents *Mathematics Education Key Learning Area – Mathematics Curriculum Guide (P1 – P6) (2000)*, *Syllabuses for Secondary Schools: Mathematics (Secondary 1-5) (1999)*,《小學數學科輔導教學》（2000）,《中學數學科輔導教學》（2000）and *Teaching Package on S1-5 Mathematics (2) – Catering for Learner Differences (2001)* can be referred to. Schools with SEN students may refer to *Information Guide to Support Services for Students with Special Educational Needs in Ordinary Schools* published by ED in 2001 and Booklet 4 of the ***Basic Education Guide – Building on Strengths (2002)***.

## 4.4 Homework

Homework assignment is an extension and consolidation of learning. Its purposes are:

1. To develop and consolidate what students have discovered or learned
2. To reinforce and maintain the concepts and skills developed
3. To encourage students to think independently, analyze, formulate and solve problems, and to assess situations and make decisions
4. To lead students to appreciate that mathematics is meaningful, enjoyable and useful.
5. To evaluate students' learning performance and progress
6. To obtain feedback on teaching objectives and teaching methods

Apart from written homework, student assignments may include oral discussions, class practice, homework exercise, group work, book reading, project work and other related activities. When setting assignments, the following should be borne in mind:

1. The quantity should be moderate because too much drill is as harmful as too little. It is undesirable to assign too many at one time. The frequency should be regulated so that students will not be unduly overburdened. Coordination among subjects is desirable.
2. Each assignment should have a specific objective. Questions selected should be geared to the objective.
3. The exercises of a good textbook should be the main part of the assignment as they are supposed to be self-contained. Those designed or derived from other sources by teachers themselves are either complementary or supplementary, e.g. fundamental questions for less able students and more demanding questions for abler students. In addition, routine selection of, say even-numbered or odd-numbered questions should be avoided.
4. Variety is essential because it maintains students' interest and also provides different ways of learning.
5. Selection of questions should cope with students' progressive ranges of ability. Core questions should be identified and assigned to students of general ability. Less should be laid on the two extremes.
6. Exercises should be designed with emphasis on practicality and closely related to daily life situations.

(School may refer to Booklet 8 of the ***Basic Education Guide – Building on Strengths (2002)*** for more ideas on meaningful homework.)