Mathematics Education
Key Learning Area

# Mathematics
Curriculum and Assessment Guide
(Secondary 4 - 6)

Jointly prepared by the Curriculum Development Council and
The Hong Kong Examinations and Assessment Authority

Recommended for use in schools by the Education Bureau
HKSARG
2007 (with updates in November 2015)

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**Membership of the CDC-HKEAA Committee on Mathematics Education and its Working Groups**

# Preamble

The Education and Manpower Bureau (EMB, now renamed Education Bureau (EDB)) stated in its report[1] in 2005 that the implementation of a three-year senior secondary academic structure would commence at Secondary 4 in September 2009. The senior secondary academic structure is supported by a flexible, coherent and diversified senior secondary curriculum aimed at catering for students' varied interests, needs and abilities. This Curriculum and Assessment (C&A) Guide is one of the series of documents prepared for the senior secondary curriculum. It is based on the goals of senior secondary education and on other official documents related to the curriculum and assessment reform since 2000, including the *Basic Education Curriculum Guide* (2002) and the *Senior Secondary Curriculum Guide* (2009). To gain a full understanding of the connection between education at the senior secondary level and other key stages, and how effective learning, teaching and assessment can be achieved, it is strongly recommended that reference should be made to all related documents.

This C&A Guide is designed to provide the rationale and aims of the subject curriculum, followed by chapters on the curriculum framework, curriculum planning, pedagogy, assessment and use of learning and teaching resources. One key concept underlying the senior secondary curriculum is that curriculum, pedagogy and assessment should be well aligned. While learning and teaching strategies form an integral part of the curriculum and are conducive to promoting learning to learn and whole-person development, assessment should also be recognised not only as a means to gauge performance but also to improve learning. To understand the interplay between these three key components, all chapters in the C&A Guide should be read in a holistic manner.

The C&A Guide was jointly prepared by the Curriculum Development Council (CDC) and the Hong Kong Examinations and Assessment Authority (HKEAA) in 2007. The first updating was made in January 2014 to align with the short-term recommendations made on the senior secondary curriculum and assessment resulting from the New Academic Structure (NAS) review so that students and teachers could benefit at the earliest possible instance. This updating is made to align with the medium-term recommendations of the NAS review made on curriculum and assessment. The CDC is an advisory body that gives recommendations to the HKSAR Government on all matters relating to curriculum development for the school system from kindergarten to senior secondary level. Its

[1] The report is *The New Academic Structure for Senior Secondary Education and Higher Education – Action Plan for Investing in the Future of Hong Kong*, and will be referred to as the *334 Report* hereafter.

membership includes heads of schools, practising teachers, parents, employers, academics from tertiary institutions, professionals from related fields/bodies, representatives from the HKEAA and the Vocational Training Council (VTC), as well as officers from the EDB. The HKEAA is an independent statutory body responsible for the conduct of public assessment, including the assessment for the Hong Kong Diploma of Secondary Education (HKDSE). Its governing council includes members drawn from the school sector, tertiary institutions and government bodies, as well as professionals and members of the business community.

The C&A Guide is recommended by the EDB for use in secondary schools. The subject curriculum forms the basis of the assessment designed and administered by the HKEAA. In this connection, the HKEAA will issue a handbook to provide information on the rules and regulations of the HKDSE Examination as well as the structure and format of public assessment for each subject.

The CDC and HKEAA will keep the subject curriculum under constant review and evaluation in the light of classroom experiences, students' performance in the public assessment, and the changing needs of students and society. All comments and suggestions on this C&A Guide may be sent to:

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# Acronyms

| | |
|---|---|
| AL | Advanced Level |
| ApL | Applied Learning |
| ASL | Advanced Supplementary Level |
| C&A | Curriculum and Assessment |
| CDC | Curriculum Development Council |
| CE | Certificate of Education |
| COC | Career-Oriented Curriculum (pilot of the Career-oriented Studies) |
| EC | Education Commission |
| EDB | Education Bureau |
| EMB | Education and Manpower Bureau |
| HKALE | Hong Kong Advanced Level Examination |
| HKCAA | Hong Kong Council for Academic Accreditation |
| HKCEE | Hong Kong Certificate of Education Examination |
| HKDSE | Hong Kong Diploma of Secondary Education |
| HKEAA | Hong Kong Examinations and Assessment Authority |
| HKSAR | Hong Kong Special Administrative Region |
| IT | Information Technology |
| KLA | Key Learning Area |
| KS1/2/3/4 | Key Stage 1/2/3/4 |
| OLE | Other Learning Experiences |
| One Committee | CDC-HKEAA Committee |
| P1/2/3/4/5/6 | Primary 1/2/3/4/5/6 |
| PDP | Professional Development Programmes |
| RASIH | Review of the Academic Structure for Senior Secondary Education and Interface with Higher Education |
| S1/2/3/4/5/6/7 | Secondary 1/2/3/4/5/6/7 |

| | |
|---|---|
| SBA | School-based Assessment |
| SEN | Special Educational Needs |
| SLP | Student Learning Profile |
| SRR | Standards-referenced Reporting |
| SSCG | Senior Secondary Curriculum Guide |
| TPPG | Teacher Professional Preparation Grant |

# Chapter 1 Introduction

This chapter provides the background, rationale and aims of Mathematics as a core subject in the three-year senior secondary curriculum, and highlights how it articulates with the junior secondary curriculum, post-secondary education and future career pathways.

## 1.1 Background

This Guide has been prepared by the Curriculum and Development Council (CDC) – Hong Kong Examinations and Assessment Authority (HKEAA) Committee on Mathematics Education (Senior Secondary) in support of the new three-year senior secondary curriculum recommended in the *334 report* on the new academic structure published in May 2005. Mathematics is a core subject for students from the primary level to the junior secondary level. In the senior secondary curriculum, Mathematics is also one of the core subjects.

The Mathematics Curriculum (S4 – 6) is a continuation of the existing Mathematics Curriculum at the junior secondary level. Its development is built on the direction set out in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 3).* Students' knowledge, skills, positive values and attitudes are further extended.

This document presents an outline of the overall aims, learning targets and objectives of the subject for the senior secondary level. It also provides suggestions regarding curriculum planning, learning and teaching strategies, assessment practices and resources. Schools are encouraged to adopt the recommendations in this Guide, taking into account their context, needs and strengths.

## 1.2 Rationale

The rationale for studying Mathematics as a core subject at the senior secondary level is presented below:

- Mathematics is a powerful means in a technology-oriented and information-rich society to help students acquire the ability to communicate, explore, conjecture, reason logically and solve problems using a variety of methods.

- Mathematics provides a means to acquire, organise and apply information, and plays an important role in communicating ideas through pictorial, graphical, symbolic, descriptive and analytical representations. Hence, mathematics at the senior secondary level helps to lay a strong foundation for students' lifelong learning, and provides a platform for the acquisition of new knowledge in this rapidly changing world.

- Many of the developments, plans and decisions made in modern society rely, to some extent, on the use of measures, structures, patterns, shapes and the analysis of quantitative information. Therefore, mathematical experiences acquired at the senior secondary level enable students to become mathematically literate citizens who are more able to cope with the demands of the workplace.

- Mathematics is a tool to help students enhance their understanding of the world. It provides a foundation for the study of other disciplines in the senior secondary and post-secondary education system.

- Mathematics is an intellectual endeavour through which students can develop their imagination, initiative, creativity and flexibility of mind, as well as their ability to appreciate the beauty of nature. Mathematics is a discipline which plays a central role in human culture.

## 1.3 Curriculum Aims

*Overall Aims*

The overall curriculum aims of the Mathematics Education Key Learning Area are to develop in students:

(a) the ability to think critically and creatively, to conceptualise, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines;

(b) the ability to communicate with others and express their views clearly and logically in mathematical language;

(c) the ability to manipulate numbers, symbols and other mathematical objects;

(d) number sense, symbol sense, spatial sense, measurement sense and the capacity to appreciate structures and patterns;

(e) a positive attitude towards the learning of mathematics and an appreciation of the aesthetic nature and cultural aspects of mathematics.

## 1.4 Interface with the Junior Secondary Curriculum and Post-secondary Pathways

### 1.4.1 Interface with the Junior Secondary Mathematics Curriculum

The Mathematics Curriculum (S4 – 6), as part of the secondary curriculum, is built on the direction for development set out in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 3).* It aims at helping students to consolidate what they have learned through basic education, broadening and deepening their learning experiences, as well as further enhancing their positive values and attitudes towards the learning of mathematics. To ensure a seamless transition between the junior and senior secondary levels, a coherent curriculum framework is designed for mathematics education at both levels.

As at the junior secondary level, the Mathematics Curriculum at the senior secondary level aims to meet the challenges of the 21st century by developing students' ability to think critically and creatively, to inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts.

A particular learning unit "Inquiry and Investigation" has been included to provide students with opportunities to improve their ability to inquire, communicate, reason and conceptualise mathematical concepts; and there is also a "Further Applications" learning unit in which they have to integrate various parts of Mathematics which they have learned, and thus recognise the inter-relationships between their experiences of concrete objects in junior forms and abstract notions in senior forms.

### 1.4.2 Interface with Post-secondary Pathways

The curriculum also aims to prepare students for a range of post-secondary pathways, including tertiary education, vocational training and employment. It consists of a

Compulsory Part and an Extended Part. In order to broaden students' choices for further study and work, two modules in the Extended Part are provided to further develop their knowledge of mathematics. These two modules are designed to cater for students who intend to:

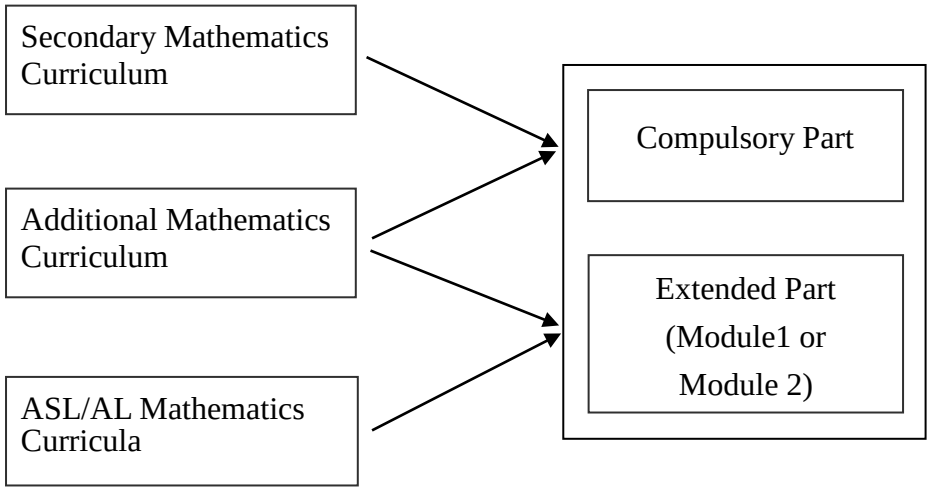
- pursue further studies which require more mathematics; or
- follow a career in fields such as natural sciences, computer sciences, technology or engineering.

**Module 1 (Calculus and Statistics)** focuses on statistics and the application of mathematics, and is designed for students who will be involved in study and work which demand a wider knowledge and deeper understanding of the application of mathematics, in particular, statistics.

**Module 2 (Algebra and Calculus)** focuses on mathematics in depth and aims to cater for students who will be involved in mathematics-related disciplines or careers.

The students' performances in the public examination in the Compulsory Part, Module 1 and Module 2 will be separately reported for the reference of different users.

The following illustration gives an indication of the migration of the former Mathematics Curricula towards the Mathematics Curriculum (S4 – 6).




The Mathematics Curriculum (S4 – 6) supports students' needs in numerous vocational areas and careers, by providing them with various learning pathways. Further details will be provided in Chapter 2.

# Chapter 2 Curriculum Framework

The curriculum framework for Mathematics embodies the key knowledge, skills, values and attitudes that students are to develop at the senior secondary level. It forms the basis on which schools and teachers can plan their school-based curricula, and design appropriate learning, teaching and assessment activities.

## 2.1 Design Principles

The following principles are used in designing the curriculum:

(a) Building on knowledge developed at the basic education level

To ensure that the curricula at different levels of schooling are coherent, the development of the Mathematics Curriculum (S4 – 6) is built on the knowledge, skills, values and attitude acquired through the Mathematics Curriculum for basic education from Primary 1 to Secondary 3.

(b) Providing a balanced, flexible and diversified curriculum

With the implementation of the senior secondary academic structure in Hong Kong, a wider range of students will gain access to Mathematics at the senior secondary level than in the past. The Mathematics Curriculum (S4 – 6) offers a Compulsory Part and an Extended Part. The Compulsory Part is a foundation for all students and provides mathematical concepts, skills and knowledge which are necessary for students' different career pathways. The Extended Part embraces two optional modules to provide add-on mathematical knowledge to suit the individual needs of students who would like to learn more mathematics and in a greater depth. The curriculum thus provides flexibility for teachers to:

- offer a choice of courses within the curriculum to meet students' individual needs, e.g. Compulsory Part, Compulsory Part with Module 1 (Calculus and Statistics) or Compulsory Part with Module 2 (Algebra and Calculus);
- organise the teaching sequence to meet individual situations; and
- make adaptations to the content.

(c) Catering for learner diversity

The curriculum provides opportunities for organising a variety of student activities to cater for learner diversity. The learning unit "Inquiry and Investigation" in the curriculum allows teachers to plan different learning activities for individual students. To further assist teachers to adapt the curriculum to suit the needs of individual groups of students, the content in the Compulsory Part is categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics constitute a set of essential concepts and knowledge which **all** students should strive to learn. Teachers can judge for themselves the suitability and relevance of the content from the Non-foundation Topics for their own students. The Extended Part comprises two modules with different orientations. Students who are more able in mathematics, more mathematically oriented or need more mathematical knowledge and skills to prepare for their future studies and careers may choose to study a module from the Extended Part. Module 1 (Calculus and Statistics) focuses more on mathematical applications, whereas Module 2 (Algebra and Calculus) places more emphasis on mathematical concepts and knowledge. Students who would like to learn more mathematics may choose the module which best suits their interests and needs.

(d) Achieving a balance between breadth and depth

The curriculum covers the important and relevant content for senior secondary students, based on the views of mathematicians, professionals in Mathematics Education and overseas Mathematics curricula at the same level. The breadth and depth of treatment in the Extended Part are intended to provide more opportunities for intellectually rigorous study in the subject.

(e) Achieving a balance between theoretical and applied learning

An equal emphasis is given on theories and applications in both real-life and mathematical contexts to help students construct their knowledge and skills in Mathematics. The historical development of selected mathematical topics is also included to promote students' understanding of how mathematical knowledge has evolved and been refined in the past.

(f) Fostering lifelong learning skills

Knowledge is expanding at an ever faster pace and new challenges are continually posed by rapid developments in technology. It is important for our students to learn how to learn, think critically, analyse and solve problems, and communicate with others effectively so that they can confront current and future challenges. The curriculum provides a suitable context for developing such abilities.

(g) Promoting positive values and attitudes to learning

Positive values and attitudes to learning, which are important in learning mathematics, permeate the Mathematics Curriculum (S4 – 6). In particular, the unit "Inquiry and Investigation" helps to develop in students an interest in learning mathematics, keenness to participate in mathematical activities, and sensitivity and confidence in applying mathematics in daily life. It also helps to foster open-mindedness and independent thinking.

## 2.2 The Mathematics Education Key Learning Area Curriculum Framework

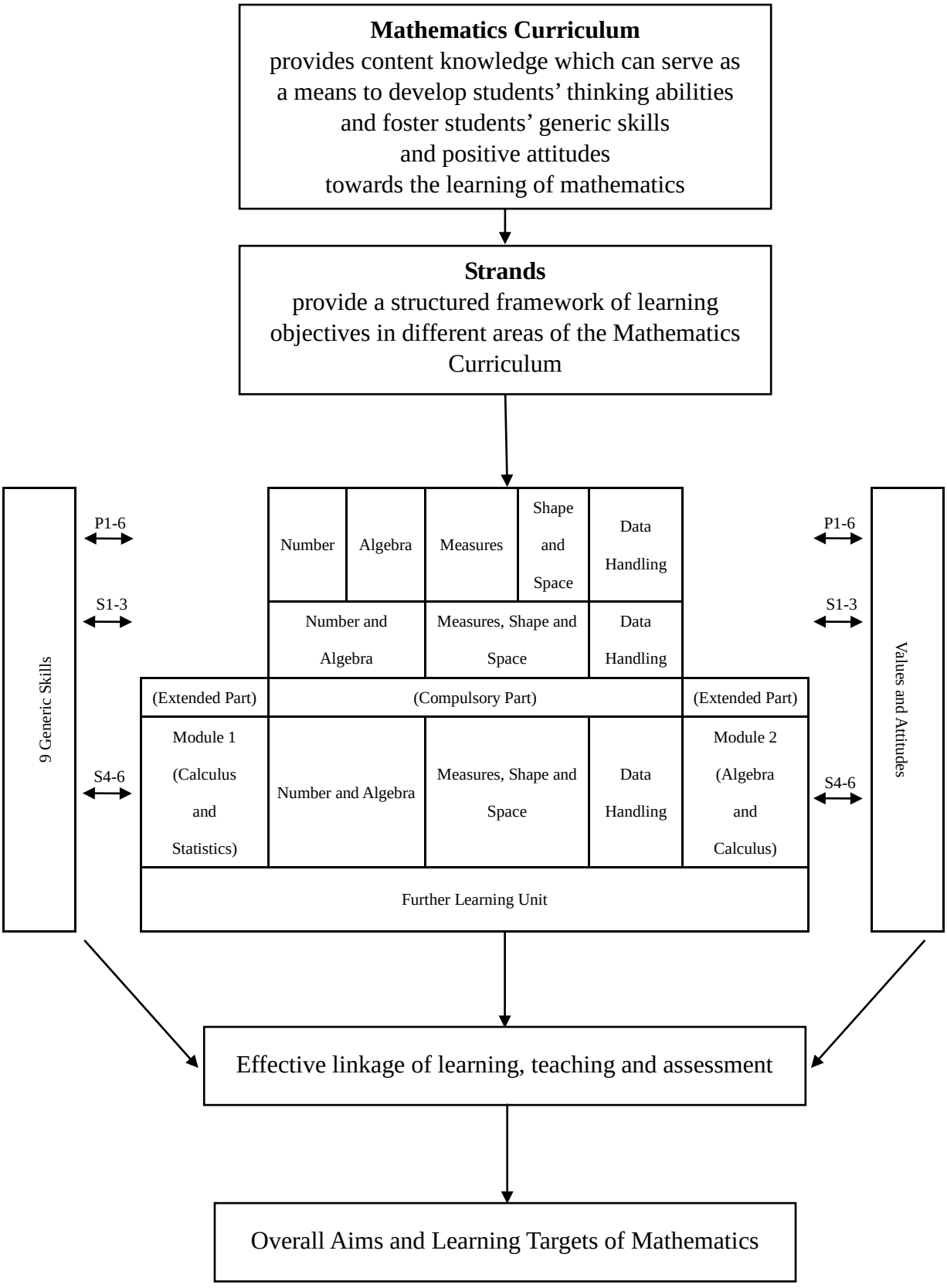
The curriculum framework for Mathematics Education is the overall structure for organising learning and teaching activities for the subject of Mathematics. The framework comprises a set of interlocking components, including:

- subject knowledge and skills, which are expressed in the form of learning targets and learning objectives within strands;
- generic skills; and
- positive values and attitudes.

The framework sets out what students should know, value and be able to do at various stages of schooling from Primary 1 to Secondary 6. It provides schools and teachers with the flexibility to adapt the Mathematics Curriculum to meet their varied needs.

A diagrammatic representation highlighting the major components of the Mathematics Curriculum framework is provided on the following page.

## Diagrammatic Representation of the Framework of the Mathematics Curriculum



### 2.2.1 Strands

Strands are categories of mathematical knowledge and concepts for organising the curriculum. Their main function is to organise mathematical content for the purpose of developing knowledge, skills, values and attitudes as a holistic process. The content of the Mathematics Curriculum is organised into five strands at the primary level and three strands at the secondary level. In particular, the Compulsory Part of the Mathematics Curriculum (S4 – 6) comprises three strands, namely "Number and Algebra", "Measures, Shape and Space" and "Data Handling". As the content of the Extended Part is interwoven, it is not categorised into strands.

### 2.2.2 Generic Skills

Generic skills can be seen as both process skills and learning outcomes in the Mathematics Education Key Learning Area. They are essential for enabling learners to learn how to learn. Nine generic skills have been identified: collaboration skills, communication skills, creativity, critical thinking skills, information technology skills, numeracy skills, problem-solving skills, self-management skills and study skills.

It should be noted that generic skills are not something to be added on to the learning and teaching of mathematical concepts, but should be embedded within them. They serve as a means to develop the acquisition and mastery of mathematical knowledge and concepts. An emphasis on communication skills, creativity and critical thinking skills in the context of mathematical activities will help to strengthen students' ability to achieve the overall learning targets of the curriculum. Daily-life applications, further applications of mathematics, inquiry and investigation are emphasised.

### 2.2.3 Values and Attitudes

Besides knowledge and skills, the development of positive values and attitudes is also important in Mathematics Education. Values and attitudes such as responsibility, commitment and open-mindedness are necessary for developing goals in life and learning. The inculcation of such positive values/attitudes through appropriate learning and teaching strategies can enhance learning, and this in turn will reinforce their development in students as part of character formation. Positive values and attitudes permeate the Mathematics

Curriculum (S4 – 6) and have been incorporated into its learning objectives, so that students can:

- develop **interest** in learning mathematics;
- show **keenness** to participate in mathematical activities;
- develop **sensitivity** towards the importance of mathematics in daily life;
- show **confidence** in applying mathematical knowledge in daily life, by clarifying one's argument and challenging others' statements;
- share ideas and experience and work **cooperatively** with others in accomplishing mathematical tasks/activities and solving mathematical problems;
- understand and take up **responsibilities**;
- be **open-minded**, willing to listen to others in the discussion of mathematical problems, respect others' opinions, and value and appreciate others' contributions;
- **think independently** in solving mathematical problems;
- be **persistent** in solving mathematical problems; and
- **appreciate** the precise, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

These values and attitudes can be fostered through the learning of mathematical content. Teachers can help students cultivate them through planned learning activities.

## 2.3 Aims of Senior Secondary Mathematics Curriculum

The Mathematics Curriculum (S4 – 6) is a continuation of the Mathematics Curriculum (S1 – 3). It aims to:

(a) further develop students' mathematical knowledge, skills and concepts;

(b) provide students with mathematical tools for their personal development and future career pathways;

(c) provide a foundation for students who may further their studies in mathematics or related areas;

(d) develop in students the generic skills, and in particular, the capability to use mathematics to solve problems, reason and communicate;

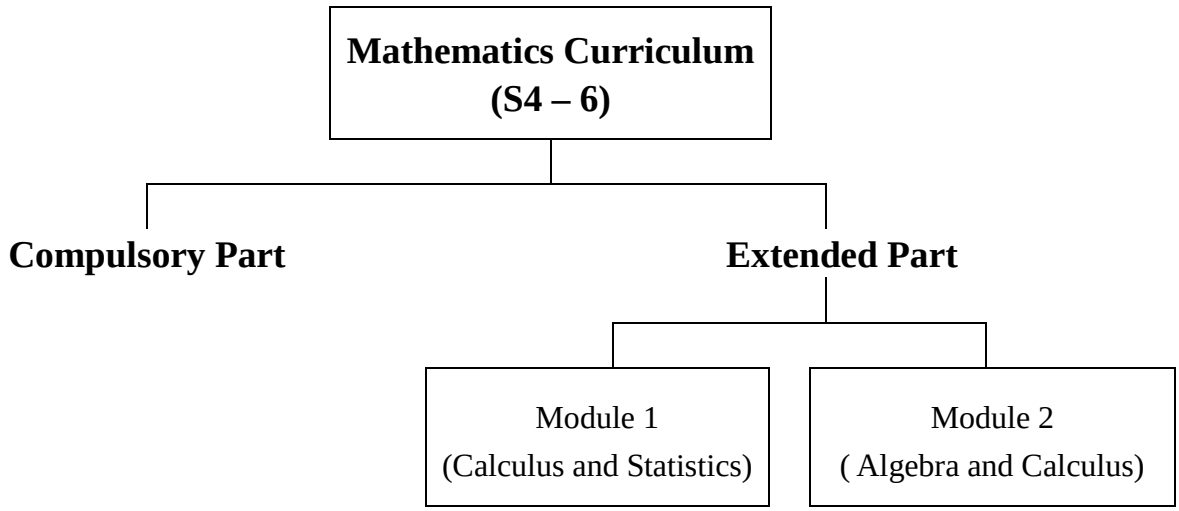
(e) develop in students interest in and positive attitudes towards the learning of mathematics;

(f) develop students' competence and confidence in dealing with mathematics needed in life; and

(g) help students to fulfil their potential in mathematics.

## 2.4 Framework of Senior Secondary Mathematics Curriculum

The structure of the Mathematics Curriculum (S4 – 6) can be represented diagrammatically as follows:



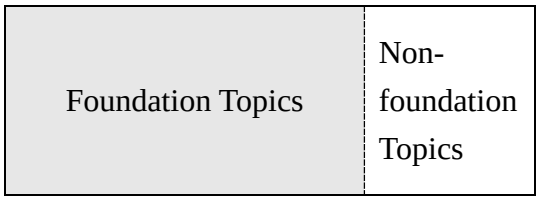

[*Note: Students may take the Compulsory Part only, the Compulsory Part with Module 1 (Calculus and Statistics) or the Compulsory Part with Module 2 (Algebra and Calculus). Students are only allowed to take at most one module from the Extended Part.*]

To cater for students who have different needs, interests and orientations, the curriculum comprises a Compulsory Part and an Extended Part. **All** students must study the Compulsory Part.

The Extended Part has two optional modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus). The inclusion of the Extended Part is designed to provide more flexibility and diversity in the curriculum. The two modules in the Extended Part provide additional mathematical knowledge to the Compulsory Part. Students, based on their individual needs and interests, are encouraged to take at most one of the two modules.

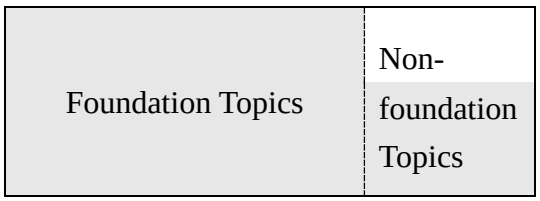
The following diagrams show the different ways in which students can progress:

(1) Students who study only the Foundation Topics in the Compulsory Part



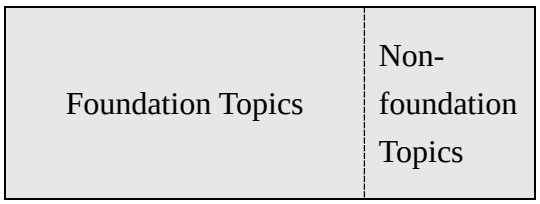

**Compulsory Part**

(2) Students who study the Foundation Topics and some Non-foundation Topics in the Compulsory Part




**Compulsory Part**

(3) Students who study all topics in the Compulsory Part




**Compulsory Part**

(4) Students who study the Compulsory Part with Module 1 (Calculus and Statistics)

| | |
|---|---|
| **Compulsory Part** | **Module 1 (Calculus and Statistics)** |

(5) Students who study the Compulsory Part with Module 2 (Algebra and Calculus)

| | |
|---|---|
| **Compulsory Part** | **Module 2 (Algebra and Calculus)** |

As a core subject, the Mathematics Curriculum (S4 – 6) accounts for up to 15% (approximately 375 hours)[1] of the total lesson time available in the senior secondary curriculum. The suggested time allocations for the Compulsory Part and the Extended Part are as follows:

| | Lesson time (Approximate number of hours) |
|---|---|
| Compulsory Part | 10% – 12.5% (250 hours – 313 hours) |
| Compulsory Part with a module | 15% (375 hours) |

## 2.5 Compulsory Part

The principles of curriculum design of the Compulsory Part comply with those of the Mathematics Curriculum (S4 – 6) as a whole, but have two distinguishing features.

First, the Compulsory Part serves as a foundation for all students and at the same time provides flexibility to cater for the diverse needs of individual students. Its content is

[1] The NSS curriculum is designed on the basis of 2,500 lesson hours. A flexible range of total lesson time at 2,400±200 hours over three years is recommended for school-based planning purposes to cater for school diversity and varying learning needs while maintaining international benchmarking standards.

As always, the amount of time spent in learning and teaching is governed by a variety of factors, including whole-school curriculum planning, learners' abilities and needs, students' prior knowledge, teaching and assessment strategies, teaching styles and the number of subjects offered. Schools should exercise professional judgement and flexibility over time allocation to achieve specific curriculum aims and objectives as well as to suit students' specific needs and the school context.

categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics constitute a coherent set of essential concepts and knowledge while the Non-foundation Topics cover a wider range of content.

Second, the topics in the Compulsory Part emphasise the relevance of mathematics to various human activities. Students are expected to engage in different activities to foster their awareness of the worldwide use of mathematical terminology, notation and strategies to solve problems. Also, to enable students to recognise and appreciate the interconnection between the different parts of mathematics they have learned at both the junior and senior secondary levels, a "Further Applications" learning unit is incorporated into the Compulsory Part.

The learning objectives of the Compulsory Part foster students' understanding of the development of mathematical knowledge and skills and their applications in the solution of various problems, including real-life ones. In addition, learning units such as "Uses and Abuses of Statistics", "Permutation and Combination" and "Further Applications" are included for students to use the mathematics learned at junior and senior secondary levels to understand and assess more sophisticated scenarios critically.

### 2.5.1 Organisation of the Compulsory Part

The most significant aspects of learning and teaching in each strand of the Compulsory Part are organised into a hierarchy from Learning Targets to specific Learning Objectives. Learning Targets set out the aims and direction for learning and teaching and, under these, Learning Objectives are identified to spell out specifically what students need to learn. In the curriculum, Learning Objectives are presented and grouped under different Learning Units.

The three strands in the Compulsory Part are "Number and Algebra", "Measures, Shape and Space" and "Data Handling". In addition, the "Further Learning Unit" is designed to integrate and apply knowledge and skills learned in the strands to solve problems in real-life as well as in mathematical contexts.

### 2.5.2 Learning Targets of the Compulsory Part

An overview of the learning targets of the three strands in the Compulsory Part is provided on the following page.

# Learning Targets in the Compulsory Part

| Number and Algebra Strand | Measures, Shape and Space Strand | Data Handling Strand |
|---|---|---|
| Students are expected to: | | |
| • extend the concepts of numbers to complex numbers;<br>• investigate and describe relationships between quantities using algebraic symbols;<br>• generalise and describe patterns in sequences of numbers using algebraic symbols, and apply the results to solve problems;<br>• interpret more complex algebraic relations from numerical, symbolic and graphical perspectives;<br>• manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve real-life problems and justify the validity of the results obtained; and<br>• apply the knowledge and skills in the Number and Algebra strand to generalise, describe and communicate mathematical ideas and solve further problems in other strands. | • use inductive and deductive approaches to study the properties of 2-dimensional shapes;<br>• formulate and write geometric proofs involving 2-dimensional shapes with appropriate symbols, terminology and reasons;<br>• inquire into and describe geometric knowledge in 2-dimensional space using algebraic relations and apply this knowledge in solving related problems;<br>• inquire and describe geometric knowledge in 2-dimensional space and 3-dimensional space using trigonometric functions and apply the knowledge in solving related problems; and<br>• interconnect the knowledge and skills in the Measures, Shape and Space strand and other strands, and apply them to formulate and solve 2-dimensional and 3-dimensional problems using various strategies. | • understand the measures of dispersion;<br>• select and use the measures of central tendency and dispersion to compare data sets;<br>• investigate and judge the validity of arguments derived from data sets;<br>• acquire basic techniques in counting;<br>• formulate and solve further probability problems by applying simple laws; and<br>• integrate the knowledge in statistics and probability to solve real-life problems. |

### 2.5.3 Foundation Topics and Non-foundation Topics in the Compulsory Part

To cater for the needs of individual students, the content of the Compulsory Part is categorised into Foundation Topics and Non-foundation Topics. The Foundation Topics of the Compulsory Part and the Foundation Part of the Mathematics Curriculum (S1 – 3) constitute a coherent set of essential concepts and knowledge. The Foundation Topics, which all students should strive to learn, are selected in accordance with the following principles in mind:

- to include basic concepts and knowledge necessary for the learning content in the Compulsory Part and for simple applications in real-life situations; and
- to cover topics from different areas to enable students to develop a coherent body of knowledge and to experience mathematics from an all-around perspective.

There are also topics beyond those in the Foundation Topics in terms of depth and breadth. They are identified as Non-foundation Topics and cover a wider range of content, to provide students who study only the Compulsory Part with a foundation for their future studies and career development. Teachers can judge for themselves the suitability and relevance of the Non-foundation Topics for their own students.

The content of Module 1 and Module 2 is built upon the study of the Foundation and Non-foundation Topics in the Compulsory Part. It is advisable for students to study both the Foundation Topics and Non-foundation Topics in the Compulsory Part if they study either one of the modules from the Extended Part.

### 2.5.4 Learning Objectives of the Compulsory Part

The time allocated to the Compulsory Part ranges from 10% to 12.5% of the total lesson time (approximately 250 hours to 313 hours), subject to the different pathways, orientations and learning speeds of students.

To aid teachers in their planning and adaptation, a suggested lesson time in hours is given against each learning unit in the following table. The learning objectives of the Non-foundation Topics are <u>underlined</u> for teachers' reference.

# The Learning Objectives of the Compulsory Part

## Notes:

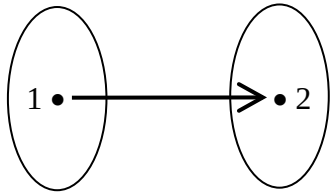
1. *Learning units are grouped under three strands ("Number and Algebra", "Measures, Shape and Space" and "Data Handling") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The learning objectives underlined are the Non-foundation Topics.*
4. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
5. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
6. *Schools may allocate up to 313 hours (i.e. 12.5% of the total lesson time) to those students who need more time for learning.*



| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Number and Algebra Strand** | | | |
| 1. Quadratic equations in one unknown | 1.1 solve quadratic equations by the factor method<br>1.2 form quadratic equations from given roots<br>1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the $x$-intercepts | 19 | The given roots are confined to real numbers. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 1.4 solve quadratic equations by the quadratic formula | | The following are **not** required for students taking only the Foundation Topics:<br>• expressing nonreal roots in the form $a \pm bi$<br>• simplifying expressions involving surds such as $2 \pm \sqrt{48}$ |
| | 1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots | | When $\Delta < 0$, students have to point out that "the equation has **no real roots**" or "the equation has **two nonreal roots**" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 1.6 solve problems involving quadratic equations | | Teachers should select the problems related to students' experiences.<br>Problems involving complicated equations such as $\frac{6}{x}+\frac{6}{x-1}=5$ are required only in the Non-foundation Topics and tackled in Learning Objective 5.4. |
| | 1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations | | The relations between the roots and coefficients include:<br>• $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$,<br>where $\alpha$ and $\beta$ are the roots of the equation $ax^2+bx+c=0$ and $a\neq 0$. |
| | 1.8 appreciate the development of the number systems including the system of complex numbers | | The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 1.9 perform addition, subtraction, multiplication and division of complex numbers | | Complex numbers are confined to the form $a \pm bi$ .<br><br>Note: The coefficients of quadratic equations are confined to real numbers. |
| 2. Functions and graphs | 2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables<br><br>2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions<br><br>2.3 understand the features of the graphs of quadratic functions | 10 | Finding the domain of a function is required but need **not** be stressed.<br><br>Representations like<br>  <br><br>are also accepted.<br><br>The features of the graphs of quadratic functions include:<br>• the vertex<br>• the axis of symmetry<br>• the direction of opening<br>• relations with the axes<br>Students are expected to find the maximum |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | and minimum values of quadratic functions by the graphical method.<br>Students are expected to solve problems relating to maximum and minimum values of quadratic functions. |
| | 2.4 find the maximum and minimum values of quadratic functions by the algebraic method | | |
| 3. Exponential and logarithmic functions | 3.1 understand the definitions of rational indices | 16 | The definitions include<br>$\sqrt[n]{a}$, $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$.<br>Students are also expected to evaluate expressions such as $\sqrt[3]{-8}$. |
| | 3.2 understand the laws of rational indices | | The laws of rational indices include:<br>• $a^p a^q = a^{p+q}$<br>• $\frac{a^p}{a^q} = a^{p-q}$<br>• $(a^p)^q = a^{pq}$<br>• $a^p b^p = (ab)^p$<br>• $\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 3.3 understand the definition and properties of logarithms (including the change of base) | | The properties of logarithms include:<br>• $\log_a 1 = 0$<br>• $\log_a a = 1$<br>• $\log_a MN = \log_a M + \log_a N$<br>• $\log_a \frac{M}{N} = \log_a M - \log_a N$<br>• $\log_a M^k = k \log_a M$<br>• $\log_b N = \frac{\log_a N}{\log_a b}$ |
| | 3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs | | The following properties and features are included:<br>• the domains of the functions<br>• the function $f(x) = a^x$ increases (decreases) as $x$ increases for $a > 1$ $(0 < a < 1)$<br>• $y = a^x$ is symmetric to $y = \log_a x$ about $y = x$<br>• the intercepts with the axes<br>• the rate of increasing/the rate of decreasing (by direct inspection) |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 3.5 solve exponential equations and logarithmic equations | | Equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log(x - 22) + \log(x + 26) = 2$ are tackled in Learning Objective 5.3. |
| | 3.6 appreciate the applications of logarithms in real-life situations | | The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed. |
| | 3.7 appreciate the development of the concepts of logarithms | | The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed. |
| 4. More about polynomials | 4.1 perform division of polynomials | 14 | Methods other than long division are also accepted. |
| | 4.2 understand the remainder theorem | | |
| | 4.3 understand the factor theorem | | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials | | The terms “H.C.F.” , “gcd”, etc. can be used. |
| | 4.5 perform addition, subtraction, multiplication and division of rational functions | | Computation of rational functions with more than two variables is **not** required. |
| 5. More about equations | 5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$ | 10 | |
| | 5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic | | |
| | 5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations | | Solutions for trigonometric equations are confined to the interval from 0° to 360° . |
| | 5.4 solve problems involving equations which can be transformed into quadratic equations | | Teachers should select the problems related to students’ experience. |
| 6. Variations | 6.1 understand direct variations (direct proportions) and inverse variations (inverse proportions), and their applications to solving real-life problems | 9 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 6.2 understand the graphs of direct and inverse variations<br><br>6.3 understand joint and partial variations, and their applications to solving real-life problems | | |
| 7. Arithmetic and geometric sequences and their summations | 7.1 understand the concept and the properties of arithmetic sequences<br><br>7.2 understand the general term of an arithmetic sequence<br><br>7.3 understand the concept and the properties of geometric sequences<br><br>7.4 understand the general term of a geometric sequence | 17 | The properties of arithmetic sequences include:<br>• $T_n = \frac{1}{2}(T_{n-1} + T_{n+1})$<br>• if $T_1, T_2, T_3, \ldots$ is an arithmetic sequence, then $kT_1 + a$, $kT_2 + a$, $kT_3 + a, \ldots$ is also an arithmetic sequence<br><br>The properties of geometric sequences include:<br>• $T_n^2 = T_{n-1} \times T_{n+1}$<br>• if $T_1, T_2, T_3, \ldots$ is a geometric sequence, then $kT_1$, $kT_2$, $kT_3, \ldots$ is also a geometric sequence |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems<br><br>7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems<br><br>7.7 solve related real-life problems | | Example: geometrical problems relating to the sum of arithmetic or geometric sequences.<br><br>Example: geometrical problems relating to infinite sum of the geometric sequences.<br><br>Examples: problems about interest, growth or depreciation. |
| 8. Inequalities and linear programming | 8.1 solve compound linear inequalities in one unknown<br><br>8.2 solve quadratic inequalities in one unknown by the graphical method<br><br>8.3 solve quadratic inequalities in one unknown by the algebraic method<br><br>8.4 represent the graphs of linear inequalities in two unknowns on a plane | 16 | Compound inequalities involving logical connectives “and” or “or” are required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 8.5 solve systems of linear inequalities in two unknowns<br><br>8.6 solve linear programming problems | | |
| 9. More about graphs of functions | 9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions<br><br>9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$<br><br>9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$<br><br>9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives | 11 | Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity. |
| **Measures, Shape and Space Strand** | | | |
| 10. Basic properties of circles | 10.1 understand the properties of chords and arcs of a circle | 23 | The properties of chords and arcs of a circle include:<br>• the chords of equal arcs are equal<br>• equal chords cut off equal arcs |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | • the perpendicular from the centre to a chord bisects the chord<br>• the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord<br>• the perpendicular bisector of a chord passes through the centre<br>• equal chords are equidistant from the centre<br>• chords equidistant from the centre are equal<br><br>Students are expected to understand why there is one and only one circle passing through given three non-collinear points.<br>Note: the property that the arcs are proportional to their corresponding angles at the centre should be discussed at Key Stage 3 when the formula for calculating arc lengths is being explicated. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 10.2 understand the angle properties of a circle | | Angle properties of a circle include:<br>• the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference<br>• angles in the same segment are equal<br>• the arcs are proportional to their corresponding angles at the circumference<br>• the angle in a semi-circle is a right angle<br>• if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter |
| | 10.3 understand the properties of a cyclic quadrilateral | | The properties of a cyclic quadrilateral include:<br>• the opposite angles of a cyclic quadrilateral are supplementary<br>• an exterior angle of a cyclic quadrilateral equals its interior opposite angle |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 10.4 understand the tests for concyclic points and cyclic quadrilaterals | | The tests for concyclic points and cyclic quadrilaterals include:<br>• if $A$ and $D$ are two points on the same side of the line $BC$ and $\angle BAC = \angle BDC$, then $A$, $B$, $C$ and $D$ are concyclic<br>• if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic<br>• if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 10.5 understand the properties of tangents to a circle and angles in the alternate segments | | The properties include:<br>• a tangent to a circle is perpendicular to the radius through the point of contact<br>• the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle<br>• the perpendicular to a tangent at its point of contact passes through the centre of the circle<br>• if two tangents are drawn to a circle from an external point, then:<br>- the distances from the external point to the points of contact are equal<br>- the tangents subtend equal angles at the centre<br>- the straight line joining the centre to the external point bisects the angle between the tangents |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 10.6 use the basic properties of circles to perform simple geometric proofs | | • if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment<br>• if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 11. Locus | 11.1 understand the concept of loci | 7 | |
| | 11.2 describe and sketch the locus of points satisfying given conditions | | The conditions include:<br>• maintaining a fixed distance from a fixed point<br>• maintaining an equal distance from two given points<br>• maintaining a fixed distance from a line<br>• maintaining a fixed distance from a line segment<br>• maintaining an equal distance from two parallel lines<br>• maintaining an equal distance from two intersecting lines |
| | 11.3 describe the locus of points with algebraic equations | | Students are expected to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $y = ax^2 + bx + c$ ). |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 12. Equations of straight lines and circles | 12.1 understand the equation of a straight line | 14 | Students are expected to find the equation of a straight line from given conditions such as:<br>• the coordinates of any two points on the straight line<br>• the slope of the straight line and the coordinates of a point on it<br>• the slope and the *y*-intercept of the straight line<br>Students are expected to describe the features of a straight line from its equation. The features include:<br>• the slope<br>• the intercepts with the axes<br>• whether it passes through a given point<br>The normal form is **not** required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 12.2 understand the possible intersection of two straight lines | | Students are expected to determine the number of intersection points of two straight lines from their equations.<br><br>Note: Solving simultaneous linear equations in two unknowns is a learning objective at Key Stage 3. |
| | 12.3 understand the equation of a circle | | Students are expected to find the equation of a circle from given conditions such as:<br>• the coordinates of the centre and the radius of the circle<br>• the coordinates of any three points on the circle<br><br>Students are expected to describe the features of a circle from its equation. The features include:<br>• the centre<br>• the radius<br>• whether a given point lies inside, outside or on the circle |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 12.4 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle | | Finding the equations of tangents to a circle is required. |
| 13. More about trigonometry | 13.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity<br><br>13.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from 0° to 360°) and other trigonometric equations (solutions in the interval from 0° to 360°)<br><br>13.3 understand the formula $\frac{1}{2} ab \sin C$ for areas of triangles<br><br>13.4 understand the sine and cosine formulae<br><br>13.5 understand Heron's formula | 21 | Simplification of expressions involving sine, cosine and tangent of $-\theta$, $90° \pm \theta$, $180° \pm \theta$, … , etc. is required.<br><br>Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and tackled in Learning Objective 5.3. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 13.6 use the above formulae to solve 2-dimensional and 3-dimensional problems | | The "above formulae" refer to those mentioned in Learning Objectives 13.3 – 13.5.<br>3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between a point and a line, and the distance between a point and a plane.<br>Note: Exploring the properties of simple 3-D figures is a learning objective at Key Stage 3. |
| **Data Handling Strand** | | | |
| 14. Permutation and combination | 14.1 understand the addition rule and multiplication rule in the counting principle | 11 | |
| | 14.2 understand the concept and notation of permutation | | Notations such as " $P_r^n$ " , " ${}_nP_r$" , " ${}^nP_r$" , etc. can be used. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 14.3 solve problems on the permutation of distinct objects without repetition | | Problems such as "permutation of objects in which three particular objects are put next to each other" are required.<br>Circular permutation is **not** required. |
| | 14.4 understand the concept and notation of combination | | Notations such as "$C_r^n$" , "${}_nC_r$" , "${}^nC_r$" , "$\binom{n}{r}$" , etc. can be used. |
| | 14.5 solve problems on the combination of distinct objects without repetition | | |
| 15. More about probability | 15.1 recognise the notation of set language including union, intersection and complement | 10 | The concept of Venn Diagram is required. |
| | 15.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events | | The addition law of probability refers to "$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ". |
| | 15.3 understand the multiplication law of probability and the concept of independent events | | The multiplication law of probability refers to "$P(A \cap B) = P(A) \times P(B)$ , where $A$ and $B$ are independent events". |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 15.4 recognise the concept and notation of conditional probability | | The rule "$P(A \cap B) = P(A) \times P(B \mid A)$" is required.<br>Bayes' Theorem is **not** required. |
| | 15.5 use permutation and combination to solve problems relating to probability | | |
| 16. Measures of dispersion | 16.1 understand the concept of dispersion | 14 | |
| | 16.2 understand the concepts of range and inter-quartile range | | |
| | 16.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data | | A box-and-whisker diagram can also be called a "boxplot". |
| | 16.4 understand the concept of standard deviation for both grouped and ungrouped data sets | | The term "variance" should be introduced.<br>Students are required to understand the following formula for standard deviation:<br>$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$. |
| | 16.5 compare the dispersions of different sets of data using appropriate measures | | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 16.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution<br><br>16.7 explore the effect of the following operations on the dispersion of the data:<br><br>(i) adding an item to the set of data<br>(ii) removing an item from the set of data<br>(iii) adding a common constant to each item of the set of data<br>(iv) multiplying each item of the set of data by a common constant | | |
| 17. Uses and abuses of statistics | 17.1 recognise different techniques in survey sampling and the basic principles of questionnaire design<br><br>17.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations | 4 | The concepts of “populations” and “samples” should be introduced.<br><br>Probability sampling and non-probability sampling should be introduced.<br><br>Students should recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 17.3 assess statistical investigations presented in different sources such as news media, research reports, etc. | | |
| **Further Learning Unit** | | | |
| 18. Further applications | Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learned in different areas<br>The main focuses are:<br>(a) to explore and solve more sophisticated real-life problems<br>(b) to appreciate the connections between different areas of mathematics | 14 | Examples:<br>• solve simple financial problems in areas such as taxation and instalment payment<br>• analyse and interpret data collected in surveys<br>• explore and interpret graphs relating to real-life situations<br>• explore Ptolemy's Theorem and its applications<br>• model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k\,a^x$ to linear relations<br>• explore the relation between the Fibonacci sequence and the Golden |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | Ratio<br>• appreciate the applications of cryptography<br>• explore the Ceva's Theorem and its applications<br>• investigate the causes and effects of the three crises in mathematics<br>• analyse mathematical games (e.g. explore the general solution of the water puzzle) |
| 19. Inquiry and investigation | Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts | 10 | This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units. |

**Grand total: 250 hours**

## 2.6 Extended Part

The Extended Part is designed for students who need more mathematical knowledge and skills for their future studies and careers, and for those whose interests and maturity have been developed to a level that enables them to benefit from further mathematical study in different areas. The Extended Part aims at extending students' mathematical horizon beyond the Compulsory Part. Students have to handle more complicated problems in the Extended Part than in the Compulsory Part.

Two modules are offered as choices for students in the Extended Part. They are Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus). Students are allowed to take at most one of the two modules.

**Module 1 (Calculus and Statistics)** is intended to cater for those students who will be involved in disciplines or careers which demand a wider scope and deeper understanding of mathematics, and for those who would like to learn more mathematical applications at the senior secondary level. It aims to:

- provide students with skills and concepts beyond the Compulsory Part;
- emphasise applications rather than mathematical rigour with a view to widening students' perspectives on mathematics; and
- provide students with intuitive concepts of calculus and statistics, related basic skills and useful tools for their future studies and careers.

**Module 2 (Algebra and Calculus)** is designed to suit the needs of students who will be involved in mathematics-related fields and careers, and those who would like to learn more in-depth mathematics at the senior secondary level. It aims to:

- provide students with skills and concepts beyond the Compulsory Part;
- emphasise understanding of mathematics for further progress in mathematically inclined disciplines; and
- provide students with a concrete foundation in algebra and calculus for their future studies and careers.

### 2.6.1 Organisation of Module 1 and Module 2

The organisation of Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus) is different from that of the Compulsory Part. The content of each of these modules is usually interwoven. Instead of categorising the content of the modules into strands as in the Compulsory Part, it is classified into different areas. Both modules have learning targets to set out the aims and directions for learning and teaching in the areas involved in the modules. The two modules are also organised into a hierarchy from Learning Targets to specific Learning Objectives.

For Module 1 (Calculus and Statistics), the three areas are Foundation Knowledge, Calculus and Statistics. For Module 2 (Algebra and Calculus), the three areas are Foundation Knowledge, Algebra and Calculus. In addition, the Further Learning Unit, independent from any of the above three areas in each module, is designed to enhance students' ability to inquire, communicate, reason and conceptualise mathematical concepts.

### 2.6.2 Learning Targets of Module 1 and Module 2

The learning targets of Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus) are provided in the following tables:

## Learning Targets of Module 1 (Calculus and Statistics)

| Foundation Knowledge | Calculus | Statistics |
|---|---|---|
| Students are expected to: | | |
| • apply binomial expansion for the study of probability and statistics;<br>• model, graph and apply exponential functions and logarithmic functions to solve problems; and<br>• understand the relationships between exponential and logarithmic functions and hence apply the two functions to solve real-life problems. | • recognise the concept of limits as the basis of differential and integral calculus;<br>• understand the idea of differentiation and integration through consideration of concrete phenomena; and<br>• find the derivatives, indefinite integrals and definite integrals of simple functions. | • understand the concepts of probability, random variables, and discrete and continuous probability distributions;<br>• understand the fundamental ideas of statistical reasoning based on the Binomial, Poisson, Geometric and Normal Distributions;<br>• use statistical ways of observing and thinking, and then make inferences; and<br>• develop the ability to think mathematically about uncertainty and then apply such knowledge and skills to solve problems. |

# Learning Targets of Module 2 (Algebra and Calculus)

| Foundation Knowledge | Algebra | Calculus |
|---|---|---|
| Students are expected to: | | |
| • rationalise surd expressions;<br>• understand the principle of mathematical induction;<br>• expand binomials using the Binomial Theorem;<br>• understand simple trigonometric functions and their graphs;<br>• understand important trigonometric identities and formulae involving compound angles; and<br>• understand the number *e*. | • understand the concepts, operations and properties of matrices and the inverses of square matrices up to order 3;<br>• solve systems of linear equations;<br>• understand the concept, operations and properties of vectors; and<br>• apply the knowledge of vectors to solve problems in 2-dimensional space and 3-dimensional space. | • understand the concept of limits as the basis of differential and integral calculus;<br>• understand the concepts and properties of derivatives, indefinite integrals and definite integrals of functions;<br>• find the derivatives, indefinite integrals and definite integrals of simple functions;<br>• find the second derivatives of functions; and<br>• apply the knowledge of differentiation and integration to solve real-life problems. |

### 2.6.3 Learning Objectives of Module 1 and Module 2

The time allocation for the Compulsory Part plus either one of the modules is 15% of the total lesson time (approximately 375 hours). To aid teachers in planning school-based curricula, a suggested lesson time in hours is provided against each learning unit.

The proposed learning objectives of the two modules are provided in the following tables:

# Learning Objectives of Module 1 (Calculus and Statistics)

## Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Foundation Knowledge Area** | | | |
| 1. Binomial expansion | 1.1 recognise the expansion of $(a+b)^n$, where $n$ is a positive integer | 3 | The use of the summation notation ($\sum$) should be introduced.<br>The following are **not** required:<br>• expansion of trinomials<br>• the greatest coefficient, the greatest term and the properties of binomial coefficients<br>• applications to numerical approximation |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 2. Exponential and logarithmic functions | 2.1 recognise the definition of the number $e$ and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ | 7 | |
| | 2.2 recognise exponential functions and logarithmic functions | | The following functions are required:<br>• $y = e^x$<br>• $y = \ln x$ |
| | 2.3 use exponential functions and logarithmic functions to solve problems | | Students are expected to know how to solve problems including those related to compound interest, population growth and radioactive decay. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 2.4 transform $y = kx^n$ and $y = ka^x$ to linear relations, where *a*, *n* and *k* are real numbers, $a > 0$ and $a \neq 1$ | | When experimental values of *x* and *y* are given, students can plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercept. |
| | Subtotal in hours | 10 | |
| **Calculus Area** | | | |
| **Differentiation and Its Applications** | | | |
| 3. Derivative of a function | 3.1 recognise the intuitive concept of the limit of a function | 5 | The concepts of continuous function and discontinuous function are **not** required.<br>Theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions should be stated without proof. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 3.2 find the limits of algebraic functions, exponential functions and logarithmic functions | | The following types of algebraic functions are required:<br>• polynomial functions<br>• rational functions<br>• power functions $x^{\alpha}$<br>• functions derived from the above ones through addition, subtraction, multiplication, division and composition, for example, $\sqrt{x^2+1}$ |
| | 3.3 recognise the concept of the derivative of a function from first principles | | Students are **not** required to find the derivatives of functions from first principles.<br>Notations including $y'$, $f'(x)$ and $\frac{dy}{dx}$ should be introduced. |
| | 3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$ | | Notations including $f'(x_0)$ and $\left.\frac{dy}{dx}\right\|_{x=x_0}$ should be introduced. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 4. Differentiation of a function | 4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation | 7 | The following rules are required:<br>• $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$<br>• $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$<br>• $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$<br>• $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions | | The following formulae are required:<br>• $(C)' = 0$<br>• $(x^n)' = nx^{n-1}$<br>• $(e^x)' = e^x$<br>• $(\ln x)' = \frac{1}{x}$<br>• $(\log_a x)' = \frac{1}{x \ln a}$<br>• $(a^x)' = a^x \ln a$<br>Implicit differentiation is **not** required.<br>Logarithmic differentiation is **not** required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 5. Second derivative | 5.1 recognise the concept of the second derivative of a function<br><br>5.2 find the second derivative of an explicit function | 2 | Notations including $y''$, $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced.<br><br>Third and higher order derivatives are **not** required. |
| 6. Applications of differentiation | 6.1 use differentiation to solve problems involving tangent, rate of change, maximum and minimum | 9 | Local and global extrema are required. |
| | Subtotal in hours | 23 | |
| **Integration and Its Applications** | | | |
| 7. Indefinite integrals and their applications | 7.1 recognise the concept of indefinite integration | 10 | Indefinite integration as the reverse process of differentiation should be introduced. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 7.2 understand the basic properties of indefinite integrals and basic integration formulae | | The notation $\int f(x)\,dx$ should be introduced.<br>The following properties are required:<br>• $\int k\,f(x)dx = k\int f(x)\,dx$<br>• $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$<br><br>The following formulae are required and the meaning of the constant of integration $C$ should be explained:<br>• $\int k\,dx = kx + C$<br>• $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$<br>• $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$<br>• $\int e^x\,dx = e^x + C$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions<br><br>7.4 use integration by substitution to find indefinite integrals<br><br>7.5 use indefinite integration to solve problems | | Integration by parts is **not** required. |
| 8. Definite integrals and their applications | 8.1 recognise the concept of definite integration | 12 | The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced.<br><br>The notation $\int_a^b f(x)\,dx$ should be introduced.<br><br>The knowledge of dummy variables, i.e. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ is required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals | | The Fundamental Theorem of Calculus refers to $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$.<br>The following properties are required:<br>• $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$<br>• $\int_a^a f(x)\,dx = 0$<br>• $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$<br>• $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$<br>• $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 8.3 find the definite integrals of algebraic functions and exponential functions<br>8.4 use integration by substitution to find definite integrals<br>8.5 use definite integration to find the areas of plane figures<br>8.6 use definite integration to solve problems | | Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves. |
| 9. Approximation of definite integrals using the trapezoidal rule | 9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals | 4 | Error estimation is **not** required. |
| | Subtotal in hours | 26 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Statistics Area** | | | |
| **Further Probability** | | | |
| 10. Conditional probability and independence | 10.1 understand the concepts of conditional probability and independent events<br>10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events $C$ and $D$ to solve problems | 3 | |
| 11. Bayes' theorem | 11.1 use Bayes' theorem to solve simple problems | 4 | |
| | Subtotal in hours | 7 | |
| **Binomial, Geometric and Poisson Distributions and Their Applications** | | | |
| 12. Discrete random variables | 12.1 recognise the concept of a discrete random variable | 1 | |
| 13. Probability distribution, expectation and variance | 13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae<br>13.2 recognise the concepts of expectation $E(X)$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems | 5 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 13.3 use the formulae $E(aX+b)=aE(X)+b$ and $\text{Var}(aX+b)=a^2\,\text{Var}(X)$ to solve simple problems | | |
| 14. Binomial distribution | 14.1 recognise the concept and properties of the binomial distribution | 5 | Bernoulli distribution should be introduced.<br>The mean and variance of the binomial distribution should be introduced (proofs are **not** required). |
| | 14.2 calculate probabilities involving the binomial distribution | | Use of the binomial distribution table is **not** required. |
| 15. Geometric distribution | 15.1 recognise the concept and properties of the geometric distribution | 4 | The mean and variance of geometric distribution should be introduced (proofs are **not** required). |
| | 15.2 calculate probabilities involving the geometric distribution | | |
| 16. Poisson distribution | 16.1 recognise the concept and properties of the Poisson distribution | 4 | The mean and variance of Poisson distribution should be introduced (proofs are **not** required). |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 16.2 calculate probabilities involving the Poisson distribution | | Use of the Poisson distribution table is **not** required. |
| 17. Applications of binomial, geometric and Poisson distributions | 17.1 use binomial, geometric and Poisson distributions to solve problems | 5 | |
| | Subtotal in hours | 24 | |
| **Normal Distribution and Its Applications** | | | |
| 18. Basic definition and properties | 18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution | 3 | Derivations of the mean and variance of the normal distribution are **not** required.<br>The formulae written in Learning Objective 13.3 are also applicable to continuous random variables. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 18.2 recognise the concept and properties of the normal distribution | | Properties of the normal distribution include:<br>• the curve is bell-shaped and symmetrical about the mean<br>• the mean, mode and median are equal<br>• the dispersion can be determined by the value of $\sigma$<br>• the area under the curve is 1 |
| 19. Standardisation of a normal variable and use of the standard normal table | 19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution | 2 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 20. Applications of the normal distribution | 20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of $x_1$, $x_2$, $\mu$ and $\sigma$, where $X \sim N(\mu, \sigma^2)$<br>20.2 find the values of $x$, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$<br>20.3 use the normal distribution to solve problems | 7 | |
| | Subtotal in hours | 12 | |
| **Point and Interval Estimation** | | | |
| 21. Sampling distribution and point estimates | 21.1 recognise the concepts of sample statistics and population parameters<br>21.2 recognise the sampling distribution of the sample mean from a random sample of size $n$ | 7 | If the population mean is $\mu$ and population variance is $\sigma^2$, then the mean of the sample mean is $\mu$ and the variance of the sample mean is $\frac{\sigma^2}{n}$. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 21.3 recognise the concept of point estimates including the sample mean, sample variance and sample proportion<br><br>21.4 recognise Central Limit Theorem | | The concept of "estimator" should be introduced.<br><br>If the population mean is $\mu$ and the population size is $N$, then the population variance is $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$.<br><br>If the sample mean is $\bar{x}$ and the sample size is $n$, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$.<br>Recognising the concept of "unbiased estimator" is required. |
| 22. Confidence interval for a population mean | 22.1 recognise the concept of confidence interval<br>22.2 find the confidence interval for a population mean | 6 | • a $100(1 - \alpha)\%$ confidence interval for the mean $\mu$ of a normal population with known variance $\sigma^2$ is given by |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | $(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$<br>• when the sample size $n$ is sufficiently large, a $100(1-\alpha)\%$ confidence interval for the mean $\mu$ of a population with unknown variance is given by<br>$(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$,<br>where $s$ is the sample standard deviation |
| 23. Confidence interval for a population proportion | 23.1 find an approximate confidence interval for a population proportion | 3 | For a random sample of size $n$, where $n$ is sufficiently large, drawn from a Bernoulli distribution, a $100(1-\alpha)\%$ confidence interval for the population proportion $p$ is given by<br>$(\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$,<br>where $\hat{p}$ is an unbiased estimator of the population proportion. |
| | Subtotal in hours | 16 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Further Learning Unit** | | | |
| 24. Inquiry and investigation | Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts | 7 | This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units. |
| | Subtotal in hours | 7 | |

**Grand total: 125 hours**

# Learning Objectives of Module 2 (Algebra and Calculus)

**Notes:**

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*



| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Foundation Knowledge Area** | | | |
| 1 Surds | 1.1 rationalise the denominators of expressions of the form $\frac{k}{\sqrt{a} \pm \sqrt{b}}$ | 1.5 | This topic can be introduced when teaching limits and differentiation. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 2. Mathematical induction | 2.1 understand the principle of mathematical induction | 3 | Only the First Principle of Mathematical Induction is required.<br><br>Applications to proving propositions related to the summation of a finite sequence are included.<br><br>Proving propositions involving inequalities is **not** required. |
| 3. Binomial Theorem | 3.1 expand binomials with positive integral indices using the Binomial Theorem | 3 | Proving the Binomial Theorem is required.<br><br>The use of the summation notation ( $\sum$ ) should be introduced.<br><br>The following are **not** required:<br>• expansion of trinomials<br>• the greatest coefficient, the greatest term and the properties of binomial coefficients<br>• applications to numerical approximation |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 4. More about trigonometric functions | 4.1 understand the concept of radian measure<br><br>4.2 find arc lengths and areas of sectors through radian measure<br><br>4.3 understand the functions cosecant, secant and cotangent and their graphs<br><br>4.4 understand the identities<br>$1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$<br><br>4.5 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine | 11 | Simplifying trigonometric expressions by identities is required.<br><br>The following formulae are required:<br>• $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$<br>• $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$<br>• $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$<br>• $\sin 2A = 2 \sin A \cos A$<br>• $\cos 2A = \cos^2 A - \sin^2 A$<br>$= 1 - 2\sin^2 A = 2\cos^2 A - 1$<br>• $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$<br>• $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | • $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$<br>• $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$<br>• $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$<br>• $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$<br>• $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$<br>• $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$<br>• $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$<br>• $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$<br>"Subsidiary angle form" is **not** required.<br>$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ and<br>$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$<br>can be considered as formulae derived from the double angle formulae. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 5. Introduction to the number *e* | 5.1 recognise the definitions and notations of the number *e* and the natural logarithm | 1.5 | Two approaches for the introduction to *e* can be considered:<br>• $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (proving the existence of this limit is **not** required)<br>• $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$<br>This section can be introduced when teaching Learning Objective 6.1. |
| | Subtotal in hours | 20 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Calculus Area** | | | |
| **Limits and Differentiation** | | | |
| 6. Limits | 6.1 understand the intuitive concept of the limit of a function | 3 | Students are **not** required to distinguish "continuous functions" and "discontinuous functions" from their graphs.<br>The theorem on the limits of sum, difference, product, quotient, scalar multiple and composite functions should be introduced but the proofs are **not** required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 6.2 find the limit of a function | | The following formulae are required:<br>• $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$<br>• $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$<br>Finding the limit of a rational function at infinity is required. |
| 7. Differentiation | 7.1 understand the concept of the derivative of a function | 14 | Students should be able to find the derivatives of elementary functions, including $C$, $x^n$ ($n$ is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, $e^x$, $\ln x$ from first principles.<br>Notations including $y'$, $f'(x)$ and $\frac{dy}{dx}$ should be introduced.<br>Testing differentiability of functions is **not** required. |
| | 7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation | | The following rules are required: |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions | | • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$<br>• $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$<br>• $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$<br>• $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$<br>The following formulae are required:<br>• $(C)' = 0$<br>• $(x^n)' = n\,x^{n-1}$<br>• $(\sin x)' = \cos x$<br>• $(\cos x)' = -\sin x$<br>• $(\tan x)' = \sec^2 x$<br>• $(\cot x)' = -\operatorname{cosec}^2 x$<br>• $(\sec x)' = \sec x \tan x$<br>• $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$<br>• $(e^x)' = e^x$<br>• $(\ln x)' = \frac{1}{x}$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 7.4 find derivatives by implicit differentiation<br><br>7.5 find the second derivative of an explicit function | | The following types of algebraic functions are required:<br>• polynomial functions<br>• rational functions<br>• power functions $x^{\alpha}$<br>• functions formed from the above functions through addition, subtraction, multiplication, division and composition, for example $\sqrt{x^2+1}$<br><br>Logarithmic differentiation is required.<br><br>Notations including $y''$, $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced.<br><br>Third and higher order derivatives are **not** required. |
| 8. Applications of differentiation | 8.1 find the equations of tangents and normals to a curve<br><br>8.2 find maxima and minima | 14 | <br><br>Local and global extrema are required. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 8.3 sketch curves of polynomial functions and rational functions<br><br>8.4 solve the problems relating to rate of change, maximum and minimum | | The following points are noteworthy in curve sketching:<br>• symmetry of the curve<br>• limitations on the values of $x$ and $y$<br>• intercepts with the axes<br>• maximum and minimum points<br>• points of inflexion<br>• vertical, horizontal and oblique asymptotes to the curve<br>Students may deduce the equation of the oblique asymptote to the curve of a rational function by division. |
| | Subtotal in hours | 31 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Integration** | | | |
| 9. Indefinite integration | 9.1 recognise the concept of indefinite integration<br><br>9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals | 16 | Indefinite integration as the reverse process of differentiation should be introduced.<br>The following formulae are required:<br>• $\int k\,dx = kx + C$<br>• $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$<br>• $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$<br>• $\int e^x\,dx = e^x + C$<br>• $\int \sin x\,dx = -\cos x + C$<br>• $\int \cos x\,dx = \sin x + C$<br>• $\int \sec^2 x\,dx = \tan x + C$<br>• $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$<br>• $\int \sec x \tan x\,dx = \sec x + C$<br>• $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$<br>For more complicated calculations, see Learning Objectives 9.4 to 9.6. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 9.3 understand the applications of indefinite integrals in real-life or mathematical contexts | | Applications of indefinite integrals in some fields such as geometry and physics are required. |
| | 9.4 use integration by substitution to find indefinite integrals | | |
| | 9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{a^2 + x^2}$ | | Notations including $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and their related principal values should be introduced. |
| | 9.6 use integration by parts to find indefinite integrals | | $\int \ln x \, dx$ can be used as an example to illustrate the method of integration by parts.<br>The use of integration by parts is limited to at most two times in finding an integral. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 10. Definite integration | 10.1 recognise the concept of definite integration | 11 | The definition of the definite integral as the limit of a sum and finding a definite integral from the definition should be introduced.<br>The use of dummy variables, including $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$, is required.<br>Using definite integration to find the sum to infinity of a sequence is **not** required. |
| | 10.2 understand the properties of definite integrals | | The following properties are required:<br>• $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$<br>• $\int_a^a f(x)\,dx = 0$<br>• $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$<br>• $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$<br>• $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions | | Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$, should be introduced. |
| | 10.4 use integration by substitution to find definite integrals | | |
| | 10.5 use integration by parts to find definite integrals | | The use of integration by parts is limited to at most two times in finding an integral. |
| | 10.6 understand the properties of the definite integrals of even, odd and periodic functions | | The following properties are required:<br>• $\int_{-a}^{a} f(x)\,dx = 0$ if $f$ is odd<br>• $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f$ is even<br>• $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$ if $f(x + T) = f(x)$, i.e. $f$ is periodic |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 11. Applications of definite integration | 11.1 understand the application of definite integrals in finding the area of a plane figure<br>11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis | 4 | Only “disc method” is required.<br>Finding the volume of a hollow solid is required. |
| | Subtotal in hours | 31 | |
| **Algebra Area** | | | |
| **Matrices and Systems of Linear Equations** | | | |
| 12. Determinants | 12.1 recognise the concept and properties of determinants of order 2 and order 3 | 3 | The following properties are required:<br>• $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$<br>• $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$<br>• $\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | • $\begin{vmatrix} a_1 & kb_1 & c_1 \\ a_2 & kb_2 & c_2 \\ a_3 & kb_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$<br>• $\begin{vmatrix} a_1 & b_1 & kb_1 \\ a_2 & b_2 & kb_2 \\ a_3 & b_3 & kb_3 \end{vmatrix} = 0$<br>• $\begin{vmatrix} a_1 + a_1' & b_1 & c_1 \\ a_2 + a_2' & b_2 & c_2 \\ a_3 + a_3' & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1' & b_1 & c_1 \\ a_2' & b_2 & c_2 \\ a_3' & b_3 & c_3 \end{vmatrix}$<br>• $\begin{vmatrix} a_1 + kb_1 & b_1 & c_1 \\ a_2 + kb_2 & b_2 & c_2 \\ a_3 + kb_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$<br>• $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$<br>Notations including $\lvert A \rvert$ and $\det(A)$ should be introduced. |
| 13. Matrices | 13.1 understand the concept, operations and properties of matrices | 9 | The addition, scalar multiplication and multiplication of matrices are required.<br>The following properties are required:<br>• $A + B = B + A$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | • $A + (B + C) = (A + B) + C$<br>• $(\lambda + \mu)A = \lambda A + \mu A$<br>• $\lambda(A + B) = \lambda A + \lambda B$<br>• $A(BC) = (AB)C$<br>• $A(B + C) = AB + AC$<br>• $(A + B)C = AC + BC$<br>• $(\lambda A)(\mu B) = (\lambda\mu)AB$<br>• $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$ |
| | 13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3 | | The following properties are required:<br>• the inverse of $A$ is unique<br>• $(A^{-1})^{-1} = A$<br>• $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$<br>• $(A^n)^{-1} = (A^{-1})^n$<br>• $(A^t)^{-1} = (A^{-1})^t$<br>• $\lvert A^{-1} \rvert = \lvert A \rvert^{-1}$<br>• $(AB)^{-1} = B^{-1}A^{-1}$<br>where $A$ and $B$ are invertible matrices and $\lambda$ is a non-zero scalar. |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| 14. Systems of linear equations | 14.1 solve the systems of linear equations of order 2 and order 3 by Cramer's rule, inverse matrices and Gaussian elimination | 6 | The following theorem is required:<br>• A system of homogeneous linear equations in three unknowns has nontrivial solutions if and only if the coefficient matrix is singular<br>The wording "necessary and sufficient conditions" could be introduced to students. |
| | Subtotal in hours | 18 | |
| **Vectors** | | | |
| 15. Introduction to vectors | 15.1 understand the concepts of vectors and scalars | 5 | The concepts of magnitudes of vectors, zero vector and unit vectors are required.<br>Students should recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$); and some notations for magnitude (including $\lvert\mathbf{a}\rvert$ and $\lvert\vec{a}\rvert$). |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | 15.2 understand the operations and properties of vectors | | The addition, subtraction and scalar multiplication of vectors are required.<br><br>The following properties are required:<br><br>• $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$<br>• $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$<br>• $\mathbf{a} + \mathbf{0} = \mathbf{a}$<br>• $0\,\mathbf{a} = \mathbf{0}$<br>• $\lambda(\mu\,\mathbf{a}) = (\lambda\mu)\,\mathbf{a}$<br>• $(\lambda + \mu)\,\mathbf{a} = \lambda\,\mathbf{a} + \mu\,\mathbf{a}$<br>• $\lambda(\mathbf{a} + \mathbf{b}) = \lambda\,\mathbf{a} + \lambda\,\mathbf{b}$<br>• If $\alpha\,\mathbf{a} + \beta\,\mathbf{b} = \alpha_1\,\mathbf{a} + \beta_1\,\mathbf{b}$ ($\mathbf{a}$ and $\mathbf{b}$ are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$ |
| | 15.3 understand the representation of a vector in the rectangular coordinate system | | The following formulae are required:<br><br>• $\lvert\overrightarrow{OP}\rvert = \sqrt{x^2 + y^2 + z^2}$ in $\mathbf{R}^3$<br><br>• $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$ and |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$ in $\mathbf{R}^2$<br><br>The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2.<br><br>The concept of direction cosines is **not** required. |
| 16. Scalar product and vector product | 16.1 understand the definition and properties of the scalar product (dot product) of vectors | 5 | The following properties are required:<br>• $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$<br>• $\mathbf{a} \cdot (\lambda\,\mathbf{b}) = \lambda\,(\mathbf{a} \cdot \mathbf{b})$<br>• $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$<br>• $\mathbf{a} \cdot \mathbf{a} = \lvert \mathbf{a} \rvert^2 \geq 0$<br>• $\mathbf{a} \cdot \mathbf{a} = 0$ if and only if $\mathbf{a} = \mathbf{0}$<br>• $\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \geq \lvert \mathbf{a} \cdot \mathbf{b} \rvert$<br>• $\lvert \mathbf{a} - \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 + \lvert \mathbf{b} \rvert^2 - 2(\mathbf{a} \cdot \mathbf{b})$ |
| | 16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$ | | The following properties are required:<br>• $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| | | | • $\mathbf{b} \times \mathbf{a} = -(\mathbf{a} \times \mathbf{b})$<br>• $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$<br>• $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$<br>• $(\lambda\, \mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\lambda\, \mathbf{b}) = \lambda\, (\mathbf{a} \times \mathbf{b})$<br>• $\lvert \mathbf{a} \times \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 \lvert \mathbf{b} \rvert^2 - (\mathbf{a} \cdot \mathbf{b})^2$<br>The following properties of scalar triple products should be introduced:<br>• $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$<br>• $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$ |
| 17. Applications of vectors | 17.1 understand the applications of vectors | 8 | Division of a line segment, parallelism and orthogonality are required.<br>Finding angles between two vectors, the projection of a vector onto another vector, the volume of a parallelepiped and the area of a triangle are required. |
| | Subtotal in hours | 18 | |

| Learning Unit | Learning Objective | Time | Remarks |
|---|---|---|---|
| **Further Learning Unit** | | | |
| 18. Inquiry and investigation | Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts | 7 | This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units. |
| | Subtotal in hours | 7 | |

**Grand total: 125 hours**

# Chapter 3 Curriculum Planning

This chapter provides guidelines to help schools and teachers to develop a flexible and balanced curriculum that suits the needs, interests and abilities of their students, and the context of their school, in accordance with the central framework provided in Chapter 2.

## 3.1 Guiding Principles

The Mathematics Curriculum (S4 – 6) allows flexibility for adaptation to cope better with the varied needs of students. To provide a rich variety of experiences and a balanced repertoire of mathematical knowledge and skills, a coherent school-based curriculum emphasising the active role of students in the learning process should be developed.

In planning their school-based Mathematics curricula, schools and teachers are encouraged to consider:

(a) The cognitive development of students

In Mathematics, there are many abstract concepts that are not always easy to understand and handle. Teachers should be aware of the cognitive development of students in planning progression across year levels and the arrangement of learning units within year levels. In the learning and teaching of the curriculum, teachers should use as many examples and illustrations as possible to help students grasp the mathematical concepts.

(b) Students' prior knowledge, abilities and inclinations

The learning and teaching of the Mathematics Curriculum (S4 – 6) should be built upon the prior knowledge of students gained from the school-based Mathematics curriculum at the junior secondary level. Before the introduction of a new concept, teachers should find out whether students have acquired sufficient prerequisite knowledge. The school-based Mathematics curriculum should address both the needs of average students and of those at both ends of the ability scale. Attention should not be focused only on academically lower achievers nor on those who perform better. Opportunities to learn should be maximised for all students.

(c) Post-secondary pathways of students

As mathematics provides the necessary language and tools for deeper understanding and study of concepts and applications in many disciplines, curriculum planning should be suited to the needs of students with different choices of elective subjects and diverse post-secondary pathways.

(d) Coherence of the curriculum

In planning the curriculum, it is important to establish coherence among the topics in the Compulsory Part and across the Compulsory Part and the modules. Essential skills and prerequisite knowledge should be covered in the Compulsory Part before the learning and teaching of particular topics in the modules.

(e) Pedagogical strategies

The curriculum should be able to help students develop a capacity for self-directed, lifelong learning. Elements of independent learning and generic skills should be embedded in relevant learning activities, and the learning and teaching activities should be suitably contextualised to promote effective learning. There should be a flexible integration of mathematical knowledge and skills in the learning and teaching of different learning units, such as 'Inquiry and Investigation'.

(f) Use of information technology

The widespread availability of computers and calculators should be taken into account when designing learning and teaching activities in Mathematics. Schools should make full use of the advantages of using information technology to assist students in understanding, visualising and exploring mathematical knowledge and skills.

(g) Assessment for learning

Assessment is more than an end-of-unit test. It informs students and teachers about what students understand and can do. Ongoing assessment is highly recommended to provide students with relevant feedback to enable them to improve their future performance. Schools and teachers are encouraged to review and plan the curriculum flexibly and make appropriate re-adjustments when necessary, taking into account the arrangements of internal assessment for the subject as specified in Chapter 5 – Assessment.

(h) Flexible time-tabling

The Mathematics Curriculum (S4 – 6) allows flexibility (e.g. the Compulsory Part and the Extended Part, Foundation Topics and Non-foundation Topics) for planning a school-based Mathematics curriculum. Schools are encouraged to make use of this flexibility in time-tabling.

## 3.2 Curriculum Planning Strategies

### 3.2.1 School-based Curriculum Planning

Schools and teachers are encouraged to adapt the central Mathematics Curriculum and develop their own school-based curricula, taking into consideration the needs, abilities and interests of their students and their school contexts, through varying:

- content, contexts and examples;
- learning and teaching strategies;
- pace of learning and teaching; and
- modes of assessment.

In designing school-based Mathematics curricula, schools are encouraged to:

(a) set and work on clear and manageable aims and targets taking into account the school contexts and the overall aims of the Mathematics Curriculum (S4 – 6);
(b) allow students to select modules in the Extended Part and group students into different classes if possible;
(c) adapt the depth of treatment of the learning objectives and the logical coordination of the learning content;
(d) allow suitable and flexible lesson time for different groups of students for the Compulsory Part and the modules across year levels;
(e) choose and adapt appropriate textbooks and other learning resources;
(f) set and work on learning activities to be carried out in the school year; and
(g) set and work on the methods and modes of assessment to enhance assessment for learning.

### 3.2.2 Cross-curricular Links

The Mathematics Curriculum (S4 – 6) provides students with a foundation for understanding concepts and carrying out investigations, with a tool for making inferences and analysing data, and with a language for presenting findings and constructing models.

The following table presents some examples of the kinds of tasks or activities that students should be encouraged to make connections with other Key Learning Areas:

| ***Key Learning Area*** | ***Examples of Activities*** |
|---|---|
| Chinese Language Education | • Compare and discuss the development of mathematics in Chinese culture<br>• Explore and discuss the applications of mathematics in ancient China<br>• Appreciate the development of mathematics in China and Chinese mathematicians' contribution to mathematics<br>• Investigate the history of Pi<br>• Discuss the uses and abuses of mathematics in discourse<br>• Invoke logical reasoning in discourse<br>• Study the biographies of mathematicians |
| English Language Education | • Compare and discuss the development of mathematics in Western culture<br>• Explore and discuss the applications of mathematics in ancient Greece and Egypt<br>• Appreciate the development of mathematics in Europe<br>• Discuss the uses and abuses of mathematics in discourse<br>• Invoke logical reasoning in discourse<br>• Study the biographies of mathematicians |
| Personal, Social and Humanities Education | • Explore strategies to solve simple financial problems<br>• Apply mathematics to finance, business, accounting and economics<br>• Discuss the uses and abuses of mathematics in different social contexts<br>• Invoke logical reasoning in discourse |

| *Key Learning Area* | *Examples of Activities* |
|---|---|
| Science Education | • Make use of the knowledge of vectors and calculus to solve problems in Physics<br>• Construct mathematical models by making use of information from Chemistry<br>• Use the necessary analytical tools in the process of investigation |
| Technology Education | • Use the necessary mathematical tools to explore, investigate and communicate in the study of modern technology<br>• Use geometrical knowledge in 2-dimensional and 3-dimensional space to explore the methods of product modelling<br>• Use the necessary mathematical tools in the workplace and in support of the subjects in Applied Learning |
| Arts Education | • Appreciate visual arts from a geometrical perspective<br>• Appreciate the applications of mathematics in music, e.g. the harmonic sequence and harmony |
| Physical Education | • Engage in learning tasks or activities that examine the pros and cons of various health and physical activities<br>• Interpret graphs related to situations encountered in human body and movement analysis |

In view of the broad cross-curricular links with other disciplines, Mathematics teachers should collaborate with teachers of other Key Learning Areas to set achievable goals, draw up schedules of work, and design learning and teaching activities, so that students can apply their mathematical knowledge in different contexts.

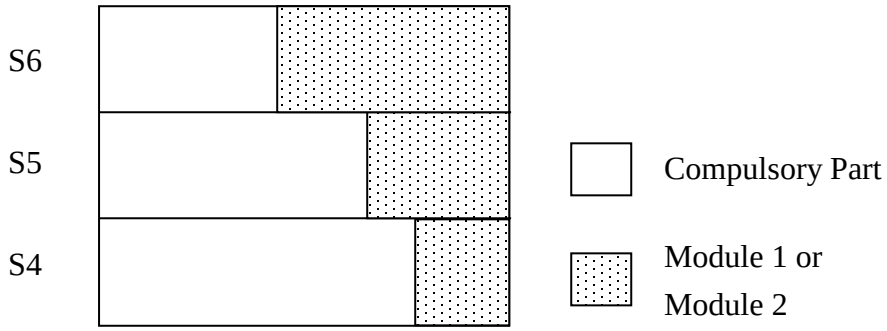
For students taking an Applied Learning course as an elective subject, Mathematics teachers are encouraged to incorporate more contextualised examples related to the Applied Learning course to assist students in the elective subject. This, in turn, will benefit the learning and teaching of Mathematics, as these students will have opportunities to apply mathematical knowledge in related Applied Learning contexts.

## 3.3 Progression

In implementing the Mathematics Curriculum (S4 – 6), schools do not have to spread the allocated lesson time uniformly over the three years. They may arrange the lesson time flexibly according to the needs of different classes.

*Flexible time allocation*

For classes taking the Compulsory Part with one module, schools may start the learning and teaching of the Compulsory Part and the module at the same time in S4. However, if teachers find it inappropriate for their students to start the module at the beginning of S4, they may consider making use of most of the allocated lesson time for Mathematics in S4 to teach the Compulsory Part first until students are more mathematically mature and are equipped with enough mathematical concepts and knowledge for the learning of the module. The figure below illustrates a possible progression pathway.




*Studying mathematics-related elective subjects and Applied Learning courses*

For students taking elective subjects or courses in Applied Learning which require a foundation knowledge of Mathematics, teachers may allocate more lesson time in S4 and relatively less lesson time in S5 and S6 on Mathematics to facilitate the learning and teaching of the elective subjects or the Applied Learning courses concerned.

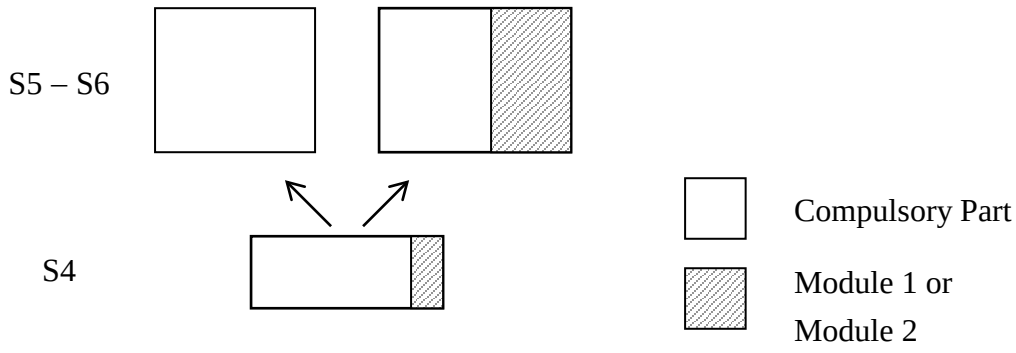
*Change of module at S5*

Some students in S5 may wish to change the module they selected in S4 (from Module 2 to Module 1 or vice versa) or to study a module they did not intend to choose at the beginning. To allow for this flexibility, teachers may focus more on the Compulsory Part in S4. In this way, students who did not select the module at the start need not spend too much time and effort in catching up on the areas of the module which have not been covered. Even for

those students who decide to drop the module in S5, the influence on their studies will be minimum.

*A "Taster" of a module in S4*

Some students may be interested in studying a module along with the Compulsory Part. However, they might like to get some ideas of what it is like first in S4 before they decide whether or not to take it. For such cases, a small portion of the lesson time may be allocated to the study of a module in S4. In S5, some students may drop the module while others may continue to study it. Alternatively, in S5, some students may even change from studying one module to the other. The figure below illustrates a possible progression pathway.



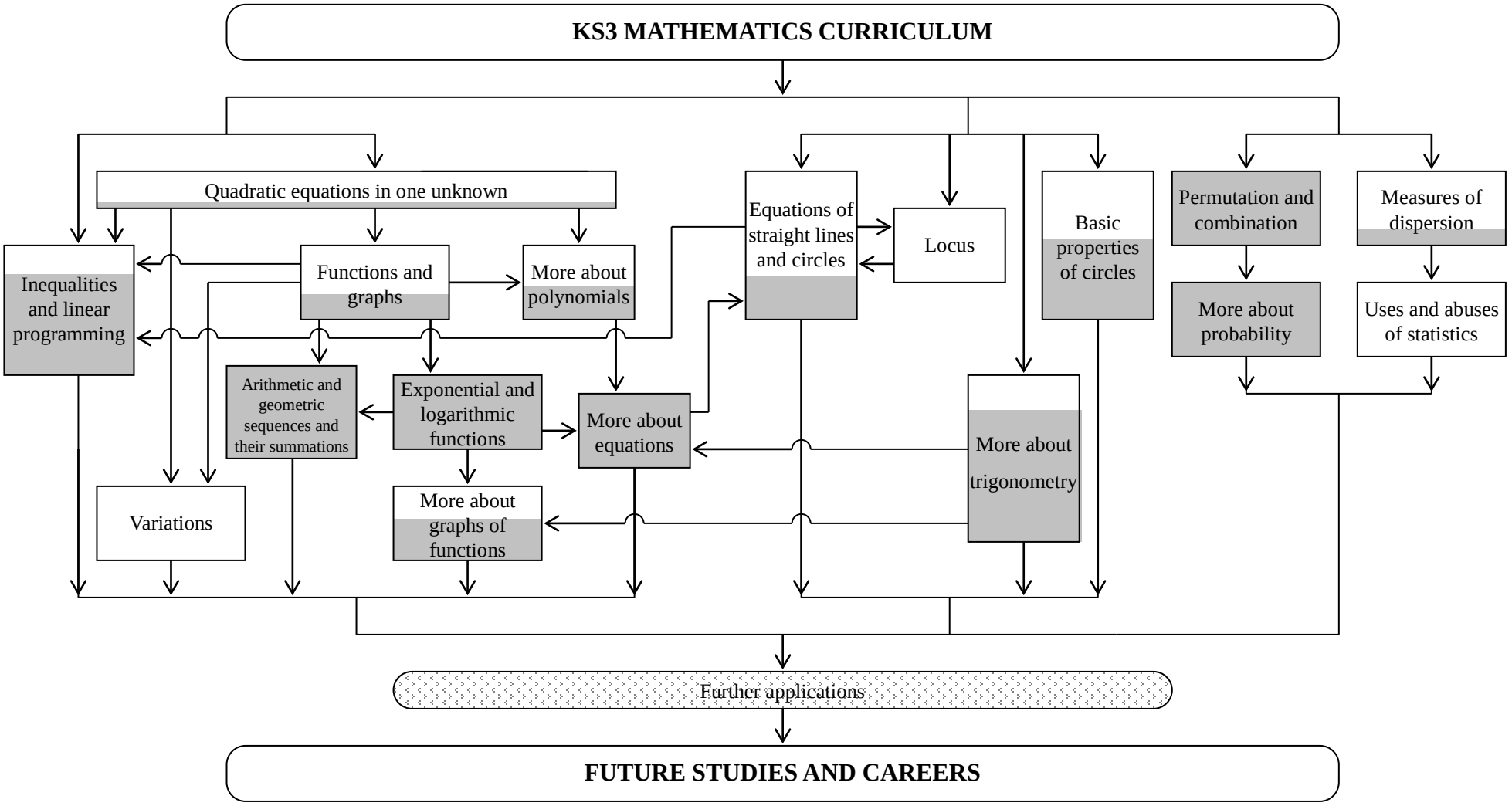

*Sequence of learning and teaching*

In planning the school-based Mathematics curriculum, teachers can refer to the learning targets and the learning objectives of the Compulsory Part, Module 1 and Module 2. It should be noted that the ordering of the targets and objectives in Chapter 2 does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequence which takes account of the needs of their students.

It should be noted also that there are many ways of sequencing the learning units. Mathematical knowledge is interrelated both within and across strands or areas. Teachers should exercise their professional judgment in arranging the sequence of learning units, paying special attention to the prerequisite knowledge required.

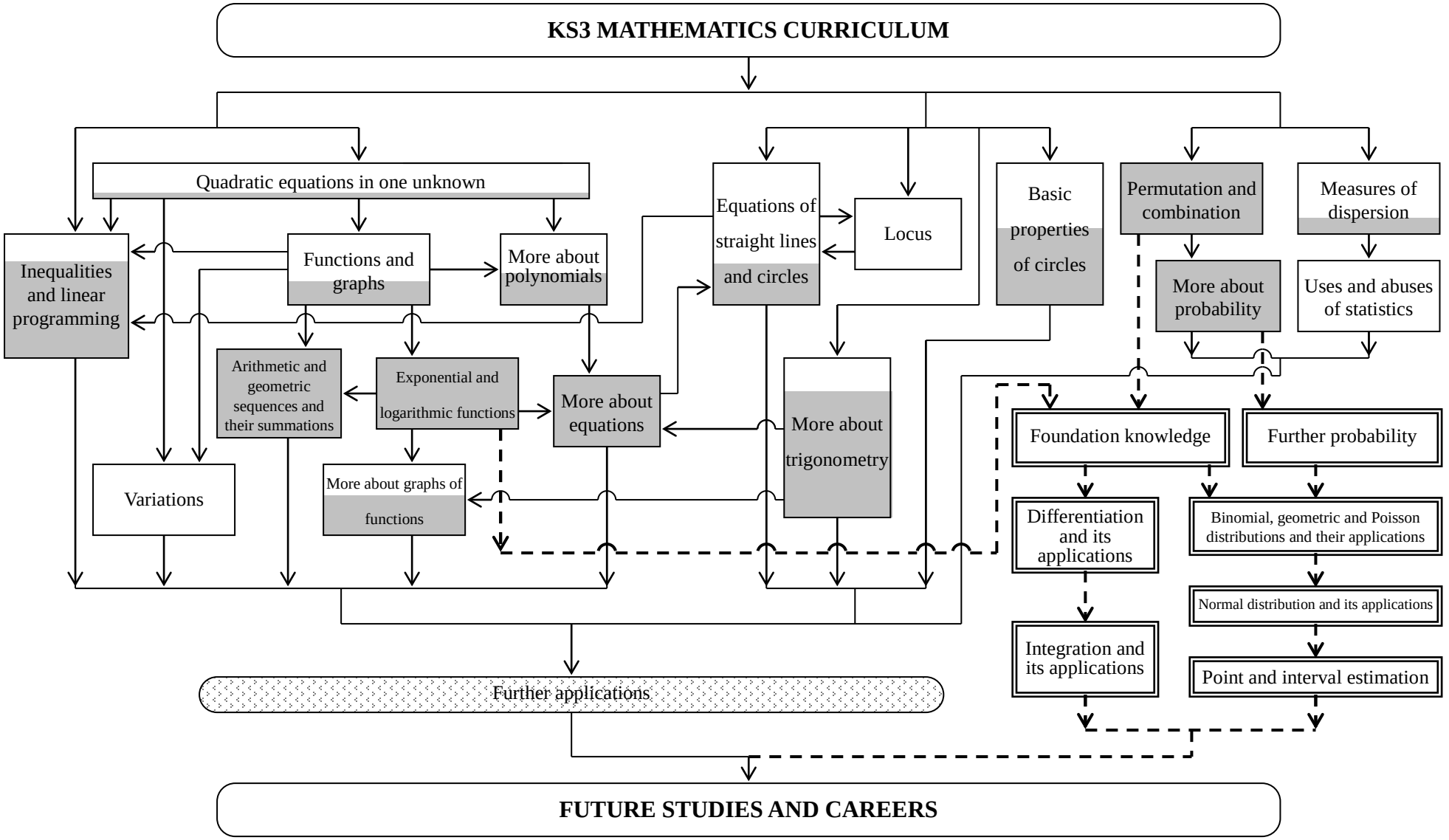
Flow charts are provided on the following pages to show possible progression pathways for the learning units in the Compulsory Part, Module 1 and Module 2. Strong links between learning units are shown. These are merely illustrative as it is not feasible to illustrate all links in the flow charts. The flow charts are included for teachers' consideration in curriculum planning.

# Flow Chart : Compulsory Part




represents Non-foundation Topics.

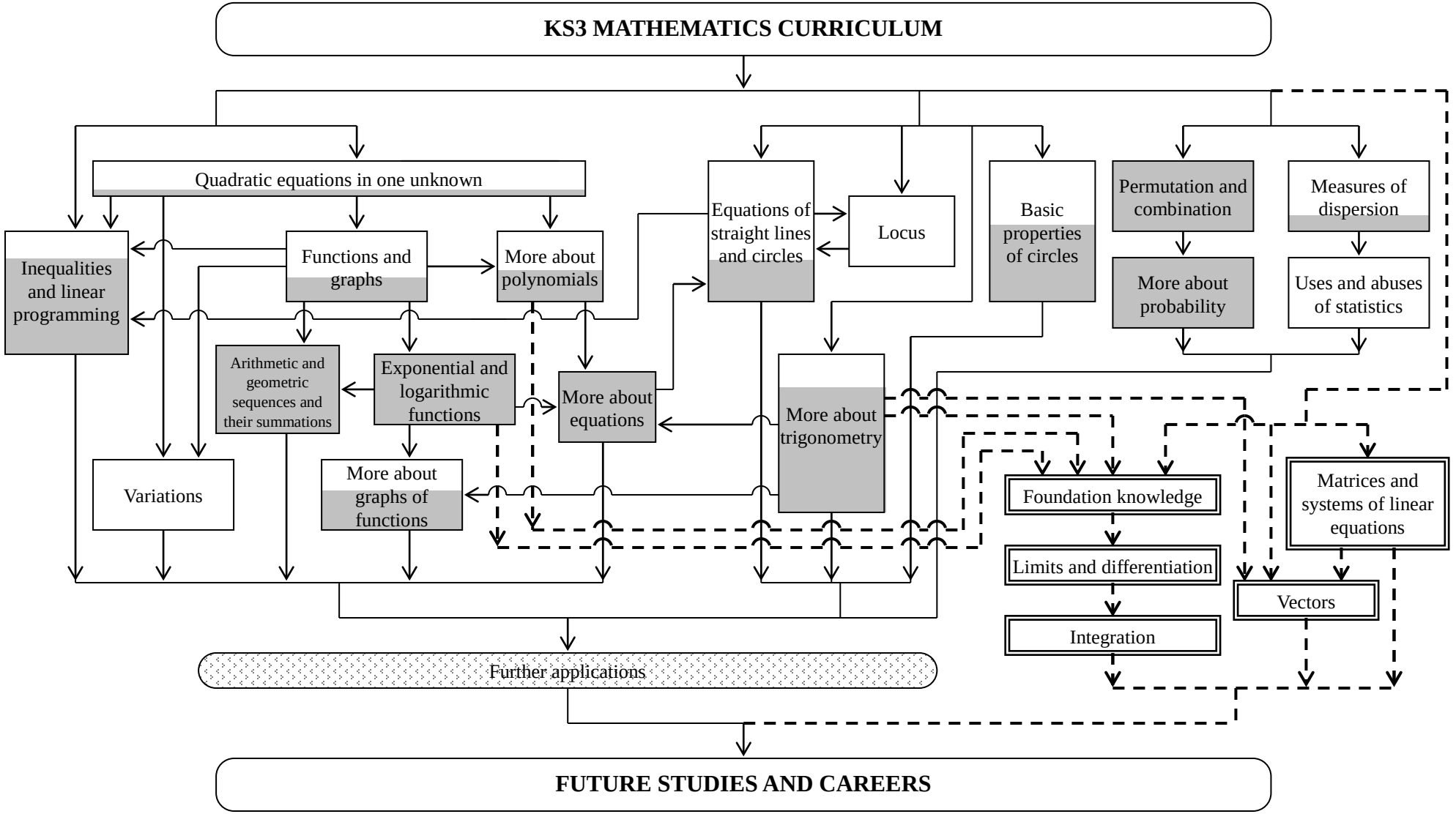
# Flow Chart : Compulsory Part with Module 1 (Calculus and Statistics)




represents Non-foundation Topics.

represents learning units in Module 1.

# Flow Chart : Compulsory Part with Module 2 (Algebra and Calculus)




represents Non-foundation Topics.

represents learning units in Module 2.

## 3.4 Curriculum Management

In managing the Mathematics Curriculum, curriculum leaders in schools are expected to:

- formulate well-defined policies for their Mathematics panels, including learning and teaching strategies, assessment and the development of school-based Mathematics curricula, and to establish effective practices for the implementation of the agreed policies;
- disseminate relevant information to Mathematics teachers and discuss matters relevant to both administration and Mathematics teaching, such as policies and guidelines, the selection of textbooks, the procurement of teaching aids and reference books, and likely teaching difficulties;
- familiarise Mathematics teachers with the Mathematics Curriculum (S1 – 6) and recent development in the teaching of mathematics;
- promote and create the time for teachers' professional development;
- encourage collaboration and maintain effective communication between teachers of Mathematics and other Key Learning Areas, so that Mathematics teachers are well aware of the developments in other Key Learning Areas;
- encourage the development of a learning culture and a learning community, including face to face communication, electronic networks and experience-sharing with other schools; and
- encourage assessment for learning and the use of evidence to adjust teaching methodologies and the curriculum.

In the planning, development and management of the school-based Mathematics curriculum, the school head, panel chairperson and Mathematics teachers must work together taking up their various roles as change agents committed to helping students learn how to learn.

(a) School Heads

School heads are key players in directing and supporting school-based Mathematics curriculum development. They should understand the direction of the central curriculum and be sensitive to contextual factors such as the needs of the students, the strengths of their Mathematics panels and the culture of their schools. They have to establish school curriculum development policies and plans, coordinate the teaching of

all subjects, and take effective measures to improve student learning in their schools. School heads should also:

- provide support for trying out new initiatives in the learning, teaching and assessment of Mathematics Curriculum (S4 – 6);
- create a supportive environment in the school, so that students have ample opportunities to think critically and creatively, to conceptualise, inquire and reason, and to use mathematics to formulate and solve problems in daily life;
- provide flexibility in the deployment of teachers and time-tabling;
- provide students with flexibility in the study and choice of modules in the Extended Part and in the grouping of students;
- help parents to understand the school's vision, rationale and practices in implementing the Mathematics Curriculum (S4 – 6); and
- encourage networking with other schools on a face to face and electronic basis to promote exchange of professional information and the sharing of good practices.

(b) Mathematics Panel Chairpersons

Mathematics panel chairpersons help to lead, plan and monitor the school-based Mathematics curricula. They are the "bridge" between the school administrative personnel and Mathematics panel members. A panel chairperson has both administrative and teaching roles. He/She is the coordinator of the subject and has to be fully aware of the needs of the teachers and their students in learning and teaching Mathematics. To implement the school-based senior secondary Mathematics curricula, Mathematics panel chairpersons should:

- set up well-defined goals for the learning, teaching and assessment of senior secondary Mathematics;
- ensure a smooth articulation between the school-based Mathematics curricula at the junior secondary and senior secondary levels;
- formulate well-defined policies and establish effective practices for the implementation of the agreed policies;
- hold regular panel meetings to discuss matters such as schemes of work, assessment policies, teaching problems which have arisen, and the selection of textbooks;
- keep abreast of the latest curriculum development and assist Mathematics teachers to implement effective learning and teaching strategies;

- enhance the effectiveness of classroom teaching by organising experience-sharing, lesson observations or collaborative lesson preparation, etc;
- monitor the quality of student learning by gathering feedback from assessment; and
- keep all documents related to the learning, teaching and assessment of senior secondary Mathematics.

(c) Mathematics Teachers

Mathematics teachers can contribute to the development of a school-based Mathematics curriculum both as individuals and in collaboration with other Mathematics teachers. They may also initiate innovative curricular changes to enable students to cope confidently with the mathematics needed in future studies, the workplace and daily life. Mathematics teachers should:

- develop in students generic skills, positive attitudes and an interest in the learning of mathematics;
- keep abreast of the latest curriculum development;
- take initiatives in trying out innovative learning and teaching strategies and assessment strategies;
- initiate the sharing of teaching ideas, knowledge and experience to foster peer support and improvement in the learning, teaching and assessment of Mathematics; and
- participate in professional development courses, workshops, seminars, etc. to enhance their professionalism.

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# Chapter 4 Learning and Teaching

This chapter provides guidelines for effective learning and teaching of the Mathematics Curriculum (S4 – 6). It is to be read in conjunction with Booklet 3 in the *Senior Secondary Curriculum Guide* (CDC, 2007), which provides the basis for the suggestions about learning and teaching set out below.

## 4.1 Knowledge and Learning

Mathematical knowledge is different from knowledge in any other disciplines. Its basic assertions have an extremely high degree of certainty, with permanent discipline structures such as axiomatic deductive systems. Unlike theories in empirical sciences, theorems in mathematics are often regarded as paradigms of necessary truths and a priori knowledge. In addition to this epistemological perspective, mathematical knowledge can be perceived as a repository of competencies in knowing certain results and in seeing connections. Viewing mathematical knowledge as products of an individual's conceptual organisation of his/her experience highlights the changing nature of the teacher's role. Teachers become collaborators and facilitators of learning. They should no longer focus on dispensing "truth" or "knowledge", but rather on helping and guiding students to develop their own personalised understanding of mathematics as a result of the activities in which they participate. The learning and teaching of mathematics, therefore, involves "doing" – that is, inquiring, experimenting, abstracting, generalising or conjecturing – not just the transmission and memorising of well constructed knowledge. Teachers are responsible for creating a learning environment in which students develop positive attitudes to learning the subject and are free to express their ideas through active participation.

Another personal or internal conceptualisation of mathematics sees mathematical knowledge as resulting from social interaction; that is, the learning of mathematics involves acquiring facts, concepts, algorithms, principles and skills as a result of social interaction that relies heavily upon context. This approach focuses on students' ability to relate mathematics to its applications and its potential use in problem-solving situations. If students learn through applying mathematics in context, they will come to appreciate the value of mathematics in social settings. It

is this “sense making” in the learning of mathematics that enables them to participate actively in “doing mathematics”.

## 4.2 Guiding Principles

The following are guiding principles and a general rationale for the effective learning and teaching of senior secondary Mathematics.

- **Knowledge**: In mathematics education, two kinds of knowledge have been widely discussed, namely procedural knowledge and conceptual knowledge. Students can solve mathematical tasks efficiently through acquiring procedural knowledge; but merely learning procedural knowledge and developing procedural skills do not constitute a balanced mathematics curriculum. It is through acquiring conceptual knowledge that students gain a deeper understanding of mathematics, which then helps them to make connections among different pieces of knowledge. It is important to focus on both kinds of knowledge in the learning of mathematics.

- **Learning**: Students in Hong Kong have a sound basic knowledge and strong procedural skills in mathematics. A number of cross-national studies have shown consistently that Hong Kong students perform very well on tasks routinely learned in schools, but are not so effective in solving exploratory problems and performing hands-on activities. In this regard, the “Inquiry and Investigation” learning units of the Compulsory Part, Module 1 and Module 2 provide opportunities for students to discover and construct knowledge, and to improve their ability to inquire, communicate, reason and conceptualise mathematical concepts through various learning activities. Students are asked to integrate what they have learned in the curriculum to explore and solve more sophisticated real-life problems in the “Further Applications” learning unit in the Compulsory Part. It should be pointed out that the learning *process* and the learning *outcomes (products)* are of equal importance. It is essential to help students move from being passive recipients of knowledge to seeing the relationships between concepts, applying ideas, and ultimately thinking critically and creatively and constructing knowledge on their own.

- **Learning targets**: It is essential to set clear learning targets. Learning will be more effective if students and their teachers have clear learning targets in mind.

- **Teaching for understanding**: A key element in a successful Mathematics lesson is the use of appropriate methods that motivate students to use what they know to act and think flexibly and to inquire critically to obtain new knowledge. To achieve this, good teaching should involve students' active participation in the learning process, so that they come to enjoy mathematics, recognise its usefulness and appreciate the subject in its own right.

- **Prior knowledge and experience**: It is necessary to pay close attention to the mathematical knowledge, skills and experience that students bring to the classroom. Teachers should therefore have a good understanding of their students' strengths, interests and needs.

- **A wide range of pedagogies**: There are many strategies for teaching Mathematics, some of which supplement each other. Teachers should consider the flexible use of a variety of teaching strategies.

- **Quality interaction**: The effective use of questioning and feedback guides students in the learning process. Students also contribute to each other's learning when they interact in different groupings to give mutual feedback. Teachers may use "scaffolds" to support them in areas that may be beyond their current reach.

- **Teaching for independent learning**: Generic skills (such as critical-thinking skills and problem-solving skills) and reflection should be nurtured through relevant learning activities. Students at the senior secondary level are more mathematically mature and should be encouraged to read mathematics-related articles or books and gather information on their own to widen their knowledge base.

- **Self-directed learning**: Students should be encouraged to take responsibility for their own learning. Open-ended questions can be used to promote their interest in learning mathematics. Opportunities should be provided to enable students to plan their own learning and to use mathematics to solve real-life problems.

- **Feedback and assessment**: Teachers need to assess the work of students, but assessment has little value without feedback. Feedback should not be confined to giving a grade, but must also provide students with a wide range of information on strengths and areas where they can improve their learning. Evidence from

assessment should also give teachers indications as to where they need to improve their teaching.

- **Resources**: In planning and conducting mathematical activities, teachers should make use of a variety of learning and teaching resources, such as textbooks, reference books, audio-visual materials and instruments for drawing shapes and making models. Well-designed learning materials play a crucial role in providing students with a means of working at their own pace, so that they can reach and, in some cases, move beyond the skills and understandings already achieved by the majority of their peers. Commonly used IT tools and hand-held technologies, such as computer-aided learning packages, dynamic geometry packages and graphing calculators, can be used to promote learning and teaching.

- **Motivation**: In general, Hong Kong students have a high regard for Mathematics as a school subject. However, a low level of confidence, a lack of perseverance in tackling difficult problems and a reluctance to discuss their learning difficulties with teachers are common barriers for them when learning mathematics. Effective motivational strategies need to be adopted to overcome these barriers. For instance, motivation can be aroused by giving students positive reinforcement and by providing them with opportunities to express their views on learning the subject. It can also be aroused by enabling them to apply their mathematical knowledge and skills successfully in real-life situations.

- **Engagement**: Students should be encouraged to participate actively in activities, collaborate closely with others, express themselves openly and treat suggestions positively, and through all of these to build up their self-confidence and positive attitudes towards learning – despite the risk of making mistakes or encountering difficulties.

- **Catering for learner diversity**: Students have a wide variety of characteristics and strengths. Teachers should employ a range of strategies to cater for such learner diversity.

## 4.3 Choosing Learning and Teaching Approaches and Strategies

The extent to which teachers are successful in facilitating learning among students depends, to some degree, on the teaching approaches and strategies they use. A

variety of learning and teaching approaches can be interwoven and deployed to suit students with different needs, interests, abilities and prior knowledge. Familiarity with the characteristics of Hong Kong students in Mathematics can help teachers adopt the most suitable learning and teaching strategies. In general, Hong Kong students:

- have a high regard for Mathematics;
- believe that diligence can help them to learn Mathematics;
- are strong in basic skills and computation, but are weaker in dealing with non-routine problems;
- value drill and practice;
- are more used to exposition than activity-based ways of learning such as the use of games or group discussion;
- seldom interrupt the flow of teaching by asking questions; and
- are usually motivated by extrinsic factors.

Three common pedagogical approaches that can be employed for the effective delivery of the Mathematics Curriculum (S4 – 6) are discussed below.

### Teaching as direct instruction

Direct instruction, a very frequently used approach in the Mathematics classroom, can make a positive contribution to the learning of mathematics if it is dynamic, and well planned and organised.

A direct instruction approach is most relevant to contexts which involve explanation, demonstration or modelling to enable learners to gain knowledge and understanding of particular concepts. In this approach, the teacher focuses on directing the learning of the students through providing information or explanations. Direct instruction, in which teachers present factual materials in a logical manner, can be employed effectively for teaching definitions and notations of mathematical terms, rigorous proofs of mathematics theorems and procedures for sketching curves. This approach can also be used with a large group of students to stimulate thinking at the start of an open discussion. Teaching Mathematics through direct instruction can produce a mathematically neat and complete lesson which contains presentations and explanation leading to an intended conclusion. Although it can be interactive, it is a more teacher-centred approach: the teacher asks questions, chooses students to answer them and evaluates the responses; and then he/she probes for more information and may ask the students to justify their answers.

Another major goal of direct instruction is to maximise student learning time. In a typical direct instruction lesson on Mathematics, the teacher usually spends some time lecturing, guides the students through a concept, provides examples and breaks complex ideas into simpler steps. The students are then given simple problems to try on their own; and the simple problems are then combined so that students can solve more complex problems. Finally, the teacher summarises what the students should have learned, and gives them assignments through which he/she can assess the amount of the content the students have learned in that lesson. Direct instruction lessons can also be facilitated by the use of audio-visual materials, when appropriate.

**Teaching the concept and notation of combination using the direct instruction approach**

Students are introduced to the concept of combination by their teacher. The teacher explains the concept and notation of combination and puts the related formulae on the board. The teacher then derives the formulae and demonstrates with some examples. The difference between the concepts of combination and permutation is also clearly explained. The teacher then alerts the students to the fact that the terminology of combination in mathematics is different from its common use in daily life – for example, the word "combination" in ***combination lock*** does not refer to combination. After that, the teacher introduces other notations for combination used in various parts of the world. At the end of the lesson, a brief note on what has been taught and some simple exercises on the topic are distributed to the students for consolidation.

The "direct instruction" approach is suitable for teaching the concept and notation of combination for the following reasons:

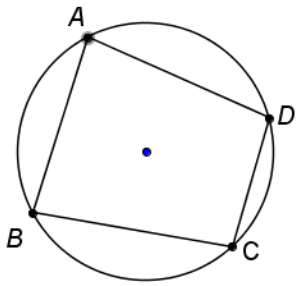
- The concept and the notation are difficult for students to understand and the proofs of the formulae are mainly procedural.
- It can be ensured that students have acquired an accurate understanding of the concept and notation.
- The notations used in the books from different countries may be different. For instance, the notations of combination $\binom{n}{r}$ and $C_n^r$ are usually found in the books written by British and Chinese scholars respectively. Students will be confused by the notations if this is not explained properly.
- The concept and notation of combination can be introduced efficiently in a short time.

## Teaching as inquiry

The Mathematics Curriculum (S4 – 6) sets out various learning activities for students to carry out investigations and includes "Inquiry and Investigation" learning units in the Compulsory Part, Module 1 and Module 2. In the inquiry approach, the emphasis is on the process and the action undertaken by the learner – inquiry tasks often focus on students' thinking and inquiring processing. Students are encouraged to help each other raise questions and develop understanding, with their experiences being seen more as a resource for learning than as a product. Students engage in complex cognitive processes requiring thoughtful discourse. They are invited, for example, to **make connections between facts, make predictions, and debate alternatives**. This relies heavily upon **extensive dialogue** among students, and can take place during **interactive whole-class teaching or during peer interaction in pairs or groups**. This approach promotes critical thinking and problem-solving skills. The topic(s) under investigation should be placed in a meaningful context, and students should be encouraged to explore and discover information through the use of **open-ended questions, group discussion, investigations, experiments and hands-on exercises**. However, there should be sufficient "wait time", so that students can explain their thinking adequately and elaborate their answers.

**Teaching properties of a cyclic quadrilateral**
**using the inquiry approach**

The teacher starts the topic "properties of a cyclic quadrilateral" in the Compulsory Part by asking students to use computers with dynamic geometry software installed and an application file with a circle and a cyclic quadrilateral drawn as in the figure below.




Students are free to:

- move the vertices of *A*, *B*, *C* and *D*;
- add lines such as diagonals *AC* and *BD*;

- extend line segments *AB*, *CD*, etc.;
- measure sizes of angles; or
- measure lengths of line segments such as *AB*, *AC*, etc.

Students are asked to record their measurements and make conjectures about the properties such as angle relations or line relations. They are encouraged to discuss their observations and conjectures with each other, and may use the software to provide evidence for them. After that, students are asked to make a presentation on their observations and conjectures, during which the teacher focuses on asking students to explain and justify their conjectures and gives them time to do so. For example, if students observe that the opposite angles of a cyclic quadrilateral are supplementary, the teacher may ask them whether they have verified their conjecture by referring to different quadrilaterals. It should be pointed out that students' observations and conjectures may differ and the teacher should provide different responses to students who have arrived at different conclusions.

As students have often encountered circles and quadrilaterals in daily life and in their previous learning experiences, they can integrate their prior knowledge and IT skills to explore new aspects of plane geometry. This topic allows students to explore a rich collection of relations, including the basic properties listed in the Compulsory Part and also aspects such as Ptolemy's Theorem* that is beyond the curriculum. Through engaging in this exploratory activity, they may come to recognise the richness of geometrical knowledge. They can construct knowledge through observing, making conjectures and testifying. Sketching a diagram using IT tools or drawing instruments are good ways for students to justify their hypotheses and conjectures. This approach is different from the familiar deductive approach to study geometric knowledge.

* Ptolemy's Theorem: For a cyclic quadrilateral, the sum of the products of the two pairs of opposite sides equals the product of the diagonals.

## Teaching as co-construction

This approach puts the focus on the class as a community of learners. It is based on the view that mathematics is a creative human activity and that social interaction in the classroom is crucial to the devgelopment of mathematical knowledge. The

teacher plays a central role in developing a problem-solving environment in which students feel free to talk about mathematics. Tasks for this approach usually involve creating knowledge and developing criteria for judging that knowledge collectively. A wide variety of linkages are made between members of the class as they contribute to the construction of knowledge. In this approach, the teacher helps students learn how to think for themselves, which may require temporary frameworks or scaffolds such as model answers, demonstrations, prompts and feedback, opportunities for students to explain their arguments to the class, or worksheets for students to work on. The teacher also requires students to check the process through which they reached a conclusion and indicate how it might be improved.

This approach relies heavily on discussion and sharing among participants. Students learn through student-student and teacher-student interactions. Through co-construction of knowledge, students develop social skills, organise their thinking and develop rational arguments.

**Teaching the quadratic formula**
**using the co-construction approach**

In teaching "solving quadratic equations by the quadratic formula", a Mathematics teacher may employ a co-construction approach in which, instead of passively accepting knowledge, students are able to construct concepts and to understand how the formula is deduced. After learning how to solve quadratic equations by the factor method, students are asked to solve quadratic equations with irrational roots. They will discover that not all quadratic expressions can be factorised into linear expressions with rational coefficients. They will realise that the factor method is limited in its usefulness. By being actively involved in well-constructed tasks – for instance, from solving $x^2 - 5 = 0$, $x^2 - 2x - 5 = 0$ to $x^2 - 2x - c = 0$ – and class discussion, students should become responsible for explaining their reasoning orally to the class, developing their own analogical thinking and discovering how the method of completing the square can be extended to solving general quadratic equations. The teacher has to sum up the discussion and refine the constraints (e.g. the coefficient of the term involving $x^2$ must be non-zero) in collaboration with students.

- In the co-construction approach, the teacher focuses on making connections between facts and fostering new understanding in students. The teacher encourages students to analyse, interpret and predict information. He/She also

promotes extensive dialogue among students, which is a very important part of learning mathematics. Communication is a way of sharing ideas and clarifying understanding. When students think, discuss and inquire about mathematical concepts, they reap a dual benefit: they communicate to learn mathematics, and they learn to communicate mathematically.

- Teaching is developmental rather than merely directive or presentational. Although algorithms provide an efficient route to getting correct answers, students do not understand how or why these procedures work. Merely directing them to follow the traditional rules or algorithms in a fixed fashion is like expecting them to arrive without having made the journey. The co-construction approach is student-oriented: the teacher serves as an expert providing guidance and does not dictate the working strategies entirely. Lessons should begin from where the students are, not from where the teacher is. Students are expected to bring the necessary skills and background knowledge to the task and to be self-motivated. Individual differences are expected, welcomed and supported.

- The classroom climate is collegial, supportive and spontaneous. Discussion, group work and active participation are encouraged and expected. Students are active creators of knowledge, not passive receivers of information. The emphasis is on understanding rather than on memorisation and repetition. Activities are problem-centred and student-driven. Lessons are built on solving problems at the appropriate level for the students. Some easier problems are dealt with early in the lesson and are used to provide paradigms. Busy work and unnecessary repetition are minimised.

As mentioned earlier, in the learning and teaching of mathematics, a single strategy is seldom adopted: an effective teacher of Mathematics very often integrates various strategies when teaching a topic. Strategies should always be used flexibly and be suited to the abilities, interests and needs of students, and the context and development of the lesson. Some common classroom practices which are effective for learning and teaching mathematics include:

- expanding concepts or definitions;
- analysing a problem in more than one way;
- using an idea in more than one setting;

- providing models that enhance understanding;
- giving examples of applications;
- providing examples and counter-examples when explaining concepts;
- requesting students to rephrase problems;
- practising and drilling;
- using scaffolding to demonstrate how to tackle a problem by, for example, induction, deduction, analogy, looking for equivalence and identities, classification, conjecturing, generalising, exhausting, formulating hypotheses, looking for patterns and regularities, charting and graphing, abstracting, intuition, analysing and synthesising, and breaking down a complicated problem into smaller parts;
- posing problems by students through changing the knowns, unknowns or constraints;
- providing opportunities for discussion and sharing of ideas; and
- brainstorming among students.

The example below illustrates how some of the approaches and strategies can be used in the Mathematics classroom:

**Teaching one of the properties of the scalar product of vectors using the direct instruction, the inquiry and the co-construction approaches**

Teachers may integrate various teaching approaches and classroom practices to introduce the properties of the scalar product of vectors so that the lessons can be more vivid and pleasurable. In this example, teaching one of the properties of the scalar product of vectors, $|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2(\mathbf{a}\cdot\mathbf{b})$, is used as an illustration.

In previous lessons, the teacher has taught the concepts of magnitudes of vectors and the scalar product of vectors using **direct instruction**. In this lesson, the students are divided into small groups to promote discussion, and the groups are asked to **explore** the geometrical meaning of the property. Here, the **inquiry approach** is adopted, with students having to carry out **investigations** with the newly acquired knowledge related to vectors. During the **exploration**, the groups may interpret the geometrical meaning differently. Some may consider one of the vectors to be a zero vector and get the above property; but others may relate it to the Pythagoras' Theorem by constructing two perpendicular vectors **a** and **b** with the same initial point. Hence, the hypotenuse is $|\mathbf{a} - \mathbf{b}|$ and $\mathbf{a}\cdot\mathbf{b} = 0$ and the result is then immediate. If some groups arrive at this conclusion, the teacher should **guide them to discover** that

their interpretation is only valid for special cases. However, the geometrical meaning of this property is related to the cosine formula learned in the Compulsory Part. If some groups can find that the property is the vector version of the cosine formula, they can be **invited to explain** how they arrived at this geometrical meaning. If none of the groups can arrive at the actual meaning, the teacher may **guide** them to find it out by **giving prompts**. Some **well-constructed prompts (or scaffolds)**, such as asking them to draw various types of triangles and **find clues** to connect $|\mathbf{a}-\mathbf{b}|$, $\mathbf{a}\cdot\mathbf{b}$, $|\mathbf{a}|$ and $|\mathbf{b}|$ with the triangles drawn, may be provided. The **co-construction approach** is adopted here.

After understanding the geometrical meaning, the result can be derived by applying the cosine formula learned in the Compulsory Part. The groups are further asked to **explore** alternative proofs. Here, the **inquiry approach** is employed. The groups may not think of proving this property with $|\mathbf{x}|^2 = \mathbf{x}\cdot\mathbf{x}$ directly. The teacher may **give some hints to guide** them. In this case, the teacher and the students are **co-constructing** knowledge. If the students still cannot prove this property, the teacher can **demonstrate** the proof on the board using the **direct instruction approach**. Whatever methods the students use, they are **invited to explain** their proofs to the class. During the explanation, the teacher and students may **raise questions and query** the reasoning.

## 4.4 Classroom Interaction

Classroom interaction helps students construct knowledge with the support of teachers and peers. Students need to participate in the discussion of concepts, procedures and proofs in the Mathematics classroom. They also have to think, listen and speak as they participate in a particular mode of inquiry. It is not enough for students simply to listen to the teacher and answer teachers' questions. Students need to develop competencies for critical thinking, and in order to develop these they need to have opportunities to think and interact with others.

Classroom interaction can be fostered in various ways, such as the following:

- *Begin each class with a problem*
  Instead of simply saying "We are going to cover the quadratic formula in this lesson", a teacher can begin by asking "Do we have a general formula for solving

quadratic equations?"

- *Be patient and use silence to encourage reflection*
  There should be pauses after questions to encourage student responses. The teacher should wait for students' replies, and resist the temptation to break the silence by answering the questions for them.

- *Arrange and use classroom space to encourage interaction*
  Putting students into small groups or arranging them to sit face to face in a circle may promote discussion and interaction among them.

- *Create a friendly environment*
  The teacher should welcome students' ideas and consider all their views without rushing into making judgments. He/She should also focus discussion on ideas rather than on judging people.

How teachers view and handle interaction is a key factor in determining the effectiveness of classroom interaction. Making use of effective questioning techniques, providing an encouraging classroom environment, scaffolding and providing appropriate feedback are important for maintaining the sort of interaction that leads to learning.

(a) *Questioning*

Appropriate questioning practices not only elicit students' thinking but also enhance their understanding. The questions the teacher asks can take various forms. Sometimes they may be simple, lower-order questions, for which students may have an immediate answer, to check whether they have learned factual information. At other times the teacher has to ask more open questions to which there may not be any one or simple answer. For example, it is helpful for the teacher to ask: "Can you explain how you got this answer?" or "Could you make use of a diagram to help with your explanation?" Instead of merely giving correct answers, the teacher has to be a good listener and give students sufficient time and opportunities to build up their understanding of mathematics through contextualised problems that make sense to them. This allows the teacher to gather useful information on students' progress in understanding mathematical ideas.

***Tips for questioning in the classroom***

- Pause (giving students time to think) after asking a question.
- Avoid asking questions which require "yes" or "no" answers.
- Follow up student responses by asking "why" or by passing the question to the class or another student for a reaction.
- Limit the use of questions that rely almost completely on memory.
- Avoid directing a question to students for disciplinary reasons.
- Avoid asking questions that contain the answer.
- Do not call on a particular student before asking a question.
- Ask open-ended questions.
- Do not label the degree of difficulty of a question.
- Avoid asking for choral responses from the whole group.

(b) *Scaffolding*

One of the teacher's roles has been conceptualised as "scaffolding". Scaffolding is the support that a teacher provides in helping students learn how to perform a task that they cannot yet manage on their own. In Mathematics, teachers' scaffolding practices can be categorised into three levels. Level 1 scaffolds refer to those prompts and stimuli that already exist in the environment, either as a result of deliberate planning or by chance, and which serve to support students' learning in Mathematics. These may include, for instance, a poster displaying useful formulae, and computer manipulatives such as dynamic geometry software and self-correcting mathematical games. Although the teacher's immediate involvement may be low, the level of support can be high depending on the time and effort he/she has expended in finding out which displays, tasks and materials are available. Level 2 scaffolds include direct interactions between teachers and students focusing on the specific tasks in hand. Practices that require students to be involved more actively in the learning process are also included in scaffolding at this level. The strategies adopted may vary from direct instruction, showing and telling, to more collaborative meaning-making. Level 3 scaffolds aim to make connections between students' prior knowledge and experience and the new mathematics topics to be learned. Students are likely to engage in longer, more meaningful discussions, and meanings are more easily shared when each student engages in the communal act of making mathematical meaning. The two Further Learning Units – "Further Applications" and "Inquiry and Investigation" – aim at providing teachers with a platform for

providing scaffolding at this level.

It is worth noting that the approach of encouraging student discussion is heavily dependent on establishing mutual trust between the teacher and the students. On the one hand, the teacher has to trust the students to persist in attempting to solve mathematical problems so that they can feel free to talk about their solutions and describe how they reached them. On the other hand, the students must trust the teacher to respect their efforts and give them opportunities to discuss their understanding and explain their attempts to solve the problems.

***Criteria for choosing a mathematical problem***

A good mathematical problem:

- has important, useful mathematics embedded in it;
- can be approached in several ways, using different solution strategies;
- has various solutions or allows different decisions or positions to be taken and defended;
- encourages student engagement and discourse;
- requires higher-order thinking and problem-solving skills;
- contributes to students' conceptual development;
- has connections with other important mathematical ideas;
- promotes the skilful use of mathematics;
- provides an opportunity to practise important skills; and
- creates an opportunity for the teacher to assess what his/her students are learning and where they are experiencing difficulties.

## 4.5 Learning Community

As noted earlier, social interaction in the classroom plays an increasingly important role for students learning mathematics. Interaction between the teacher and students, and among the students, influences what is learned and how it is learned. It is important for the teacher to cultivate a "classroom culture" in which helping one's peers to learn is not a marginal activity, but a central concern. Once such a culture has been established, opportunities for learning arise (which are not usually present in traditional classrooms) as students collaborate to solve problems. Thus, building a "learning community" provides a new way of seeing and managing the classroom to

improve the quality of learning and teaching. Presentations to the class after project work or reading tasks can be exciting and challenging, and giving presentations enables students to connect knowledge, skills, values and attitudes when constructing their mathematical knowledge.

In this type of classroom setting, students learn a lot more than mathematical content. They develop beliefs about mathematics and about their own and the teacher's role. A sense of what is valued is also developed, along with attitudes and forms of motivation. This approach is designed to foster the belief that persisting in figuring out personally challenging problems is valued more than a page of correct answers, and that cooperation and negotiation have greater value than competition and conflict. Students are encouraged to talk about their methods of solution without their correctness being evaluated. As a result, students share a sense of participation in a learning community, in which, collectively, they are more willing and able to engage in activities and accept the responsibility to co-construct and judge knowledge and understanding within the class.

## 4.6 Catering for Learner Diversity

Attention should not be focused only on academically lower achievers or those who are more capable in mathematics. The needs of all students, whatever their ability level, are equally important.

Measures to cater for learner diversity at the curriculum level and the school level have been described in Chapters 2 and 3 respectively. In daily classroom teaching, teachers can cater for learner differences by providing students with different assignments or activities graded according to levels of difficulty, so that students work on exercises that match their progress in learning. For less able students, assignments may be relatively basic and straightforward in nature; and for more able students, assignments can be more open-ended and challenging enough to cultivate and sustain their interest in learning. Alternatively, teachers can provide students with the same exercises, but with variations in the amount and style of support given – for example, by providing more clues, or by breaking a complicated problem into several simpler but connected parts for students with weaker performance.

In interacting with the whole class, teachers can make adjustments to suit the needs of students. For example, they can ask less able students simple questions (e.g. If this

polynomial is divided by $x - 3$, what value of $x$ should be put into the polynomial to obtain the remainder?) and reserve more challenging questions for the more able ones (e.g. If you are not allowed to use long division, how can you find the remainder when this polynomial is divided by $x^2 - 3x + 2$?).

Besides whole-class teaching, teachers can consider different grouping strategies to cater for the needs of students and enhance peer-to-peer interaction. Students can also be encouraged to express themselves openly and share their work in groups to build up their self-confidence and knowledge.

Information technology provides a viable means for teachers to cater for learner differences. For some topics, such as graphs of trigonometric functions, curve sketching and definite integrals, the use of appropriate software packages can promote vivid and rigorous learning and teaching, as well as enhancing students' interest in the subject. IT tools can enhance learning for able students by letting them extend their horizons in mathematics beyond their current practice; and they can allow less able students to focus more on strategies for tackling problems, or on drawing conclusions from a large set of data from daily-life contexts, without going through tedious computations and mathematical procedures.

## 4.7 Use of Information Technology (IT) in Learning and Teaching

Technology has had a fundamental impact on the learning and teaching of mathematics. The memorisation of static facts and the drilling of skills in mathematics are not as important as in the past. Mathematics is now generally considered to be primarily a search for patterns and relationships. When students are exploring regularities, looking for recurring themes and making conjectures, they are doing mathematics.

Commonly used IT tools in the learning and teaching of mathematics include computers, hand-held technologies (such as graphing calculators) and mathematics-oriented software packages. These tools have the potential to support students in their learning and help them construct knowledge in such a way that they gradually become independent and confident in demonstrating the required skills and understanding. Powerful mathematics-related software packages can also help to engage students who at first do not like Mathematics by letting them experience the excitement and enjoyment of learning the subject.

Students in Hong Kong have generally been equipped with basic IT skills in their junior secondary schooling. Teachers can make good use of their proficiency in technology to enhance the learning and teaching of Mathematics by:

- extending the experience of learning mathematics, and encouraging active student participation in exploratory and investigatory activities;
- supporting, supplementing and extending learning and teaching activities such as exercises and tutorials, charting, graphical analysis, simulation, modelling, information retrieval and data processing;
- developing new teaching strategies and practices in the classroom, such as providing students with an interactive learning environment for contextual and situational learning; and
- contributing to students' self-regulated learning with the help of computer-assisted learning packages and the resources available on the Internet, and extending students' learning outside the classroom.

It is nevertheless necessary to make appropriate decisions on when to use technology (e.g. to know when to use a graphing software or to work by hand to sketch the graph of a rational function at different learning and teaching stages). Teachers have to decide when to use computers or hand-held devices to demonstrate a mathematical concept (e.g. to explore the features of the graphs of quadratic functions using an interactive approach).

In some cases, IT tools may provide shortcuts which are undesirable, especially in a learning process in which mental development is required for concept-building. Exaggerated use of such tools may lead to a de-emphasis on skills. It should be stressed, therefore, that Mathematics should be taught in its own right, with its own educational objectives, for the information age.

# Chapter 5 Assessment

This chapter discusses the roles of assessment in learning and teaching Mathematics, the principles that should guide assessment of the subject and the need for both formative and summative assessment. It also provides guidance on internal assessment and details of the public assessment of Mathematics. Finally, information is given on how standards are established and maintained and how results are reported with reference to these standards. General guidance on assessment can be found in the *Senior Secondary Curriculum Guide* (SSCG) (CDC, 2009).

## 5.1 The Roles of Assessment

Assessment is the practice of collecting evidence of student learning. It is a vital and integral part of classroom instruction, and serves several purposes and audiences.

First and foremost, it gives feedback to students, teachers, schools and parents on the effectiveness of teaching and on students' strengths and weaknesses in learning.

Second, it provides information to schools, school systems, government, tertiary institutions and employers to enable them to monitor standards and to facilitate selection decisions.

The most important role of assessment is in promoting learning and monitoring students' progress. However, in the senior secondary years, the more public roles of assessment for certification and selection come to the fore. Inevitably, these imply high-stake uses of assessment since the results are typically used to make critical decisions about individuals.

The Hong Kong Diploma of Secondary Education (HKDSE) provides a common end-of-school credential that gives access to university study, work, and further education and training. It summarises student performance in the four core subjects (including Mathematics) and in various elective subjects, including both discipline-oriented subjects and the new Applied Learning courses. It needs to be interpreted in conjunction with other information about students as shown in the Student Learning Profile.

## 5.2 Formative and Summative Assessment

It is useful to distinguish between the two main purposes of assessment, namely "assessment *for* learning" and "assessment *of* learning".

"Assessment *for* learning" is concerned with obtaining feedback on learning and teaching, and utilising this to make learning more effective and to introduce any necessary changes to teaching strategies. We refer to this kind of assessment as "formative assessment" because it is all about forming or shaping learning and teaching. Formative assessment should take place on a daily basis and typically involves close attention to small "chunks" of learning.

Formative assessment is designed to measure what students know and are learning as they go along, and the information gathered is used as feedback to plan the future learning and teaching activities in which students and teachers are to be engaged. It can be integrated into normal classroom activities and carried out in various ways including observation of students' performance during lessons, class discussion, oral presentations and project work, and examining the written work of students done in class or at home.

"Assessment *of* learning" is concerned with determining progress in learning, and is referred to as "summative" assessment, because it is all about summarising how much learning has taken place. Summative assessment is normally undertaken at the conclusion of a significant period of instruction (e.g. at the end of the year, or of a key stage of schooling) and reviews much larger "chunks" of learning.

In practice, a sharp distinction cannot always be made between formative and summative assessment, because the same assessment can in some circumstances serve both formative and summative purposes. Teachers can refer to the SSCG for further discussion of formative and summative assessment.

Formative assessment should be distinguished from continuous assessment. The former refers to the provision of feedback to improve learning and teaching based on formal or informal assessment of student performance, while the latter refers to the assessment of students' ongoing work, and may involve no provision of feedback that helps to promote better learning and teaching. For example, accumulating results in class tests carried out on a weekly basis, without giving students constructive feedback, may neither be effective formative assessment nor meaningful summative assessment.

There are good educational reasons why formative assessment should be given more attention and accorded a higher status than summative assessment, on which schools tended to place a greater emphasis in the past. There is research evidence on the beneficial effects of formative assessment, when used for refining instructional decision-making in teaching, and generating feedback to improve learning. For this reason, the CDC report *Learning to Learn – The Way Forward in Curriculum Development* (CDC, 2001) recommended that there should be a change in assessment practices, with schools placing due emphasis on formative assessment to make assessment *for* learning an integral part of classroom teaching.

Another distinction to be made is between internal assessment and public assessment. Internal assessment refers to the assessment practices that teachers and schools employ as part of the ongoing learning and teaching process. In contrast, public assessment refers to the assessment conducted as part of the assessment processes in place for all schools. On balance, internal assessment should be more formative, whereas public assessment is more summative.

## 5.3 Assessment Objectives

The assessment objectives for Mathematics are closely aligned with the curriculum framework and the broad learning outcomes presented in earlier chapters.

The assessment objectives of the Compulsory Part are to test the candidates':

- knowledge of the mathematical facts, concepts, skills and principles presented in this guide;
- familiarity with and use of mathematical symbols;
- ability to use appropriate mathematical techniques for solving a variety of problems; and
- ability to communicate ideas and to present arguments mathematically.

The assessment objectives of Module 1 (Calculus and Statistics) are to test the candidates':

- understanding of the concepts, principles and methods in Calculus and Statistics presented in this guide; and

- ability to apply appropriate techniques in Calculus and Statistics for solving a variety of problems.

The assessment objectives of Module 2 (Algebra and Calculus) are to test the candidates':

- understanding of the concepts, principles and methods in Algebra and Calculus presented in this guide; and
- ability to apply appropriate techniques in Algebra and Calculus for solving a variety of problems.

## 5.4 Internal Assessment

This section presents the guiding principles that can be used as the basis for designing the internal assessment and some common assessment practices for Mathematics for use in schools. Some of these principles are common to both internal and public assessment.

### 5.4.1 Guiding Principles

Internal assessment practices should be aligned with curriculum planning, teaching progression, student abilities and local school contexts. The information collected will help to motivate, promote and monitor student learning, and will also help teachers to find ways of promoting more effective learning and teaching.

(a) *Alignment with the learning objectives*

A range of assessment practices should be used to assess the achievement of different learning objectives. These include testing candidates' ability to: think critically and creatively; conceptualise, investigate and reason mathematically; use mathematics to formulate and solve problems in real-life as well as in mathematical contexts and other disciplines; and communicate with others and express their views clearly and logically in mathematical language. The weighting given to different areas in assessment should be discussed and agreed among teachers. The assessment purposes and criteria should also be made known to students so that they have a full understanding of what is expected of them.

(b) *Catering for the range of student ability*

Assessment practices incorporating different levels of difficulty and diverse modes should be used to cater for students with different aptitudes and abilities. This helps to ensure that the more able students are challenged to develop their full potential and the less able ones are encouraged to sustain their interest and succeed in learning.

(c) *Tracking progress over time*

As internal assessment should not be a one-off exercise, schools are encouraged to use practices that can track learning progress over time (e.g. portfolios). Assessment practices of this kind allow students to set their own incremental targets and manage their own pace of learning, which will have a positive impact on their commitment to learning.

(d) *Timely and encouraging feedback*

Teachers should provide timely and encouraging feedback through a variety of means, such as constructive verbal comments during classroom activities and written remarks on assignments. Such feedback helps students sustain their momentum in learning, and to identify their strengths and weaknesses.

(e) *Making reference to the school's context*

As learning is more meaningful when the content or process is linked to a setting which is familiar to students, schools are encouraged to design some assessment tasks that make reference to the school's own context (e.g. its location, relationship with the community, and mission).

(f) *Making reference to current progress in student learning*

Internal assessment tasks should be designed with reference to students' current progress, as this helps to overcome obstacles that may have a cumulative negative impact on learning. Teachers should be mindful in particular of concepts and skills which form the basis for further development in learning.

(g) *Feedback from peers and from the students themselves*

In addition to giving feedback, teachers should also provide opportunities for peer assessment and self-assessment in student learning. The former enables students to learn among themselves, and the latter promotes reflective thinking which is vital for students' lifelong learning.

(h) *Appropriate use of assessment information to provide feedback*

Internal assessment provides a rich source of data for providing evidence-based feedback on learning in a formative manner.

### 5.4.2 Internal Assessment Practices

A range of assessment practices suited to Mathematics, such as tests, examinations, homework assignments, oral questioning, projects and exploratory tasks can be used to promote the attainment of the various learning outcomes. However, teachers should note that these practices should be an integral part of learning and teaching, not "add-on" activities.

Among the most widely used methods for internal assessment are tests, examinations and homework assignments:

***Tests*** can be used for:

- determining what students have mastered and whether they are ready to proceed to the next teaching unit; and
- providing information to teachers so that they can make adjustments in their teaching.

***Examinations*** can be used for:

- deciding whether students have progressed satisfactorily over a school term; and
- providing information about students' learning to other schools, educational institutions and employers.

***Homework assignments*** can help:

- students to consolidate concepts in mathematics; and
- teachers to assess the performance of their students.

It is important to ensure that the number of homework assignments given is not excessive and that they are at a suitable level of difficulty and related appropriately to specific objectives. Also, they should not be confined to routine mathematical problems. When marking homework assignments, specific, clear, constructive and supportive comments, and suggestions for improvement, should be given as this helps students to identify their strengths and weaknesses and to know what is required for improvement.

Other possible practices are:

***Oral questioning***

Oral questioning need not be seen as a test of spoken language only – it can be helpful in other subjects also. It is a flexible approach which allows teachers to discuss matters in depth with able students, to tease out the meaning of obscure statements, and to find out the reasons for conclusions. Teachers are encouraged to try using oral assessment as it can be a valuable supplement to conventional assessment methods.

***Projects***

A project is any piece of assigned or mutually agreed work from which the constraints of lesson time have been largely removed. Asking students to carry out project work provides them with an opportunity to study a topic of interest in depth. Teachers may wish to draw the following steps in the process to their students' attention:

- Clarify the areas of interest
- Establish a framework for enquiry
- Find and select resource materials
- Organise data
- Present findings

***Exploratory tasks***

Exploratory tasks can be very useful in learning and teaching mathematics as a way of monitoring students' investigative abilities, higher-order skills and achievements on a continuing basis, and the scores on the tasks can be used to form part of the record of student progress. The use of appropriate tasks which are aligned with learning objectives can help to reduce the pressure of summative assessment; and the results on the tasks can also reflect the effectiveness of teaching and so lead teachers to make reasonable adjustments to their teaching strategies.

## 5.5 Public Assessment

### 5.5.1 Guiding Principles

Some principles guiding public assessment are outlined below for teachers' reference.

(a) *Alignment with the curriculum*

The outcomes that are assessed and examined through the HKDSE should be aligned with the aims, objectives and intended learning outcomes of the senior secondary curriculum.

(b) *Fairness, objectivity and reliability*

Students should be assessed in ways that are fair and are not biased against particular groups. A characteristic of fair assessment is that it is objective and under the control of an independent examining authority that is impartial and open to public scrutiny. Fairness also implies that assessments provide a reliable measure of each student's performance in a given subject, so that, if they were to be repeated, very similar results would be obtained.

(c) *Inclusiveness*

The HKDSE examinations need to accommodate the full spectrum of student aptitude and ability. The written papers in the public examination will contain multiple-choice and short questions which test candidates' basic knowledge of mathematics and long questions testing higher-order thinking skills.

(d) *Standards-referencing*

The reporting system is "standards-referenced", i.e. student performance is matched against standards which indicate what students have to know and be able to do to merit a certain level of performance. For Mathematics, a set of written descriptors has been developed to provide information on the typical performance of candidates at each level.

(e) *Informativeness*

The HKDSE qualification and the associated assessment and examinations system provide useful information to all parties. First, it provides feedback to students on their performance and to teachers and schools on the quality of the teaching provided. Second, it communicates to parents, tertiary institutions, employers and the public at large what it is that students know and are able to do, in terms of how their performance matches the standards. Third, it facilitates selection decisions that are fair and defensible.

### 5.5.2 Assessment Design

Table 5.1 below outlines the assessment design of the Compulsory Part, Module 1 and Module 2 with effect from the 2016 HKDSE Examination. The assessment design is subject to continual refinement in the light of feedback from live examinations. Full details are provided in the Regulations and Assessment Frameworks for the year of the examination and other supplementary documents, which are available on the HKEAA website (www.hkeaa.edu.hk/en/hkdse/assessment/assessment_framework/).

**Table 5.1** An outline of the assessment design

***Compulsory Part***

| Component | | | Weighting | Duration |
|---|---|---|---|---|
| Public examination | Paper 1 | Conventional questions | 65% | 2¼ hours |
| | Paper 2 | Multiple-choice questions | 35% | 1¼ hours |

***Module 1 (Calculus and Statistics)***

| Component | | Weighting | Duration |
|---|---|---|---|
| Public examination | Conventional questions | 100% | 2½ hours |

***Module 2 (Algebra and Calculus)***

| Component | | Weighting | Duration |
|---|---|---|---|
| Public examination | Conventional questions | 100% | 2½ hours |

### 5.5.3 Public Examinations

Various kinds of items, including multiple-choice questions, short questions and long questions, are used to assess students' performance in a broad range of skills and abilities. Multiple-choice questions permit a more comprehensive coverage of the curriculum; short questions can be used to test basic knowledge and concepts. Longer questions aim to test candidates' higher-order skills. Schools may refer to the live examination papers regarding the format of the examination and the standards at which the questions are pitched.

The content to be examined is based on the learning objectives outlined in Chapter 2 of this guide. For both the Compulsory and Extended Parts, knowledge of the subject matter in the Foundation and Non-Foundation Parts of the Mathematics Curriculum (S1 – 3) is assumed, and, for the Extended Part, knowledge of the subject matter in the Compulsory Part is also assumed.

### 5.5.4 Standards and Reporting of Results

Standards-referenced reporting is adopted for the HKDSE. What this means is that candidates' levels of performance are reported with reference to a set of standards as defined by cut scores on the mark scale for a given subject. Standards referencing relates to the way in which results are reported and does not involve any changes in how teachers or examiners mark student work. The set of standards for a given subject can be represented diagrammatically as shown in Figure 5.1.

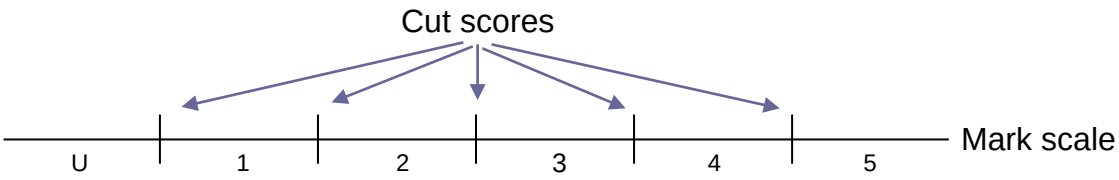



**Figure 5.1** Defining levels of performance via cut scores on the mark scale for a given subject

Within the context of the HKDSE there are five cut scores, which are used to distinguish five levels of performance (1–5), with 5 being the highest. A performance below the cut score for Level 1 is labelled as "Unclassified" (U).

For each of the five levels, a set of written descriptors has been developed to describe what the typical candidate performing at this level is able to do. The principle behind these descriptors is that they describe what typical candidates *can* do, not what they *cannot* do. In other words, they describe performance in positive rather than negative terms. These descriptors represent "on-average" statements and may not apply precisely to individuals, whose performance within a subject may be variable and span two or more levels. Samples of students' work at various levels of attainment are provided to illustrate the standards expected of them. These samples, when used together with the level descriptors, will help to clarify the standards expected at the various levels of attainment.

In setting standards for the HKDSE, Levels 4 and 5 are set with reference to the standards achieved by students awarded grades A–D in the HKALE. It needs to be stressed, however, that the intention is that the standards will remain constant over time – not the percentages awarded different levels, as these are free to vary in line with variations in overall student performance. Referencing Levels 4 and 5 to the standards associated with the old grades A–D is important for ensuring a degree of continuity with past practice, for facilitating tertiary selection and for maintaining international recognition. Secure monitoring tests are used to ensure maintenance of standards over time.

To provide finer discrimination for selection purposes, the Level 5 candidates with the best performance have their results annotated with the symbols ** and the next top group with the symbol *. The HKDSE certificate itself records the level awarded to each candidate. The levels awarded to candidates in the Extended Part will be reported separately from the Compulsory Part.

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# Chapter 6 Learning and Teaching Resources

This chapter discusses the importance of selecting and making effective use of learning and teaching resources, including textbooks, to enhance student learning. Schools need to select, adapt and, where appropriate, develop the relevant resources to support student learning.

## 6.1 Purpose and Function of Learning and Teaching Resources

Learning and teaching resources provide students with a basis for enriching their learning experiences. Effective use of resources will help students to: consolidate what they have learned; extend and construct knowledge for themselves; and develop the learning strategies, generic skills, values and attitudes they need – and thus lay a solid foundation for lifelong learning. The following are some examples of learning and teaching resources for Mathematics:

- textbooks and workbooks;
- reference books;
- newspapers/journals/periodicals/advertising leaflets/maps;
- audio-visual materials including ETV programmes;
- learning and teaching packages;
- software packages;
- resources on the Internet;
- community resources; and
- instruments and equipment for drawing shapes and making models.

All of these resources should be drawn upon to help students learn and broaden their learning experiences. The available resources need to be adapted to meet the different needs and abilities of students. In addition, where necessary, teachers are advised to develop their own learning and teaching materials for this purpose.

## 6.2 Guiding Principles

The basic considerations when selecting learning and teaching resources are as follows:

- They should be in line with the curriculum aims and contain core elements included in it.
- They should take students' prior knowledge and experience into consideration.
- They should present knowledge, concepts and ideas in an accurate and effective way.
- They should arouse students' interest and engage them actively in learning.
- They should be written in simple language to convey information clearly.
- They should provide access to knowledge, as well as scaffolds, to help students progress in their learning.
- They should cater for students' individual differences by providing a variety of learning activities at different levels of difficulty.
- Learning resources used in addition to textbooks should promote independent learning by complementing and extending what students have learned in class.
- They should facilitate discussion and further inquiry.

## 6.3 Types of Resources

### 6.3.1 Textbooks

In Hong Kong, textbooks are the main resources that Mathematics teachers use in deciding what and how to teach. Most of the instructional time is related to them in some way. Very often, they are the only resources to which all students have access during lessons, other than the teacher. Most problems for students' exercises and homework are taken from them. Textbooks, therefore, need to be selected with great care.

There is no single textbook which can suit the learning needs of all students in Hong Kong. In choosing textbooks, schools need to take into account the mathematical abilities, needs and interests of their students, as well as the quality of the books. In evaluating their suitability, teachers' comments on the extent to which the textbooks will need to be adapted should also be taken into consideration. It is also advisable to consult the Textbook Information and Recommended Textbook List provided on the EDB's homepage when selecting textbooks for the Mathematics Curriculum (S4 – 6).

Noted below in Table 6.1 is a list of the key characteristics of good Mathematics

textbooks.

**Table 6.1** The main features of good Mathematics textbooks

**Good Mathematics textbooks**

A good Mathematics textbook should:

- be in line with the curriculum aims and targets of the Mathematics Curriculum (S4 – 6);
- help students develop the mathematical concepts, thinking abilities, generic skills, values and attitudes promoted in the curriculum;
- stress the importance of the learning process as much as finding the correct answers;
- provide opportunities for students to investigate tasks, discuss findings, make conjectures, test hypotheses and defend their solutions;
- provide learning activities and task-oriented problems at the students' level of understanding to engage them in exploratory work and encourage high level thinking;
- provide adequate examples and illustrations to help students understand the mathematical concepts and skills;
- introduce concepts and skills logically in clear language;
- allow flexibility for teachers in selecting materials that engage students in solving problems in real world;
- be written in the language that is simple, clear, easy to understand and appropriate to students' level; and
- be accurate in content and the use of mathematical terms and symbols.

Many textbooks contain some excellent ideas. However, they are limited by the linear presentation of ideas in print form, and so need to be adapted, as suggested in Table 6.2.

**Table 6.2** Ways of using textbooks

| **Points to note to get the best out of Mathematics textbooks** |
|---|
| Teachers should:<br><br>• decide on the content and sequence of teaching based on their students' prior knowledge and abilities and their own view of progression – and not be bound by a textbook's content or sequence. This will involve them in exercising their professional judgment on whether all the materials in a textbook should be covered for their students;<br>• make use of the activities and tasks provided in textbooks, so that students have opportunities to gain first-hand experience in exploring mathematics, as this can enhance their interest in the subject. However, once again, adjustments should be made to cope with the needs, interests and abilities of their students;<br>• select suitable problems for classwork and homework. Students may be informed clearly about the difficulty levels of the problems assigned;<br>• provide students with relevant supplementary materials if there are topics that are not fully explained; and<br>• make use of any supporting resources provided (e.g. websites and computing programs) to enhance learning and teaching. |

### 6.3.2 Reference Books

Reference books are valuable resources for both teachers and students. It is excellent if schools can afford to have a good collection of such books, but teachers and students can also make use of the resources in the community, for example the resources in the public libraries. The principles for selecting Mathematics reference books are much the same as for textbooks, but it is still worthwhile taking the following points about them into account:

- They are for complementing or further extending the content of textbooks, to enable students to make connections and gain a better understanding.

- The main concepts presented in them should be the same as those teachers expect students to learn.
- They should be challenging, but not frustrating or discouraging. They should promote students' interest and active involvement in learning.
- They should be used to encourage students to define problems or issues, conduct research, and draw conclusions that reflect their learning.

### 6.3.3 The Internet and Technologies

The emergence of the Internet has had a great impact on the learning and teaching of many subjects, including Mathematics. It is an invaluable source of information, and very often there are learning and teaching materials which can be downloaded for use. For example, there is freeware, such as *GeoGebra*, which requires no payment, and there are also trial versions of software packages, such as *Geometer's Sketchpad*, which allow free trials for a short period. The Internet also includes materials for self-directed learning which engage learners actively with mathematical ideas in an interactive environment. In addition, there are graphical and interactive tools through which students can acquire learning experiences that are very difficult to provide by traditional means.

In contrast to school libraries, whose collections cannot be accessed easily at home, the Internet provides many fascinating multimedia elements and hyperlinks that can be easily accessed. Moreover, it provides users with a variety of references such as articles and electronic journals. However, teachers must be aware of possible ethical, behavioural and privacy issues in using the Internet, and students should be alerted to the fact that not all information on the Internet is reliable.

The Internet also offers platforms for discussions. For instance, there are helpful mathematics websites, such as "Ask Dr Math" and "Ask NRICH", which allow students to discuss and ask questions freely. A list of the addresses of relevant websites for the learning and teaching of mathematics is available at

http://www.edb.gov.hk/en/curriculum-development/kla/ma/link/index.html

Technology has not only vastly increased the amount of information available but has also dramatically changed the way that Mathematics is taught and learned. Teachers need to adopt new teaching approaches and use technologies strategically to promote learning and teaching. Technology also allows students to engage more easily in

exploration, make generalisations and take a more active role in the learning process. For example, when they explore the concept of functions, they may use graphing calculators or computer algebra systems to graph functions easily and quickly, thus leaving more time for them to explore suitable mathematical models to explain real-life phenomena or make discoveries and conjectures.

While technology has great potential for improving learning, there is also the possibility of wasting a great deal of time and resources. Teachers and students must ensure that they do not become immersed in technical details, forgetting the aims of learning and teaching mathematics. It should also be borne in mind that students vary in the extent to which they are comfortable in learning through IT.

### 6.3.4 Community Resources

Materials such as advertising leaflets, statistical reports and articles in the press can supply up-to-date information of interest to students. Various associations or organisations in the community also provide valuable seminars and forums to familiarise teachers and students with current issues in mathematics, e.g.

- The Hong Kong Association for Mathematics Education
  http://www.hkame.org.hk/

- The Hong Kong Association for Science and Mathematics Education
  http://www.hkasme.org/

- The Hong Kong Mathematical Society
  http://www.hkms.org.hk/

- The Hong Kong Statistical Society
  http://www.hkss.org.hk/

## 6.4 Use of Learning and Teaching Resources

With the greater emphasis on inquiry learning in mathematics education, teachers have to make use of a wide range of learning resources: textbooks; concrete materials such as blocks and geometric models; audio-visual materials; written resources such

as magazines and journals; and IT resources such as dynamic geometry software and computer algebra systems. These resources must be used flexibly. However, it is important to be aware of the availability and limitations of different types of resources, and note that the ultimate goal of using them is to improve learning and teaching of mathematics.

## 6.5 Resource Management

Proper use of resources can make learning and teaching more interesting and effective. It is advisable, therefore, to build up a suitable stock of resources, including teaching aids, reference books/materials, audio-visual aids and computer software packages.

If space is available, new resources and learning packages should be displayed for teachers' information and perusal. An up-to-date list of the teaching aids, audio-visual aids and computer software available in the school should be easily accessible for teachers, for example by putting it on the school intranet.

Teachers should be encouraged to use resources which have been newly acquired or developed; and they should be invited to make suggestions for procurement. Small workshops, demonstrations or experience-sharing sessions can be conducted in school to provide teachers with a better understanding of the resources available. It is also helpful and cost-effective if teachers regularly share what they have developed (e.g. supplementary materials, illustrations, graphs, mathematical diagrams, as well as learning and teaching activities and strategies).

In implementing the senior secondary curriculum, schools may use resources flexibly to cater for their needs. Schools are advised to refer to the relevant circulars issued by the EDB from time to time.

To help schools in managing curriculum change, the EDB has produced a curriculum resources directory at

http://www.edb.gov.hk/cr

which offers a central pool of ready-to-use learning and teaching resources and useful references developed by the EDB and other parties.

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Appendix 1

# Reference Books for Learning and Teaching

## Reference Books for Teachers


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# Useful Websites

(For an updated list of websites, you may visit the homepage of the Mathematics Education Section at “http://www.edb.gov.hk/cd/maths”)

## A. Learning and Teaching

(1) All Elementary Mathematics- The Mathematical Web High School
http://www.bymath.com/stuff/aboutus.html

(2) Ask Dr. Math
http://forum.swarthmore.edu/dr.math/dr-math.html

(3) Association of Teachers of Mathematics
http://www.atm.org.uk/

(4) Centre for Innovation in Mathematics Teaching
http://www.cimt.plymouth.ac.uk/

(5) Centre for Teaching Mathematics, University of Plymouth
http://www.tech.plym.ac.uk/research/mathematics_education/

(6) EDB Mathematics Education Website
http://www.edb.gov.hk/cd/maths

(7) Explorelearning
http://www.exploremath.com/index.cfm

(8) HK Association for Mathematics Education
http://www.hkame.org.hk/

(9) HK Association for Science and Mathematics Education
http://www.hkasme.org/

(10) Java Applets on Mathematics
http://www.walter-fendt.de/m14e/index.html

(11) Manipula Math with Java
http://www.ies-math.com/math/java/

(12) Math in Daily Life
http://www.learner.org/exhibits/dailymath/

(13) Mathematical Association of America
http://www.maa.org/

(14) Mathematics Lessons that are fun! fun! fun!
http://math.rice.edu/~lanius/Lessons/

(15) Maths online
http://www.univie.ac.at/future.media/moe/galerie.html

(16) MathsNet
http://www.mathsnet.net/

(17) MSTE Online Resource Catalog
http://www.mste.uiuc.edu/resources.php

(18) National Council of Teachers of Mathematics
http://www.nctm.org/

(19) Open-ended Assessment in Mathematics
http://www.heinemann.com/math/register.cfm

(20) Project Interactivate
http://www.shodor.org/interactivate/

(21) Schools of California Online Resources for Education (SCORE) – Mathematics
http://www.score.k12.ca.us/

(22) Shapescape
http://www.shapescape.com/

(23) Support Measure for the Exceptionally Gifted Students
http://www.edb.gov.hk/cd/ge

(24) Teaching Ideas
http://www.teachingideas.co.uk/maths/contents.htm

(25) The Math Forum@Drexel
http://mathforum.org/t2t/faq/gail/index.html

(26) 初中數學網
http://www.czsx.com.cn/

(27) 母語教學支援中心 — 數學
http://www.cmi.hku.hk/Teaching/math.html

(28) 高中數學網
http://www.pep.com.cn/gzsx/

## B. Recreational Mathematics

(29) Interactive Mathematics Miscellany and Puzzles
http://www.cut-the-knot.org/

(30) Living Mathematics
http://sunsite.ubc.ca/LivingMathematics/

(31) Mathematical Excalibur
http://www.math.ust.hk/mathematical_excalibur/

(32) Mathematical Stamp Collecting
http://www.math.wfu.edu/~kuz/Stamps/stamppage.htm

(33) Mathpuzzle
http://mathpuzzle.com/

(34) NRICH Enriching Mathematics
http://nrich.maths.org/

(35) Origami and Mathematics
http://www.paperfolding.com/math/

(36) Probability Games
http://www.betweenwaters.com/probab/probab.html

(37) Online Integrator
http://integrals.wolfram.com/

(38) The National Math Trail
http://www.nationalmathtrail.org/

## C. Statistical Data

(39) Agriculture, Fisheries and Conservation Department - Country & Marine Parks – Useful Statistics
http://www.afcd.gov.hk/English/country/cou_lea/cou_lea_use/cou_lea_use.html

(40) Business-Stat Online
http://bso.hktdc.com/bso/jsp/bso_home.jsp

(41) Census & Statistics Department
http://www.censtatd.gov.hk

(42) Environmental Protection Department
http://www.epd.gov.hk/epd/tc_chi/environmentinhk/waste/data/waste_data.html

(43) European Commission: Eurostat
http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/

(44) Food and Agriculture Organization of the United Nations (FAO)
http://www.fao.org/waicent/portal/statistics_en.asp

(45) Hong Kong International Airport-International Air Traffic Statistics at HKIA
http://www.hongkongairport.com/chi/aboutus/statistics.html

(46) Hong Kong Statistical Society
http:/www.hkss.org.hk/

(47) Macau - Statistics and Census Service
http://www.dsec.gov.mo/c_index.html

(48) Narcotics Division, Security Bureau
http://www.nd.gov.hk/tc/statistics_list.htm

(49) Organisation for Economic Co-operation and Development (OECD)
http://www.oecd.org/

(50) Singapore Department of Statistics
http://www.singstat.gov.sg/

(51) Statistics Glossary
http://www.statsoft.com/textbook/glosfra.html

(52) The Land Registry
http://www.landreg.gov.hk/tc/home/index.htm

(53) The World Bank Group
http://data.worldbank.org/

(54) United Nations Development Programme (UNDP)
http://www.undp.org/content/undp/en/home/operations/procurement/statistics.html

(55) United Nations Economic and Social Commission for Asia and the Pacific (ESCAP) -- Statistics Division
http://www.unescap.org/stat/data/index.asp

(56) United Nations Economic Commission for Europe (UNECE) -- Statistical Division
http://www.unece.org/stats/stats_h.html

(57) United Nations Educational, Scientific and Cultural Organization (UNESCO) Institute of Statistics
http://stats.uis.unesco.org/ReportFolders/ReportFolders.aspx

(58) United Nations Statistics Division (UNSD) Statistical Databases
http://unstats.un.org/unsd/databases.htm

(59) United States Census Bureau
http://www.census.gov/main/www/access.html

(60) World Health Organization (WHO) Statistical information System
http://www3.who.int/whosis/mort/text/download.cfm?path=whosis,mort,mort_download&language=english

## D. Dictionaries and Glossaries

(61) A Dictionary of Measures, Units and Conversions
http://unafold.math.rpi.edu/dictunit.html

(62) Interactive Mathematics Dictionary
http://intermath.coe.uga.edu/dictnary/homepg.asp

(63) Math Glossary, Math Terms
http://www.cut-the-knot.com/glossary/atop.shtml

(64) Mathematical Quotations Server
http://math.furman.edu/~mwoodard/mquot.html

(65) The Encyclopedia of Polyhedra
http://www.georgehart.com/virtual-polyhedra/vp.html

(66) The Internet Glossary of Statistical Terms
http://www.animatedsoftware.com/statglos/statglos.htm

(67) Visual Dictionary of Special Plane Curves
http://www.xahlee.org/SpecialPlaneCurves_dir/specialPlaneCurves.html

(68) Wikipedia - Mathematics
http://www.wikipedia.org/wiki/Mathematics

(69) Wolfram MathWorld
http://mathworld.wolfram.com/

(70) 數學科詞彙表
http://www.cmi.hku.hk/Ref/Glossary/Mat/k.htm

## E. History of Mathematics

(71) Chronological List of Mathematicians
http://aleph0.clarku.edu/~djoyce/mathhist/chronology.html

(72) Mathematicians who were born or died today
http://www-history.mcs.st-and.ac.uk/~history/Day_files/Now.html

(73) The MacTutor History of Mathematics archive
http://www-gap.dcs.st-and.ac.uk/~history/

(74) 中國古代數學
http://www.chiculture.net/0803/html/index.html

## F. Software Packages

(75) Cabri Geometry
www.cabri.com

(76) Geometer's Sketchpad
http://www.keycurriculum.com/products/sketchpad

(77) Gnuplot
http://www.gnuplot.info/

(78) Math WWW Virtual Library – Mathematics Software
http://www.math.fsu.edu/Science/Software.html

(79) NCTM Illuminations – Resources for Teaching Math
http://illuminations.nctm.org/Default.aspx

(80) Peanut Software (Winplot, Wingeom, Winstats,..)
http://math.exeter.edu/rparris/default.html

(81) Poly
http://www.peda.com/poly/

(82) QuickMath
http://www.quickmath.com/

(83) Scilab
http://www.scilab.org/

## G. Mathematics Competitions

(84)Hang Lung Mathematics Award
http://www.hkedcity.net/article/special/hanglung/news.phtml

(85)Hong Kong Mathematics Olympiad (HKMO)
http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/sa/hkmo-index.html

(86 International Mathematical Olympiad
http://www.imo-official.org/

(87)International Mathematics Olympiad Hong Kong Preliminary Selection Contest
http://ge.hkage.org.hk/en/competitions/detail/858

(88)Mathematics Challenge for Young Australians
http://www.amt.edu.au/mcya.html

(89)Mathematics Project Competition and Mathematics Book Report Competition for Secondary Schools
http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/sa/mpc-mbrc.html

(90)Web Sites with information about Mathematics Competitions
http://www.mathpropress.com/competitions.html

(91)World Class Arena
http://www.worldclassarena.hk/tc/home/home.php

## H. Miscellaneous

(92) American Mathematical Society
http://www.ams.org/

(93) London Mathematical Society
http://www.lms.ac.uk/

(94) Mathematical Database
http://eng.mathdb.org/

(95) Mathematics WWW Virtual Library
http://www.math.fsu.edu/Science/math.html

(96) The Math Forum@Drexel
http://mathforum.org/

(97) Wolfram Research
http://www.wolfram.com/

(98) 數學傳播
http://w3.math.sinica.edu.tw/media/default.jsp

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# Glossary

| Term | Description |
|---|---|
| **Applied Learning (ApL, formerly known as Career-oriented Studies)** | Applied Learning (ApL, formerly known as Career-oriented Studies) is an essential component of the senior secondary curriculum. ApL uses broad professional and vocational fields as the learning platform, developing students' foundation skills, thinking skills, people skills, values & attitudes and career-related competencies, to prepare them for further studies and / or for work as well as for lifelong learning. ApL courses complement 24 NSS subjects, diversifying the senior secondary curriculum. |
| **Assessment objectives** | The outcomes of the curriculum to be assessed in the public assessments. |
| **Biliterate and trilingual** | Capable of reading and writing effectively in Standard Written Chinese, English and to use Cantonese, Putonghua and spoken English. The language education policy of Hong Kong is to enable the Hong Kong students to become biliterate (in written Chinese and English) and trilingual (in Cantonese, Putonghua and spoken English). |
| **Co-construction** | Different from the direct instruction and construction approaches to learning and teaching, the co-construction approach emphasises the class as a community of learners who contribute collectively to the creation of knowledge and the building of criteria for judging such knowledge. |
| **Core subjects** | Subjects recommended for all students to take at senior secondary level: Chinese Language, English Language, Mathematics and Liberal Studies. |

| Term | Description |
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| **Curriculum and Assessment (C&A) Guide** | A guide prepared by the CDC-HKEAA Committee. It embraces curriculum aims / objectives / contents and learning outcomes, and assessment guidelines. |
| **Curriculum interface** | Curriculum interface refers to the interface between the different key stages/educational stages of the school curriculum (including individual subjects), e.g. the interface between Kindergarten and Primary; Primary and Secondary; and Junior Secondary and Senior Secondary. The Hong Kong school curriculum, made up of eight key learning areas (under which specific subjects are categorised), provides a coherent learning framework to enhance students' capabilities for whole-person development through engaging them in the five essential learning experiences and helping them develop the nine generic skills as well as positive values and attitudes. Thus when students move on to senior secondary education, they will already have developed the basic knowledge and skills that the study of various subjects requires. When designing the learning and teaching content and strategies, teachers should build on the knowledge and learning experiences students have gained in the previous key stages. |
| **Elective subjects** | A total of 20 subjects in the proposed new system from which students may choose according to their interests, abilities and aptitudes. |

| Term | Description |
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| **Generic skills** | Generic skills are skills, abilities and attributes which are fundamental in helping students to acquire, construct and apply knowledge. They are developed through the learning and teaching that take place in different subjects or key learning areas, and are transferable to different learning situations. Nine types of generic skills are identified in the Hong Kong school curriculum, i.e. collaboration skills, communication skills, creativity, critical thinking skills, information technology skills, numeracy skills, problem solving skills, self-management skills and study skills. |
| **Hong Kong Diploma of Secondary Education (HKDSE)** | The qualification to be awarded to students after completing the three-year senior secondary curriculum and taking the public assessment. |
| **Internal assessment** | This refers to the assessment activities that are conducted regularly in school to assess students' performance in learning. Internal assessment is an inseparable part of the learning and teaching process, and it aims to make learning more effective. With the information that internal assessment provides, teachers will be able to understand students' progress in learning, provide them with appropriate feedback and make any adjustments to the learning objectives and teaching strategies they deem necessary. |

| Term | Description |
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| **Key Learning Area (KLA)** | Organisation of the school curriculum structured around fundamental concepts of major knowledge domains. It aims at providing a broad, balanced and coherent curriculum for all students in the essential learning experiences. The Hong Kong curriculum has eight KLAs, namely, Chinese Language Education, English Language Education, Mathematics Education, Personal, Social and Humanities Education, Science Education, Technology Education, Arts Education and Physical Education. |
| **Knowledge construction** | This refers to the process of learning in which learners are involved not only in acquiring new knowledge, but also in actively relating it to their prior knowledge and experience so as to create and form their own knowledge. |
| **Learning community** | A learning community refers to a group of people who have shared values and goals, and who work closely together to generate knowledge and create new ways of learning through active participation, collaboration and reflection. Such a learning community may involve not only students and teachers, but also parents and other parties in the community. |
| **Learning differences** | This refers to the gaps in learning that exist in the learning process. Catering for learning differences does not mean rigidly reducing the distance between the learners in terms of progress and development but making full use of their different talents as invaluable resources to facilitate learning and teaching. To cater to learners' varied needs and abilities, it is important that flexibility be built into the learning and teaching process to help them recognise their unique talents and to provide ample opportunities to encourage them to fulfil their potential and strive for achievement. |

| Term | Description |
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| **Learning outcomes** | Learning outcomes refer to what learners should be able to do by the end of a particular stage of learning. Learning outcomes are developed based on the learning targets and objectives of the curriculum for the purpose of evaluating learning effectiveness. Learning outcomes also describe the levels of performance that learners should attain after completing a particular key stage of learning and serve as a tool for promoting learning and teaching. |
| **Learning targets and learning objectives** | • Learning targets set out broadly the knowledge/concepts, skills, values and attitudes that students need to learn and develop.<br>• Learning objectives define specifically what students should know, value and be able to do in each strand of the subject in accordance with the broad subject targets at each key stage of schooling. They are to be used by teachers as a source list for curriculum, lesson and activity planning. |
| **Level descriptors** | A set of written descriptions that describe what the typical candidates performing a certain level is able to do in public assessments. |
| **Other learning experiences** | For whole person development of students, 'Other Learning Experiences' (OLE) is one of the three components that complement the examination subjects and Applied Learning (formerly named as Career-oriented Studies) under the Senior Secondary Curriculum. It includes Moral and Civic Education, Aesthetics Development, Physical Development, Community Service and Career-related Experiences. |
| **Public assessment** | The associated assessment and examination system for the Hong Kong Diploma of Secondary Education. |

| Term | Description |
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| **SBA Moderation Mechanism** | The mechanism adopted by HKEAA to adjust SBA marks submitted by schools to iron out possible differences across schools in marking standards and without affecting the rank order determined by the school. |
| **School-based assessment (SBA)** | Assessments administered in schools as part of the learning and teaching process, with students being assessed by their subject teachers. Marks awarded will count towards students' public assessment results. |
| **School-based curriculum** | Schools and teachers are encouraged to adapt the central curriculum to develop their school-based curriculum to help their students achieve the subject targets and overall aims of education. Measures may include readjusting the learning targets, varying the organisation of contents, adding optional studies and adapting learning, teaching and assessment strategies. A school-based curriculum, hence, is the outcome of a balance between official recommendations and the autonomy of the schools and teachers. |
| **Standards-referenced Reporting** | Candidates' performance in public assessment is reported in terms of levels of performance matched against a set of standards. |
| **Student diversity** | Students are individuals with varied family, social, economic and cultural backgrounds and learning experience. They have different talents, personalities, intelligence and interests. Their learning abilities, interests and styles are, therefore, diverse. |

| Term | Description |
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| **Student learning profile** | It is to provide supplementary information on the secondary school leavers' participation and specialties during senior secondary years, in addition to their academic performance as reported in the Hong Kong Diploma of Secondary Education, including the assessment results for Applied Learning courses, thus giving a fuller picture of the student's whole person development. |
| **Values & attitudes** | Values constitute the foundation of the attitudes and beliefs that influence one's behaviour and way of life. They help form principles underlying human conduct and critical judgment, and are qualities that learners should develop. Some examples of values are rights and responsibilities, commitment, honesty and national identity. Closely associated with values are attitudes. The latter supports motivation and cognitive functioning, and affects one's way of reacting to events or situations. Since both values and attitudes significantly affect the way a student learns, they form an important part of the school curriculum. |

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# Membership of the CDC-HKEAA Committee on Mathematics Education (Senior Secondary)

(From December 2003 to September 2013)

| | | |
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| **Chairperson:** | Mr LAI Chu-kuen | |
| **Members:** | Dr AU Kwok-keung | (from January 2004) |
| | Mr CHAN Chi-hung | (until August 2012) |
| | Prof CHENG Shiu-yuen | |
| | Mr CHIU Kwok-sing | (from January 2013) |
| | Mr CHOY Chung-kau | (from January 2013) |
| | Dr FUNG Chi-yeung | (from January 2004 to August 2005) |
| | Mr FUNG Tak-wah | (from January 2004) |
| | Mr HUNG Chun-may | (from January 2004 to November 2007) |
| | Mr KWOK Ka-keung | |
| | Mr LAI Chi-keung | |
| | Mr LEE Kwok-man | (from January 2013) |
| | Dr LEUNG Yuk-lun Allen | (from January 2013) |
| | Mr LUNG Tak-yee | (from June 2009) |
| | Mr SIU Yuk-cheung | (from January 2004 to August 2008) |
| | Dr TANG Kwok-chun | (until September 2012) |
| | Dr TSING Nam-kiu | (from January 2004) |
| | Mr TSO Wai-chuen | (from January 2013) |
| | Mr WAN Chi-keung | (from December 2010 to May 2012) |
| | Dr WU Ka-ho | |
| | Ms YAN Pui-lung | (until August 2012) |
| | Dr YU Leung-ho | |
| **Ex-officio Members:** | Mr CHU Kan-kong (HKEAA) | (from April 2006 to May 2009) |
| | Mr FONG Chong-sun (HKEAA) | (from June 2009) |
| | Mr LEE Pak-leung (EDB) | (until March 2011) |
| | Mr NG Siu-kai (EDB) | (from April 2011) |
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| **Secretary:** | Mr LEUNG Kwong-shing (EDB) | |

# CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (Compulsory Part)

(From February 2005 to September 2013)

| | | |
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| **Convenor:** | Dr NG Yui-kin | |
| **Members:** | Mr CHAN Sai-hung | (from September 2006) |
| | Mr CHENG Chun-chor, Litwin | (from September 2005) |
| | Dr CHEUNG Siu-hung | |
| | Mr CHU Kan-kong | |
| | Dr FUNG Chi-yeung | (until August 2005) |
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| | Mr SIU Yuk-cheung | (until August 2008) |
| | Ms TANG Mei-yue | |
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| **Recorders:** | Mr LEUNG Chi-kit | (until August 2005) |
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# CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (Module 1)

(From February 2005 to September 2013)

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| **Convenor:** | Mr CHAN Sau-tang | |
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| | Ms CHEUNG Kit-yuk | |
| | Mr CHU Kan-kong | |
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| | Mr POON Wai-keung | |
| | Mr TSO Wai-chuen | |
| | Dr WU Ka-ho | |
| **Recorders:** | Mr CHENG Sze-man | (until August 2005) |
| | Mr KOON Kin-hin | (from September 2005 to August 2006) |
| | Mr CHENG Sze-man | (from September 2006) |

# CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (Module 2)

(From February 2005 to September 2013)

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| | Mr CHONG Hoi-ming | |
| | Mr CHU Kan-kong | |
| | Mr CHU Lap-foo | |
| | Mr KWOK Ka-keung | |
| | Ms LAW Wing-chung | (until August 2005) |
| | Mr LEE Kwok-chu | |
| | Mr LEUNG Kwong-shing | (from September 2005) |
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| **Recorders:** | Ms LAW Wing-chung | (until August 2005) |
| | Mr LEUNG Chi-kit | (from September 2005 to August 2006) |
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# CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (School-based Assessment)

(From February 2005 to September 2013)

| | | |
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| | Ms CHAK May | (from September 2005 to August 2006) |
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(From September 2013 – August 2015)

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| | Dr NG Yui-kin (EDB) | (from January 2014) |

# Membership of the CDC-HKEAA Committee on Mathematics Education

(From September 2015 – August 2017)

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| **Chairperson:** | Mr LAM Ka-yiu |
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