Pattern Lock of Mobile Phones

Key Stage: 4

Strand: Data Handling

Learning Units: Permutation and combination

Objective: Apply mathematical knowledge and coding to solve daily-life problems

Pre–requisite Knowledge:

(i) understand the addition rule and multiplication rule in the counting principle

(ii) understand the concepts of permutation and combination

(iii) recognise reflective and rotational symmetries in 2-D figures

Relationship with other KLA(s) in STEM Education:

(i) The learning element "Programming concepts" within the Information & Communication Technology knowledge context in the Technology Education KLA Curriculum (S1 to 3)

(ii) The module "Basic Programming Concepts" of Information and Communication Technology Curriculum (S4 to 6) of Technology Education KLA.

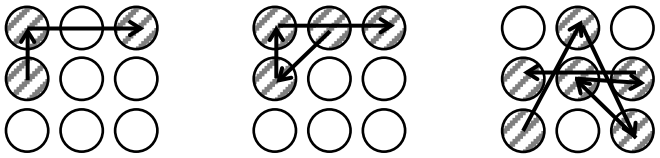
Description of the Activity:

Activity 1

(i) Teacher set a lock pattern on a mobile phone or tablet before the lesson. Then, the teacher asks the students to unlock the pattern lock by trial and error method. Students can try THREE times to unlock by guessing the pattern. If no further information is provided, it seems quite hard for students to unlock by sole guessing.

(ii) The teacher then introduces the basic requirement for setting the lock patterns:

1. Only straight lines can be used to connect the dots.
2. Horizontal, vertical and diagonal lines are allowed.
3. Once a dot is connected, it cannot be connected again.
4. If you connect 2 dots directly, with an unconnected dot in between, it will also be connected.
5. You must connect the dots continuously.



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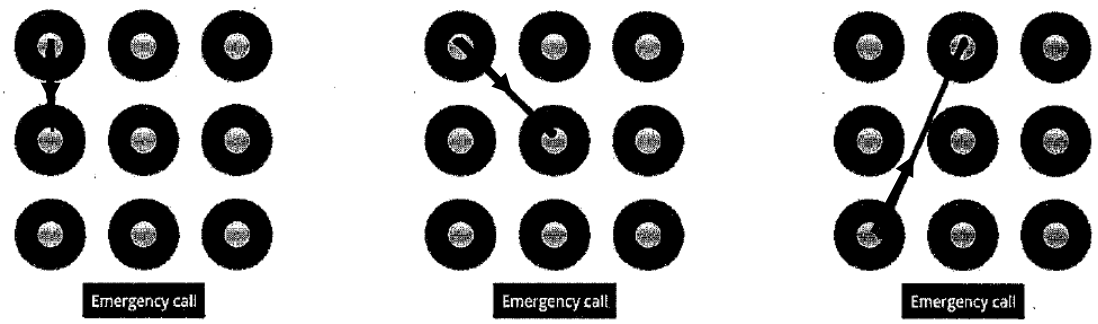
(Note: The teacher may remind students at a later stage that lock patterns used in mobile phones require four or more dots)

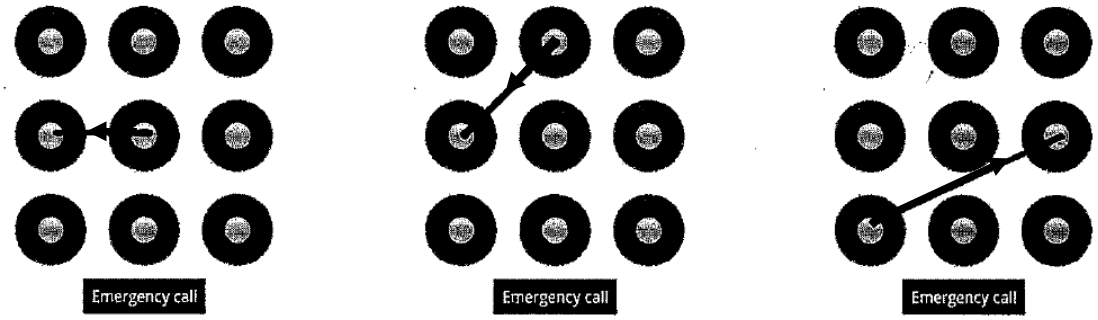
(iii) The teacher then asks the students to calculate the number of the lock patterns they can create for different number of connecting dots. For example, if the lock pattern consists of one dot only, there are 9 combinations on the screen. By trial and error method for at most 9 times, students can unlock the pattern lock. It seems too simple to have only one dot as the lock pattern.

(iv) **Patterns formed by connecting 2 dots**

Students are then asked to connect only two dots by straight lines to set the lock pattern. The order of the two dots does matter. When the two dots' connecting sequence is reversed, the patterns are different. Students can try to list out all the patterns by connecting two dots in the supplementary page.

Some of the patterns are shown below:

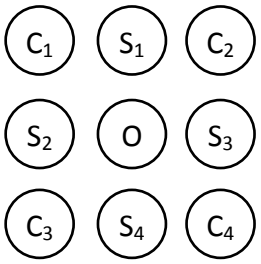




Students are required to find the total number of patterns of two dots.

(v) The teacher may introduce the following labelling of dots to students and ask students to complete the table to facilitate the counting of the number of patterns of two dots.

Firstly, we group the nine points into three categories, four corner dots C_i , one center dot O and four side dots S_i, where i = 1, 2, 3, 4.



Different sequences are listed in the table below for students' completion:

Combination	Sequence	Total number
O , C_i	O–C_i	1 × 4 = 4
	C_i–O	4 × 1 = 4
O , S_i	O–S_i	1 × 4 = 4
	S_i–O	
C_i , S_i	C_i–S_i	
	S_i–C_i	
Si , Sj	Si–Sj	

(vi) The teacher may also introduce another way of counting by considering the total number of permutations of choosing 2 dots from 9 dots and subtract from it the number of connections that do not work (e.g. C_1 to C_2 , S_1 to S_4)

Notes for Teachers:

(i) Counting by analysing different cases:

Combination	Sequence	Total number
O , C_i	O–C_i	$1 \times 4 = 4$
	C_i–O	$4 \times 1 = 4$
O , S_i	O–S_i	$1 \times 4 = 4$
	S_i–O	$4 \times 1 = 4$
C_i , S_i	C_i–S_i	$4 \times 4 = 16$
	S_i–C_i	$4 \times 4 = 16$
Si , Sj	Si–Sj	$4 \times 2 = 8$

Total number of patterns = 4 + 4 + 4 + 4 + 16 + 16 + 8 = 56

(ii) Counting by subtracting invalid cases:

We choose 2 dots from 9 dots without any restriction and order them, we have P_2^9 combinations. However, some connection do not work, for example, we cannot connect C_i–C_j directly. We need to eliminate those patterns. There are 16 cases about that (permutation of the four corners and permutation of opposite sides).

Number of patterns by connecting 2 dots

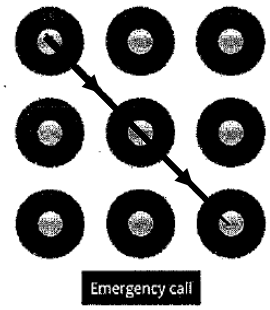
$= 9 \times 8 - (3 \times 4 + 1 \times 4)$

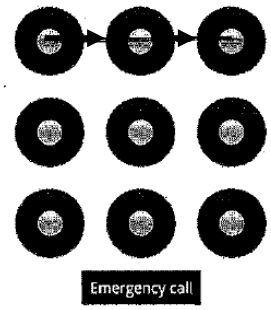
$= 56$

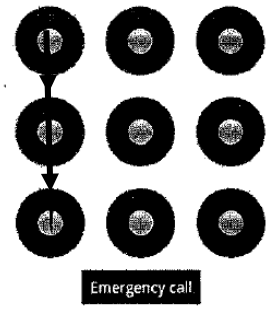
<u>Activity 2</u>

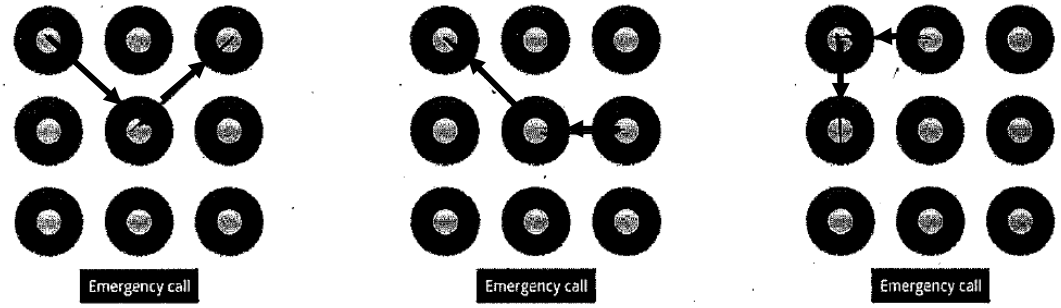
(i) **Patterns formed by connecting 3 or more dots**

Students may try to draw patterns of 3 dots or 4 dots and list them out. The teacher should remind students that unconnected dots cannot be crossed without being connected and there should be no repeated dots. Some of the patterns are as follows:









Students should recognise that there are already many different patterns even when there are only 3 dots.

(ii) **Guessing Patterns**

1. Students are required to form groups of four. Each group needs to design their lock patterns in the Android phone or tablet by connecting **four or five dots**.

2. The teacher randomly select **one** group (Group A) and assign another group (Group B) to guess the pattern of Group A.

3. Group B can raise at most **THREE** "YES" or "NO" questions and require group A to answer them.

   Some example of the questions are listed below:

   - Does your pattern pass through 5 dots?
   - Does your pattern have axes of symmetry?
   - Does your pattern contain a slant edge?
   - Does your pattern contain a cross?
   - Does your pattern have a vertical line?
   - Does your pattern passes through upper right dot?

     ……..

   (Based on students' pre-requisite mathematical knowledge, each group can raise any creative "YES" or "NO" questions)

4. According to the answers from Group A, students of Group B try to guess the pattern at most **TWO** times. They may mark their guessed patterns in the supplementary sheets.

5. Any group who gets the correct answer can receive a gift.

(Note: Teachers can adjust the difficulty of the patterns by setting restrictions or providing more hints. To minimise the complexity and to increase the chance of correct guesses, the number of dots in the lock patterns shall be restricted.)

(iii) **Counting the number of patterns involving 3 dots**

1. The teacher may guide students to analyse different cases by completing the following table

Combination	Sequence	Total number
C_i , C_j , O	O–C_i–C_j C_i–O–C_j	$1 \times 4 \times 1 = 4$ $4 \times 1 \times 3 = 12$
C_i , C_j , S_i	S_i–C_i–C_j C_i–S_i–C_j	$4 \times 2 \times 1 = 8$ $4 \times 4 \times 3 = 48$
C_i , S_i , S_j	C_i–S_i–S_j S_i–C_i–S_j S_i–S_j–C_i	
C_i , S_j , O	S_i–C_j–O C_i–S_i–O C_i–O–S_i S_i–O–C_j …	
S_i , S_j , O		
S_i , S_j , S_k		

2. The teacher may also guide students to consider subtracting the number of wrong connections (those wrong in the 1st connection and wrong in the 2nd connection with the 1st correct) from P_3^9.

Notes for Teachers:

(i) The total number of lock patterns for 3 dots is 320. The methods are as follows:

(a) Counting by analysing different cases:

Combination	Sequence	Total number
C_i , C_j , O	O–C_i–C_j C_i–O–C_j	$1 \times 4 \times 1 = 4$ $4 \times 1 \times 3 = 12$

Combination	Sequence	Total number
C_i , C_j , S_i	S_i–C_i–C_j	$4 \times 2 \times 1 = 8$
	C_i–S_i–C_j	$4 \times 4 \times 3 = 48$
C_i , S_i , S_j	C_i–S_i–S_j	$4 \times 4 \times 2 = 32$
	S_i–C_i–S_j	$4 \times 4 \times 3 = 48$
	S_i–S_j–C_i	$4 \times 2 \times 4 = 32$
C_i , S_j , O	S_i–C_j–O	$4 \times 4 \times 1 = 16$
	C_i–S_i–O	$4 \times 4 \times 1 = 16$
	C_i–O–S_i	$4 \times 1 \times 4 = 16$
	Si–O–Cj	$4 \times 1 \times 4 = 16$
	O–Si–Cj	$1 \times 4 \times 4 = 16$
	O–Ci–Si	$1 \times 4 \times 4 = 16$
S_i , S_j , O	O–Si–Sj	$1 \times 4 \times 3 = 12$
	Si–O–Sj	$4 \times 1 \times 3 = 12$
	Si–Sj–O	$4 \times 2 \times 1 = 8$
S_i , S_j , S_k	Si–Sj–Sk	$4 \times 2 \times 1 = 8$

Total number of patterns = (4+12) + (8+48×2+32×2) + (16×6) + (12×2+8) + 8 = 320

(b) Counting by subtracting invalid cases:

Number of lock patterns by connecting 3 dots

= P_3^9

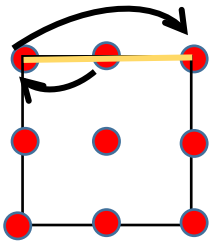
– number of wrong ways in the 1st connection (C to C, S to opposite S)

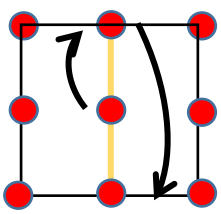
– numbers of wrong ways in the 2nd connection but correct in the 1st connection (1st correct connection and then C to C, 1st correct connection and then S to opposite S, but eliminate those correct straight line patterns of 3 dots: SCC, OSS and OCC)

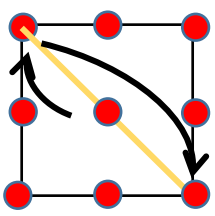
= $9 \times 8 \times 7$
$-(3 \times 4 \times 7 + 1 \times 4 \times 7)$
$-(5 \times 3 \times 4 + 7 \times 1 \times 4 - 4 \times 2 \times 1 - 1 \times 4 \times 1 - 1 \times 4 \times 1)$

= $504 - 112 - 72$

= 320

SCC: 

OSS: 

OCC: 

Extension Activities:

(i) **Coding**

To find the total number of lock patterns used in usual mobile phones, which involve four or more dots, is more difficult as it becomes very complicated to list out all the combinations of lock patterns with four dots or above. Students may write computer programme to complete the task. The idea is to label the dots as

1 2 3

4 5 6

7 8 9

and count all permutation of these 9 numbers of length four, five, six, … up to nine, e.g. 1 2 3 4, 4 7 3 5, 5 1 3 2 4, 2 7 8 9 6 3, …, but cross out those with sequences that are not allowed within (e.g. …1 3 2…, …3 9 7…, …7 3 5…).

The total number of combinations of lock patterns consisting of four dots or above, are recorded as follows:

Number of Dots	Number of Patterns
4	1 624
5	7 152
6	26 016
7	72 912
8	140 704
9	140 704

The total number of lock patterns with four dots or above
= 1624 + 7152 + 26016 + 72912 + 140704 + 140704 = 389 112

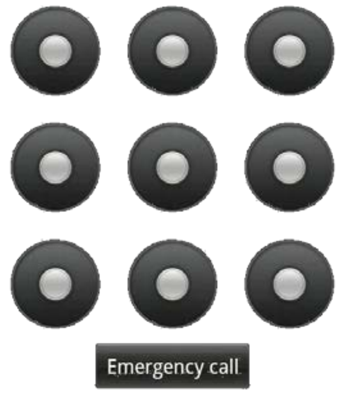
(ii) **Application of statistics**

Lock patterns for mobile phone normally need to involve four dots or more. We may ask the students to design their lock patterns freely under the rules of setting the graphic lock in the mobile phones. Then, we record their patterns of the involvement about different points. According to the research outcomes done by a graduate from Norwegian University of Science and Technology about 4000 lock patterns, 77% of them starts at the four corner points, where 44% starts at the top left-most corner.

Usually, our lock patterns start at the top left-hand corner, moving to the right side and then downwards. We may check students' patterns to compare the results with that of the research.

Reference:

1. https://arstechnica.com/security/2015/08/new-data-uncovers-the-surprising-predictability-of-android-lock-patterns/
2. http://www.guokr.com/article/49408/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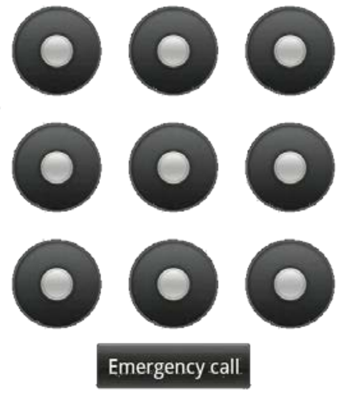
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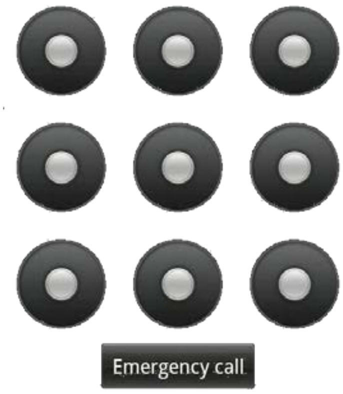
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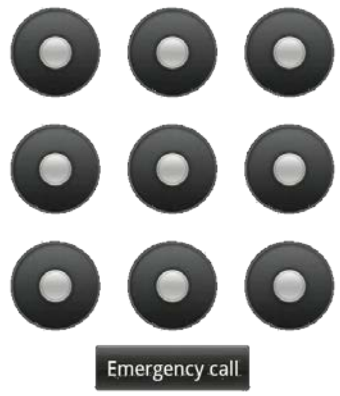
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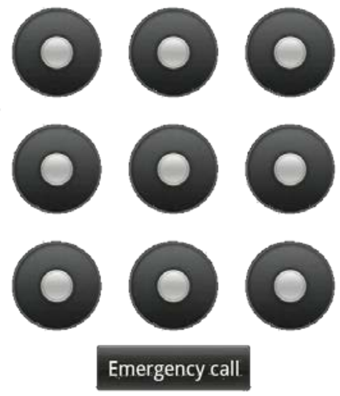
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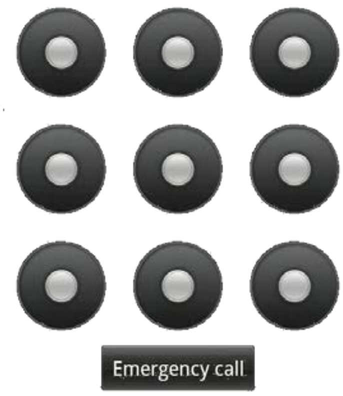
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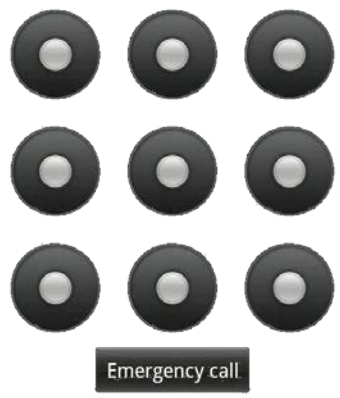
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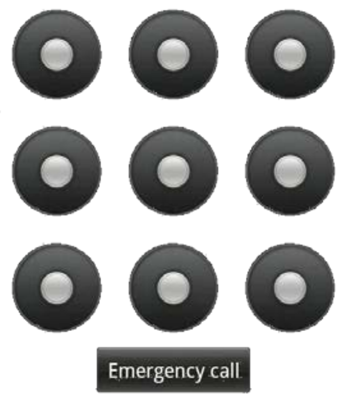
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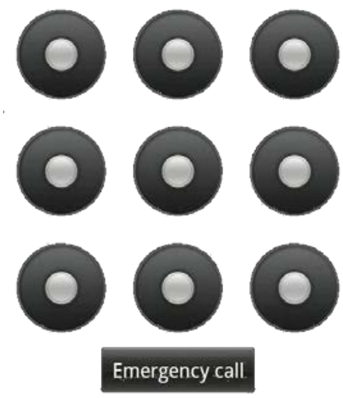
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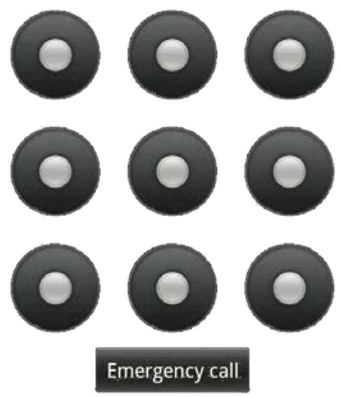
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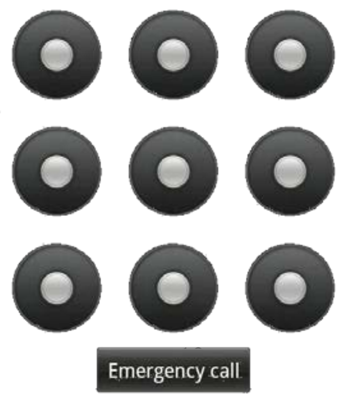
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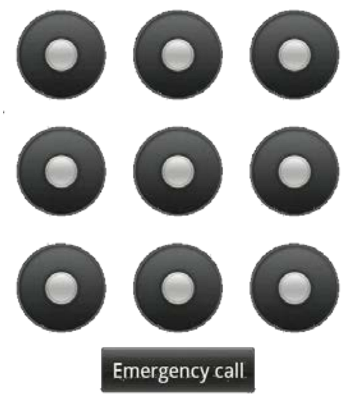
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