Topic Overview	
Topic	Strategies and Management E4: Resources Management – Simple interest, Compound Interests & Time Value of Money
Level	S3
Duration	3 lessons (40 minutes per lesson)

Learning Objectives:

1. Understand the concept of simple and compound interests,
2. Understand the concept of time value of money, present value and future value, and
3. Apply the concept of time value of money and calculate present value and future value.

Overview of Contents:	
Lesson 1	Simple Interest
Lesson 2	Compound Interest
Lesson 3	Time Value of Money

Resources:

- Topic Overview and Teaching Plan
- PowerPoint Presentation

Suggested Activities:

- Class Discussion
- In-class exercise

Lesson 1	
Theme	Simple Interest
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. explain the concept of interest,
2. calculate interest by using simple interest formula,
3. explain and apply the concept of discount, and
4. name the financial products using simple interest.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher introduces the concept of interest	PPT #2	3 minutes
Part II: Content		
✧ Teacher explains how to calculate simple interest with example	PPT #3 – 5	8 minutes
✧ **Activity 1: In-class exercise** ■ Students are required to calculate the principal by using simple interest formula. ✧ Teacher goes through the answers with students and makes conclusion.	PPT #6 PPT #7	3 minutes 2 minutes
✧ Teacher introduces the concept of borrowing and discount. ✧ Teacher provides further example of simple interest calculation for bank deposit.	PPT #8 – 11 PPT #12 – 13	8 minutes 4 minutes
✧ Teacher describes financial products using simple interest.	PPT #14	5 minutes
✧ **Activity 2: In-class Exercise** ■ Revision for simple interest calculation. ✧ Teacher goes through the answers with students and makes conclusion.	PPT #15 PPT #16	3 minutes 2 minutes
Part III: Conclusion		
✧ Teacher concludes the lesson by reviewing the key points covered.		2 minutes

Lesson 2	
Theme	Compound Interest
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. explain the concept of compound interest,
2. calculate interest by using compound interest formula, and
3. describe the concept of present value.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher introduces the concept of compound interest and compare it with simple interest.	PPT #2 – 3	3 minutes
Part II: Content		
✧ Teacher explains how to calculate compound interest with examples.	PPT #4	5 minutes
✧ **Activity 1: In-class exercise** ■ Students are required to calculate compound interest. ✧ Teacher goes through the answers with students and makes conclusion.	PPT #5 PPT #6	5 minutes 5 minutes
✧ Teacher demonstrates the use of formula for calculating compound interest with an example. ✧ Teacher explains daily and continuous compound interest calculation. ✧ Teacher introduces the concept of present value and its calculation.	PPT #7 – 9 PPT #10 – 12 PPT #13 – 16	5 minutes 5 minutes 10 minutes
Part III: Conclusion		
✧ Teacher concludes the lesson by reviewing the key points covered.		2 minutes

Lesson 3	
Theme	Time Value of Money
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. apply the concept of time value of money; and
2. calculate present value and future value.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher recaps the nature of present value and discusses with students why one dollar today is worth more than one dollar in the future say 1 year later.	PPT #2 - 3	8 minutes
Part II: Content		
✧ Teacher introduces the concept of future value and its calculation.	PPT #4 – 7	9 minutes
✧ Teacher explains the application of present value and discounting cash flows calculations.	PPT #8 – 9	4 minutes
✧ Teacher introduces time value terminology denoted on a timeline.	PPT #10	2 minutes
✧ Teacher continues the discussion of the application of present value and discounting cash flows calculations.	PPT #11 – 12	4 minutes
✧ **Activity 1: In-class exercise** ■ Students are required to make decision by using the concept of time value of money.	PPT #13	6 minutes
✧ Teacher goes through the answers with students and makes conclusion.	PPT #14	5 minutes
Part III: Conclusion		
✧ Teacher concludes the lesson by reviewing the key points covered.		2 minutes

Simple interest, Compound Interests & Time Value of Money

Lesson 1 – Simple Interest

1

The Concept of Interest

- When you deposit money into a savings account at a bank you expect the bank to pay you additional money on top of your saving. The extra money is called **interest.**
- Interest is also paid by a borrower to a lender for the loan borrowed.
- There are two methods to calculate interest:
  - Simple interest
  - Compound interest

2

Simple Interest Formula

- The simple interest (I) is calculated as:
- Principal x Interest Rate x Time
- We can use short form to interpret this: I = P x r x t
  - P = Principal (amount invested or borrowed)
  - r = Interest rate (per annum)
  - t = time (measured in years or converted to years by dividing by 12 months)

$$I = P \times r \times t$$

3

Simple Interest Formula – Example

- You have $10,000 in a savings account that pays 3% simple interest per annum.

a) How much interest will you earn in 2 years?

b) How much interest will you earn if you withdraw the money after 9 months?

4

Simple Interest Formula – Example (Solutions)

a) Interest = $10,000 x 3% x 2 = $600
b) Interest = $10,000 x 3% x 9/12 = $225

5

Activity 1 – Find the Principal Amount Invested

- What is the amount of principal that gives an interest of $3,000 in 4 years at 5% simple interest?

6

Activity 1 – Find the Principal Amount Invested

I = \$3,000, r = 5%, t = 4

I = P x r x t

\$3,000 = P x 5% x 4

P = \$3,000 ÷ 5% ÷ 4

= \$15,000

7

Borrowing and Repayment

- You have bought some goods from a shop at \$8,000 on credit. The interest rate charged by the shop is 14% per annum.
- How much will you have to pay to the shop after 6 months?

8

Borrowing and Repayment

- Answer:
- $8,000 + ($8,000 x 14% x 6/12) = $8,560

9

Discount

- The simple interest formula can also help you calculate the money you are actually spending when the shop is offering discount on the goods.
- For example, your favourite brand fashion is on sale for 20% off the regular price. You find the T-shirt you like at a price of $380 originally.
- What is the amount of the discount?
- What is the sale price?

10

Teacher discuss the relationship between interest and discount. It is important as the discount rate to calculate present value is in fact the interest rate.

Discount

- Answer:
- Discount = $380 x 20% = $76
- Sale price = $380 - $76 = $304 or $380 x (1 – 20%) = $304

11

Simple interest calculation

- Banks are currently paying an interest rate of 6% per year on deposits.
- Using simple interest calculation, how much you can get after one year if you deposit $1,000 into the bank? How about after 5 years?

12

Simple interest calculation

Example - Simple Interest

Interest earned at a rate of 6% for five years on a principal balance of $1,000.

Year	Balance at the start of year	Interest earned during year	Balance at the end of year
1	$1,000	$60	$1,060
2	$1,060	$60	$1,120
3	$1,120	$60	$1,180
4	$1,180	$60	$1,240
5	$1,240	$60	$1,300

13

Financial Products Using Simple Interest

- Bonds/debentures
  - Interest is calculated based on the face value of the bonds and normally paid half-yearly.
  - i.e. if the face value of a bond is $10,000, interest rate is 5% per annum. Then interest for the every six months =
  - $10,000 x 5% x 6/12 = $250

- iBond
  - iBond is different as the interest rate is not fixed.
  - Its interest rate is varied for each payment and is linked with the Consumer Price Index (CPI).
  - However, it is still using simple interest calculation.

14

Activity 2 – Exercise

- Your elder brother invested $50,000 in bonds for 2 years at interest rate of 6% per annum. How much interest has he earned if the interest is paid annually, and what is the balance in his account?

15

Activity 2 – Answer

- Your elder brother invested $50,000 in bonds for 2 years at interest rate of 6% per annum. How much interest has he earned if the interest is paid annually, and what is the balance in his account?

P = $50,000, r = 6%, t = 2

Interest = $50,000 x 6% x 2 = $6,000

Balance = $50,000 + $6,000 = $56,000

16

The End

17

Simple interest, Compound Interests & Time Value of Money

Lesson 2 – Compound Interest

1

Compound Interest

- The usual commercial procedure for dealing with interest earned is to add it to the investment at the end of each interest period.
- i.e. Re-investing your interest income from an investment makes your money grow faster over time!
- This system of adding interest to the investment is called a **compound interest system**.
- Compound interest uses the same information as simple interest, but what is new is the frequency of compounding ***n***.

2

Simple Interest vs Compound Interest

- Simple interest – interest is paid only on the principal
- Compound interest – interest is paid on both principal and interest, compounded at regular intervals

3

Calculation of Compound Interest

- Let us begin our analysis by considering a sum of $1,000 invested at 10% per annum over a five-year period. It is easiest to tabulate the calculation as below:

Year	Start of Year Investment	Annual Interest Earned	End of Year Investment
1	1000.00	1000.00x0.1 = 100.00	1000.00 + 100.00
2	1100.00	1100.00x0.1 = 110.00	1100.00 + 110.00
3	1210.00	1210.00x0.1 = 121.00	1210.00 + 121.00
4	1331.00	1331.00x0.1 = 133.10	1331.00 + 133.10
5	1464.10	1464.10x0.1 = 146.41	1610.51

4

Activity 1

- Banks are currently paying an interest rate of 6% per year on deposits.
- Using compound interest calculation, how much can you get after one year if you deposit $1,000 into the bank? How about after 5 years?

5

This question is the same as the exercise in previous lesson except for the use of compound interest instead of simple interest. Teacher can compare these two exercises with students and explain the difference and the impact of compound interest.

Activity 1 – Answers

Example – Compound Interest

Interest earned at a rate of 6% for five years on a previous year outstanding balance.

Year	Balance at the start of year	Interest earned during year	Balance at the end of year
1	$1,000	$60	$1,060
2	$1,060	$63.6	$1,123.6
3	$1,123.6	$67.42	$1,191.02
4	$1,191.02	$71.46	$1,262.48
5	$1,262.48	$75.75	$1,338.23

6

Results as Series

We can now summarise the results so far as

Sum at the end of period 1 = $P(1 + r)$

Sum at the end of period 2 = $P(1 + r)^2$

Sum at the end of period 3 = $P(1 + r)^3$

Sum at the end of period 4 = $P(1 + r)^4$

Sum at the end of period 5 = $P(1 + r)^5$

7

General Equation

- We can write down the general result which will give the sum invested, A, by the end of period n, as

$$A = P(1 + r)^n$$

8

Example

Let us now return to our original problem of investing $1,000 at 10% pa for 5 years. In our general terminology we have here that

$P = 1{,}000, \quad r = 0.1, \quad n = 5$

and wish to find ***A***. Substituting these numbers into our formula we have

$$A = 1{,}000(1 + 0.1)^5$$
$$= 1{,}000(1.1)^5$$

The calculation can now be completed using your calculator, to give $A = \$1{,}610.51$

9

Daily and Continuous Compounding

- Daily compound interest formula: divide r by 365 and multiply n by 365

$$A = P(1+\tfrac{r}{365})^{365n} \quad and \quad I = A - P$$

10

Teacher can mention the interest of our bank deposit is calculated on daily basis.

Daily and Continuous Compounding

- Example: Find the compound amount if $5,000 is deposited at 5% interest for 10 years if interest is compounded daily.

$$A = P(1+\tfrac{r}{365})^{365n}$$
$$= \$5{,}000(1+\tfrac{5\%}{365})^{3650}$$
$$= \$8{,}243{,}32$$

11

Summary

- Under the compound interest calculation, the amount is larger due to the power of interest on interest.
- In finance, compound interest calculation is implicitly assumed. That is, interest or return on investment earned will be re-invested at the interest rate assumed.

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Present Value of a Future Sum

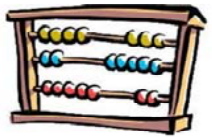
- Your uncle has give you a sum of $1,000,000 but you are not to receive the money until you have reached the age of 28 (15 years later). However, you would like to have the money now so you consult your bank manager. He is willing to buy your fund now, but the problem is what it is worth today.

13

Present Value

- It is known that the sum of $1,000,000 is guaranteed in 15 years and a fixed interest rate of 3.5% p.a. over the 15 years is assumed. The sum that he should give you now is called the present value of the future sum. If this sum of money is invested for 15 years at 3.5% p.a., it would amount to $1,000,000. In our previous notation we have

A = 1,000,000 r = 0.035 n = 15



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Present Value

We wish to find ***P***. Using the compound interest formula, we have

$1{,}000{,}000 = P(1 + 0.035)^{15}$

or, on transposing or solving this equation for P

$$P = \frac{1{,}000{,}000}{(1.035)^{15}} = \$596{,}891$$

15

Generalising

This means that the present value of $1,000,000 payable in 15 years with a fixed interest rate of 3.5% is $596,891. This is the sum that the bank manager should be offering to you.

In general, the present value of a sum **A** payable after *n* periods with a rate *r* per period is given by

$$P = \frac{A}{(1 + r)^n}$$

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The End

17

Simple interest, Compound Interests & Time Value of Money

Lesson 3 – Time Value of Money

1

Let's discuss:

- One dollar today is worth more than one dollar in the future, say 1 year later. Why?

2

Reason

- The reason is simple. If you get $1,000 today, you can deposit the money into the bank and earn the interest over 1 year. After 1 year, your wealth is, for sure, more than $1,000. Thus, compared with getting $1,000 in one year, getting $1,000 today is worth more!
- Suppose the annual interest rate is 6% and you are asked to give up $1,000 now. How much will you require to get back in one year to break even?

3

Teacher explains the reason for larger value in future is because of interest not inflation. Otherwise the value will be lower in future when there is a period of deflation. Interest is always true because this is the cost of borrowing. Borrower must pay more when he repays to the lender!

Future Value

- What is Future Value?
  - Future value is the value of a present amount at a future date, found by applying compound interest over a specified period of time.
  - In the above example, $1,060 is the 1 year's future value of $1,000 now [$1,000 x (1 + 6%)]. $1,338.2 is the 5 year's future value of $1,000 now [$1,000 x $(1 + 6\%)^5$].
  - If the principal is fixed at $1,000, what is the general formula of the T-year's future value?

4

Future Value

Future Value of the principal of $1,000 = FV

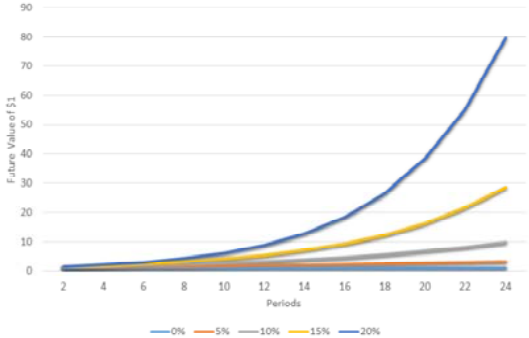
$$FV = \$1000 \times (1+r)^t$$

$(1+r)^t$ is known as future value interest factor.

- What does the future value depend on?
  1. Number of years of investment, t
  2. The interest rate, r
  3. The principal amount, $1,000
- The larger the above three values, the larger the future value is.

5

Future Value of $1



6

How can we find the future value?

1. Using scientific calculator
2. Using future value table
3. Using financial calculator
4. Using excel

7

Teacher can demonstrate how to use scientific calculator and future value table.
Financial calculator will not be used in Secondary schools but in universities.
Excel is suggested to be used in senior secondary stage.

Present Value

- The above questions can be asked in another way. If we want to have a certain sum of money in the future, say at the end of five years, given a certain interest rate, how much should we save today? This is one of the examples of present value calculation.
- Present value is the current dollar value of a future amount; the amount of money that would have to be invested today at a given interest rate over a specified period to equal the future amount.
- The process of finding present values or the inverse of compounding interest is called **discounting cash flows.**
- The interest rate used in converting future value into present value is called **the discount rate.**
- **$1/(1+r)^t$** is known as the **discount factor.**

8

Present Value

- The relationship between the present value and future value can be shown by the following formula:

$$\text{Present Value} = PV$$

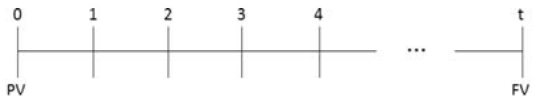
$$\boxed{PV = \frac{\text{Future Value after t periods}}{(1+r)^t}}$$

$\frac{1}{(1+r)^t}$ is the discounting factor or present value interest factor.

9

Time Value Terminology Denoted on a Timeline

- Consider the time line below:



- PV is the Present Value, that is, the value today.
- FV is the Future Value, or the value at a future date.
- The number of time periods between the Present Value and the Future Value is represented by "t".
- The rate of interest is called "r".
- All time value questions involve the four values above: PV, FV, r, and t. Given three of them, it is always possible to calculate the fourth.

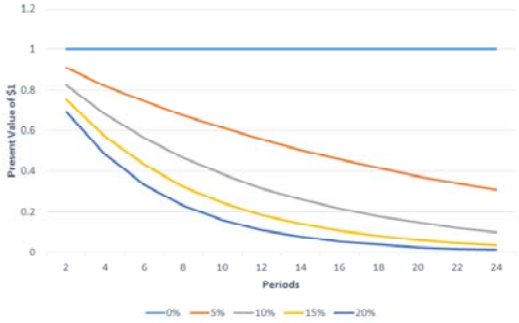
10

What does the present value depend on?

1. The larger the discount rate, the lower the present value is. This property is very important in finance as it can explain why when the interest rate goes up, usually the asset prices fall.
2. The larger the t, the lower the present value is. The present value of more distant cash flows is worth less.
3. The larger the future value, the higher the present value is.

11

Present Values with Compounding



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Activity 1 – Making Decision by Using the Concept of Time Value of Money

Your auto dealer gives you the choices of paying $155,000 by cash now, or making three payments: $80,000 now and $40,000 at the end of the following two years. If the interest rate is 8%, which method do you prefer?

13

Activity 1 – Answer

- Present values can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots$$

- PV (by instalment) = $80,000/(1.08)^0 + $40,000/(1.08)^1 + $40,000/(1.08)^2 = $151,331
- PV (full payment) = $155,000
- *Which method do you prefer?*

14

Answer: 'by instalment' because the cost is lower than full payment.

The End

15

Section A: Multiple Choice Questions (@1, total 10 marks)

1. You have $22,000 in a savings account that pays 3% simple interest per annum. How much interest will you earn in 2 years?

A. $660.
B. $679.80.
C. $1,320.
D. $1,339.80.

Level of difficulty: *

2. You have $27,000 in a savings account that pays 4% simple interest per annum. How much interest will you earn if you withdraw the money after 8 months?

A. $720.
B. $810.
C. $1,080.
D. $1,620.

Level of difficulty: **

3. What is the principal that gives an interest of $6,000 over 3 years at 5% simple interest per annum?

A. $18,000.
B. $40,000.
C. $80,000
D. $120,000.

Level of difficulty: **

4. The face value of a bond is $10,000 with a simple interest rate of 4% per annum and is paid every six months. How much is the interest for each payment?

A. $100.
B. $200.
C. $400.
D. $800.

Level of difficulty: ***

5. Your deposit of $25,000 is currently paying a compound interest rate of 6% per year. What will be the interest earned (to the nearest dollar) 4 years later?

A. $6,562.
B. $9,600.
C. $10,499
D. $49,600.

Level of difficulty: **

6. Find the compound interest (to the nearest dollar) for a bond with face value $10,000 at 3% interest for 6 years if interest is compounded biannually.

A. $10,900.
B. $11,800.
C. $11,941.
D. $11,956.

Level of difficulty: ***

7. What is the present value (to the nearest dollar) of the principal that gives $50,000 at a compound interest rate of 4% in 4 years?

A. $42,467.
B. $42,740.
C. $43,122.
D. $44,740.

Level of difficulty: **

8. What is the reason for one dollar today which is worth more than that in future?

A. Appreciation.
B. Foreign exchange.
C. Deflation.
D. Interest.

Level of difficulty: **

9. What is the future value (to the nearest dollar) of a principal of $18,000 at an interest rate of 6% p.a. after 5 years?

A. $23,400.
B. $24,400.
C. $24,088.
D. $26,024.

Level of difficulty: ***

10. The larger the discount rate, the _____ the present value is.

A. lower.
B. higher.
C. same.
D. unknown.

Level of difficulty: **

Section B: Short Questions (20 marks)

** 1. An item is on sale with 25% off the regular price. After discount the price is $2,820.

a) What is the regular price? (3 marks)

b) What is the amount of the discount? (3 marks)

*** 2. A term deposit of $36,000 is deposited in bank at 4% interest rate for 5 years and the interest is compounded daily. What is the total amount (to the nearest dollar) in the bank after 5 years later? (4 marks)

*** 3. You want to buy a smartphone and the seller offers you either to pay $5,000 by cash now or make three payments: $3,000 now, $1,100 at the end of year 1 and $1,150 at the end of year 2. If the interest rate is 4%, which method do you prefer? Why? (10 marks)

Suggested Solutions

Section A: MCQs

1. C	2. A	3. B	4. B	5. A
6. D	7. B	8. D	9. C	10. A

Section B: Short Questions.

Question 1 (@3, total 6 marks)

a) Regular price = $2,820 / (1 – 25%) = $3,760.
b) Discount = $3,760 - $2,820 = $940.

Question 2 (4 marks)

$36,000 x (1 + 4%/365)^1,825 = $43,970

Question 3

PV (by instalments) = $3,000 + $1,100/1.04 + $1,150/1.04^2 = $5,121 (4 marks)
PV (full payment) = $5,000 (1 mark)
It is better to pay now because it is $121 more if paying by instalments. (5 marks)