2016 年國際數學奧林匹克通訊 IMO 2016 NEWSLETTER

第二期

APR 15

2nd ISSUE

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挑戰園地

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笑一笑

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2016 年第五十七屆國際數學奧林匹克籌備委員會

Organising Committee of the 57th International Mathematical Olympiad 2016

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2016 年第五十七屆
國際數學奧林匹克籌備委員會

香港將於 2016 年 7 月主辦第五十七屆國際數學奧林匹克（IMO），迎接來自超過 100 個國家的中學生數學精英。希望《數聞》可在我們邁向 2016 年 IMO 期間帶動同學和公眾對數學的興趣，更希望這種氣氛歷久不衰。

歡迎讀者向《數聞》投稿。文章須為原著，以中文或英文寫成（或兩種文本兼備），長度為一至四頁（就一種語言而言），並以電郵附件方式傳送至 info@imohkc.org.hk，或郵寄至 九龍油麻地 405 號九龍政府合署 4 樓 403 室 教育局數學教育組《數聞》編輯，標題為「Submission to IMOment」。

Hong Kong is proud to be hosting the brightest secondary school mathematics talents from over 100 countries at the 57th International Mathematical Olympiad (IMO) in July 2016. We hope that IMOment will promote interest in mathematics among students and the public in this period leading up to IMO 2016, and beyond.

Readers are welcome to submit articles on mathematics and/or mathematical Olympiad to IMOment. Submissions should be original, one to four pages in length in either Chinese or English (or both), and should be sent by attachment to an email to info@imohkc.org.hk, or be mailed to Rm . 403, 4/F, Kowloon Government Offices, 405 Nathan Road, Kowloon, titled "Submission to IMOment."

Organising Committee of the 57th International Mathematical Olympiad 2016

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WWW.IMO2016.ORG

IMO 2014 後記 (上)
IMO 2014 AND BEYOND (I)

/梁達榮 LEUNG TAT-WING

我寫這篇專題文章有三個目的：（1）敘述IMO 2014的情況；（2）為培訓本地隊員提供建議；（3）希望為香港主辦IMO 2016盡一分綿力。

I write this article with three goals in mind: (1) to report on IMO 2014; (2) to give some idea how we can further train our team members; and (3) finally and hopefully provide us some help of how to organize IMO 2016.

行程

第55屆國際數學奧林匹克由2014年7月3日至7月13日於南非開普敦舉行。我們坐了13個小時飛機從香港到達約翰內斯堡，等了一段時間後再轉乘兩小時的短途機抵達開普敦。當然，相比起阿根廷和哥倫比亞的比賽旅程，這已算是十分輕鬆了。由於香港將主辦IMO 2016，因此代表隊今年在觀察員（連同領隊和副領隊）的陪同下出賽。我們在這次旅途中蒐集了不少資料，以助日後的籌備工作。今年IMO舉行期間正值世界杯，我們有幸在更好的時段（晚上六時或十時）觀賞了幾場賽事。只可惜，我們坐回程航班時錯過了由德國對阿根廷的總決賽，直至下了飛機才得知賽果。

南非的天氣不錯。我們去的時候是當地的冬天，白晝溫度約有20°C，晚間則為10°C左右。下雨時，天氣會變得更寒冷。我們下榻一間位於山腳的酒店，我想那大概是桌山山脈的一部分。酒店的風景宏偉，城市的結構美觀，就像一個英式小鎮。我們在那所酒店住了六天後，再與參賽學生一起入住開普敦大學。其實在我們到達三天後，學生們才抵達開普敦，他們一直住在大學宿舍。雖然大學的食宿比不上酒店，但我覺得還可以接受。唯一一點是，宿舍的每個入口都安裝了鐵閘，並由門衛看守，這令我有點擔憂。我不禁想到南非的治安問題──這個國家有相當高的失業率（25%）及堅尼系數（0.63），當然還有種族及其他社會問題。

Itinerary

The 55th International Mathematical Olympiad was held in Cape Town, South Africa, from 3 July to 13 July, 2014. It took us 13 hours flying from Hong Kong to Johannesburg, waiting for a couple of hours, then another 2 hours' flight to Cape Town. Surely when compared with Argentina and Colombia, it was a much easier trip. Because we have to host IMO 2016, this year several observers (with leaders or deputy leaders) came with us. We have gathered a lot of information in this trip, which will help us tremendously in our preparation. This IMO was held when World Cup matches were going on, and we are lucky that we still managed to watch several games, and at better times (6 pm or 10 pm). We missed only the final game, Germany vs Argentina, when we were exactly in our return flight, and I managed to get the result only when we got off the plane.

Weather in South Africa was nice. It was winter, and usually 20°C during the day time and about 10°C during the night. If it was raining, then it got a bit cooler. We first stayed in a hotel, right below a mountain, I believe it belongs to the Table Mountain range. And the view, if I may say, is simply majestic. The city structure looks nice, and it simply looks like a decent English town. The hotel is pretty normal and we stayed there for 6 days, and we then moved to the University of Cape Town (UCT) and stayed with the students. Our students arrived Cape Town three days after us, and they were stationed in dormitories of the University all the time. Though accommodation and food were not as good as in the hotel, I believe I can bear it. Only thing is, every entrance of a dormitory in the University is equipped with heavy iron gate and is watched by a security guard, and that I found it a bit scary. This reminds me of the security issue in South Africa. Of course it is a country with high unemployment rate (25%), high Gini coefficient (0.63), and there are the race problem and other things.

眾領隊用了三天時間在30道題目中挑選出6題，然後修改用字，再編寫英文及其他官方語言的版本。 接着他們便以討論的方式通過由選題委員會及協調員所建議的評分準則。不久，參賽學生陸續抵達，與領隊一同參與翌日的開幕典禮。為免領隊與參賽者有任何交流，他們被安排分開出席典禮。學生在緊接的兩個早上分別進行兩次4.5小時的比賽。領隊便趁着這段時間到處觀光遊覽，待比賽完畢後便入住大學，而學生們也有空閑的時間去大開眼界。我得知我的學生參觀了好望角，又有機會乘坐纜車登上桌山山頂。身為領隊的我因為要參與協調的工作，所以無暇觀賞以上兩個景點。協調的過程是由領隊、副領隊以及兩位主辦國的協調員一起商討學生各題的得分。幸好，在詳細的評分準則和經驗豐富的協調員幫助下，我們的工作進行得十分順利。最後一天，我們抓緊時間到處遊覽後就出席了閉幕典禮，翌日便返回香港。

選題

2014年3月底，主辦國（南非）收到141份來自43個國家的題目建議書。雖然我不知道負責小組從甚麼時候開始工作，但可以肯定，他們用了超過一個月時間篩選30條題目，並提供不同的解答方法及意見，然後印成小冊子讓評審委員會過目。順帶一提，選題委員會由六名國際成員組成。有人告訴我，他們在南非見面前後均負責了一些工作。經過篩選的題目當然極具水準，但我對是次的挑選仍有所保留。我認為今年的題型較偏頗:一共有6道代數題、9道組合數學題、7道幾何題及8道數論題。其中一些代數及數論題更涉及相當多組合數學的成份。再加上，有幾道組合數學題實在太深奧，連領隊大會也擔心沒有參賽者能解答出來。

Leaders spent three days to select the 6 problems from a shortlist of 30 problems, then refined the wordings and wrote the English version and other official versions. Then they discussed marking schemes proposed by the Problem Selection Committee and coordinators, and approved the marking schemes. The students then arrived, and the next day leaders and contestants together participated in the Opening Ceremony, with leaders and contestants still separated so that they could not communicate during the Ceremony. Students then wrote the two 4.5 hours contests on the mornings of the next two days, while leaders had the time to do a bit of sightseeing and the like. After the two contests, leaders were then moved to the University. On the other hand, students were free, and they had the chances to see further things. I knew my students got the chances to see the Cape of Good Hope, and took a cable car to the top of the Table Mountain. Because I, as a leader, had to participate in the coordination process, had to miss both events. Coordination is a process in which leader and deputy leader, plus two coordinators of the host country, come together to decide how many points are to be awarded to a particular problem submission of a student. Given that we had nice and detailed marking schemes, and the coordinators are generally very experienced, we encountered little trouble in deciding points. We then had a final day excursion and the Closing Ceremony, on the same day. The next day we were then heading home.

Problem Selection

By the end of March 2014, the host country (South Africa) received 141 problem proposals from 43 countries. I don't know when the problem selection group did start working, but surely, it took them more than 1 month to select 30 shortlist problems. Furthermore they modified them, supplied alternative solutions and comments, and prepared a booklet for Jury members to consider. Incidentally the problem group was composed of 6 international members, and I was told, they managed to do something before they formally met in South Africa, and also after they left. The selected problems are of course of high quality. However I cannot say I am totally happy with the selection. Indeed I think the problems selected were rather skewed, there were 6 algebra problems, 9 combinatorics problems, 7 geometry problems and 8 number theory problems. Some algebra problems and number theory problems in fact have quite a bit of combinatorics flavor. Moreover, several hard combinatorics problems were simply too hard. The Jury worried very much if one of them was selected, no one would be able to solve it.

問題 1. 設 $a_0 < a_1 < a_2 < \cdots$ 是無窮正整數數列。證明：存在唯一的整數 $n \geq 1$，滿足

$$a_n < \frac{a_0 + a_1 + \cdots + a_n}{n} \leq a_{n+1}.$$

問題 2. 設 $n \geq 2$ 為整數。考慮一個由 n^2 個單位方格所組成的 $n \times n$ 棋盤。將 n 只城堡擺在棋盤的方格中，使得每一列及每一行都恰有一只城堡，如此稱為和平擺法。試找出最大的正整數 k，使得對每一種 n 只城堡的和平擺法，都能找到 $k \times k$ 的正方形，它的 k^2 個單位方格中都沒有城堡。

問題 3. 在凸四邊形 $ABCD$ 中，$\angle ABC = \angle CDA = 90^\circ$。設 H 點是由 A 點向 BD 引垂線的垂足。令 S,T 兩點分別位於 AB 邊與 AD 邊上，滿足：H 落在三角形 SCT 內部，且

$$\angle CHS - \angle CSB = 90^\circ, \quad \angle THC - \angle DTC = 90^\circ.$$

證明：直線 BD 是三角形 TSH 外接圓的切線。

問題 4. 設 P,Q 兩點落在銳角三角形 ABC 的 BC 邊上，滿足 $\angle PAB = \angle BCA$ 及 $\angle CAQ = \angle ABC$。而 M,N 兩點分別落在直線 AP 與 AQ 上，使得 P 為 AM 的中點，Q 為 AN 的中點。證明：直線 BM 與 CN 的交點落在三角形 ABC 的外接圓上。

問題 5. 對每個正整數 n，開普敦銀行都發行幣值為 $\frac{1}{n}$ 的硬幣。今給定有限多個這樣的硬幣(其幣值不一定不同)，其總值最多為 $99 + \frac{1}{2}$。證明：可以將這些硬幣分成 100 堆或更少堆，使得每一堆硬幣的總值最多為 1。

問題 6. 平面上的一組直線，若其中任兩條不平行、任三條不共點，則稱這組直線位於一般位置。位於一般位置的直線組，將平面分割成若干區域，其中有些區域的面積是有限的；這些區域稱為此直線組的有限區域。證明：對任意足夠大的 n，皆可以在位於一般位置的 n 條直線組裡，選取至少 $\sqrt{n}$ 條直線著上藍色，使得此直線組沒有任何有限區域的邊界完全是藍色。

註：證出的結果中，如果 $\sqrt{n}$ 換成了 $c\sqrt{n}$，會依常數 c 之值給予分數。

IMO 2014 試題

在領隊大會上，有人建議先從4道較易的題目中選出2題，而那4題分別涉及四大範疇：代數、組合數學、幾何及數論。同樣地，我們在四大範疇中分別再選出4道難度適中的題目。當挑選出較易的2題後，難度適中的2題便自動取自另外兩個範疇，然後再隨機挑選出較難的2題（題三及題六）。最終這項建議被採納。我們選出了2道較易的代數及幾何題，2道組合數學及數論的適中題。但我認為那條代數題並非真正的代數題，解答時當然需要一些代數的運算，可是答案始終取決於整數的離散結構。總之，那道題不屬於不等式，也不算是函數方程。那道組合數學題是關於在一個特定的棋局中棋子之間的「洞」。其實數論題也並非完全有關數論，解答時不需要運用同餘或其他數論的知識，只需要把不同面值的硬幣組合，所以它應該是組合數學類型的題目。最後，我們選出了較難的幾何題及組合數學題。幾乎可以肯定，近年的比賽都有2道幾何題被選中，參賽者在解答這些題目時很少會用到高深的理論或技巧。但基於領隊們的喜好，立體幾何題通常不會出現。加上這次比賽有3道題涉及組合數學，所以我覺得現在選題的模式有機會令題目範疇分佈不勻。

(待續...)

(To be continued...)

Problem 1. Let $a_0 < a_1 < a_2 < \cdots$ be an infinite sequence of positive integers. Prove that there exists a unique integer $n \geq 1$ such that

$$a_n < \frac{a_0 + a_1 + \cdots + a_n}{n} \leq a_{n+1}.$$

Problem 2. Let $n \geq 2$ be an integer. Consider an $n \times n$ chessboard consisting of n^2 unit squares. A configuration of n rooks on this board is *peaceful* if every row and every column contains exactly one rook. Find the greatest positive integer k such that, for each peaceful configuration of n rooks, there is a $k \times k$ square which does not contain a rook on any of its k^2 unit squares.

Problem 3. Convex quadrilateral $ABCD$ has $\angle ABC = \angle CDA = 90^\circ$. Point H is the foot of the perpendicular from A to BD. Points S and T lie on sides AB and AD, respectively, such that H lies inside triangle SCT and

$$\angle CHS - \angle CSB = 90^\circ, \quad \angle THC - \angle DTC = 90^\circ.$$

Prove that line BD is tangent to the circumcircle of triangle TSH.

Problem 4. Points P and Q lie on side BC of acute-angled triangle ABC so that $\angle PAB = \angle BCA$ and $\angle CAQ = \angle ABC$. Points M and N lie on lines AP and AQ, respectively, such that P is the midpoint of AM, and Q is the midpoint of AN. Prove that lines BM and CN intersect on the circumcircle of triangle ABC.

Problem 5. For each positive integer n, the Bank of Cape Town issues coins of denomination $\frac{1}{n}$. Given a finite collection of such coins (of not necessarily different denominations) with total value at most $99 + \frac{1}{2}$, prove that it is possible to split this collection into 100 or fewer groups, such that each group has total value at most 1.

Problem 6. A set of lines in the plane is in *general position* if no two are parallel and no three pass through the same point. A set of lines in general position cuts the plane into regions, some of which have finite area; we call these its *finite regions*. Prove that for all sufficiently large n, in any set of n lines in general position it is possible to colour at least $\sqrt{n}$ of the lines blue in such a way that none of its finite regions has a completely blue boundary.

Note: Results with $\sqrt{n}$ replaced by $c\sqrt{n}$ will be awarded points depending on the value of the constant c.

Problems of IMO 2014

When the Jury members met, it was suggested that first we selected 2 out of 4 easy problems, with one problem from each of the topics algebra, combinatorics, geometry and number theory. Again 4 medium problems from the four topics were selected. When the two easy problems were chosen, the two medium problems from the other two categories were automatically selected. Then the hard problems (problem 3 and 6) were chosen arbitrarily. The suggestion was adapted. Finally two easy problems of algebra and geometry were selected, and so were two medium problems of combinatorics and number theory. However I am not sure if the easy algebra problem is really an algebra problem, of course it involves some algebraic manipulations, but I think the result very much depends on the discrete structure of integers. It is neither an inequality problem nor a functional equation problem anyway. The medium combinatorics problem concerns "holes" within a distribution of rooks in a checkerboard. The number theory problem again is not really number theory, there is no need of congruence or other number theory things, it basically involves merging or grouping of coins of different values, so is more like a combinatorics problem. Finally a hard geometry problem and a hard combinatorics problem were selected. It is quite certain in these days two geometry problems are to be selected. Those are the problems contestants cannot easily quote high power theorems or use more specialized techniques. However due to the preference of leaders, in general there are no 3D geometry problems. And in this contest, three problems are really of combinatorial flavor. So I think the new method of choosing problems is no guarantee of the good distribution of problems.

MUIRHEAD 不等式
MUIRHEAD'S INEQUALITY

盧安迪 ANDY LOO

堅持及毅力
PERSISTENCE AND WILLPOWER

我們在學校學過**解**不等式。例如，對於以下不等式：

In school we have learned to **solve** inequalities. Given the inequality

$$\frac{x-3}{2}+7>5$$

我們可解得：當且僅當 $x > -1$ 時，不等式成立。

we can find that this inequality holds if and only if $x > -1$.

但在數學比賽中，我們往往需要**證明**一條不等式在某給定範圍內成立。例如，證明對於**所有**實數 a、b、c,

But in mathematics competitions, we are often asked to **prove** that an inequality holds for **all** values of the variables in certain ranges. For example, prove that for all real numbers a, b and c, we have

$$a^2+b^2+c^2 \geq ab+bc+ca$$

證明這不等式有很多不同方法。最簡單的方法，可能是使用「任意實數的平方都是非負實數」這一性質。我們知道

There are many different ways to prove this. Perhaps the simplest way is to use the fact that the square of any real number is nonnegative. So we have

$$(a-b)^2 \geq 0$$

於是有

which yields

$$a^2+b^2 \geq 2ab$$

同理，我們知道

Similarly, we know

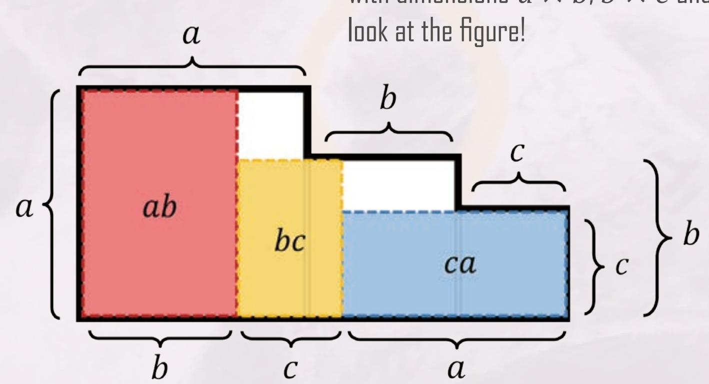
$$b^2+c^2 \geq 2bc \qquad c^2+a^2 \geq 2ca$$

把這三條不等式疊加起來，再除以二，我們便得到所需的不等式！

Adding these three inequalities and dividing by two, we get the desired inequality!

有趣的是，當 a、b、c 是正數時，這條不等式也有一個幾何證明。暫且假設 $a \geq b \geq c$。我們知道是 $a^2+b^2+c^2$ 邊長為 a、b、c 的三個正方形的面積之和，而 $ab+bc+ca$ 則是尺寸為 $a \times b$、$b \times c$、$c \times a$ 的長方形的面積之和。現在，看看下圖！

Interestingly, for the case that a, b and c, are positive, this inequality also has a geometric proof. Without loss of generality, let us assume $a \geq b \geq c$. Note that $a^2+b^2+c^2$ is the sum of the areas of three squares with side lengths a, b and c respectively, while $ab+bc+ca$ is the sum of the areas of three rectangles with dimensions $a \times b$, $b \times c$ and $c \times a$ respectively. Now, look at the figure!



不失其一般性，我們可假設若 a、b、c 有其他的大小次序，我們也可以用類似的方法構圖來證明。

If a, b and c are in some other order, we can use a similar method to prove the inequality.

現在，我們看看一個非常好的、用來證明不等式的工具，就是由蘇格蘭數學家 Robert Franklin Muirhead（1860–1941）發現的 **Muirhead 不等式**。

Now we will introduce a powerful tool for proving the above inequality, and many others. It is named **Muirhead's inequality** after Scottish mathematician Robert Franklin Muirhead (1860–1941).

要使用 Muirhead 不等式，我們首先要定義「**優化**」這個概念。如果兩個**單調遞減**的實數列 $(a_1,a_2,\ldots,a_n)$ 和 $(b_1,b_2,\ldots,b_n)$（即滿足 $a_1 \geq a_2 \geq \ldots \geq a_n$ 及 $b_1 \geq b_2 \geq \ldots \geq b_n$）滿足以下關係，我們便說 $(a_1,a_2,\ldots,a_n)$ 比 $(b_1,b_2,\ldots,b_n)$ 優化，並寫成 $(a_1,a_2,\ldots,a_n) \succ (b_1,b_2,\ldots,b_n)$:

To understand Muirhead's inequality, we first have to define the concept of **majorization**. Given two **monotonic decreasing** sequences $(a_1,a_2,\ldots,a_n)$ and $(b_1,b_2,\ldots,b_n)$ of real numbers (i.e. $a_1 \geq a_2 \geq \ldots \geq a_n$ and $b_1 \geq b_2 \geq \ldots \geq b_n$), we say that $(a_1,a_2,\ldots,a_n)$ majorizes $(b_1,b_2,\ldots,b_n)$ if the following relations hold, and denoted by $(a_1,a_2,\ldots,a_n) \succ (b_1,b_2,\ldots,b_n)$:

$$\begin{aligned} a_1 &\geq b_1 \\ a_1+a_2 &\geq b_1+b_2 \\ &\vdots \\ a_1+a_2+\ldots+a_{n-1} &\geq b_1+b_2+\ldots+b_{n-1} \\ a_1+a_2+\ldots+a_n &= b_1+b_2+\ldots+b_n \end{aligned}$$

舉例說，$(4, 3, 1) \succ (3, 3, 2)$，因為 $4 \geq 3$、$4+3 \geq 3+3$、$4+3+1 = 3+3+2$。

For example, $(4, 3, 1) \succ (3, 3, 2)$ because $4 \geq 3$, $4+3 \geq 3+3$ and $4+3+1 = 3+3+2$.

Muirhead 不等式指出，如果 $(a_1,a_2,\ldots,a_n) \succ (b_1,b_2,\ldots,b_n)$，那麼對於所有正實數 $x_1,x_2,\ldots,x_n$，粗略而言，$x_i^{a_j}$ 之積的**對稱總和**大於或等於 $x_i^{b_j}$ 之積的**對稱總和**。以 $(4, 3, 1) \succ (3, 3, 2)$為例，$(4, 3, 1) \succ (3, 3, 2)$對於任何正實數 x、y、z 有

Muirhead's inequality states that if $(a_1,a_2,\ldots,a_n) \succ (b_1,b_2,\ldots,b_n)$, then for all positive real numbers $x_1,x_2,\ldots,x_n$, the **symmetric sum** of the products of $x_i^{a_j}$ is larger than or equal to the **symmetric sum** of the products of $x_i^{b_j}$. What does this mean? Take as an example. As $(4, 3, 1) \succ (3, 3, 2)$, according to Muirhead's inequality, we can claim that for any positive real numbers x, y and z,

$$x^4y^3z+x^4z^3y+y^4x^3z+y^4z^3x+z^4x^3y+z^4y^3x \geq x^3y^3z^2+x^3z^3y^2+y^3x^3z^2+y^3z^3x^2+z^3x^3y^2+z^3y^3x^2$$

同樣地，因為 $(2, 0, 0) \succ (1, 1, 0)$，$a^2+b^2+c^2 \geq ab+bc+ca$ 這條不等式可以立即由 Muirhead 不等式推論出來！

Likewise, as $(2, 0, 0) \succ (1, 1, 0)$, the inequality $a^2+b^2+c^2 \geq ab+bc+ca$ is just simple case of Muirhead's inequality!

Muirhead 不等式亦指出，不等式中等號成立的充分必要條件是所有變數的數值都相等。所以當且僅當 $a = b = c$，$a^2+b^2+c^2 = ab+bc+ca$ 才成立。讀者可否看出怎樣從先前的兩個證明（即配方法及幾何方法）得出這個等號成立的條件呢？

Muirhead's inequality also states that the equality of the inequality holds when all the variables are equal. So $a^2+b^2+c^2 = ab+bc+ca$ if $a = b = c$. Can you see how this equality condition can also be derived from the two proofs we gave earlier (completing the square and the geometric proof)?

相比起配方法和幾何方法，Muirhead 不等式的一個優勢是它較容易用於證明複雜的不等式。它是繼上一期討論的坐標幾何之後，港隊的第二張王牌。在大部分情況下，要運用 Muirhead 不等式，仍需堅持及毅力，進行大型及繁複的運算、將算式展開等工作。在本期的**挑戰園地**，你也有機會一嘗運用 Muirhead 不等式！

One advantage of Muirhead's inequality over completing the square and geometric methods is that Muirhead's inequality can be more readily applied to proving more complicated inequalities. It is the second trump card of the Hong Kong team, after coordinate geometry, which was discussed in the last issue. In most cases, massive expansion is needed to write the inequality in a form to which Muirhead's inequality can be applied, and it needs persistence and willpower. In the **Challenge Corner** of this issue, you will also get a taste of Muirhead's inequality!

「與實力相若、志同道合的朋輩比拼是一件令人興奮的事。」

"There is a level of excitement in participating with peers with similar interests and talents in a competitive activity."

IMO 獎牌得主

AN IMO MEDALLIST

/梁哲雲 LEUNG CHIT-WAN

許多成功的學者曾經參加過 IMO。同時，不少 IMO 的參賽者在日後從事數學研究中亦獲得了學界最高榮譽的獎項，例如菲爾茲獎、奈望林納獎和沃爾夫數學獎等。其中以現今全球最頂尖數學家之一**陶哲軒**的事蹟最引人注目。他除了極力支持 IMO 外，最近更承諾擔任 **IMO 基金會**的贊助人。陶哲軒於 1986 年首次參加 IMO，後來在 1988 年，當時只有十二歲的他更成為了最年輕的 IMO 金牌得主。陶哲軒與香港有着密切的關係，他的父母由香港移民至澳洲，而他的母親則在香港當過數學教師。

陶哲軒獲獎無數。為了讚揚他對分析學（當中包括 Kakeya conjecture 和 wave maps）的貢獻，他在 2000 年、2002 年和 2003 年分別獲頒賽勒姆獎、博謝紀念獎和克萊數學研究獎。在 2005 年，他與 Allen Knutson 同獲得美國數學學會頒發的 Levi L. Conant Prize。他在 2006 年奪得拉馬努金獎。

2006 年 8 月，在馬德里舉行的第二十五屆國際數學家大會上，陶哲軒獲頒發菲爾茲獎，成為史上最年輕的得獎者。在 2010 年，他與 Enrico Bombieri 一同拿下費薩爾國王國際獎。同年，他個人奪得 the Nemmers Prize in Mathematics 和波里亞獎。在 2012 年，瑞典皇家科學院將克拉福德獎頒給陶哲軒和 Jean Bourgain。最近，陶哲軒在 2013 年度獲得數學突破獎。

陶哲軒教授說過：「IMO 留給我許多美好的回憶。正如參與學校的運動項目一樣，與實力相若、志同道合的朋輩比拼是一件令人興奮的事。我強烈建議每個高中生都爭取到國內和世界各地旅遊及見識的機會，IMO 可以正面地改變一班極具天分的年青數學家一生，因此我會全力支持 IMO 基金會。」

A close study of the IMO history confirms and reiterates the success of numerous academicians who were en route from IMO to celebrity, many of whom have won top honours in the field of mathematics, the Fields Medal, the Nevanlinna Prize and the Wolf Prize, for instance. Among them, the case of **Terence Tao**, one of the world's leading mathematicians, is of peculiar interest. He is a strong supporter of the IMO, and has recently been appointed **Patron of the IMO Foundation**. Tao first competed in the IMO in 1986 and was the youngest ever gold medallist, at the age of 12 in 1988. Tao has a strong affiliation with Hong Kong. His parents are immigrants from Hong Kong to Australia and his mother was formerly a mathematics teacher in Hong Kong.

Tao has won numerous honours and awards. He received the Salem Prize in 2000, the Bôcher Memorial Prize in 2002, and the Clay Research Award in 2003, for his contributions in analysis including work on the Kakeya conjecture and wave maps. In 2005, he received the American Mathematical Society's Levi L. Conant Prize with Allen Knutson, and in 2006 he was awarded the SASTRA Ramanujan Prize.

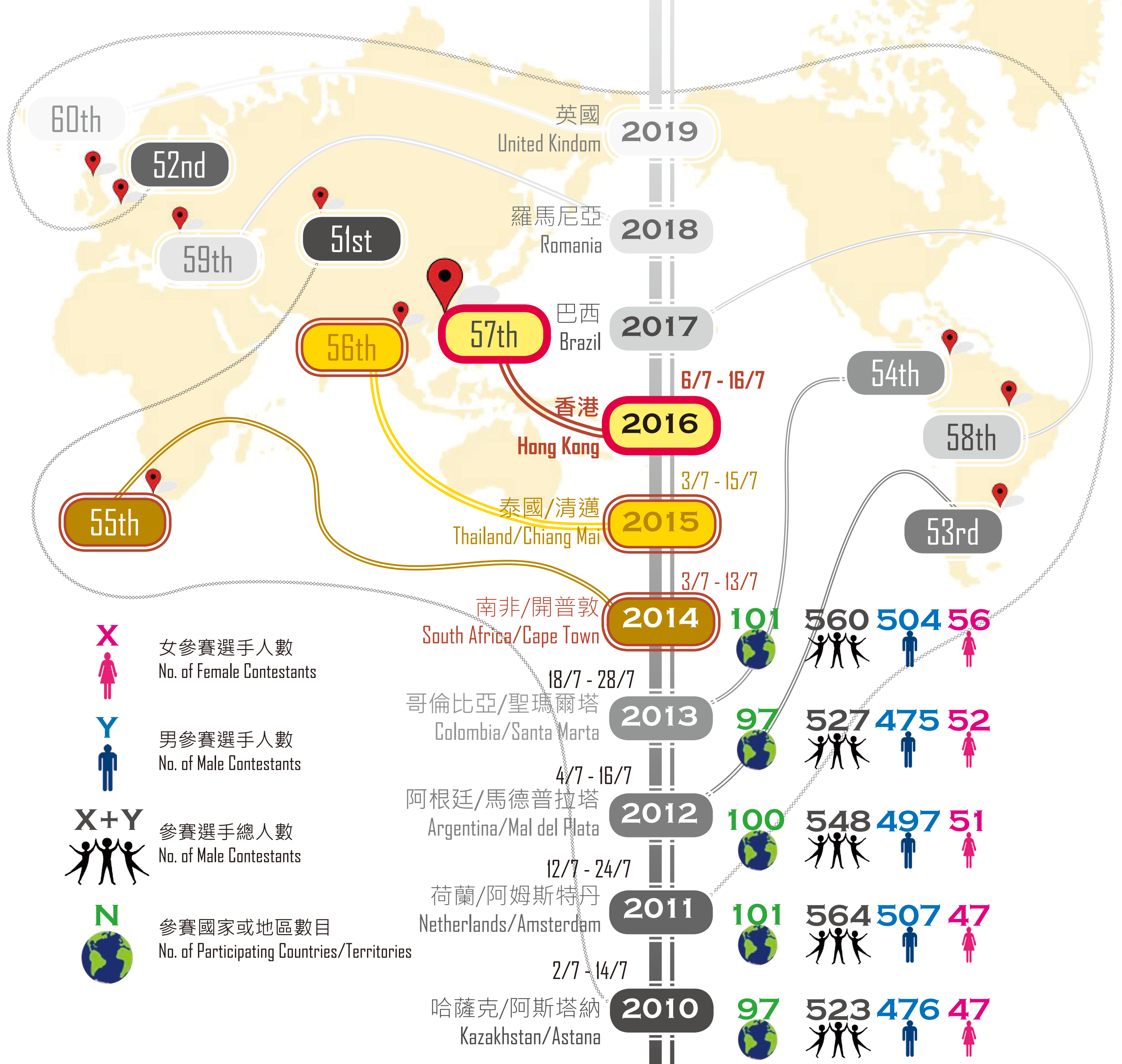
In August 2006, at the 25th International Congress of Mathematicians in Madrid, he became one of the youngest persons ever to be awarded a Fields Medal. In 2010, he received the King Faisal International Prize jointly with Enrico Bombieri. Also in 2010, he was awarded the Nemmers Prize in Mathematics and the Polya Prize. In 2012 he and Jean Bourgain received the Crafoord Prize in Mathematics from the Royal Swedish Academy of Sciences. Recently, he was awarded the Breakthrough Prize in Mathematics in 2013.

Professor Terence Tao said, "I have very fond memories of taking part in IMO. Like any other school sporting event, there is a level of excitement in participating with peers with similar interests and talents in a competitive activity. The opportunity to travel nationally and internationally is an experience I strongly recommend for all high-school students. Taking part in IMO can be a life-changing event for young, gifted mathematicians. I therefore wholeheartedly support the IMO foundation."

附錄 ANNEX

有關歷屆 IMO 的詳情，可參閱 IMO 官方網站（http://www.imo-official.org/organizers.aspx）。為方便參考，以下列出近年及未來數屆IMO的資料:

Details of past IMOs can be found on the IMO's official website (http://www.imo-official.org/organizers.aspx). For ease of reference, recent and prospective IMOs are listed as follows:



時間表 TIMELINE

附錄ANNEX

LIST OF AWARD-WINNING IMO MEDALLISTS

IMO獎牌得主的獲獎名單

菲爾茲獎被視為數學界的最高榮譽，在四年一度的國際數學家大會上頒發給四十歲或以下的數學家，每次最多四名得獎者。

The Fields Medal is often seen as the greatest honour a mathematician can receive. Every four years at the International Congress of Mathematicians, up to four Fields medals are awarded to young mathematicians of at most 40 years old.

塞勒姆獎每年頒給一位在拉斐爾•塞勒姆研究領域——傅立葉分析方面作出傑出貢獻的年輕數學家。

The Salem Prizes are awarded annually to a young mathematician who has done outstanding work in the field of interest of Raphaël Salem, namely, in Fourier Analysis.

參賽者 (年份) Contestants (Year)	IMO 獎牌 (年份) IMO Medal (Year)
菲爾茲獎 The Fields Medal	
Margulis, Grigorij (1978)	銀牌 Silver (1962)
Drinfeld, Vladimir (1990)	金牌 Gold (1969)
Yoccoz, Jean Christophe (1994)	銀牌 Silver (1973)、 金牌 Gold (1974)
Borcherds, Richard Ewen (1998)	銀牌 Silver (1977)、 金牌 Gold (1978)
Gowers, William Timothy (1998)	金牌 Gold (1981)
Lafforgue, Laurent (2002)	銀牌 Silver (1984 + 1985)
Perelman, Grigori (2006)	金牌 Gold (1982)
Tao, Terence (2006)	銅牌 Bronze (1986)、 銀牌 Silver (1987)、 金牌 Gold (1988)
Lindenstrauss, Elon (2010)	銅牌 Bronze (1988)
Ngô Bảo Châu (2010)	金牌 Gold (1988 + 1989)
Smirnov, Stanislav (2010)	金牌 Gold (1986 + 1987)
Mirzakhani, Maryam (2014)	金牌 Gold (1994 + 1995)
Avila, Artur (2014)	金牌 Gold (1995)
塞勒姆獎 The Salem Prize	
Dahlerg, Bjorn E. (1978)	沒有得獎 No award (1967)
Aleksandrov, Alexei B. (1982)	銀牌 Silver (1970 + 1971)
Yoccoz, Jean Christophe (1988)	銀牌 Silver (1973)、 金牌 Gold (1974)
Konyagin, Sergei (1990)	金牌 Gold (1972 + 1973)
Nazarov, Fedor (1999)	金牌 Gold (1984)
Tao, Terence (2000)	銅牌 Bronze (1986)、 銀牌 Silver (1987)、 金牌 Gold (1988)
Smirnov, Stanislav (2001)	金牌 Gold (1986 + 1987)
Lindenstrauss, Elon (2003)	銅牌 Bronze (1988)
Soundararajan, Kannan (2003)	銀牌 Silver (1991)
Green, Ben Joseph (2005)	銀牌 Silver (1994 + 1995)
Avila de Melo, Artur (2006)	金牌 Gold (1995)

參賽者 (年份) Contestants (Year)	IMO 獎牌 (年份) IMO Medal (Year)
奈望林納獎 The Nevanlinna Prize	
Razborov, Aleksandr (1990)	金牌 Gold (1979)
Shor, Peter (1998)	銀牌 Silver (1977)
沃爾夫數學獎 Wolf Prizes for Mathematics	
Lovasz, Laszlo (1999)	銀牌 Silver (1963)、 金牌 Gold (1964 + 1965 + 1966)
Margulis, Grigori (2005)	銀牌 Silver (1962)
哥德爾獎 The Gödel Prize	
Babai, Laszlo (1993)	銀牌 Silver (1966 + 1967)、 金牌 Gold (1968)
Hastad, Johan (1994)	銅牌 Silver (1976)、 金牌 Gold (1977)
Shor, Peter (1999)	銀牌 Silver (1977)
Lovasz, Laszlo (2001)	銀牌 Silver (1963)、 金牌 Gold (1964 + 1965 + 1966)
Razborov, Aleksandr (2007)	金牌 Gold (1979)

奈望林納獎授予在計算機科學的數學領域中有卓越成就的研究人士，由 1982 年始，每隔四年在國際數學家大會上頒給一名四十歲或以下的得獎者。

The Nevanlinna Prize honours great achievements in mathematical aspects of computer science. It was first awarded in 1982, subsequently once every four years at the International Congress of Mathematicians. It is awarded to a single recipient of age at most 40.

沃爾夫數學獎自 1978 年起，每年頒給兩位數學家。在 1999 年成立阿貝爾獎之前，沃爾夫數學獎一直被視為數學界的諾貝爾獎。

Wolf Prizes in Mathematics have been awarded annually since 1978 to two mathematicians. Until the creation of the Abel Prize in 1999, the Wolf Prize was generally seen as the mathematical analogue to the Nobel Prize.

哥德爾獎是以庫爾特•哥德爾命名的獎項，自 1993 年每年頒發給在理論計算機科學方面創作出優秀論文的作者。

The Gödel Prize is awarded for outstanding papers in theoretical computer science, named after Kurt Gödel. It has been awarded annually since 1993.

笑一笑 Laugh Out Loud

挑戰園地 Challenge Corner

第一期挑戰園地的解答及得獎名單，可見:
For the solutions and list of awardees of the Challenge Corner of the 1st issue, please see:
http://www.edb.gov.hk/tc/curriculum-development/kla/ma/IMO/IMOment.html

1. 希淳和彥琪打開了 n 包糖果，每包有 n 顆糖果。希淳先拿 10 顆，然後彥琪拿 10 顆，然後希淳拿 10 顆，然後彥琪拿 10 顆，如此類推。最後，輪到彥琪拿糖果時，只剩下少於 10 顆，於是彥琪便拿了所有剩下的糖果。希淳總共比彥琪多拿多少顆糖果？

Helsa and Kiki open n bags of candies. Each bag has n candies. Helsa takes 10 candies, then Kiki takes 10 candies, then Helsa takes 10 candies, then Kiki takes 10 candies, and so on. Finally, when it is Kiki's turn to take candies, there are fewer than 10 candies left, so Kiki just takes all the candies left. In total, how many more candies does Helsa take than Kiki?

2. 已知 n 為大於 1 的正整數，而邊長為 n、$n+1$、$n+2$ 的三角形的面積為整數。求 n 的兩個可能值（包括證明）。

For a positive integer n greater than 1, the area of a triangle with side lengths n, $n+1$ and $n+2$ is an integer. Find, with proof, two possible values of n.

3. 在社交網站 Mathbook，每名用戶都有一張個人資料圖片和一張封面圖片。有些用戶的個人資料圖片相同，但不是所有用戶的個人資料圖片都相同。有趣的是，如果兩名用戶的個人資料圖片不同，他們的的封面圖片便相同。求證：所有用戶的封面圖片都相同。

On the social networking website Mathbook, each user has a profile picture and a cover picture. Some users have identical profile pictures, but not all users have identical profile pictures. Interestingly, any two users who don't have identical profile pictures have identical cover pictures. Prove that all users have identical cover pictures.

4. 設 x、y、z、w 為正實數。求證以下不等式。（提示：試用本期有關 Muirhead 不等式的文章中介紹的方法！）

If x, y, z and w are positive real numbers, prove the following inequality. (Hint: Try the method introduced in the article on Muirhead's inequality in this issue!)

$$\left(x+y+z+w\right)^4 \geq 64\left(x^4+y^4+z^4+w^4\right)$$

歡迎香港中學生讀者電郵至 info@imohkc.org.hk 提交解答（包括證明），並於電郵中列明學生中英文姓名、學校中英文名稱及學生班級。每一名學生只可發送一份電郵。首 20名 答對最多題目的同學將獲贈紀念品，但每間學校最多有 3 名同學得獎。解答可以中文或英文提交。打字及掃描文件皆可接受。得獎者將於下一期公布。2016 年第五十七屆國際數學奧林匹克籌備委員會對本活動安排有最終決定權。如有疑問，可電郵至 info@imohkc.org.hk 查詢。

Hong Kong secondary school student readers are welcome to submit solutions (with proofs) via email to info@imohkc.org.hk, specifying the student's name in Chinese and in English, the school's name in Chinese and in English, and the student's class in the email. Each student may send at most one email. Souvenirs will be awarded to the first 20 students solving the most questions on the condition that each school can have at most 3 awardees. Solutions can be submitted in Chinese or English. Both typed and scanned files are acceptable. The awardees will be announced in the next issue. The decision of the Organising Committee of the 57th International Mathematical Olympiad on any matter of this activity is final. Enquiries may be emailed to info@imohkc.org.hk.