

Solutions to Challenge Corner of 5th issue of IMOMent

1. If x , y and z are real numbers such that $\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} = 2015$, find all possible values of $\frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y}$.

Solution: Since $\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} + \frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y} = 0$, the answer is -2015 .

2. Find the area of a triangle whose side lengths are $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{13}$ respectively. (Hint: Construct a square.)

Solution: Let $ABCD$ be a square with side length 3. Let E be a point on AB such that $AE = 2$. Let F be a point on AD such that $AF = 1$. Then EFC is a triangle whose side lengths are $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{13}$. Its area is

$$3^2 - \frac{1 \times 2}{2} - \frac{1 \times 3}{2} - \frac{2 \times 3}{2} = \frac{7}{2}.$$

3. Find all integer solutions to the following equation:

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{2015}$$

Solution: Multiplying through by $2015xy$, the equation becomes $2015y - 2015x = xy$; rearranging, $(x - 2015)(y + 2015) = -2015^2$. From this we can obtain the various solutions, not forgetting to reject $(x, y) = (0, 0)$.

4. Refer to the discussion of **correlated equilibria** in the article *Nash and Beyond* in this issue. Given Harold's recommendation habit described in the article, find a correlated equilibrium of the game other than the correlated equilibrium stated in the article.

Solution: Plainly, it is a correlated equilibrium for both John and Alicia to always do the opposite of Harold's recommendation. (Other possibilities, if any, are acceptable.)

List of awardees:

Name of Student	Name of School
CHUI Ho Man	China Holiness Church Living Spirit College
CHOY Yu Hin	Valtorta College
FAN Man Hon	HKUGA College
YEUNG Ka Lok	Evangel College
LEE Cheuk Yiu	Evangel College

Awardees will also be notified with a separate e-mail.