## Solutions to Challenge Corner of 6th issue of IMOment

1. How many positive divisors does one googol have? (Please see our interview with Professor Jacob Lurie in the current issue for the definition of a googol.)

   **Solution:** Recall that one googol is $10^{100} = 2^{100} \times 5^{100}$. Each positive divisor of it is uniquely determined by the exponent of 2 and the exponent of 5 in the divisor. Each of these two choices admits 101 possibilities. Hence the answer is $101 \times 101 = 10201$.

2. Consider a $2015 \times 2015$ chessboard. The $1 \times 1$ square at the top-left corner is removed. Can the remaining part be tiled with $1 \times 4$ and $4 \times 1$ rectangles without overlap?

   **Solution**: Write the complex number $i^{x+2y}$ in the square in the $x^{\text{th}}$ row and $y^{\text{th}}$ column, for all *x*, *y*. Then the numbers covered by each $1 \times 4$ or $4 \times 1$ rectangle sum to 0. However, one may check that the numbers in the remaining part of the chessboard do not sum to 0.

3. Find the radius of the circumcircle of a triangle with side lengths 5, 5, 6.

   **Solution:** Let the triangle be *ABC*, with *BC* being the side of length 6. Let *D* be the midpoint of *BC*. By Pythagoras' theorem, $AD = 4$. By symmetry, the circumcenter *O* of the triangle lies on the line *AD*, and is inside the triangle since angle *BAC* is acute. Let the circumradius of the triangle be *R*. Since $OD^2 + BD^2 = OB^2$, we have $(4-R)^2 + 3^2 = R^2$, which gives $R = \dfrac{25}{8}$.

4. If *x*, *y*, *z* are positive real numbers such that $xyz = x + y + z$, prove that there exists a triangle *ABC* such that $x = \tan A$, $y = \tan B$, and $z = \tan C$.

   **Solution:** Since *x*, *y*, *z* are positive, there exist real numbers *a*, *b*, *c* between 0 and $\pi / 2$ such that $x = \tan a$, $y = \tan b$, and $z = \tan c$. From the given condition we have $\tan c = \dfrac{\tan a + \tan b}{\tan a \tan b - 1} = -\tan(a+b) = \tan(-a-b)$ and hence $c = k\pi - a - b$ for some integer *k*. Since $0 < a, b, c < \pi / 2$, we have $c = \pi - a - b$ and the desired result follows.

## List of awardees:

| Name of Student | Name of School |
|---|---|
| LAI Chun Yin | St. Bonaventure College & High School |
| Jeffrey TSE | Pui Ching Middle School |
| HUI Chun Wai | Y.O.T. Tin Ka Ping Primary School |
| CHAU Wing Ho | Tsuen Wan Government Secondary School |
| NG Ting Chun | Christian Alliance S C Chan Memorial College |

*Awardees will also be notified with a separate e-mail.*