Solutions to Challenge Corner of 7th issue of IMOment

1. Any soccer formation consists of *x* defenders, *y* midfielders and *z* forwards for some non-negative integers *x*, *y* and *z*, such that the total number of defenders, midfielders and forwards is 10. For example, $(x,y,z)=(4,4,2)$ and $(x,y,z)=(10,0,0)$ are soccer formations, but $(x,y,z)=(3.5,6,0.5)$ and $(x,y,z)=(4,4,1)$ are not. How many soccer formations are there?

   Solution: If there are *x* defenders, the number *y* of midfielders can take $11-x$ different values (namely, 0, 1, …, $10-x$). After *x* and *y* are fixed, there remains one possible value of *z*. So the answer is
   $$11+10+9+...+1=66.$$

2. Refer to Example 3 in the article on *The devil in the detail* in the current issue. Try to obtain different conclusions about the ranking of the teams through different applications of the rules.

   Solution: Yellow got 9 points, 10 goals scored and 1 goal conceded, and all the other three teams got 3 points, 3 goals scored and 6 goals conceded. Hence, Yellow ranked top, but as the other three teams had same points, numbers of goals scored and conceded, their ranks should be determined by the results from the matches between the teams in question.

   Consider the match results between Red, Green and Blue, which include the match Red 2:1 Blue, Blue 2:0 Green, and Green 3:0 Blue. Hence, all teams got 3 points in these match results.

   Coach of Red would argue, ‘Since the goal difference and the match results are all the same in three teams, we should draw lots to determine the rank.’

   Coach of Green would argue, ‘No, we should also consider the goal differences of each match.’

   Coach of Blue would argue, ‘Agree! Amongst these matches, Red scored 2 but conceded 4 goals, while Blue and Green both scored 3 but conceded 2 goals.’

   Coach of Green would say, ‘Hence Red should rank 4th, and Blue and Green should draw lots.’

   Coach of Blue would further argue, ‘But then we should consider the match result between Blue and Green, in which Blue defeated Green by 2:0. Hence, we should be the 2nd, and Green being 3rd.’

3. Given that the product of the lengths of the three sides of a triangle is 480 and its circumradius is of length 5, find its area.

Solution: Recall that the area of a triangle is given by the formula $\frac{1}{2}ab\sin C$. By sine law, we have $\sin C = \frac{c}{2R}$. Hence the area of the triangle is $\frac{abc}{4R} = 24$.

4. Prove that for any positive real numbers *a*, *b* and *c*, we have the following inequality:

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$$

Solution: Without loss of generality, assume $a \geq b \geq c$. Then $b+c \leq c+a \leq a+b$ and hence $\frac{1}{b+c} \geq \frac{1}{c+a} \geq \frac{1}{a+b}$. By the rearrangement inequality, we have

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{b}{b+c} + \frac{c}{c+a} + \frac{a}{a+b}$$

and

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{c}{b+c} + \frac{a}{c+a} + \frac{b}{a+b}.$$

Adding the last two inequalities, we get the desired result.

Alternatively, this inequality can be proved by expansion and applying Muirhead's inequality, which was introduced in the 2nd issue of IMOment.

List of awardees:

Name of Student	Name of School
Jeffrey TSE	Pui Ching Middle School
LAI Chun Yin	St. Bonaventure College & High School
FAN Man-hon	HKUGA College
NG Ting Chun	Christian Alliance S C Chan Memorial College
LAM Chun-san	Carmel Devine Grace Foundation Secondary School

Awardees will also be notified with a separate e-mail.