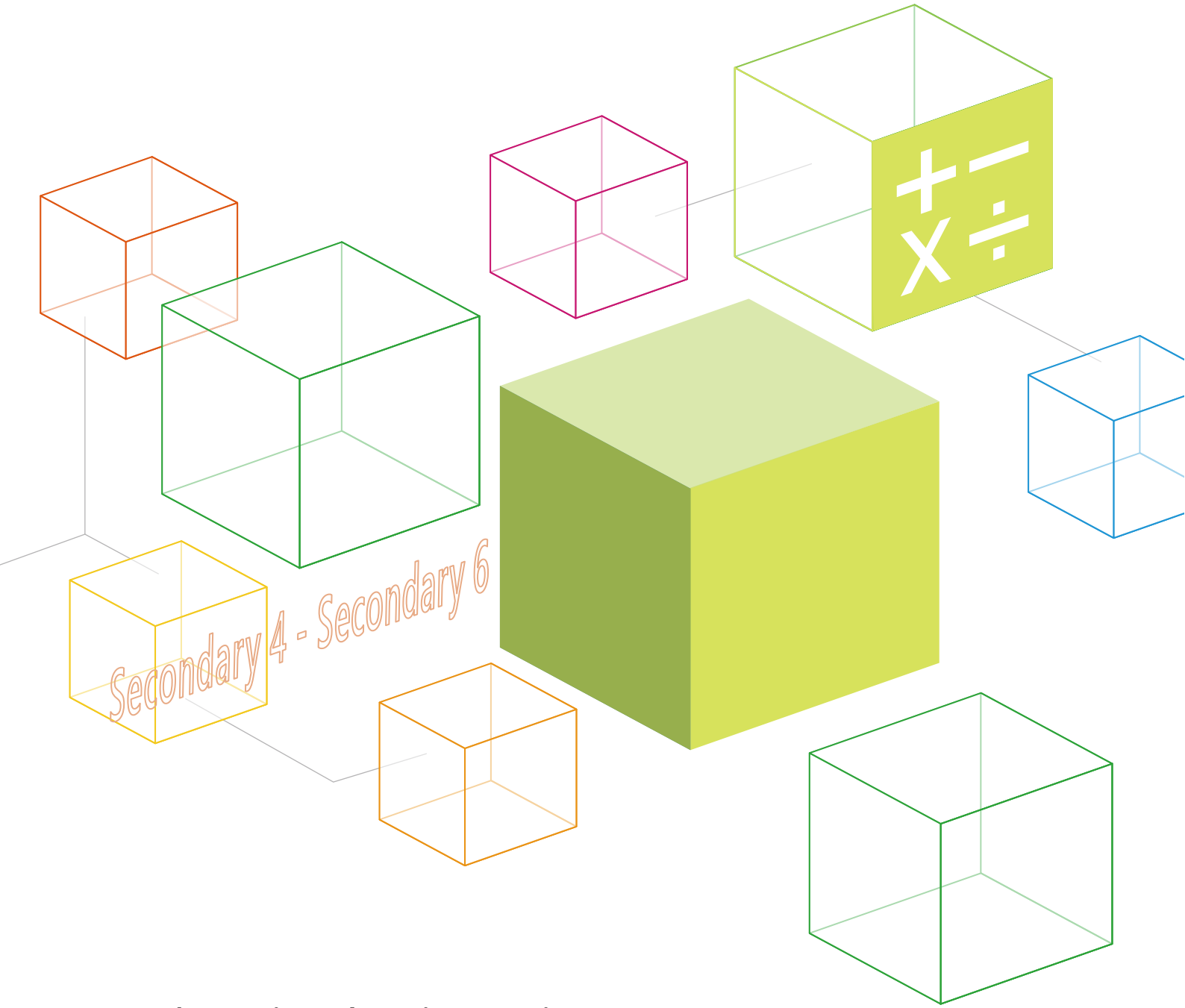
Explanatory Notes to Senior Secondary Mathematics Curriculum — Compulsory Part



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Foreword

To keep abreast of the ongoing development of school curriculum, *Mathematics Curriculum and Assessment Guide (Secondary 4 - 6)*, hereinafter referred to as "C&A Guide", was updated in December 2017. Senior Secondary Mathematics Curriculum comprises a Compulsory Part and an Extended Part. The Extended Part has two modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus).

In the C&A Guide, the Learning Objectives of the Compulsory Part are grouped under different Learning Units in the form of a table. The notes in the "Remarks" column of the table in the C&A Guide provide supplementary information about the Learning Objectives. The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of the Compulsory Part;
2. the strategies suggested for the teaching of the Compulsory Part;
3. the connections and structures among different Learning Units of the Compulsory Part;
4. the context of development from different key stages, such as Key Stage 3, to the Compulsory Part; and
5. the curriculum articulation between the Compulsory Part and the Extended Part.

The explanatory notes in this booklet together with the "Remarks" column and the suggested lesson time of each Learning Unit in the C&A Guide are to indicate the breadth and depth of treatment required. Teachers are advised to teach the contents of the Compulsory Part as a connected body of mathematical knowledge and develop in students the capability to use mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the C&A Guide does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequence which takes account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They should be sent to:

Chief Curriculum Development Officer (Mathematics)
Mathematics Education Section
Education Bureau
4/F, Kowloon Government Offices
405 Nathan Road, Kowloon

Fax: 3426 9265
E-mail: ccdoma@edb.gov.hk

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method 1.2 form quadratic equations from given roots 1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts 1.4 solve quadratic equations by the quadratic formula 1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots 1.6 solve problems involving quadratic equations 1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations 1.8 appreciate the development of the number systems including the system of complex numbers 1.9 perform addition, subtraction, multiplication and division of complex numbers	19

Explanatory Notes:

In Learning Unit 8 “Linear equations in one unknown” at Key Stage 3 (KS3), students learnt how to formulate and solve linear equations in one unknown. They also learnt how to formulate simultaneous linear equations in two unknowns, and solve them by both the algebraic method and the graphical method in Learning Unit 9 “Linear equations in two unknowns”. In the Compulsory Part, students are required to solve quadratic equations and other more complicated algebraic equations.

In this Learning Unit, students are required to use:

- the factor method
- the quadratic formula
- the graph of the parabola $y = ax^2 + bx + c$

to solve the quadratic equation $ax^2 + bx + c = 0$, and to form a quadratic equation from given roots. Students should be able to choose an appropriate strategy in solving quadratic equations.

The graphical method will be further elaborated in Learning Objective 9.2, and teachers should pay attention to the Explanatory Notes to Learning Unit 9 “More about graphs of functions”.

Students should be able to solve the problems involving quadratic equations. The problems should be related to their experiences as far as possible. Solving the equations which can be transformed into quadratic equations, such as $\frac{6}{x}+\frac{6}{x-1}=5$ and related problems, and solving higher degree equations by using the factor theorem or the graphs of functions will be treated in Learning Unit 4 “More about Polynomials”, Learning Unit 5 “More about Equations”, and Learning Unit 9 “More about Graphs of Functions” accordingly.

In this Learning Unit, all the coefficients of quadratic equations and the given roots in Learning Objective 1.2, are confined to real numbers.

Regarding the expression of the solutions to quadratic equations, it should be noted that performing mixed arithmetic operations of simple quadratic surds in Learning Objective 4.3 at KS3 belong to the Non-foundation Topics. In this connection, when using the quadratic formula to solve quadratic equations such as $x^2-4x-4=0$, the students who have not studied the above-mentioned topics in the Non-foundation Topics are **not required** to simplify surds like $2\pm\frac{\sqrt{32}}{2}$.

Students are required to understand the relations between the discriminant of a quadratic equation and the nature of its roots in addition to solving quadratic equations. In the C&A Guide, the term “understand” usually implies a more demanding Learning Objective than the term “recognise” does. For example, in Learning Objective 1.5 “understand the relations between the discriminant of a quadratic equation and the nature of its roots” means that students are required to know the details of the relations, the justification of why the relations hold, and how to use these relations to perform further operations and solve problems.

The concept of complex numbers is introduced in Learning Objective 1.8. When students are asked to determine the nature of the roots of a quadratic equation with a negative discriminant, they are required to point out that "the equation has **no real roots**" or, more precisely, "the equation has **two nonreal** roots" instead of "the equation has no roots" or "the equation has two complex roots". The students who do not study Learning Objective 1.9 (Non-foundation Topics) are **not required** to express the nonreal roots in the form of $a \pm bi$ when using the quadratic formula to solve quadratic equations.

Students are required to understand the relations between the roots and the coefficients, which includes:

- $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$, where α and β are the roots of the equation $ax^2 + bx + c = 0$ and $a \neq 0$.

Teachers may discuss with students or let them explore other relations between the roots and coefficients, such as $\alpha^2 + \beta^2$. Nevertheless, memorisation of these results is **not required**.

Mathematical concepts have come a long way and their development has often been influenced by cultures and other humanistic factors. The development of number systems makes no exception. By organising various activities, such as decorating display boards or reading projects, teachers may get students to appreciate how number systems were developed from the natural number system to the rational number system, the real number system and the complex number system. For example, "Why did Pythagoreans in ancient Greece not accept the existence of irrational numbers?", "Why were imaginary numbers not well-articulated until the sixteenth century?", etc. are some interesting topics for discussion. Moreover, teachers may ask students to discuss topics such as the hierarchy of the number system, the representation of terminating decimals and recurring decimals in fractional form, and the proof of irrationality of numbers such as $\sqrt{2}$, $\frac{1+\sqrt{5}}{2}$, etc.

The students who study Learning Objective 1.9 (Non-foundation Topics) should be able to perform the addition, subtraction, multiplication and division of complex numbers, of which the form is confined to the standard form $a \pm bi$. The polar form of complex numbers, Argand diagram and De Moivre's theorem are **not required** in the Curriculum.

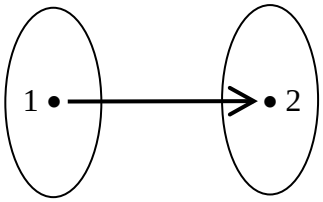
Teachers may refer to the C&A Guide (pp. 101 – 102) for using the co-construction approach to teach the quadratic formula.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables 2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions 2.3 understand the features of the graphs of quadratic functions 2.4 find the maximum and minimum values of quadratic functions by the algebraic method	10

Explanatory Notes:

In Learning Objectives 7.3 and 7.4 at KS3, students had a preliminary recognition of the concept of sequences, which may be considered as an embryonic form of functions. In this Learning Unit, students are required to recognise the intuitive concepts of functions, dependent and independent variables. They should also be able to distinguish between the examples of functions and non-functions. Furthermore, students are required to recognise the intuitive concepts of domain and co-domain as these two concepts are indispensable for defining functions. Students are required to recognise that even with the same relations between variables, different choices of domains and co-domains result in different functions, and in some cases non-function. Knowing these two concepts can help students compare different functions in detail (see the Remarks of Learning Objective 3.4), but using set language to rigorously define functions or express ranges are **not required** in the Curriculum. Besides, compositions of functions are **not required** in the Curriculum. When students encounter functions such as $f(x) = \sin x^2$, they may simply regard the calculation of the functional value as a sequence of consecutive manipulations, say in the above-mentioned example: taking the square of x and the sine of the square.

When explicating the notations of function, teachers may introduce the concept of dummy variables. There are various ways to represent functions. Under different circumstances, teachers may adopt different methods, i.e. tabular, algebraic, graphical methods, or even the following intuitive method to represent functions:



Having acquired the concept of functions, students are required to consolidate the concept by studying their familiar quadratic functions, and to understand the following features of the graphs of quadratic functions.

- The vertex
- The axis of symmetry
- The direction of opening
- Its relations with the axes

Students should be able to determine the direction of opening of the graph of a quadratic function from the coefficient of x^2, which is the leading coefficient, to find the *y*-intercept from the constant term, and to use the discriminant to determine whether the graph cuts the *x*-axis. On the other hand, students should also be able to read, from the graph of a quadratic function, the information about its axis of symmetry and vertex. They are also required to understand the relation between the maximum/minimum value of the quadratic function and the vertex of its graph, and hence be able to find the maximum/minimum value of the quadratic function by the graphical method.

The students who study Learning Objective 2.4 (Non-foundation Topics) should be able to use the algebraic method to find the maximum/minimum value of a quadratic function, and to solve related problems. Apart from the method of completing the square, teachers may introduce other algebraic methods. For instance, teachers may guide the more able students to recognise, from the characteristics of the graphs of quadratic functions, that the x-coordinate of the vertex of $y = x^2 - 2x$ is $\frac{\alpha + \beta}{2} = 1$. By substituting $x = 1$, they can obtain the minimum value of the function, i.e. $1^2 - 2(1) = -1$. Using differentiation to find maximum and minimum values is **not required** in the Compulsory Part, and belongs to a Learning Objective of the Learning Unit "Applications of Differentiation" of Module 1 or Module 2 of the Extended Part.

Teachers may lead students to further explore other relations between the coefficients of the quadratic functions and the corresponding graphs. For example, to determine, from the sign of the value of $\alpha\beta$, whether the two x-intercepts are on the same side or opposite sides of

the y-axis; to determine whether the values of a, b and c are 0, positive, or negative from the graph of the quadratic function $y = ax^2 + bx + c$.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
3. Exponential and logarithmic functions	3.1 understand the definitions of rational indices 3.2 understand the laws of rational indices 3.3 understand the definition and properties of logarithms (including the change of base) 3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs 3.5 solve exponential equations and logarithmic equations 3.6 appreciate the applications of logarithms in real-life situations 3.7 appreciate the development of the concepts of logarithms	16

Explanatory Notes:

In Learning Unit 10 "Laws of Integral indices" at KS3, student understood the concepts of integral indices, including the definitions of a^n, a^0 and a^{-n} (where n is a positive integer). In this Learning Unit, students are required to understand how the definition of indices is extended from integral indices to rational ones so that the laws of indices will still hold for most cases. Students are required to understand the definitions of rational indices: $\sqrt[n]{a}$, $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$, where a is a positive real number, m is an integer and n is a positive integer. Though students at KS3 learnt how to evaluate simple expressions $\sqrt[n]{a}$ (e.g. $\sqrt[3]{-8}$), when n is an odd and a is negative, the written form $(-8)^{\frac{1}{3}}$ should be avoided. Furthermore, students are required to understand that the laws of indices do not hold when a is negative.

The laws of rational indices in Learning Objective 3.2 include:

- $a^p a^q = a^{p+q}$

- $\dfrac{a^p}{a^q} = a^{p-q}$
- $(a^p)^q = a^{pq}$
- $a^p b^p = (ab)^p$
- $\dfrac{a^p}{b^p} = \left(\dfrac{a}{b}\right)^p$

The properties of logarithms in Learning Objective 3.3 include:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a MN = \log_a M + \log_a N$
- $\log_a \dfrac{M}{N} = \log_a M - \log_a N$
- $\log_a M^k = k \log_a M$
- $\log_b N = \dfrac{\log_a N}{\log_a b}$

When studying the laws of rational indices and the properties of logarithms, students are required to clearly understand the necessary conditions for the assertion. For example, when $\log_a 1 = 0$, a must be positive and $a \neq 1$.

Once students understand the formula for the change of base, they can make use of calculators to obtain the value of any logarithm (e.g. $\log_2 3$). However, the study of natural logarithm, a Learning Objective in the Extended Part, is **not required** in the Compulsory Part.

Although indices in Learning Objective 3.2 are confined to rational numbers, students are required to recognise that the definition of real indices is a further extension of rational indices. The details of extension are **not required** in the Curriculum, but it may be an interesting topic for further exploration. On the other hand, students should be aware that the domain of exponential functions is the set of real numbers, while the domain of logarithmic functions is the set of positive real numbers. The domain of the latter is different from that of the quadratic functions with which students are familiar. Teachers may guide their students to discuss the differences between the graphs of the exponential

functions (and the logarithmic functions) for $a > 1$ or $0 < a < 1$. Students are required to recognise that when $a > 1$（or $0 < a < 1$）and the real number x increases, the function $f(x) = a^x$ and $f(x) = \log_a x$ (where $x > 0$) increases (or decreases), and the graphs of $y = a^x$ and $y = \log_a x$ (where $x > 0$) are symmetric to $y = x$. Since the concept of inverse functions is **not required** in the Curriculum, the term "inverse function" **need not be** introduced when discussing the symmetric relation between the graphs of $y = a^x$ and $y = \log_a x$. When students understood the relation between exponential functions and logarithmic functions, they can deduce the corresponding features of logarithmic functions from those of exponential functions. Through various examples such as $y = 2^x$, $y = x^2$ and $y = x^3$, teachers may discuss the rate of increasing/decreasing of the functions.

Learning Objective 3.5 mainly covers simple equations such as $2^x = 5$ or $\log_3(x+4) = 2$. The equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log_2(x+1) + \log_2(x-3) = 3$ are tackled in Learning Objective 5.3.

In this Learning Unit, students are required to understand the concepts of exponential and logarithmic functions. Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels, to let students appreciate the applications of logarithms in real-life situations and understand why logarithms have to be involved in these formulae, to enhance students' ability and confidence in applying mathematical knowledge or skills as well as processes of mathematical modelling in real-life situations or STEM related scenarios. Students may try to use different formulae to calculate the earthquake intensity, but memorisation of such formulae is **not required**.

By organising various activities for students to experience the difficulties in performing complicated operations without any calculation tools, teachers may guide students to discuss the topics such as the history of development of the concepts of logarithm and its applications to the design of some calculation tools in the past, such as slide rules and the logarithmic table.

Students studying Module 1 or Module 2 of the Extended Part will continue the study of other properties and applications of exponential functions and logarithmic functions.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
4. More about polynomials	4.1 perform division of polynomials 4.2 understand the remainder theorem 4.3 understand the factor theorem 4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials 4.5 perform addition, subtraction, multiplication and division of rational functions	14

Explanatory Notes:

In Learning Unit 11 "Polynomials" at KS3, students learnt addition, subtraction and multiplication of polynomials, and factorisation of simple polynomials. In Learning Objective 13.1 at KS3, they also learnt the operations of algebraic fractions having denominators as the product of linear factors. In this Learning Unit, students are required to further their study on the division of polynomials, more complicated problems on factorisation of polynomials, and manipulations on rational functions with denominators of degree higher than one. Addition, subtraction, multiplication and division of rational functions can be considered as mixed arithmetic operations of polynomials.

Long division is a standard procedure for performing the division of polynomials. Teachers may introduce other procedures, such as synthetic division. It should however be noted that complicated computations of polynomials are **not** the objective of the Curriculum.

Students are required to understand the significance of the division algorithm $f(x) = g(x)Q(x) + R(x)$ and understand how to deduce the remainder theorem under the condition $g(x) = ax + b$. The factor theorem may be considered as a special case of the remainder theorem. When using the factor theorem to factorise polynomials, teachers may lead students to appreciate its usefulness (e.g. to solve some equations with degree greater than two) and observe its limitations (e.g. not all higher degree equations can be solved efficiently by this method).

Students are required to use the factor theorem to decompose polynomials such as $x^3 \pm a^3$, but memorisation of identities involving factorisation of $x^3 \pm a^3$ is **not required**.

To articulate with the Extended Part, the term “rational function” is used in this Learning Unit to replace the term “algebraic fraction” used at KS3. Nevertheless, the study of the properties of rational functions is **not required** in the Compulsory Part.

The concepts of the greatest common divisor and the least common multiple play a crucial role in performing addition, subtraction, multiplication, division and simplification of rational functions. In this connection, students are required to understand these two concepts comprehensively. Teachers may review with students the concepts of greatest common divisor and least common multiple of numbers before introducing the similar concepts for polynomials. When teaching the greatest common divisor (sometimes called “the highest common divisor”) and the least common multiple (sometimes called “the lowest common multiple”), teachers should feel free to choose any commonly used short form, such as “H.C.F.”, “gcd” or “(a , b)”, to denote the greatest common divisor of a and b. To facilitate students to read other reference books, teachers should introduce other commonly used short forms in addition to the chosen one. When performing the operations of addition, subtraction, multiplication and division of rational functions, the number of variables should not be more than two to avoid over-complicated computations. Division of rational functions includes computations like “$\frac{1}{x^2 - y^2}$ is divided by $\frac{1}{x + y}$”, but it should be noted that complicated computations are **not** the objective of this Learning Unit.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$ 5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic 5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations 5.4 solve problems involving equations which can be transformed into quadratic equations	10

Explanatory Notes:

In Learning Unit 9 "Linear equations in two unknowns" at KS3, students learnt how to formulate simultaneous linear equations in two unknowns, and use the algebraic method and the graphical method to solve simultaneous linear equations in two unknowns. In this Learning Unit, one of the simultaneous equations will be quadratic, and the problems will be extended from those in Learning Objective 1.6 to the problems involving equations which can be transformed into quadratic equations. Students should note the differences the number of solutions of simultaneous equations, between one linear and one quadratic equation, and simultaneous linear equations.

Students are **not required** to study the graphs of quadratic equations such as $x = y^2 - 3y + 6$ or $xy + y^2 = 1$ in the Compulsory Part. Hence, when using the graphical method to solve simultaneous equations in Learning Objective 5.1, the quadratic equations in two unknowns should be confined to the form $y = ax^2 + bx + c$. However, there are no such restrictions when using the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic.

Students are required to note whether the solutions are reasonable or not when solving equations which can be transformed into quadratic equations. For example, when solving the equation $2\sin^2\theta - 5\sin\theta + 2 = 0$, students are required to note that $\sin\theta = 2$ does not

give a real solution. Moreover, the solutions of equations involving trigonometric functions are confined to the interval from 0° to 360° (see Learning Objective 14.2).

In order to arouse students' interest, teachers may select the problems related to students' daily-life experience when teaching Learning Objective 5.4. Through discussion, teachers may lead students to discover the diversity of methods of solving equations. Students should explore various problem solving strategies and be able to select the most appropriate one.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
6. Variations	6.1 understand direct variations and inverse variations, and their applications to solving real-life problems 6.2 understand the graphs of direct and inverse variations 6.3 understand joint and partial variations, and their applications to solving real-life problems	7

Explanatory Notes:

In Learning Unit 6 “Rates, ratios and proportions” at KS3, students recognised direct proportion, inverse proportion and some relations between quantities. In Learning Unit 2 “Functions and graphs”, they are required to learn the basic concept of functions and thus recognised that relations may hold between variables. In this Learning Unit, different forms of the relations between variables will be further elaborated, including direct variation, inverse variation, joint variation, partial variation, and their daily-life applications. Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as applying variations to solve problems about mathematical modelling, to enhance students’ ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios.

Although students at KS3 learnt direct proportion and inverse proportion, and how to solve problems involving proportions, they might not understand how to deal with direct proportion and inverse proportion by using the concept of variations, i.e. direct variation $y = kx$ and inverse variation $y = \frac{k}{x}$. When discussing the graphs of the direct variation $y = kx$ and inverse variation $y = \frac{k}{x}$, teachers may remind their students that the domains of the two functions may contain negative real numbers.

When solving problems involving partial variation, students usually have to solve quadratic or higher degree equations. Therefore, students should study Learning Unit 1 “Quadratic equation in one unknown” and Learning Unit 4 “More about polynomials” before this Learning Unit.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
7. Arithmetic and geometric sequences and their summations	7.1 understand the concept and the properties of arithmetic sequences 7.2 understand the general term of an arithmetic sequence 7.3 understand the concept and the properties of geometric sequences 7.4 understand the general term of a geometric sequence 7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems 7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems 7.7 solve related real-life problems	17

Explanatory Notes:

In Learning Objective 7.3 at KS3, students learnt the concept of sequences of numbers when studying number patterns. In this Learning Unit, they are required to further understand the concepts, properties, the general formulae of summations of the two common sequences (arithmetic sequences and geometric sequences) and their applications.

Teachers may guide students to investigate the following property: for any real numbers a and k, the sequences $T_1 + a,\ T_2 + a,\ T_3 + a, \ldots$ and $kT_1,\ kT_2,\ kT_3, \ldots$ must be arithmetic if $T_1,\ T_2,\ T_3, \ldots$ is an arithmetic sequence, and hence the sequence $kT_1 + a,\ kT_2 + a,\ kT_3 + a, \ldots$ is also arithmetic. Moreover, students should be able to discover that, when the sequence $T_1,\ T_2,\ T_3, \ldots$ is geometric and $k \neq 0$, the sequence $kT_1,\ kT_2,\ kT_3, \ldots$ is geometric. The discussion on whether the sequence 0, 0, 0,… is geometric is **not required** in the Compulsory Part.

Teachers may introduce examples such as $a_n = a_{n-1}+k$, $a_n = ra_{n-1}$, $a_{n+2} = a_{n+1}+a_n$ to let students recognise how to represent a sequence by the relation between its terms.

When solving geometrical problems related to the sum of arithmetic or geometric sequences, students may sometimes need to apply the knowledge of Learning Unit 14 “More about Trigonometry”. Teachers should thus note the order of learning and teaching of these related Learning Units.

The terms “arithmetic mean” and “geometric mean” **need not be** introduced when the property of the arithmetic sequence $T_n = ½ (T_{n–1} + T_{n+1})$ and the property of the geometric sequence $T_n^2 = T_{n-1} \times T_{n+1}$ are being discussed.

Learning Objective 7.5 may involve geometric problems about finding the sum to a finite number of terms of an arithmetic sequence or a geometric sequence. Learning Objective 7.6 may involve the geometric problems about finding the sum to infinity for a geometric sequence. Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as problems related to interest, growth or depreciation in Learning Objective 7.7, to enhance students’ ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown 8.2 solve quadratic inequalities in one unknown by the graphical method 8.3 solve quadratic inequalities in one unknown by the algebraic method 8.4 represent the graphs of linear inequalities in two unknowns in the rectangular coordinate plane 8.5 solve systems of linear inequalities in two unknowns 8.6 solve linear programming problems	16

Explanatory Notes:

In Learning Unit 14 “Linear Inequalities in one unknown” at KS3, students learnt how to solve linear inequalities in one unknown and to represent the solution on the number line. In this Learning Unit, students are required to solve compound linear inequalities in one unknown involving logical connectives “and” or “or”, quadratic inequalities in one unknown by using the graphical method and algebraic method, linear inequalities in two unknowns by the graphical method, and linear programming problems.

In Learning Objective 8.1, teachers should let students discover that, in solving compound linear inequalities in one unknown, the solutions of the two linear inequalities may, in general, divide the number line into three regions. The solutions of the compound inequalities can thus be obtained. Students are required to note that the solutions of “$x > 3$ and $x < 5$” and “$x > 3$ or $x < 5$” are different.

In Learning Unit 3S2 "Triangles" at KS1, students recognised that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side, but they do not know how to express these relations by inequalities. In this Learning Unit, students are required to consolidate their understanding of solving compound linear inequalities in one unknown by solving the problems of simple triangle inequalities.

When students are familiar with the features of quadratic graphs, they should be able to solve quadratic inequalities in one unknown by the graphical method. They are also

required to use the algebraic method to solve quadratic inequalities in one unknown.

In Learning Unit 9 “Linear equations in two unknowns” at KS3, students learnt how to draw the graphs of linear equations on the coordinate plane. Students understood that the graph of a linear equation in two unknowns ($ax+by+c = 0$, where a and b are not all zero) is a straight line; all the coordinates of points lying on the straight line satisfy the linear equation in two unknowns; all the coordinates of points not lying on the straight line do not satisfy the linear equation in two unknowns. Teachers may thus further discuss with students the solution of linear inequalities in two unknowns. When solving linear inequalities in two unknowns, students are required to determine which region(s) bounded by the straight lines will correspond to the solutions of the linear inequalities. The method of testing values is one that can be easily mastered by students. Teachers may use information technology, such as dynamic geometry software, to discuss with students which points on the plane do/do not satisfy the linear inequalities in two unknowns.

In Learning Objective 8.5, students are required to use the graphical method to solve systems of linear inequalities in two unknowns. However, using the algebraic method to solve systems of linear inequalities in two unknowns is **not required** in the Curriculum.

When teaching Learning Objective 8.6, teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as using linear programming to solve optimisation problems in mathematical modelling, to enhance students’ ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios. Teachers should also provide their students with opportunities to discuss the problem solving strategies.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions 9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$ 9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$ 9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives	11

Explanatory Notes:

In Learning Unit 2, "Functions and graphs", students had a preliminary recognition of the concept of function. In this Learning Unit, they are required to compare the graphs of different functions, to solve equations and inequalities by using the graphical method, and to understand the concept of transformations of function.

Some students may regard expressions like $y = 4$ as merely solutions of some equations and even do not know how to draw the graph of $y = 4$ on the coordinate plane. Teachers should introduce the concept of "constant function" to their students. The students who do not study Learning Unit 3 "Exponential and logarithmic functions" (Non-foundation Topics) are **not required** to discuss the graphs of exponential functions and logarithmic functions. When comparing the graphs of different functions, students are required to compare their domains, existence of maximum or minimum values, symmetry and periodicity.

In Learning Objective 1.3, students learnt how to solve the quadratic equation $ax^2 + bx + c = 0$ by reading the x-intercept(s) of the graph of $y = ax^2 + bx + c$. In Learning Objective 9.2, students are required to use the parabola $y = ax^2 + bx + c$ and the straight line $y = k$ to solve the quadratic equation $ax^2 + bx + c = k$. For example, from the graph of $y = 2x^2 - 5x - 1$, students should be able to solve not only the quadratic equation $2x^2 - 5x - 1 = 0$, but also the quadratic equation $2x^2 - 5x - 4 = 0$ by making use of the

graph of the straight line $y = 3$. However, using the graph of $y = 2x^2 - 5x - 1$ to solve quadratic equations such as $2x^2 - 6x + 1 = 0$ is **not required** in the Curriculum.

Students are required to extend the method learnt in Learning Objective 8.2 to the functions other than quadratic functions. In other words, even though the function $f(x)$ is not of the form $ax^2 + bx + c$, students are still required to know how to read the solution(s) of $f(x) = k$ from the graph of $y = f(x)$. They should be able to make use of the graph of $y = f(x)$ and the straight line $y = k$ to obtain the solutions of inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$.

When exploring the transformations of functions, students may first observe the change of the relation between the independent and dependent variables by making use of the tabular form, and then use graphing software to compare the changes of the graphs of the functions after transformations. Teachers may encourage their students to adopt the approach of mathematical thinking from particular to general in obtaining the relations between the graphs of $y = f(x) + k$, $f(x + k)$, $k f(x)$, $f(kx)$ and the original function $y = f(x)$. During discussions, teachers should ask their students to apply the concepts and terminologies, e.g. translation and reflection, learnt in Learning Unit 26 "Rectangular coordinate system" at KS3, to describe the changes of the graphs. When discussing with students the transformation involving dilation such as $k f(x)$ or $f(kx)$, teachers should note that dilation is **not required** in the curriculum at KS3. On the other hand, teachers should guide their students to discuss the changes of functions in their algebraic forms resulting from the transformations of their graphs. For example, students are required to understand that the reflection of the graph of $y = f(x)$ along the x-axis can be represented by $y = -f(x)$. The changes of the algebraic forms of the functions are confined to $f(x) + k$, $f(x + k)$, $k f(x)$, $f(kx)$ or their combinations. That is to say, the rotation of the graphs of functions is **not required** in the Curriculum.

The concept of composition of functions is **not required** in the Curriculum. However, the graph of functions such as $y = -x^2 + 4$ can be considered as a reflection of the graph of $y = x^2$ in the x-axis, and a subsequent upward translation of 4 units along the y-axis.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
10. Equations of straight lines	10.1 understand the equation of a straight line 10.2 understand the possible intersection of two straight lines	7

Explanatory Notes:

In Learning Unit 9 “Linear equations in two unknowns” at KS3, students understood the graph of a linear equation in two unknowns. In Learning Objective 10.1, students are required to find the equation of a straight line from given conditions, such as:

- the coordinates of any two points on the straight line
- the slope of the straight line and the coordinates of a point on it
- the slope and the *y*-intercept of the straight line

In Learning Unit 12 “Locus”, students should have an initial grasp of the concept of locus and solve simple problems on loci. Teachers may guide students to understand the relation between an equation and its graph from the perspective of locus, then to formulate the equation of a given graph, and understand the properties of the graph from its equation.

Based on the abilities and needs of their students, teachers may decide if the terms “Two-Point Form”, “Point-Slope Form”, “Slope-Intercept Form”, etc. would be introduced or not. The conversions among different forms of linear equations are **not** objectives of the Curriculum. However, given the equation of a straight line, students are required to describe the features, including:

- the slope
- the intercepts with the axes
- whether it passes through a given point

In Learning Objective 10.1, students are required to recognise the relation between slope and inclination. Since the inclination may involve the tangent of an obtuse angle, teachers should note that students must have the pre-requisite knowledge in Learning Unit 14 "More about trigonometry" before learning this relation. The equation of a straight line in normal form is **not required** in the Curriculum.

In Learning Unit 9 “Linear equations in two unknowns", students are required to solve simultaneous linear equations in two unknowns. Students are required to recognise how to

tackle those simultaneous equations with no solutions, only one solution, and more than one solution by algebraic methods. Hence, students can go further in Learning Unit 10.2 to tackle those simultaneous equations with no solutions, and more than one solution by the graphical method.

The content of this Learning Unit may be used as the fundamental basis of other senior secondary level subjects, such as Physics and Economics. It is hence recommended that teachers should teach this Learning Unit in the first semester of Secondary Four for achieving alignment with other subjects smoothly. Teachers may use real-life examples or related learning elements in Science Education or Technology Education KLAs to enhance students' ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
11. Basic properties of circles	11.1 understand the properties of chords and arcs of a circle 11.2 understand the angle properties of a circle 11.3 understand the properties of a cyclic quadrilateral 11.4 understand the tests for concyclic points and cyclic quadrilaterals 11.5 understand the properties of tangents to a circle and angles in the alternate segments 11.6 use the basic properties of circles to perform simple geometric proofs	23

Explanatory Notes:

In Learning Units 19 – 24 at KS3, students learnt the basic concept and proof of geometry. The content were mainly about rectilinear figures. In the Compulsory Part, the scope of study is extended to circles, and the learning process may still follow the same sequence of approaches, i.e. from intuitive to deductive. For example, to develop students' exploratory spirit and the capability to reason logically, teachers may ask their students to explore the basic geometric properties of circles by using dynamic geometry software, and subsequently to attempt to prove their conjectures under teachers' guidance. Teachers may refer to the C&A Guide (pp 99 – 100) for employing an inquiry approach to teach the properties of a cyclic quadrilateral. In this Learning Unit, teachers should discuss the concepts of tangency, incircle and circumcircle to articulate with the learning of Learning Unit 24 "Centres of triangles" at KS3.

In Learning Objectives 11.1– 11.3, students are required to understand the basic properties of circles. Students not only have to know the properties and to use the properties to perform calculations, but also have to understand the justifications or proofs for these properties. However, using these properties to perform other simple geometric proofs belongs to the Non-foundation Topics.

The property that the length of an arc of a circle is proportional to the angle subtended at the centre was discussed in Learning Unit 16.1 at KS3, but teachers may remind their students that the length of a chord of a circle is not proportional to the angle subtended at

the centre.

In Learning Objective 11.1, students are required to understand the following properties of chords and arcs of a circle:

- the chords of equal arcs are equal
- equal chords cut off equal arcs
- the perpendicular from the centre to a chord bisects the chord
- the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord
- the perpendicular bisector of a chord passes through the centre
- equal chords are equidistant from the centre
- chords equidistant from the centre are equal

Students are required to understand that there is one and only one circle passing through given three non-collinear points.

In Learning Objective 11.2, students are required to understand the following angle properties of a circle:

- the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference
- angles in the same segment are equal
- the arcs are proportional to their corresponding angles at the circumference
- the angle in a semi-circle is a right angle
- if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter

In Learning Objective 11.3, students are required to understand the following properties of a cyclic quadrilateral:

- the opposite angles of a cyclic quadrilateral are supplementary
- an exterior angle of a cyclic quadrilateral equals its interior opposite angle

In Learning Objective 11.4 (Non-foundation Topics), students are required to understand the following tests for concyclic points and cyclic quadrilaterals:

- if A and D are two points on the same side of the line BC and $\angle BAC = \angle BDC$, then A , B , C and D are concyclic
- if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic
- if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic

In Learning Objective 11.5 (Non-foundation Topics), students are required to understand the following properties of tangents to a circle and angles in the alternate segments:

- a tangent to a circle is perpendicular to the radius through the point of contact
- the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle
- the perpendicular to a tangent at its point of contact passes through the centre of the circle
- if two tangents are drawn to a circle from an external point, then:
  - the distances from the external point to the points of contact are equal
  - the tangents subtend equal angles at the centre
  - the straight line joining the centre to the external point bisects the angle between the tangents
- if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment
- if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle

In Learning Objective 11.6 (Non-foundation Topics), students are required to use the basic properties of circles to perform simple geometric proofs. The problems involved may not be restricted to the contents of this Learning Unit. According to students' abilities, teachers may add some problems involving knowledge of geometry such as quadrilaterals, centres of triangles, etc. which students learnt at KS3.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
12. Loci	12.1 understand the concept of loci 12.2 describe and sketch the locus of points satisfying given conditions 12.3 describe the locus of points with algebraic equations	6

Explanatory Notes:

To introduce the concept of loci, teachers may make use of daily examples such as long exposure photographs of headlights of moving cars and star trails. Students may make use of dynamic geometry software to explore the locus of a point moving under the given condition, but they are required to understand that a locus does not necessarily involve the movement of points. For example, all points which are at a fixed distance from a fixed point constitute the locus of a circle.

In Learning Objective 12.2, students are required to describe a locus in words, and to sketch its graph. In Learning Objective 12.3, students are required to use algebraic equations to represent locus of points. When finding the equations of loci, students are also required to make use of the knowledge learnt at KS3, e.g. the related knowledge of using the distance formula in Learning Unit 26 “Rectangular coordinate system” at KS3 to find the equation of the locus of points maintaining an equal distance from two given points.

In Learning Objective 12.2, students are required to describe and sketch the following locus of points satisfying given conditions:

- maintaining a fixed distance from a fixed point
- maintaining an equal distance from two given points
- maintaining a fixed distance from a line
- maintaining an equal distance from two parallel lines
- maintaining an equal distance from two intersecting lines

In this Learning Unit, students are required to find the equations of loci of straight lines, circles, and parabolas of the form $y = ax^2 + bx + c$ under given conditions. The equations of straight lines and circles will be discussed in detail in Learning Unit 10 “Equations of straight lines” and Learning Unit 13 “Equations of circles”.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
13. Equations of circles	13.1 understand the equation of a circle 13.2 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle	7

Explanatory Notes:

In this Learning Unit, when handling questions related to the equations of circles, the learning content covered in Learning Unit 11 "Basic properties of Circles" may be involved.

Students are required to find the equation of a circle from given conditions such as:

- the coordinates of the centre and the radius of the circle
- the coordinates of any three points on the circle

Students are required to describe the features of a circle from its equation. The features include:

- the centre
- the radius
- whether a given point lies inside, outside or on the circle

Students taking Learning Objective 5.2 (Non-foundation Topics) have an understanding of using algebraic methods to solve simultaneous equations in two unknowns, one linear and one quadratic. In Learning Objective 13.2 (Non-foundation Topics), students may apply this knowledge to deduce that there are three possible cases to describe the intersections of a circle and a straight line. As a result, they may use the discriminant of a quadratic equation to determine the number of points of intersection of a straight line and a circle, and to find the equations of tangents to the circle. On the other hand, teachers may guide students to use plane geometry to prove that no straight line can cut a circle at more than two points.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
14. More about trigonometry	14.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity 14.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from 0° to 360°) and other trigonometric equations (solutions in the interval from 0° to 360°) 14.3 understand the formula $\frac{1}{2} ab \sin C$ for areas of triangles 14.4 understand the sine and cosine formulae 14.5 understand Heron's formula 14.6 understand the concept of projection 14.7 understand the angle between a line and a plane, and the angle between 2 planes 14.8 understand the theorem of three perpendiculars 14.9 solve related 2-dimensional and 3-dimensional problems	25

Explanatory Notes:

Students understood the sine, cosine and tangent ratios for an acute angle in a right-angled triangle in Learning Unit 27 "Trigonometry" at KS3. Teachers may now use a unit circle in the rectangular coordinate plane to define trigonometric functions, and to introduce the concepts of positive angles and negative angles. Students should be able to find the maximum and minimum values of trigonometric functions. They should also be able to obtain the periodicity of trigonometric functions from their graphs, and to simplify sine, cosine and tangent expressions involving $-\theta$, $90° \pm \theta$, $180° \pm \theta$ …etc., according to the periodicity of the functions. Radian measure, a Learning Objective in Module 2 of the Extended Part, is **not required** in the Compulsory Part.

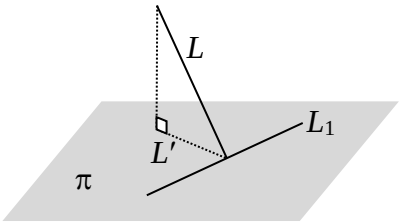
In Learning Objective 14.2, solving the trigonometric equations $a\sin\theta = b$, $a\cos\theta = b$ and $a\tan\theta = b$ belongs to the Foundation Topics, while solving other trigonometric equations such as $\sin 2\theta = 0.5$, $\sin\theta = 2\cos\theta$ and $\tan\theta - \cos(90° - \theta) = 0$ belongs to

the Non-foundation Topics. Moreover, solving the equations that can be transformed into quadratic equations, such as $6\sin^2\theta + 5\sin\theta + 1 = 0$ and $\tan\theta = \cos\theta$, belongs to the Non-foundation Topics (see Learning Objective 5.3).

In Learning Unit 18 "Mensuration" at KS3, students recognised the projection of a point on a plane and the concept of height of a 3-D figure. Students recognised the definition of a straight line being perpendicular to a plane, which is: "The straight line is perpendicular to any straight lines on the plane passing through the intersecting point of the original straight line and the plane". Students also recognised the condition for a straight line being perpendicular to a plane, which is "a straight line is perpendicular to two straight lines on a plane passing through the intersecting point of the original straight line and the plane". The concept of projection would further be discussed in Learning Objective 14.6 of the Compulsory Part, including the projection of a straight line on a plane. These concepts help students understand Learning Objective 14.7 and 14.8. The concept of inclination/angle of inclination is required in Learning Objective 14.7. Teachers may introduce the concept of the line of greatest slope.

As the theorem of three perpendiculars is involved to solve a variety of 3-dimensional problems, students are required to understand the following theorem of three perpendiculars and their proofs in Learning Objective 14.8:

The projection of a straight line L on a plane π is a straight line L' , and L and L' intersect a straight line L_1 on π at the same point. If L' is perpendicular to L_1, then L is perpendicular to L_1; vice versa.



In Learning Objective 14.9, students are required to solve related 3-dimensional problems, including the angle of intersection between two straight lines, the angle between a line and a plane, the distance between two points, the distance between a point and a line, and the distance between a point and a plane.

Learning Unit	Learning Objective	Time
Data Handling Strand		
15. Permutations and combinations	15.1 understand the addition rule and multiplication rule in the counting principle 15.2 understand the concept and notation of permutation 15.3 solve problems on the permutation of distinct objects without repetition 15.4 understand the concept and notation of combination 15.5 solve problems on the combination of distinct objects without repetition	11

Explanatory Notes:

Through different key stages, students acquired an intuitive idea on counting. In this Learning Unit, students are required to have an in-depth understanding on the basic counting principle, including knowing when to apply the addition rule and the multiplication rule. Besides, students should also be able to solve more complicated real-life problems through permutation and combination.

When explicating the concepts of permutation and combination, teachers may feel free to choose any common notations, such as P_r^n, ${}_nP_r$, ${}^nP_r$ and C_r^n, ${}_nC_r$, ${}^nC_r$, $\binom{n}{r}$. To facilitate students to read reference books, teachers should introduce other commonly used notations in addition to the chosen ones.

In this Learning Unit, students are required to understand the differences between permutation and combination. They are also required to understand the relations $C_r^n = \frac{P_r^n}{r!}$ and $C_r^n = C_{n-r}^n$. More complicated properties of permutation or combination, such as $C_r^n + C_{r-1}^n = C_r^{n+1}$, are **not required** in the Compulsory Part.

There is a wide variety of problems on permutation and combination. Only solving simple problems on permutation and combination, such as “permutation of objects in which three particular objects are put next to each other”, are required in this Learning Unit. Solving problems involving circular permutation, and permutation or combination of identical

objects or distinct objects with repetitions, are **not required** in the Curriculum.

Learning Unit	Learning Objective	Time
Data Handling Strand		
16. More about probability	16.1 recognise the notation of set language including union, intersection and complement 16.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events 16.3 understand the multiplication law of probability and the concept of independent events 16.4 recognise the concept and notation of conditional probability 16.5 use permutation and combination to solve problems related to probability	10

Explanatory Notes:

Having learnt the basic concept of probability and how to use counting methods to calculate probabilities in Learning Unit 31 "Probability" at KS3, students are required, in the Compulsory Part, to use the addition law and the multiplication law of probability to solve more complicated probability problems. Further problems on probability will be treated in Module 1 of the Extended Part.

In order to express the relation between different events (including mutually exclusive, complementary and independent events) and the addition law and multiplication law of probability, the events in this Learning Unit are all expressed in set notations. In this connection, before learning the contents of Learning Objectives 16.2 – 16.5, students are required to recognise the basic concepts of sets, which include expressing sets by listing the elements, using descriptions to characterise the elements and Venn diagrams. Students are also required to recognise the concepts and notations of empty set, universal set, union, intersection and complement which often appear in probability problems, but the rigorous definitions of these concepts and rules of set operations, such as De Morgan's Law, are **not required** in the Compulsory Part.

In Learning Objective 16.2, students are required to understand the addition law of probability, i.e. "$P\ (A \cup B) = P\ (A) + P\ (B) - P\ (A \cap B)$".

In Learning Objective 16.3, students are required to understand the multiplication law of probability, i.e. "$P(A \cap B) = P(A) \times P(B)$, where A and B are independent events". Students are also required to understand that "$P(A \cap B) = P(A) \times P(B)$" may be regarded as the definition of independent events. Students should not determine whether two events are independent by intuition.

When applying the addition rule and multiplication rule of probability, students are required to understand how the special cases of the rules under particular conditions, say when A and B are mutually exclusive events. Furthermore, when the multiplication rule is being introduced, students are required to recognise the concept and the notation of conditional probability, and be able to solve problems related to simple conditional probability. However, Bayes' Theorem is **not required** in the Compulsory Part, and will be treated in Module 1 of the Extended Part.

In Learning Objective 16.4, students are required to recognise the rule "$P(A \cap B) = P(A) \times P(B \mid A)$".

Students who learnt Learning Unit 15 "Permutation and Combination" (Non-foundation Topics) are required to apply the techniques of counting to solve problems related to probability.

Learning Unit	Learning Objective	Time
Data Handling Strand		
17. Measures of dispersion	17.1 understand the concept of dispersion 17.2 understand the concepts of range and inter-quartile range 17.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data 17.4 understand the concept of standard deviation for both grouped and ungrouped data sets 17.5 compare the dispersions of different sets of data using appropriate measures 17.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution 17.7 understand the effect of the following operations on the dispersion of the data: (i) adding a common constant to each item of the set of data (ii) multiplying each item of the set of data by a common constant	13

Explanatory Notes:

At KS2, students learnt a simple way to measure the central tendency of a set of discrete data – average. At KS3, they learnt other ways of measuring the central tendency for both ungrouped and grouped data. In the Compulsory Part, they are required to know further that, in many cases, only central tendency is not enough for describing the distribution of a set of data. Students are required to understand the concepts of dispersion, range and inter-quartile range, and they should be able to construct and interpret the box-and-whisker diagrams, sometimes called “boxplots”. Given a set of ungrouped or grouped data, students should be able to find its standard deviation, and understand its meaning. Moreover, they should be able to choose an appropriate measure to compare the dispersions of different sets of data.

As “variance” is a commonly used term, students are required to recognise this term and that variance equals to the square of standard deviation. Further calculations involving variance will be studied in Module 1 of the Extended Part. In the Compulsory Part, the formula for standard deviation is confined to population standard deviation only, i.e. $\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$. The formula for the estimate of the standard deviation of the population from which a sample has been taken will be treated in Module 1 of the Extended Part.

Students who learnt Learning Objective 17.6 (Non-foundation Topics) are required to understand the simple applications of standard deviation on real-life problems involving standard scores and the normal distribution. Teachers may use real-life examples or related learning elements in Science Education or Technology Education KLAs to enhance students’ ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios. During the calculation process, students are **not required** to look up the table of normal distribution or memorise the percentages of data lying within 1, 2, or 3 standard deviations from the mean. In Learning Objective 17.7 (Non-foundation Topics), students are required to explore how the changes of some of the data within the group will affect the dispersion. As a result, students are expected to have a more in-depth understanding of the properties of different statistics.

Learning Unit	Learning Objective	Time
Data Handling Strand		
18. Uses and abuses of statistics	18.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 18.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations 18.3 assess statistical investigations presented in different sources such as news media, research reports, etc.	4

Explanatory Notes:

At primary and junior secondary levels, students had experience in statistical methods, in particular, data collection, presentation and interpretation of statistical diagrams and statistical charts. They had a basic understanding of the concept of statistic. At the senior secondary level, they are required to have a more thorough recognition of real-life statistics.

The concepts of populations and samples may be introduced by quoting several daily examples. Students are required to know why survey sampling is almost inevitable in daily statistics, and recognise different techniques in survey sampling. Regarding sampling techniques, students are required to recognise the basic concepts of both probability sampling and non-probability sampling. However, it should be noted that calculations involving survey sampling, such as the calculation of standard deviations of samples, are **not required** in the Compulsory Part. Since questionnaire is a very commonly used means to collect data, students are required to recognise how the factors such as the types, wording and ordering of questions and response options can influence the reliability and validity of questionnaires when constructing questionnaires.

Students acquainted themselves at KS3 with the uses and abuses of statistical diagrams/charts and measures of central tendency. In the Compulsory Part, students are required to discuss further the uses and abuses of statistical methods in various daily-life activities or investigations. Discussions including the purpose of survey, the sampling method adopted, the way of collecting data and the method of analysis are required. Moreover, students should be able to have a more thorough analysis of statistical investigations presented in different sources such as news media and research reports. Analyses of the sampling method of data collection, design of questionnaires, organisation and presentation of data, statistical analysis and inferences should be included. Students are expected to be able to integrate the statistical knowledge learnt at different stages.

Learning Unit	Learning Objective	Time
Further Learning Unit		
19. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learned in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	14

Explanatory Notes:

This Learning Unit is different from the application topics in other Learning Units. It does not aim at learning some particular mathematical knowledge, but it allows students to appreciate the connections between different areas of mathematics through exploring and solving more sophisticated real-life problems and to develop their ability to integrate and apply what they have learnt in different Learning Units. Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, including mathematical modelling, to enhance students' ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios, such as the examples suggested in the C&A Guide (pp. 38 – 39):

- solve simple financial problems in areas such as taxation and instalment payment
- analyse and interpret data collected in surveys
- explore and interpret graphs related to real-life situations
- explore Ptolemy's Theorem and its applications
- model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k\ a^x$ to linear relations
- explore the relation between the Fibonacci sequence and the Golden Ratio
- explore the Ceva's Theorem and its applications
- analyse mathematical games (e.g. explore the general solution of the water puzzle)

Teachers may, based on their students' abilities and needs, choose other more appropriate topics for their students. Moreover, teachers should allow their students to search the information for clues, explore different strategies and avoid giving too many hints to students.

Learning Unit	Learning Objective	Time
Further Learning Unit		
20. Inquiry and investigation	Through various inquiry activities and mathematical modelling learning activities, construct and apply mathematical knowledge, further improve the ability to inquire, communicate, reason, conceptualise mathematical concepts and apply mathematics	10

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling learning activities. In other words, this is not an independent and isolated Learning Unit. Teachers may make use of the suggested lesson time to engage students in various learning activities, such as activities related to enrichment topics, cross-learning unit activities, and cross-Key Learning Area activities based on mathematical topics. Among the learning activities, teachers should plan about 2–3 hours of mathematical modelling learning activities[1] for students in each school year, enabling them to analyse and make decisions with mathematical thinking when encountering problems in different real-world contexts, and strengthening their ability to apply mathematics to solve STEAM and other real-world problems. Teachers can make good use of Mathematics lessons, lessons designated for project learning or cross disciplinary learning, or other suitable lessons, together with other learning time, such as after-school or post-examination activity periods, to conduct mathematical modelling learning activities. These activities should be regarded as STEAM learning activities of Mathematics.

1. The Process of Mathematical Modelling

Mathematical modelling is the process of applying mathematical knowledge and tools to solve real-world problems. It emphasises on handling problems and analysing data in a mathematical way. Teachers should guide students to **analyse the real-world problem**, clarify key factors of the problem, identify related information/data and the underlying mathematical elements, simplify the problem by putting forward suitable assumptions, convey it into a mathematical problem, and determine relations between relevant quantities and variables. Students apply their prior mathematical knowledge and skills to **formulate a mathematical model** and **solve it for a mathematical solution**, and then **interpret the**

[1] Schools may adopt flexible arrangements at Secondary 6, including reducing the lesson time for mathematical modelling learning activities to 1 to 2 hours, or making own decision on whether to arrange such learning activities, in consideration of school-based situations.

mathematical solution by considering the meaning of the solution in the context of the real-world problem to obtain **a real-world solution**. Subsequently, students **evaluate the mathematical model** through validation and reflection to suggest refinements of the model. Figure 1 and Table 1 show the five steps of the mathematical modelling process and the relevant elements of each step respectively:

Figure 1: The process of mathematical modelling

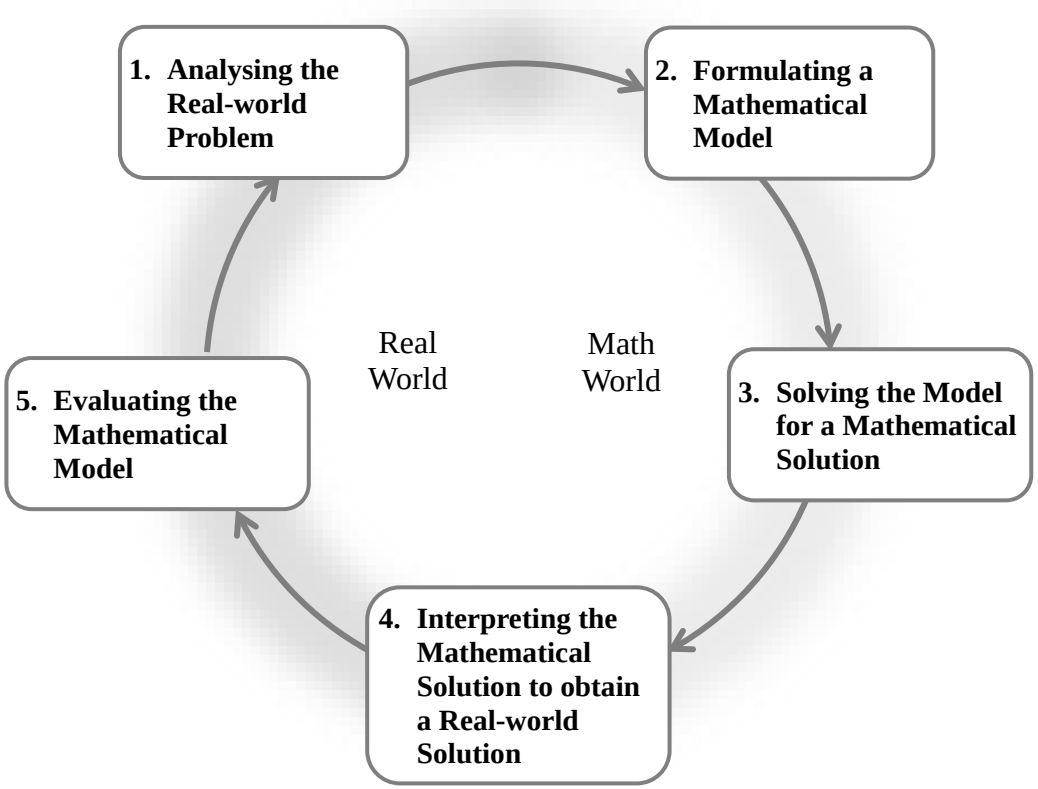


Table 1: Elements of the steps of mathematical modelling

Steps	Elements
1. Analysing the Real-world Problem	• Understand the real-world context • Clarify key factors of the problem • Identify related information/data and mathematical elements of the problem
2. Formulating a Mathematical Model	• Put forward assumptions to simplify the real-world problem • Convey the problem in a mathematical way • Determine relations between key quantities/variables
3. Solving the Model for a Mathematical Solution	• Apply mathematical knowledge, skills and different tools to solve the model
4. Interpreting the Mathematical Solution to obtain a Real-world Solution	• Consider the meaning of the mathematical solution in the context of the real-world problem
5. Evaluating the Mathematical Model	• Validate the model against the real-world context • Reflect on the strengths and limitations of the model • Compare different models • Suggest refinements of the model

The steps and elements of the mathematical modelling process shown on previous page aim to provide teachers with a general reference for designing mathematical modelling learning activities. Teachers may adjust individual steps or omit certain elements to suit the actual teaching circumstances and students' abilities. For example, the element "Comparing different models" in Step 5 "Evaluating the mathematical model" can be omitted if the activity does not involve multiple mathematical models.

In Key Stage 3, students have gained understanding of the five steps of mathematical modelling process and their related elements. Students should realise that, after completing Step 5 "Evaluating the mathematical model", the model could be refined, leading to a new cycle of the mathematical modelling process. In Key Stage 4, students are expected to apply proficiently the modelling cycle comprising the five steps, to formulate several models or a series of models to solve real-world problems. For details of the learning of mathematical modelling at different key stages, please refer to Table 3 "Notes on the Learning and Teaching of Mathematical Modelling across Key Stages".

2．Design of Mathematical Modelling Learning Activities

Problems in mathematical modelling learning activities are different from conventional mathematical word problems. Contexts of word problems are usually simplified. The conditions and information needed for solving the problems are given, complete, and just sufficient. They are mainly designed to consolidate students' understanding of mathematical concepts and their application of specific skills. In contrast, problems in mathematical modelling learning activities are generally un-simplified real-world problems. They are open-ended and do not lead to a specified or explicit solving approach, nor a unique or optimal solution. Furthermore, the relevant conditions and information may not be fully provided. Students need to make suitable assumptions to simplify the problem, as well as collect and select the necessary information/data, to facilitate the formulation of a mathematical model that fits the context.

For students at Key Stage 4, teachers should take note of the following when designing mathematical modelling learning activities:

- The learning activities should be structed in alignment with the five steps of mathematical modelling;
- Apart from selecting real-life contexts familiar to students for designing the problems, teachers may also use those less familiar real-life contexts, such as infectious diseases, land use, and plant growth. Furthermore, relevant information / data related to the problems may be unstructured or scattered in multiple data sources;

- In selecting real-world problems, teachers should ensure they align with students' mathematical knowledge at this Key Stage, such as equations, graphs of functions, variations, inequalities, trigonometry, and probability;
- Teachers may incorporate the application of digital technology into mathematical modelling learning activities to foster students' ability to formulate and solve mathematical models using suitable digital tools (including spreadsheets, graphing software, dynamic geometry software, and artificial intelligence tools, etc.);
- Teachers may refer to the curriculum documents of other subjects, or collaborate with teachers of them to design cross-subject mathematical modelling learning activities, thereby enhancing students' ability to integrate and apply multi-disciplinary knowledge to solve problems;
- Teachers may incorporate elements of values education into mathematical modelling learning activities based on the real-life situations involved, where appropriate;
- Teachers may consider students' abilities and learning needs to conduct mathematical modelling activities in diverse modes such as classroom activities or project learning.

3．Learning and Teaching of Mathematical Modelling

At this Key Stage, students should be able to apply the full mathematical modelling process to solve real-world problems. Throughout the modelling process, teachers should provide timely guidance and support to help students learn to complete the mathematical modelling learning activities independently. Regarding the learning and teaching of mathematical modelling, teachers should take note of the following:

<u>Step 1: Analysing the Real-world Problem</u>

- With reference to the materials provided for mathematical modelling activities and under teachers' guidance, students develop an understanding of more complex real-world problems and identify the factors relevant to the problems. For example, when addressing a problem on the selection of the optimal lunch route, students may consider factors such as travelling route, walking speed and the number of suitable restaurants. Students should determine which factors are mathematically relevant and more significant for solving the problem, thereby identifying key factors involved;
- Students should learn to identify information / data that are useful for solving the problem, and to collect, filter and integrate such information / data in accordance with key factors identified.

<u>Step 2: Formulating the Mathematical Model</u>

- With reference to the materials provided for mathematical modelling activities and under teachers' guidance, students can formulate appropriate assumptions to simplify real

problems. Students may simplify a problem through different assumptions and should consider the relevance and reasonableness of the assumptions;

- When using mathematical representations to express relationships among variables for constructing mathematical models, students may apply different concepts of functions, as well as direct variation, inverse variation and partial variation. They should also use graphs of functions to interpret and understand the models;
- Teachers may guide students taking the Extended Part to apply relevant mathematical knowledge, such as differentiation, matrices and probability distributions, in the formulation of mathematical models;
- Students may apply digital tools during model formulation. For example, students with a preliminary or intuitive understanding of the "line of best fit" may use spreadsheets or other tools incorporating the related function to generate trend lines for data prediction, without requiring knowledge of the underlying mathematical proofs.

<u>Step 3: Solving the Model for a Mathematical Solution</u>

- With teachers' guidance, students may apply their mathematical knowledge flexibly during the problem-solving process and make appropriate use of various digital tools, such as spreadsheets and mathematical graphing software, to obtain solutions to the models, and reduce workload on repetitive calculations and graph plotting. For example, when a model involves more complex functions, students can use digital tools to plot the corresponding graphs, investigate extreme values, or identify approximate values from the graphical representations;
- Students may also present their formulated mathematical models from multiple perspectives by using diagrams, tables, functions and graphs.

<u>Step 4: Interpreting the Mathematical Solution to Obtain a Real-world Solution</u>

- After obtaining solutions of a model, students should consider the meaning of model solutions in the context of the real-world problem, thereby solving the real-world problem. For example, when interpreting the output of a scoring model, students should assess whether a higher score necessarily represents a better outcome. Similarly, when a rainfall prediction model produces a probability of 60%, students should consider whether this implies that it will rain 60% of the time;
- Teachers should pay attention to whether students have independently assessed the reasonableness of model solutions in the context of the real-world problem, and whether they are able to deal appropriately with solutions that are not meaningful in practice, such as rejecting results that fall outside the specified range of values.

Step 5: Evaluating the Mathematical Model

- Students should compare different models from the perspective of modelling method, and analyse their strengths and limitations, such as the predictive accuracy, the reasonableness of assumptions, and computational complexity. Based on this analysis, they should suggest refinements of the models. Teachers may encourage students to express their views, fostering an open and rational atmosphere for discussion, for cultivating their critical thinking skills and creativity.

Teachers should guide students to rerun the modelling process based on their proposed refinements, thereby constructing an optimised mathematical model. Students may revisit Step 1 “Analysing the Real-World Problem”, to update key factors of the problem or collect additional data. Alternatively, they may bypass Step 1 and proceed directly to Step 2 “Constructing the Mathematical Model” to revisit their assumptions and update the mathematical model accordingly.

4. Assessment of Mathematical Modelling

Regarding the assessment of mathematical modelling, teachers should take note of the following:

- Teachers can assess students” performance in each modelling step by reviewing both the process and outcomes of the modelling process, such as what reflected in classroom discussions, oral presentations, worksheets, etc. Teachers can set appropriate assessment criteria to help understand students learning outcomes across different steps of the modelling process, for providing appropriate feedback.
- Teachers can refer to Table 2 as a reference to construct an assessment form for mathematical modelling learning activities, to assess students’ performance in consideration of both modelling process and outcomes. An assessment form may be designed based on the modelling steps and related elements involved in the modelling activity. Teachers can then give comments and suggestions for improvement based on students’ performance in each step and in the whole activity. For instance, when evaluating performance in Step 1 “Analysing the real-world problem”, teachers may assess whether students can clarify key factors of the real-world problem and collect appropriate data. These criteria enable teachers to gain a more concrete understanding of students’ mastery of the various modelling steps.
- Teachers can provide timely oral or written feedback at different stages of mathematical modelling learning activities. Peer assessment can also be conducted to help students understand their performance and improve their learning. Teachers can also use the assessment results to improve the delivery of mathematical modelling learning activities.

Table 2: Example of an Assessment Form

Step	Element*	Assessment / Feedback
1. Analysing the Real-world Problem	- Able to understand the real-world context - Able to clarify key factors of the problem - Able to identify related information/data and mathematical elements of the problem	Teachers may provide comments on student performance at different stages of the activity based on the modelling steps and elements involved. They can also assign marks or grades accompanied by corresponding descriptions.
2. Formulating a Mathematical Model	- Able to put forward assumptions to simplify the real-world problem - Able to convey the problem in a mathematical way - Able to determine relations between key quantities/variables	
3. Solving the Model for a Mathematical Solution	- Able to apply mathematical knowledge, skills and different tools to solve the model	
4. Interpreting the Mathematical Solution to obtain a Real-world Solution	- Able to consider the meaning of the mathematical solution with respect to the real-world problem	
5. Evaluating the Mathematical Mode	- Able to validate the model against the real-world context - Able to reflect on the strengths and limitations of the model - Able to compare different models - Able to suggest refinements of the model	
Overall Feedback:		

* The elements in the table could be adapted to suit the mathematical modelling learning tasks by adding, removing, or further specifying individual elements. For example, if a task incorporates digital tools for computation, the element in Step 3 “Able to apply mathematical knowledge, skills, and tools to find model solutions” may be changed to “Able to utilise spreadsheets to organise data from different scenarios and calculate the required results”.

5. Effective Use of Artificial Intelligence in the Learning and Teaching of Mathematical Modelling

Digital tools have served as important aids in enhancing effectiveness of learning and teaching. Making effective use of artificial intelligence (AI) in the learning and teaching of mathematical modelling can further assist teachers and students in exploring modelling problems efficiently. Teachers may consider the following when incorporating AI elements into mathematical modelling learning activities:

- Teachers may guide students to utilise AI for facilitating them to understand the real-world problems. For instance, when evaluating the degree of comfort of a study room environment, students may first identify and list factors that may affect the comfort level, and then prompt the AI to provide feedback or generate questions that inspire them to explore other potential factors. This helps students deepen their thinking and clarify key factors of the problem;
- Teachers may guide students to utilise AI to facilitate data organisation and analysis. By reducing time spent on routine calculations and data processing, students can focus more on higher-order cognitive processes, such as problem comprehension, formulation of assumptions, exploration of variable relationships, and interpretation of results. For instance, in a decision-making activity on purchasing hand warmers, students may input data from different brands of hand warmers for AI to compute average values, rapidly generating comparison outputs or graphical representations;
- Teachers must be cautious about preventing AI from replacing student thinking. Teachers should guide students to think independently, leading them to view AI as a supporting tool for exploration and reflection. As AI-generated content may not always be accurate or appropriate, students need to examine AI outputs critically and cultivate an attitude towards use AI with care;
- Teachers should also guide students on how to use AI, preventing them from directly submitting AI-generated results and cultivating an awareness of responsible use of AI.

Teachers may utilise AI to assist in designing mathematical modelling learning activities, organising data, predicting students' learning difficulties, and producing learning resources. Teachers must carefully review the outputs of AI to ensure teaching quality and learning outcomes.

6. Learning and Teaching Resources

Regarding the examples of mathematical modelling learning activities, please refer to the learning and teaching resources on the website of the Mathematics Education Section of Education Bureau.

Table 3: Notes on the Learning and Teaching of Mathematical Modelling across Key Stages

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
Problem	1. Nature and complexity of problems	• Based on daily life contexts that students are familiar with • Related information/data are given • Guide students to formulate a simple model	• Based on real-life contexts that students are familiar with • The given information/data can be surplus or insufficient • Require students to formulate one or several models	• Based on real-life contexts that student may or may not be familiar with • The given information/data can be unstructured or scattered in diverse data sources • Require student to formulate several models or a series of models
Teacher	2. Teacher instruction	• Provide well-structured mathematical modelling learning activities along with clear and comprehensive instructions, as well as close guidance and support	• Provide sufficient guidance and support while allowing students to have the space to explore the problem themselves	• Facilitate students to conduct mathematical modelling activities themselves by providing timely guidance and support throughout the process
Student	3. Students' recognition of modelling process	• Be guided to experience the mathematical modelling process, with relatively less emphasis on model evaluation • Be aware that mathematics can be used to model real-world scenarios for solving problems, while not required to identify nor memorise modelling steps	• Be guided to recognise the modelling process according to the five steps of the modelling process	• Learn to conduct mathematical modelling activities themselves according to the five steps of the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
	4. Students' modelling capability	• Put forward or recognise the assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem and be able to provide an explanation on the relevance and reasonableness
		• Express real-world problems as simple mathematical problems and formulate related mathematical models after considering the key quantities/variables provided	• Express the real-life problem as a mathematical problem and formulate the related model(s) through identifying key quantities/variables and represent the relationship between them mathematically	• Express the more complicated real-life problem as a mathematical problem and formulate the related models through defining, identifying and organising key quantities/variables mathematically
		• Put forward or recognise possible strengths and limitations of the models	• Perform evaluation of model, such as comparing the results with real-world data, to identify the strengths and limitations of the model	• Understand the importance of model evaluation and perform evaluation of the model to identify the strengths and limitations of the model
			• Propose possible refinements to the model	• Make refinements to the model
	5. Students' application of mathematical knowledge and tools during the modelling process	• Apply mathematical knowledge and skills of lower and upper primary, such as those about arithmetic operations, areas, percentages, averages, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at junior secondary, such as linear equations, polynomial, trigonometry, probability, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at senior secondary, such as functions and graphs, variations, 3-D geometry, probabiliyt, differentiation*, matrices* and probability distribution* in the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
		• Use manipulatives, diagrams, tables and simple mathematical expressions to present the modelling results • Use calculators and simple digital tools, etc. as appropriate in the modelling process	• Use diagrams, tables, algebraic expressions and graphs to present the modelling results • Use digital tools, such as spreadsheet, graphing software, dynamic geometry software and artificial intelligence tools, etc. as appropriate in the modelling process	• Use diagrams, tables, functions and their graphical representations to present the modelling results • Use digital tools, such as spreadsheet, graphing software, dynamic geometry software, calculus calculator*, matrix calculator* and artificial intelligence tools, etc. as appropriate in the modelling process

* Applicable to students taking Module 1 or Module 2.

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Ad Hoc Committee for Integrating Mathematical Modelling into Secondary Mathematics Curriculum

