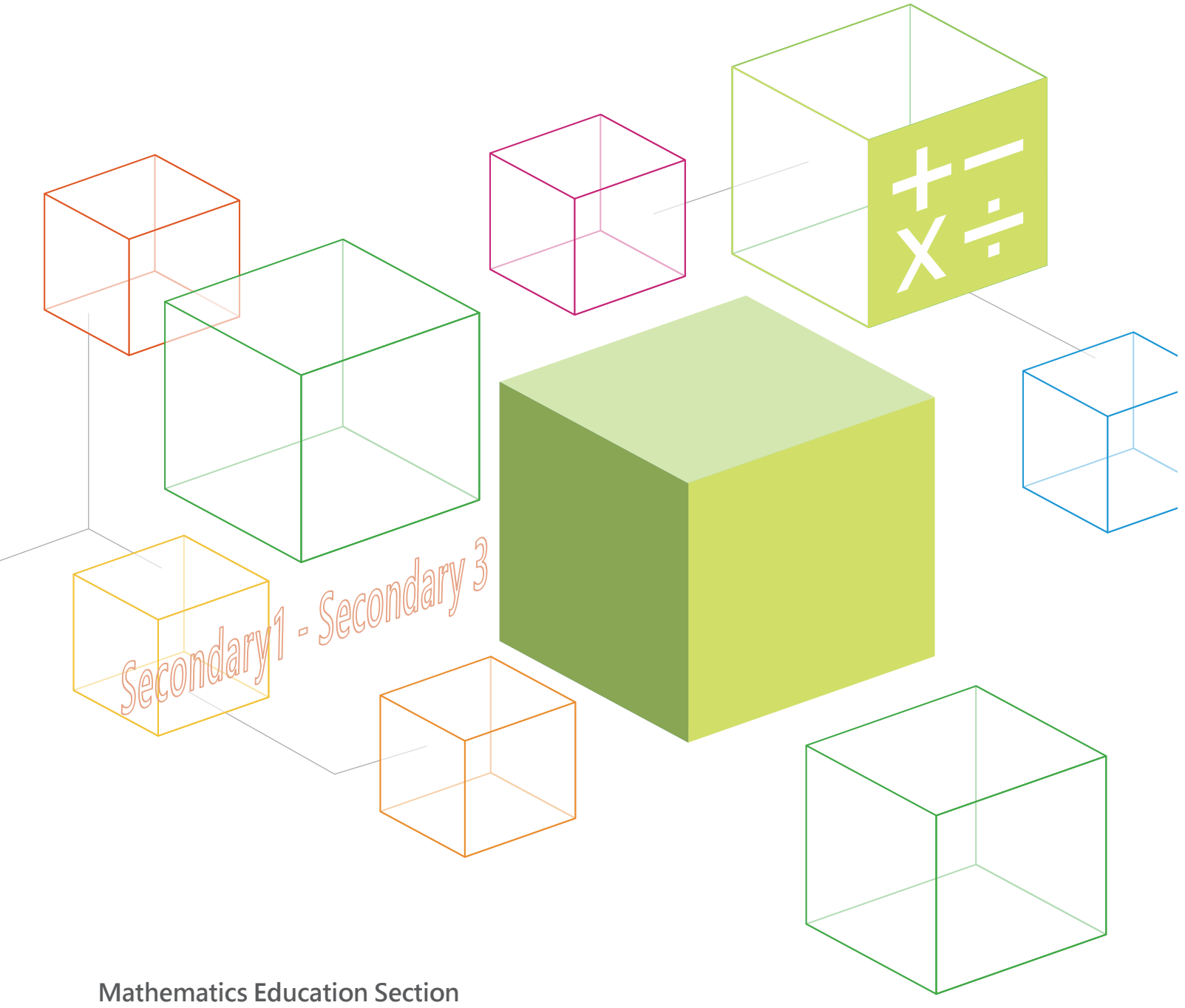
Explanatory Notes to Junior Secondary Mathematics Curriculum



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Foreword

To keep abreast of the ongoing renewal of school curriculum at primary and secondary levels, the revised *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6)* (2017) and its supplements setting out the learning content at each key stage have been prepared by the Curriculum Development Council and released in late 2017. Among these documents, the *Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Junior Secondary Mathematics (2017)* (hereafter referred to as "*Supplement*") aims at elucidating in detail the learning targets and content of the revised junior secondary Mathematics curriculum.

In the *Supplement*, the Learning Objectives of the junior secondary Mathematics curriculum are grouped under different Learning Units in the form of a table. The notes in the "Remarks" column of the table provide supplementary information about the Learning Objectives.

The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of the junior secondary Mathematics curriculum;
2. the strategies suggested for the teaching of the junior secondary Mathematics curriculum;
3. the connections and structures among different Learning Units of the junior secondary Mathematics curriculum; and
4. the context of development between the junior secondary Mathematics curriculum and other key stages, such as Key Stage 1,2 and 4.

Teachers may refer to the "Remarks" column and the suggested lesson time of each Learning Unit in the *Supplement*, with the explanatory notes in this booklet being a supplementary reference, for planning the breadth and depth of treatment in learning and teaching. Teachers are advised to teach the content of the junior secondary Mathematics as a connected body of mathematical knowledge and develop in students the capability for using mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the *Supplement* does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequences which take account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They may be sent to:

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Learning Unit	Learning Objective	Time
Number and Algebra Strand		
1. Basic computation	1.1 recognise the tests of divisibility of 4, 6, 8 and 9 1.2 understand the concept of power 1.3 perform prime factorisation of positive integers 1.4 find the greatest common divisor and the least common multiple 1.5 perform mixed arithmetic operations of positive integers involving multiple levels of brackets 1.6 perform mixed arithmetic operations of fractions and decimals	8

Explanatory Notes:

The purpose of this Learning Unit is to improve the interface between the Mathematics curriculum of Key Stage 2 and Key Stage 3, so as to strengthen the vertical continuity of the curriculum. All Learning Objectives of this Learning Unit are basic knowledge of Key Stage 3, which extend students' learning outcomes in Key Stage 2 and prepare students for learning the other Learning Units in Key Stage 3. Therefore, teachers are suggested to teach this Learning Unit at the start of Key Stage 3.

In Learning Objective 1.1, students are required to recognise tests of divisibility of 4, 6, 8 and 9. Tests of divisibility mean the methods of identifying whether a positive integer is divisible by the specified positive integer, which include both of the conditions of being divisible and not divisible. Students have recognised the tests of divisibility of 2, 3, 5, and 10 intuitively without proof in Learning Unit 4N2 "Division (II)" of the primary Mathematics curriculum. While in this Learning Objective, students are required to further recognise the tests of divisibility of 4, 6, 8 and 9. According to students' abilities and interests, teachers may explain why the tests of divisibility hold, but the explanations are not required in the curriculum. The test of divisibility of 6 usually refers to whether a number can pass both tests of divisibility of 2 and 3. Teachers may point out this test of divisibility holds not solely because $6 = 2 \times 3$. Teachers may emphasise that the key factor for this test to hold lies on the fact that 2 and 3 do not have a common factor larger than 1. Teachers may make use of different examples such as 4, 12, 20, 28, 36, … which are divisible by both 2 and 4 but not divisible by 8, to help students recognise that not all tests of divisibility of composite numbers

may be constructed in similar sense as the test of divisibility of 6. However, the related explanation or proof is **not required** in the curriculum. This Learning Objective helps students handle the prime factorisation (or factorization) of positive integers in Learning Objective 1.3, and find the greatest common divisor and the least common multiple in Learning Objective 1.4.

In the primary Mathematics curriculum, students are **not required** to learn the concept of power. In Learning Objective 1.2, the related concept in numerical computation is introduced to help students manage the content in other Learning Objectives. This Learning Objective only requires students to compute a given power of any positive integers. Students are also required to recognise $3^4 = 3\times3\times3\times3 = 81$, and express 81 as 3^4. Computations involving powers, such as $7^2 \times 7^3 = 7^5$ are **not required** in this Learning Objective. Teachers may also introduce the concept of exponents.

Students recognised the concept of prime numbers and composite numbers in Learning Unit 4N3 "Multiples and factors" of the primary Mathematics curriculum. They also recognised that 1 is neither a prime number nor a composite number. In Learning Objective 1.3, through identifying prime numbers and composite numbers, students are required to decompose positive integers into the product of its prime factors, and represent the results using the notation of powers.

In Learning Unit 4N4 "Common multiples and common factors" of the primary Mathematics curriculum, students recognised how to find the least common multiples and highest common factors of two numbers by listing their multiples and factors, and by using short division. They also recognised the short forms L.C.M. and H.C.F. In Learning Objective 1.4, students are required to apply prime factorisation in Learning Objective 1.3, and to extend the use of short division, to find the greatest common divisor and the least common multiple of two or more numbers. Teachers may consider whether or not the term "index notation" would be introduced during the discussion of prime factorisation. In this Learning Objective, students are also required to recognise H.C.F., gcd, etc. are short forms of the greatest common divisor. Learning prime factorisation helps students understand the concepts of the greatest common divisor and the least common multiple of polynomials in Learning Objective 4.4 of the Compulsory Part of senior secondary Mathematics.

Learning Objective 1.4 does not limit to finding the greatest common divisor and the least common multiple of two numbers. Hence, teachers are advised to use appropriate examples and counter examples to illustrate that in finding L.C.M. of more than two numbers, when the divisors are all prime numbers, correct results would be obtained regardless of the order

of division and whether the divisors are the common divisors of all numbers or just some of them. Since this Learning Objective serves to assist students in handling simplifications and operations of algebraic expressions in later stages, the exercises involving over-complicated operations or exceedingly large numbers should be avoided.

In Learning Unit 3N4 “Four arithmetic operations (I)” of the primary Mathematics curriculum, students recognised and used brackets for mixed operations, in which more than one pair of brackets may be involved. However, operations involving multiple levels of brackets, such as $(4 - (2 - 1)) \times 3$, were **not required**. This restriction is no longer applicable to Key Stage 3. Hence in Learning Objective 1.5, students are required to perform mixed arithmetic operations of positive integers involving multiple levels of brackets, such as $12 + (7 – (5 – 2))$, $((35 – 20) – (5 + 7)) \times 2$, etc. Teachers may introduce different types of brackets for this Learning Objective to let students recognise the various notations of brackets, such as ( ), [ ] and { }, etc.

In Learning Units 4N5 “Four arithmetic operations (II)”, 5N5 “Fractions (V)”, and 6N1 “Decimals (IV)” of the primary Mathematics curriculum, students performed mixed arithmetic operations of three numbers (including integers, fractions and decimals). For operations and comparison involving three fractions with different denominators, all denominators should not exceed 12. The above restrictions are no longer applicable to Key Stage 3. Through Learning Objectives 1.5 and 1.6, students should be able to perform mixed arithmetic operations of integers, fractions and decimals involving multiple levels of brackets, but over-complicated operations are **not required**.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
2. Directed numbers	2.1 understand the concept of directed numbers 2.2 perform mixed arithmetic operations of directed numbers 2.3 solve problems involving directed numbers	9

Explanatory Notes:

This Learning Unit covers some basic concepts and knowledge of Key Stage 3, which are based on what students have learnt in primary Mathematics and prepares them for learning other Learning Units of Key Stage 3

In Learning Objective 2.1, teachers may use daily life examples such as a thermometer and a level indicator of a lift to introduce the concept of negative numbers, and discuss with students the meanings of negative numbers in daily life, such as debts, the temperature below zero, floor levels underground. All these examples carry the similar meaning of a measure less than or smaller than a certain reference point. Teachers may also use directed numbers as a kind of numerical representation in cases such as temperature, profit and loss, etc. to help students understand and accept the concept and applications of negative numbers, as well as the concept of directed numbers.

Teachers should use the number line to help students understand the concept of directed numbers. Teachers may guide students to understand that different points on a number line represent different numbers, with the point representing 0 on the number line may be regarded as the reference point of the number line. The distance between "0" and a point on the number line is the value of the number represented by the point. A number line may extend along the two opposite ends of "0" so that there exist a pair of points, each on one side of "0", which are equidistant from "0". The pair of numbers represented by this pair of points are opposite numbers to each other, and the number on the right side of "0" is usually known as a positive number, and the other one on left is usually known as a negative number. Teachers may use examples to illustrate the above abstract concept, such as the opposite number of 1 is -1, and the opposite number of -2 is 2. Moreover, 0 is the origin that it is neither a positive nor a negative number. Directed numbers include negative numbers, 0 and positive numbers. In this Learning Objective, students are required to represent directed numbers on a number line, and to compare the magnitude of directed numbers. Students are required to recognise

that, in general, a number on the right of another is larger. Teachers may guide students to use "<" and ">" signs to represent relationships such as $-7 < -5$ and $7 > 5$.

In Learning Objective 2.2, students are required to understand mixed arithmetic operations involving negative numbers, and perform mixed arithmetic operations of directed numbers involving multiple levels of brackets. Teachers may illustrate the addition and subtraction of directed numbers by moving the points on the number line or using other methods. Starting from moving points on the number line or other illustrative methods, students are also required to perform arithmetic operations gradually using common notations and expressions involving directed numbers, such as 3 + (–4), (–5) – (–7) and so on.

Teachers may use a multiplication table like below to help students construct the concepts of multiplication and division of directed numbers through observing patterns:

	+3	+2	+1	0	−1	−2	−3
+3	+9	+6	+3	0	−3	−6	−9
+2	+6	+4	+2	0	−2	−4	−6
+1	+3	+2	+1	0	−1	−2	−3
0	0	0	0	0	0	0	0
−1	−3	−2	−1	0	1	2	3
−2	−6	−4	−2	0	2	4	6
−3	−9	−6	−3	0	3	6	9

Teachers may guide students to observe the pattern by filling in the blanks, for example, students may first fill in the products of positive numbers, observe the patterns in the rows and columns, and then fill in the products of a positive and a negative number and the products of two negative numbers. Similar tables of division may also be created.

While performing mixed arithmetic operations of directed numbers, students are required to extend the concept of mixed arithmetic operations of positive numbers learnt in Key Stage 2 and Learning Unit 1 "Basic computation to mixed arithmetic operations of directed numbers, including the rules of using brackets which are the basic computation skills students have to manage. Performing mixed arithmetic operations of directed numbers may familiarise students with the above skills, but over-complicated computations should be avoided.

In Learning Objective 2.3, students are required to solve problems involving directed numbers, such as using directed numbers to describe real-life or mathematical problem situations. The problems are suggested to be related to students' daily-life experience or real-life scenarios.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
3. Approximate values and numerical estimation	3.1 recognise the concept of approximate values 3.2 understand the estimation strategies 3.3 solve related real-life problems 3.4 **design numerical estimation strategies according to the contexts and judge the reasonableness of the results obtained	6

Explanatory Notes:

This Learning Unit is formed by combining two Learning Units "Numerical estimation" and part of "Approximation and errors" in the original curriculum. In this Learning Unit, students are required to recognise the concept of approximate values, understand the estimation strategies, and solve related real-life problems. Designing numerical estimation strategies according to the contexts and judge the reasonableness of the results obtained is an enrichment topic. The concepts of maximum absolute errors, relative errors and percentage errors will be covered in Learning Objective 15.2 while the concept of scientific notations will be covered in Learning Objective 10.4.

In Learning Unit 4D1 "Bar charts (II)", 5N1 "Multi-digit numbers" and 6N2 "Decimals (V)" of the primary Mathematics curriculum, students recognised the concepts of rounding off a whole number to a certain place, and rounding off a decimal to the nearest tenth or hundredth.

In Learning Objective 3.1, students are required to recognise further the concept of approximate values, including rounding off a number to a certain number of significant figures, a certain place and a certain number of decimal places. Teachers may use daily-life examples, such as estimating the total numbers of teachers and students of a school, the distance between the school and an MTR station, and an area of a stadium, to illustrate the needs for approximate values, and consolidate the concept of approximate values. Teachers may discuss with students the reasons for taking approximate values. Students are required to recognise the concept of significant figures, such as the more number of significant figures an approximate value is correct to, the closer it is to the actual value. Similarly, teachers should discuss with students the concept of rounding off decimals to a certain number of decimal places. The concept of approximate values in Learning Objective 3.1 helps students

recognise the concept of errors in measurement in Learning Objective 15.1.

Learning Objective 3.2 includes three estimation strategies, namely rounding off, rounding up and rounding down. Students should understand the similarities and differences between these three estimation strategies, including the fact that for rounding up and rounding down, the approximated values obtained are respectively not less than and not larger than the actual value, but this does not hold for rounding off. Teachers may use vocabularies like “about”, “near”, “a bit more than” or “a bit less than”, etc. to describe the result of estimation.

Building on Learning Objective 3.2, in Learning Objective 3.3, students are required to solve related real-life problems by using appropriate estimation strategies. Teachers may use a variety of real-life examples to help students identify under what daily life scenarios, it is suitable to use estimation, and determine which estimation strategies should be employed and how accurate the estimation should be under specific situations.

In Learning Objective 3.4, teachers may arrange suitable enrichment learning and teaching activities according to students’ abilities and interests to discuss with students how to design numerical estimation strategies according to the contexts. Teachers may discuss and analyse with students whether results obtained from the estimation strategies are reasonable.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
4. Rational and irrational numbers	4.1 recognise the concept of *n*th root 4.2 recognise the concepts of rational and irrational numbers 4.3 perform mixed arithmetic operations of simple quadratic surds $a\sqrt{b}$ 4.4 **explore the relation between constructible numbers and rational and irrational numbers	7

Explanatory Notes:

This Learning Unit is an extension of students' recognition of integers and fractions. It introduces the concepts of rational and irrational numbers. In this Learning Unit, students are required to recognise the definition of rational and irrational numbers as well as some of their examples. Students are **not required** to prove that a certain number is an irrational number, but they are required to recognise some common irrational numbers, such as $\sqrt{2}$, $\sqrt{3}$ and π.

In Learning Unit 1 "Basic computation", students understood the concept of power, and performed prime factorisation of positive integers with results presented in the form of powers. In Learning Objective 4.1, students are required to recognise the concept of *n*th root and its notation. Teachers may help students recognise the relationship between square and square root by the examples such as: Given 2 squares, one of them with known length of sides while the other with known area, and the area and length of sides are to be found respectively. Similarly, students may recognise the relationship between cube and cube root by calculating the length of side from its volume and the volume from its length of side of cubes, and consequently recognise the concept of *n*th root and its notation. Students are required to calculate the *n*th root of a given number, such as $\sqrt[3]{-8}$, $\sqrt[4]{81}$. However, this Learning Objective does not require computations such as $\sqrt[3]{2}$ $\sqrt[3]{4} = \sqrt[3]{8}$. Students are **not required** to use fractional power to represent *n*th root in this Learning Objective.

Students are required to recognise that $\sqrt{a}$ means the positive square root of a through examples such as $2^2 = (-2)^2 = 4$ but $\sqrt{4}$ only equals 2. According to students' abilities and interests, teachers may discuss with students why a in $\sqrt{a}$ has to be non-negative. Discussions related to complex numbers will be covered in Learning Unit 1 "Quadratic equations in one unknown" in the Compulsory Part of senior secondary Mathematics

curriculum.

In Learning Objective 4.2, students are required to recognise the concepts of rational and irrational numbers, including that rational numbers can be written as fractions of numerators and denominators both being integers (where denominators are non-zero), but irrational numbers cannot be written as fractions in the aforementioned format. Students are only required to recognise examples of irrational numbers such as $\sqrt{2}$, $\sqrt{3}$ and π. Students are **not required** to prove the above numbers as irrational numbers, but they are required to represent rational and irrational numbers on a number line by comparing the values, for example locating $\sqrt{2}$ between 1 and 2. Students are **not required** to locate irrational numbers on a number line by methods such as constructions using compasses and straightedge. Students are only required to mark the approximate location of the numbers, and not to mix up the order of different numbers with respect to their magnitude.

In Learning Objective 4.3, students are required to perform simplification and mixed arithmetic operations of simple quadratic surds. In this Learning Objective, simple quadratic surds are limited to those in the form of $a\sqrt{b}$, where a is a rational number and b is a positive rational number. Students are required to use formulae $\sqrt{cd} = \sqrt{c}\sqrt{d}$ (where c and d are non-negative rational numbers) and $\sqrt{\frac{c}{d}} = \frac{\sqrt{c}}{\sqrt{d}}$ (where c is a non-negative rational number and d is a positive rational number) to perform simplification and mixed arithmetic operations of simple quadratic surds, such as $\sqrt{12}=\sqrt{4\times3}=2\sqrt{3}$, $\sqrt{3}+\sqrt{12}=\sqrt{3}+2\sqrt{3}=3\sqrt{3}$ and $\frac{8}{3\sqrt{2}} = \frac{8\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}} = \frac{4\sqrt{2}}{3}$. However, more complicated mixed arithmetic operations such as $\frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$ will be covered in Module 2 of the Extended Part of the senior secondary Mathematics curriculum, and therefore are **not required** in this Learning Objective.

While performing mixed arithmetic operations of simple quadratic surds, students are required to recognise how to simplify the surds into the simplest form, and to operate with like surds. Teachers may help students understand the concept of like surds by recalling the concept of like terms. However, over-complicated operations should be avoided.

In Learning Objective 4.4, teachers may introduce appropriate enrichment learning and teaching activities on the exploration of the relation between constructible numbers and rational and irrational numbers according to students' abilities and interests. Teachers may explore with students through related materials of mathematical history that the meaning of

the discovery of $\sqrt{2}$ being not a rational number in mathematical development, in order to raise students' interest towards the development of the number system. Teachers may let students with higher ability briefly recognise the proof of $\sqrt{2}$ being an irrational number. After students have recognised the basic knowledge of construction with compasses and straightedge in Learning Objective 20.5, and Pythagoras' theorem in Learning Unit 25, teachers may also explore with students on the methods of constructing line segments with lengths of rational numbers or some irrational numbers with a given unit length using compasses and straightedge. Teachers may stimulate students to think about whether there are some lengths which are not constructible using similar methods, and thereby bring out the basic concept of constructible numbers. Students should be able to point out the square root of all rational numbers are constructible numbers. Teachers may let students further explore what numbers apart from the square root of rational numbers are constructible (such as $\sqrt{\sqrt{2}}$, $\sqrt{1+\sqrt{2}}$, etc. which are the square roots of constructible numbers).

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
5. Using percentages	5.1 understand the concept of percentage changes 5.2 solve related real-life problems	15

Explanatory Notes:

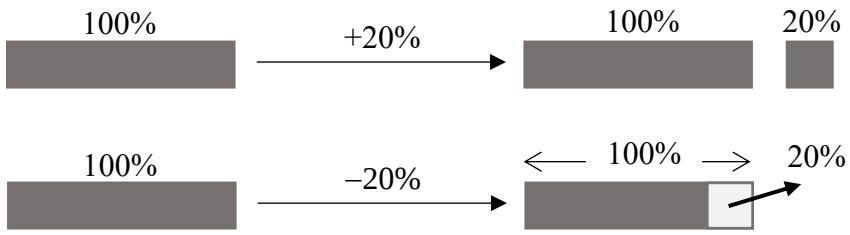
In Learning Unit 6N3 "Percentages (I)" and 6N4 "Percentages (II)" of the primary Mathematics curriculum, students recognised the basic concept of percentages, performed the interconversion between a percentage and a decimal, and the interconversion between a percentage and a fraction. At the primary level, students were only required to solve simple problems related to percentages and percentage changes. More complicated problems, such as those listed below, are **not required**:

- what percent is 100 more than 80
- what is the percentage increase from 100 to 120

This Learning Unit aims at helping students understand further the concept of percentage changes.

In Learning Objective 5.1, students are required to calculate the percentage change from the given original value and new value. Students are required to understand that it is an increase of 20% (can be expressed as "the percentage change is +20%") from 100 to 120, but not a decrease of 20% from 120 to 100. Students are required to master the relation amongst the original value, the new value and the percentage change. They are required to recognise that percentage change can also be called "percentage of change".

Teachers may use the following diagrams to help students understand the concept of percentage change.



Students are required to make use of the following formulae flexibly to calculate the original value, the new value and the percentage change.

- new value = original value $\times$ (1 + percentage change)
- percentage change $= \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$

Remark: Teachers should note that these formulae hold for positive quantities. When the original value is negative, slightly different formulae are needed.

Students are required to note that percentage changes could be positive or negative. They are required to understand the meaning of positive and negative percentage changes in real-life situations.

The problems in Learning Objective 5.2 include those about discount and profit or loss, growth and depreciation, simple and compound interests, successive and component changes, and salary tax. As the problems of percentage may lead to many formulae, teachers should let students use the two formulae above flexibly in different situations to relieve students from the burden of reciting excessive formulae with similar meaning. When introducing the problem of discount and profit or loss, students are required to recognise the terms cost, marked price and selling price, and the relations of these terms. When calculating the problem of discount, students are required to recognise the meaning of common terms such as 20% off. Teachers may also introduce the meaning of the Chinese terms "八折", "八五折" used in daily life according to the needs and ability of students. When introducing the problem of simple and compound interests, students are required to recognise principal, interest rate, period, interest and amount, and the relations among them. Students are required to distinguish between simple interest and compound interest. Students are required to calculate salary tax which can be considered as a real-life application of percentage. However, problems involving complicated calculations, such as finding annual salary from a known amount of salary tax and tax allowance, should be avoided.

Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as the growth rate of a certain species in nature or depreciation rate of parts in machines, to design learning and teaching activities or as class examples to let students recognise how percentage change is applied to solve real-life problems, and how percentage is used to describe real-life situations quantitatively.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
6. Rates, ratios and proportions	6.1 understand the concepts of rates, ratios and proportions 6.2 solve problems involving rates, ratios and proportions	8

Explanatory Notes:

In Learning Unit 6M4 "Speed" of the primary Mathematics curriculum, students recognised the basic concept of rates. Students also recognised how to use the unitary method to solve problems involving direct proportion in Learning Units 4N5 "Four arithmetic operations (II)" and 5N5 "Fractions (V)" of the primary Mathematics curriculum, but the term "direct proportion" was not introduced. This Learning Unit further discusses rates, ratios and proportions, including the concepts of direct and inverse proportion.

In Learning Objective 6.1, teachers may use daily-life examples, such as typing speed, ratio between the numbers of boys and girls in the class, to help students understand the meaning of rates, ratios and proportions, as well as their relations. When teachers introduce the concept of rate, teachers are advised to emphasise that rate represents the relation of the amount of one quantity per unit of the other quantity. Teachers may use examples such as speed (i.e. the distance travelled in each unit of time interval) to explain this concept. Teachers may let students understand how to perform the interconversion between different units of rates, such as the interconversion between km/h and m/s.

Students are required to understand the concept and notation of two-term ratios. In introducing $a:b$, it can be represented as $\frac{a}{b}$, where $b \neq 0$. Teachers may use daily-life examples, such as mixing household bleach and water in the ratio of 1:99, and the aspect ratio "16:9" of movie screens and televisions, to introduce ratios. Teachers should also clarify that the concept of ratios in mathematics and the notation of ratios commonly used in daily-life are not necessarily the same. Teachers may cite non-examples such as the score of 1:0 in a football match as an illustration.

Students are required to understand the following properties of ratios:

- $a:b$ and $b:a$ are different.
- $a:b = 2:7$ does not mean that $a = 2$ and $b = 7$.

- $a:b = ka:kb$, where k is any non-zero real number.

Teachers may introduce to students the *k*-method (for example, if $a:b = 2:7$, one can let $a = 2k$, $b = 7k$ where $k \neq 0$) to solve the problems of ratio.

Students are required to understand how to extend the two-term ratios to ratios of three-term or more. In solving problems related to ratios, students would frequently face equations involving fractions such as $\frac{x}{3} = \frac{x+1}{5}$, etc. Teachers may help students revise the relevant techniques in solving these kinds of equations specifically.

Teachers may introduce direct proportion through recalling the unitary method to students. Teachers may also use different daily-life examples to discuss direct and inverse proportions with students. As students have recognised speed in the Learning Unit 6M4 "Speed" of the primary Mathematics curriculum, teachers may consider to introduce the concept of inverse proportion by discussing with students that the time and the speed are in inverse proportion if the distance is fixed. Students may understand direct proportion from the concept of equal ratios. They may also understand the concept of inverse proportion through direct proportion. Teachers may introduce this concept by using a table to investigate the relation between x and y if x and $\frac{1}{y}$ are in direct proportion.

Teachers should also use the concept of equal ratios to clarify some common misunderstandings of direct proportion and inverse proportion, such as "if y increases (decreases) when x increases, then x and y must be in direct (inverse) proportion". Teachers may use a counterexample to disprove the above assertions, for example,

x	1	2	3	4
y	1	4	9	16

where x and y do not satisfy the relation of direct proportion.

In Learning Objective 6.2, students are required to solve problems involving rates, ratios and proportions. Using direct and inverse proportion to solve related problems in different scenarios are also required. Teachers should note that this Learning Unit focuses on tackling problems of direct and inverse proportions by using ratios. Using variation relations (i.e. direct and inverse variations) to solve direct and inverse proportions, and understanding direct

and inverse proportions through graphical representation belong to the learning content of Learning Unit 6 “Variations” of the Compulsory Part of the senior secondary Mathematics curriculum. Students are required to solve problems about plan diagrams involving scales. While solving problems by direct and inverse proportions, the related equations should involve one unknown only. Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs, such as common examples of maps and scale plans, discount, interest rate, exchange rate, density and concentration, to enhance learning and teaching. Teachers may also use the cell diagram under a microscope, online maps, or other real-life scenarios, to introduce the concept of scale, and to design classroom examples or exercises through these scenarios to enhance students’ ability and confidence in applying mathematical knowledge or skills in real-life situations or STEM related scenarios. This Learning Unit is connected with Learning Objectives 18.3 and 22.3 so that students may solve problems involving similar figures by the knowledge of ratios and proportions.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
7. Algebraic expressions	7.1 represent word phrases by algebraic expressions 7.2 represent algebraic expressions by word phrases 7.3 recognise the concept of sequences of numbers 7.4 recognise the preliminary idea of functions	7

Explanatory Notes:

At the secondary level, students are required to use algebraic symbols to represent more abstract mathematical concepts, in which algebraic expression is an important foundation of mathematical language. In this regard, teachers should select different examples to let students build up a solid foundation through this Learning Unit, so that they could learn the related topics afterward more effectively.

In Learning Unit 5A1 "Elementary algebra" of the primary Mathematics curriculum, students recognised the use of letters to represent numbers, including recognising the meaning of the representations such as $3x$, $\frac{2x}{3}$ (the algebraic expressions should involve only one unknown quantity), where $3x$ is $3\times x$, $x\times 3$ or $x+x+x$; $\frac{x}{3}$ is $x\div 3$, $\frac{1}{3}\times x$ or $x\times\frac{1}{3}$. Students also used algebraic expressions to represent the operations of and relations between quantities that are described in words and involve unknown quantities. Students then adopt this as a foundation to solve problems of simple equations in Learning Unit 5A2 "Simple equations (I)" and 6A1 "Simple equations (II)" of the primary Mathematics curriculum. In Learning Unit 7 "Algebraic expressions", students are required to further their learning in algebraic expressions and the related concepts, but the concepts of like terms and unlike terms would be dealt with in Learning Unit 11 "Polynomials".

In Learning Objective 7.1, students learn how to represent word phrases by algebraic expressions which are not confined to one unknown quantity, but are restricted to those expressions which are combinations of addition, subtraction, multiplication, division, and power of numbers and variables. Students should recognise the meanings of the representations such as ab being $a\times b$, and $\frac{a}{b}$ being $a\div b$, and teachers should remind

students the difference between the representation ab and the number such as 53 (that is 5 × 10 + 3). Students should also recognise such as $-\frac{2x}{3} = \frac{-2x}{3} = \frac{2x}{-3}$ and that the expressions $4 \div (2a)$ should not be written as $4 \div 2a$. As students will come across some more complicated algebraic expressions in this Key Stage, to avoid ambiguity in presentation, students are required to replace the use of the division sign "÷" gradually by fractional notations in handling algebraic expressions.

The formulae of areas of 5M1 "Area (II)" at Key Stage 2 are described in words, for example, the formula of area of a triangle is the base times the height and divided by 2. In this Learning Unit, students are required to represent formulae by algebraic expressions. For instant, the said formula of area of a triangle may be represented by $A = \frac{bh}{2}$. Students are required to recognise some common word phrases in mathematics, including mathematical terms such as "sum", "product" and "square". They are also required to recognise the importance of bracket. For example, "$(a + b)^2$" and "$a^2 + b^2$" represent different algebraic expressions with different meanings. In order to let students recognise the algebraic expressions in-depth, students are required to represent algebraic expressions by word phrases in Learning objective 7.2.

Students are required to guess the next term of a sequence from some given terms and give explanations in Learning Objective 7.3. For example, in the sequence 1, 3, 5, ?, students may guess that the next term is 7 because the terms of the sequence are consecutive odd numbers; or in the sequence 1, 2, 3, 5, 8, ?, students may guess that the next term is 13 because starting from the third term of the sequence, the terms are the sum of the previous two terms. Teachers should emphasise that the term is not unique when guessing merely from some given terms of a sequence. Therefore, this Learning Objective stresses that students should present their guesses by explanation. However, students are **not required** to present their guesses and explanations using algebraic method. In this Key Stage, teachers are also **not required** to introduce the algebraic expressions on recurrence relations between terms of sequences.

In this Learning Objective, students are required to find a particular term from the general term of a sequence, such as finding the third term a_3 from the general term of a sequence $a_n = n^2 + 1$. Sequences of odd numbers, even numbers, square numbers, and triangular numbers are required. Teachers may discuss with students the difference between the representations of a sequence with a given general term and that of a sequence with some leading terms. Students are required to recognise that the values of all terms can be found through the given general term, and the value of each term is unique.

Teachers may introduce Learning Objective 7.4 “recognise the preliminary idea of functions” through the discussion of the general term of, for example, sequence of square numbers n^2, including the concept of input-processing-output. The input of this example is a positive integer and the output is a square number. But the formula of the area of a square x^2 is different as the input is not confined to positive integers. The rigorous definition of functions, including domains, co-domains, independent and dependent variables are dealt with in Learning Unit 2 “Functions and graphs” of the Compulsory Part of the senior secondary Mathematics curriculum.

The algebraic expressions discussed in this Leaning Unit are confined to expressions involving addition, subtraction, multiplication, division and powers of numbers and variables.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
8. Linear equations in one unknown	8.1 solve linear equations in one unknown 8.2 formulate linear equations in one unknown from a problem situation 8.3 solve problems involving linear equations in one unknown	7

Explanatory Notes:

In Learning Unit 5A2 “Simple equations (I)” and 6A1 “Simple equations (II)” of the primary Mathematics curriculum, students learnt solving special types of simple equations and related problems. Students also recognised the “balance principle” used in solving equations. This Learning Unit further requires students to solve general linear equations in one unknown.

In Learning Objective 8.1, students should understand the meaning of “solutions” .

As students have learnt representing word phrases by algebraic expressions in Learning Unit 7 “Algebraic expressions”, they should be equipped with sufficient foundation to formulate linear equations in one unknown from a problem situation in Learning Objective 8.2, and use linear equations in one unknown to solve problems in Learning Objective 8.3.

This Learning Unit discusses the linear equations in one unknown having one solution only. Students may come across linear equations in one unknown having infinitely many solutions or no solutions in Learning Unit 9 “Linear equations in two unknowns” and Learning Unit 12 “Identities”.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
9. Linear equations in two unknowns	9.1 understand the concept of linear equations in two unknowns and their graphs 9.2 solve simultaneous linear equations in two unknowns by the graphical method 9.3 solve simultaneous linear equations in two unknowns by the algebraic methods 9.4 formulate simultaneous linear equations in two unknowns from a problem situation 9.5 solve problems involving simultaneous linear equations in two unknowns	12

Explanatory Notes:

At junior secondary level, apart from mastering how to formulate and solve linear equations in one unknown in Learning Unit 8 “Linear equations in one unknown”, students are required to understand and solve linear equations in two unknowns in this Learning Unit. Using two unknowns to formulate equations and hence simultaneous linear equations in two unknowns helps students extend their understandings on equations, as well as solving more complex problems that could not be described by linear equation in one unknown easily. This Learning Unit also introduces the relation between algebra and graphs to students: After students learnt solving simultaneous linear equations in two unknowns by the graphical method and algebraic methods, they could extend the related concept to learn related topics in the senior secondary Mathematics curriculum, including Learning Unit 5 “More about equations”, Learning Unit 6 “Variations”, Learning Unit 9 “More about graphs of functions”, Learning Unit 10 “Equations of straight lines”, Learning Unit 13 “Equations of circles” of the Compulsory Part of the senior secondary Mathematics curriculum, and Learning Unit 14 “Systems of linear equations” of the Extended Part Module 2 of the senior secondary Mathematics curriculum. This Learning Unit discusses linear equations in two unknowns in the form of $ax + by = c$, where a and b cannot be both 0.

Students learnt using algebraic expressions to represent unknowns, and recognising the basic concept of input-processing-output of functions in Learning Unit 7 “Algebraic expressions”. In Learning Objective 9.1, students are required to understand the concept that two algebraic

symbols representing two unknowns in the same equation. Students also learnt using substitution to find values of unknowns in formulae in Learning Unit 13 “Formulae”. Therefore, teachers may explain that a linear equation in two unknowns $ax + by = c$ has infinitely many solutions using substitution after students have understood the concept of solutions of a linear equation in two unknowns. But students are **not required** to learn the concept of solution sets. Students should then understand the relation between the solutions of a linear equation in two unknowns and its graph: Through the concept of coordinates of a point in Learning Unit 26 “Rectangular coordinate system”, students should understand that each solution of a linear equation in two unknowns could be regarded as the coordinates of a point, and points corresponding to all the solutions constitute the graph of that equation. Regarding the graph of a linear equation in two unknowns, students are required to understand that:

- the graph of a linear equation in two unknowns is a straight line
- all the coordinates of points lying on the straight line satisfy the linear equation in two unknowns
- all the coordinates of points not lying on the straight line do not satisfy the linear equation in two unknowns

Students are required to recognise that the graphs of the equations $x = c$ and $y = d$ are a vertical line and horizontal line respectively. Teachers may use Information Technology to assist students in understanding the graphs of linear equations in two unknowns in-depth. Nevertheless, students are required to plot the graphs of linear equations in two unknowns on graph papers by paper and pencil.

In Learning Objective 9.2, students are required to solve simultaneous linear equations in two unknowns by the graphical method. Students are required to understand that solving simultaneous linear equations in two unknowns means to find the solutions satisfying all the equations of simultaneous linear equations. Therefore, after students understood that the graph of a linear equation in one unknown is a straight line in Learning Objective 9.1, they should understand the solution of simultaneous linear equations in two unknowns is the coordinates of the point(s) lying on both straight lines. Appropriate use of Information Technology, including the feature of magnification of graphs, can increase the accuracy of the values of the solution obtained. Nevertheless, students should recognise that the exact values may not necessarily be found by the graphical method in Learning Objective 9.2. The simultaneous linear equations in two unknowns included in Learning Objective 9.2 are confined to those equations that have only one solution. The use of graphical methods in solving simultaneous linear equations in two unknowns that have no solutions or more than one solution are dealt with in the content of Learning Objective 10.2 of the Compulsory Part

of the senior secondary Mathematics curriculum.

In Learning Objective 9.3, students are required to solve simultaneous linear equations in two unknowns by algebraic methods which include substitution and elimination. Students are required to recognise how to tackle those simultaneous equations with no solutions, only one solution, and more than one solution by algebraic methods. Students are required to use "The equations have no solutions." and "The equations have infinitely many solutions." to describe the conclusion of the above two special cases "no solutions" and "more than one solutions" respectively. Students are **not required** to write down the general solutions of simultaneous linear equations in two unknowns that have infinitely many solutions.

After learning the methods of solving simultaneous linear equations in two unknowns in Learning Objectives 9.2 and 9.3, students are required to formulate and solve simultaneous linear equations in two unknowns from problem situations, and then solve problems in Learning Objectives 9.4 and 9.5. Teachers may emphasise that although some problem situations could be expressed by linear equations in one unknown only, it is in general clearer to describe the relations between variables in simultaneous linear equations in two unknowns.

Teachers may select problems related to students' daily-life experience when teaching Learning Objective 9.5. They may discuss the meanings of solutions of equations from different scenarios to let students master the ways to solve problems involving simultaneous linear equations in two unknowns.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
10. Laws of integral indices	10.1 understand the laws of positive integral indices 10.2 understand the definitions of zero exponent and negative exponents 10.3 understand the laws of integral indices 10.4 understand scientific notations 10.5 understand the binary number system and the denary number system 10.6 **understand other numeral systems, such as the hexadecimal number system	11

Explanatory Notes:

In this Learning Unit, students start with the laws of positive integral indices and further understand that these laws are also applied to integral indices. Important applications of the laws of integral indices include scientific notations and the expanded form of different numeral systems. These applications strengthen the lateral coherence between Mathematics and other disciplines, including Science and Computer Literacy.

In Learning Objective 1.2, students only understand the concept of power and apply it to known numbers. In Learning Objective 10.1, students are required to understand the representations of the laws of positive integral indices by algebraic expressions and apply them on algebraic expressions, the laws include:

- $a^p a^q = a^{p+q}$
- $\frac{a^p}{a^q} = a^{p-q}$
- $(a^p)^q = a^{pq}$
- $a^p b^p = (ab)^p$
- $\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$

The first three laws are related to computations with the same base, whereas the other laws are related to computations with the same index. In the calculation process, students are required to distinguish whether the case is of the same base or the same index, and then choose the appropriate law to reduce common mistakes such as:

- $m^3 n^2 = (mn)^{3+2}$
- $\frac{6^4}{3^2} = \left(\frac{6}{3}\right)^{4-2}$
- $(x^3)^2 = x^{3+2}$

Teachers may place emphasis on clarifying these mistakes in Learning Objective 10.1 to help students apply the laws of positive integral indices skillfully to build up a solid foundation to learn the laws of integral indices.

In Learning Objective 10.2, teachers may let students recognise that the laws of positive integral indices could be extended to the laws of integral indices through the appropriate definition of zero exponent and negative exponents. Students would further understand the laws of rational indices in Learning Unit 3 “Exponential and logarithmic functions” of the Compulsory Part of the senior secondary Mathematics curriculum. Teachers should emphasise to students that 0^0 is undefined. After understanding the definitions of zero exponent and negative exponents, students are required to understand the laws of indices listed in Learning Objective 10.1 are also applied to integral indices, and therefore these laws are also the laws of integral indices, but the base involved should be non-zero.

In Learning Objective 10.3, students are required to extend Learning Objectives 10.1 and 10.2 to understand the laws of integral indices. Students are required to apply the laws to manipulate numerical and algebraic expressions involving integral indices. However, the focus of this Learning Objective is on students’ understanding of the laws of integral indices. Over-complicated operations should therefore be avoided.

In Learning Objective 10.4, students are required to understand the advantage of scientific notations, that is, to express a very large number or a number that is very close to zero in a more concise form. Expressing these numbers in scientific notations is very common in scientific calculators. Teachers may use real-life examples or related learning elements in Science Education or Technology Education KLAs such as the distance between the sun and the earth, micro measurements under microscopes, the speed of light (3×10^8 m/s), manipulation speed of a computer microprocessor, the amount of the greenhouse gas emissions produced by power stations etc. to enhance students’ interest and facilitate learning

and teaching.

Learning Objective 10.5 involves only the understandings of non-negative integers in binary and denary number system including the interconversion between binary numbers and denary numbers. Students are **not required** to learn the computations in different numeral systems except in denary number system. After mastering the concept of place value in Learning Objective 10.5, teachers may discuss with students the enrichment topics of other numeral systems in Learning Objective 10.6 according to the ability and interest of students.

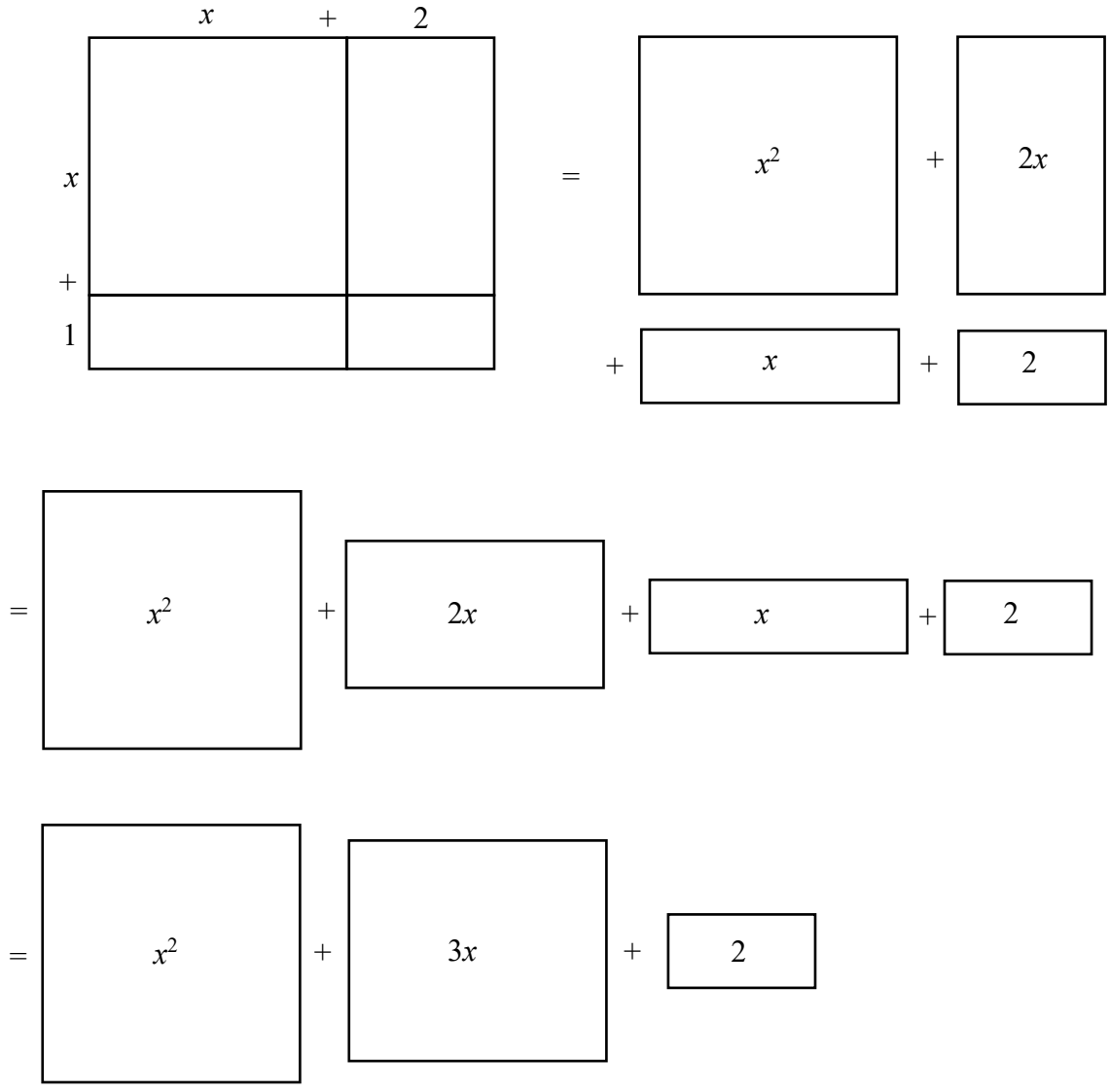
Learning Unit	Learning Objective	Time
Number and Algebra Strand		
11. Polynomials	11.1 understand the concept of polynomials 11.2 perform addition, subtraction, multiplication and their mixed operations of polynomials 11.3 factorise polynomials	15

Explanatory Notes:

In Learning Unit 6A1 "Simple Equation (II)" of the primary Mathematics curriculum, students recognised the basic concepts of like terms and unlike terms. Students learnt the rules of addition and subtraction of like terms to handle relation such as $8x+3x=11x$, but they were **not required** to learn the terms "like terms" and "unlike terms" at primary level. In this Learning Unit, students are further required to handle the operations of polynomials.

In Learning Objective 11.1, students are required to understand the concept of polynomials, including the concepts of terms, monomials, binomials, orders, powers, constant terms, like terms, unlike terms and coefficients. Teachers may clarify some common mistakes for students, for example, the terms of the polynomials x^3-2x being mistakenly regarded as x^3 and $2x$. Besides, students are required to arrange the terms of polynomials in ascending order or descending order.

In Learning Unit 10 "Laws of integral indices", students understand the laws of positive integral indices, such as $x^2x^3=x^5$, etc., and recognised the distributive property of multiplication in 4N5 "Four arithmetic operations (II)" of the primary Mathematics curriculum. In Learning Objective 11.2, students are required to use the aforementioned knowledge to perform addition, subtraction, multiplication of polynomials and their mixed operations. They are required to perform operations of polynomials with more than one variable. Performing division of polynomials belongs to the learning content of Learning Unit 4 "More about polynomials" of the Compulsory Part of the senior secondary Mathematics curriculum. Students are required to understand the concept of expansion of polynomials through multiplication of polynomials. Teachers may consider using the following area-related diagrams to elucidate the related concept.



In Learning Objective 11.3, students are required to understand that factorisation can be regarded as a reverse process of expansion of polynomials. In order to understand the meaning of factorisation, students are required to understand the meaning of one polynomial being a factor of another polynomial. Students are required to perform factorisation by extracting common factors (and grouping of terms) and cross-method. Teachers may provide appropriate examples to let students recognise that not all quadratic polynomials can be factorised by the above methods. Students use identities to factorise (or factorize) polynomials in Learning Objective 12.3, and factor theorem in Learning Unit 4 “More about polynomials” in the Compulsory Part of the senior secondary Mathematics curriculum to factorise more general polynomials. Teachers may consider using the following diagrams to help students understand the meaning of factorisation of polynomials.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
12. Identities	12.1 understand the concept of identities 12.2 use identities to expand algebraic expressions 12.3 use identities to factorise polynomials	8

Explanatory Notes:

In Learning Objective 12.1, students are required to understand the difference between equations and identities as well as proving identities. An identity can be considered as an equation that the solutions are all numbers. Teachers may use a linear equation in one unknown with infinitely many solutions as an example of identities to explain the related concept.

Teachers are advised to help students differentiate the variables of identities with the unknown coefficients, and introduce the method of comparing corresponding coefficients of polynomials to find the unknown coefficients. For example, by comparing the coefficients of x and constant terms respectively in the identity $A(2x+1)+B(x-1)\equiv 5x-2$, we have $2A+B=5$ and $A-B=-2$, and the values A and B can then be solved. Students may also solve A and B by substituting specific values of x, for example, putting $x=-\frac{1}{2}$ and $x=1$.

Learning Objective 12.2 involves using identities to expand algebraic expressions, including the identities of difference of squares $(a-b)(a+b)\equiv a^2-b^2$ and perfect squares $(a\pm b)^2\equiv a^2\pm 2ab+b^2$. The expansion of algebraic expressions are not confined to polynomials, for example, $(a\pm\frac{1}{a})^2\equiv a^2\pm 2+\frac{1}{a^2}$.

In Learning Objective 12.3, students are required to perform factorisation by using the identities of Learning Objective 12.2, but the objects for factorisation are restricted to polynomials only.

The identities of difference and sum of cubes are **not required**. Students may use factor theorem to factorise polynomials such as $x^3\pm a^3$ in Learning Unit 4 "More about polynomials" of the Compulsory Part of the senior secondary Mathematics curriculum.

This Learning Unit can also help students understand trigonometric identities to be introduced in Learning Objective 27.2.

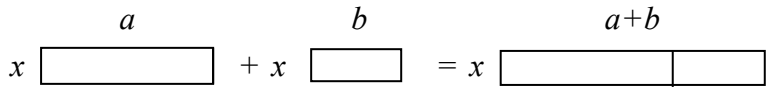
Learning Unit	Learning Objective	Time
Number and Algebra Strand		
13. Formulae	13.1 perform operations of algebraic fractions 13.2 use substitution to find the values of unknowns in the formulae 13.3 change the subject of formulae not involving radical signs	9

Explanatory Notes:

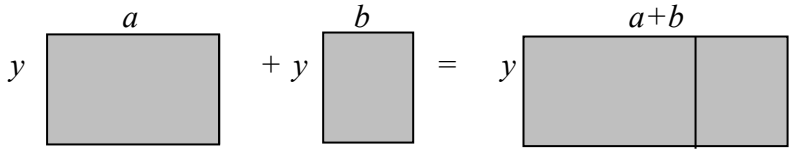
In Learning Unit 7 "Algebraic Expressions", students are required to represent formulae (or formulas) by algebraic expressions. In Learning Unit 11 "Polynomials", students are also required to perform addition, subtraction, multiplication and their mixed operations, and factorisation of polynomials. In this Learning Unit, students are required to further apply the above knowledge to algebraic expressions for the operations including moving terms, grouping terms and simplification, and finding values of unknowns by the method of substitution and solving equations.

In Learning Objective 13.1, students are required to simplify algebraic fractions through reduction of algebraic fractions. In order to avoid over complicated operations, algebraic fractions involved in this Learning Objective are confined to those having denominators as the product of linear factors with rational coefficients, for example: $\frac{1}{xy}$, $\frac{x^3}{xy^2}$, $\frac{1}{x(x+1)}$, $\frac{xy}{x(x+1)^2}$, $\frac{3x+6}{x(x+2)^2}$, etc. As such, even if students have not learnt how to factorise polynomials in general and find the greatest common divisor of polynomials (which will be handled only in the learning content of the Compulsory Part of the senior secondary Mathematics curriculum), they can still reduce algebraic fractions to the lowest form. Teachers may also compare the methods of simplifying algebraic fractions with reducing fractions to help students clarify some common mistakes. For example, some students may mistakenly simplify the algebraic fraction $\frac{3x}{3x+y}$ to $\frac{1}{1+y}$ by cancelling $3x$ in the numerator and denominator.

In this Learning Objective, students are **not required** to do over complicated operations of algebraic expressions. They are **not required** to do operations involving finding the greatest



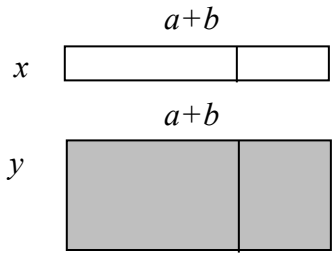
$$xa + xb = x(a + b)$$



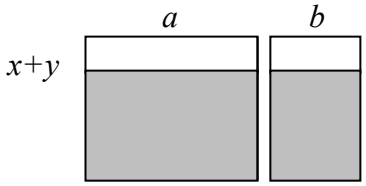
$$ya + yb = y(\ a + b)$$

Teachers may combine the diagrams to demonstrate how to factorise by using the method of grouping of terms.

$$\begin{aligned} xa + xb + ya + yb &= x(a + b) + y(\ a + b) \\ &= (x + y)(a + b) \end{aligned}$$



By allowing students to combine the diagrams in different ways, teachers may let students explore other ways of grouping of terms for factorisation.



$$\begin{aligned} xa + xb + ya + yb &= (xa + ya) + (xb + yb) \\ &= (x + y)a + (x + y)b \\ &= (x + y)(a + b) \end{aligned}$$

Factorisation by using the identities of difference of two squares or perfect squares is not included in this Learning Unit. The related content is covered in Learning Unit 12 “Identities”.

common divisor in algebraic fractions with different denominators in this Key Stage. They are also **not required** to decompose fractions by partial fraction.

Learning Objective 13.2 aims to help students understand that a formula is an algebraic equality to describe the relation between variables. By substituting different values into the same variables of a formula, the corresponding values of the unknown variable can be obtained through the same procedures of arithmetic operations. Teachers may consider using real-life examples or related learning elements in Science Education and Technology Education to strengthen the integration of students' knowledge, such as the density formula $D = \frac{m}{v}$ or the formula for conversion of temperature units $F = \frac{9C}{5} + 32$.

In Learning Objective 13.3, students are required to change the subject of formulae not involving radical signs by moving terms. In the process of changing subject of formulae, students may need to factorise part of the algebraic expressions in the formulae. Teachers may also summarise and compare the steps and sequences of mixed arithmetic operations, solving equations and change of subject of formulae to consolidate students' knowledge and skills in algebraic manipulations, so that students can be equipped to apply algebra to handle mathematical problems in more complex real-life scenarios, or in Science Education and Technology Education Key Learning Areas in the future.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
14. Linear inequalities in one unknown	14.1 understand the concept of inequalities 14.2 recognise the basic properties of inequalities 14.3 solve linear inequalities in one unknown 14.4 solve problems involving linear inequalities in one unknown	6

Explanatory Notes:

At primary level, students recognised the inequality signs “>” and “<”, and used them to express the relation of the magnitudes of two numbers. In Learning Objective 7.1, students are also required to represent word phrases by algebraic expressions. On these basis, in this Learning Unit, students are required to extend the concepts of algebraic expressions and equations to the use of inequality signs “>”, “<”, “≥” and “≤” to formulate and solve linear inequalities in one unknown.

The following are required in Learning Objective 14.1:

- Use inequalities to represent word phrases
- Represent the following inequalities on the number line:
  $x > a$, $x \geq a$, $x < a$ and $x \leq a$

Students are required to understand the meanings of signs “>”, “<”, “≥” and “≤” in Mathematics. Teachers may discuss with students the real-life scenarios involving the terms such as “at most”, “at least”, “not more than” and “not less than” to deepen students' understanding the meaning of signs “>”, “<”, “≥” and “≤” to avoid confusion. Teachers may also help students establish the preliminary concept of solution sets (students are **not required** to recognise the term “solution sets” and its definition). For example, the solutions of the inequality $x \geq 3$ are all the values that satisfy this numerical relation, such as 3, 3.1, 4, 100, ... , so that students may understand the mathematical meaning of the graphical representations of the solutions of inequalities on the number line. They may also get prepared for linking it to the learning of the graphical representations of solutions of linear equations in two unknowns and linear inequalities in two unknowns in the future.

Students are required to recognise and apply the basic properties of inequalities specified in

Learning Objective 14.2 for the operations used in solving linear inequalities in one unknown in Learning Objective 14.3. These properties include:

- If $a > b$ and $b > c$, then $a > c$
- If $a > b$, then $a \pm c > b \pm c$
- If $a > b$ and c is positive, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$
- If $a > b$ and c is negative, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

  where "$>$" and "$<$" in the above properties can be replaced by "$\geq$" and "$\leq$" respectively.

Teachers may compare the basic properties of linear equations in one unknown with linear inequalities in one unknown through concrete examples to emphasise and explain the differences between equations and inequalities, such as the four relations of "$>$", "$<$", "$\geq$" and "$\leq$" in inequalities are only transitive but not symmetric (students are **not required** to recognise mathematical terms such as "transitive relation" and "symmetric relation"); and "if $x > y$, then $ax > ay$" is not always true, etc., to reduce the common mistakes of students may make in solving inequalities.

In Learning Objective 14.3, students are required to solve linear inequalities in one unknown, and represent the solutions of inequalities on the number line. However, compound inequalities involving logical connectives "and" or "or" are **not included**. Those compound inequalities will be handled in Learning Objective 8.1 of the Compulsory Part of senior secondary Mathematics curriculum.

Students are also required to apply the above concepts and knowledge in Learning Objective 14.4 to solve problems involving linear inequalities in one unknown.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
15. Errors in measurement	15.1 recognise the concept of errors in measurement 15.2 recognise the concepts of maximum absolute errors, relative errors and percentage errors 15.3 solve problems related to errors 15.4 **design estimation strategies in measurement according to the contexts and judge the reasonableness of the results obtained	6

Explanatory Notes:

In the Measures Strand of the primary Mathematics curriculum, students learnt how to choose and use appropriate measuring tools and standard units to measure the length, weight, capacity of objects and distance between objects. In Learning Objective 15.1, students are required to further recognise that the actual value of a measure in all real measurements are unknown; and all measurement results obtained by using different measuring tools and measuring methods, such as reading the designated value of the measurement from digital measuring tools or traditional analogue measuring tools, are regarded as the approximate values of the measures. In Learning Objective 15.2, though teachers are **not required** to introduce the term and the symbol of absolute value at this Key Stage, students are required to recognise that the absolute value of the difference between a result of measurement and the actual value is the absolute error of the measurement. Teachers may guide students to recognise the relation between the absolute error and the scale interval of the measuring tool, and hence find the range of the actual value, as well as the maximum absolute error of the measurement. During the discussion on maximum absolute error, teachers should introduce the concepts of upper limit and lower limit of actual value, and how the degree of accuracy of the measurement could be described by using the maximum absolute error.

Students are required to recognise the relative error and percentage error by extending the concept of maximum absolute error. Teachers may use various real-life examples to illustrate the use of maximum absolute error, relative error and percentage error and the description of the degree of accuracy of the measurement. Nevertheless, students are **not required** to recognise how arithmetic operations on measurement would affect the values of the errors. They are **not required** to deal with problems of cumulative errors when using formulae to manipulate measurement either.

In Learning Objective 15.3, students are required to calculate the various types of errors of measurement from the known measurement results of an object and the scale interval of the measurement tool or the range of the actual measure. Students are also required to find the upper and lower limits of the actual measure of an object from known measurement result and the accuracy of measurement. Hence, students are required to deal with problems such as finding the upper and lower limits of the area of a rectangle from the given measurement results of the length and width of the rectangle and the accuracy of measurement.

In Learning Objective 15.4, teachers may also introduce appropriate enrichment topics according to students' abilities and interests, such as guiding them to explore some measurement strategies that can reduce errors in different situations. For example, students may measure the total weight of a large number of identical objects and then take the average as the weight of a single object. The above strategy can reduce the relative error and percentage error of the measurement, assuming that the weights of the identical objects are the same.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
16. Arc lengths and areas of sectors	16.1 understand the formula for arc lengths of circles 16.2 understand the formula for areas of sectors of circles 16.3 solve problems related to arc lengths and areas of sectors of circles 16.4 **recognise the Circle Dissection Algorithm of the ancient Chinese mathematician Liu Hui and further recognise Huilu and Tsulu (approximations of π)	8

Explanatory Notes:

In Learning Units 6M3 “Perimeter (II)” and 6M5 “Area (III)” of the primary Mathematics curriculum, students recognised and used the formulae for circumferences and areas of circles. However, problems on finding the diameter and radius of a circle from its given area were **not included**.

In this Learning Unit, students are required to understand the mathematical concepts of angles at the centre, arcs and sectors of a circle, etc.

In Learning Objective 16.1, students are required to understand the property that angles at the centre are proportional to the corresponding arc lengths in a circle. Teachers may consider first guiding students to discover that equally dividing the round angle at the centre of a circle means equally dividing the circle, and hence to understand that if the corresponding angle at the centre of an arc is obtained by taking *m* parts from *n* equal parts of the round angle at the centre of the circle, the arc length of the arc is also obtained by taking *m* parts from *n* equal parts of the circumference of the circle. From this, students are required to understand that the ratio of the corresponding angle at the centre of an arc to a round angle is equal to the ratio of the arc length to the circumference of the circle. Teachers are **not required** to further explain this proportional relationship when irrational numbers are involved. However, the formula for arc lengths of circles derived from the proportional relation as stated could be used for all arcs of circles. Applying the same concept, students should understand the formula for areas of sectors of circles in Learning Objective 16.2, by understanding the ratio of the area of a sector to the area of the corresponding circle is equal to the ratio of the angle of the sector to a round angle.

In Learning Objective 16.3, students are required to use the formulae for areas of circles, arc lengths and areas of sectors of circles to find unknowns such as areas, radii, diameters and arc lengths, etc. from given information. Teachers should include problems of finding the radius and diameter of a circle from given area in this Learning Objective as an interface between the primary and secondary Mathematics curricula. Students are also required to solve real-life problems related to arc lengths and areas of sectors of circles, including problems involving the perimeters and areas of composite figures which compose of figures such as circles, sectors and rectangles, etc.

In Learning Objective 16.4, teachers may include appropriate enrichment topics according to students' abilities and interests. For example, students may recognise how *Liu Hui* (劉徽) used the Circle Dissection Algorithm ("*ge yuan shu*", 割圓術) to find a more accurate approximations of the area of a circle and π ("*Huilu*", 徽率) by using regular polygons with numbers of sides as multiples of 6 to approximate their circumscribed circle. Teachers may also introduce to students the achievements of *Zu Chongzhi* (or *Tsu Chung-chi*, 祖沖之) in calculation of the approximation of π, that he proposed the "*yuelu*" (約率, approximate ratio) $\frac{22}{7}$ and the "*milu*" (密率, "close ratio") $\frac{355}{113}$ (also known as "*Tsulu*", 祖率). Indeed, *Zu*'s "*milu*" $\frac{355}{113}$ is even correct to 7 significant figures of π.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
17. 3-D figures	17.1 recognise the concepts of right prisms, right circular cylinders, right pyramids, right circular cones, regular prisms, regular pyramids, polyhedra and spheres 17.2 recognise the sections of prisms, circular cylinders, pyramids, circular cones, polyhedra and spheres 17.3 sketch the 2-D representations of 3-D figures 17.4 **recognise the three orthographic views of 3-D figures 17.5 **recognise Euler's formula and explore the number of regular polyhedra (Platonic solids)	5

Explanatory Notes:

In Learning Units 1S1 "3-D shapes (I)", 2S1 "3-D shapes (II)" and 5S2 "3-D shapes (III)" of the primary Mathematics curriculum, students preliminarily recognised the concepts of prisms, circular cylinders, pyramids, circular cones and spheres. However, teachers should note that at primary level, teachers are **not required** to use prisms, circular cylinders, pyramids and circular cones which are not right 3-D shapes as examples. Hence, students' recognition of these figures may be limited to the examples of right prisms, right circular cylinders, right pyramids and right circular cones only. As such, students are required to compare the examples of right prisms versus oblique prisms, and right pyramids versus oblique pyramids in Learning Objective 17.1, so as to recognise the concepts of right prisms, right circular cylinders, right pyramids and right circular cones. Students may first intuitively observe that all the lateral faces of a right prism are rectangles and a prism which is not right has lateral faces of parallelogram but not rectangle, as well as the line segment joining the centres of the two bases of a right circular cylinder are perpendicular to the two bases. Students may also intuitively recognise the concepts of right pyramids and right circular cones through the intuitive concept of taking the centre of gravity as the centroid of the base, that for all right pyramids and right circular cones, the line segment joining the vertex with the centroid of the base is perpendicular to the base. Teachers may illustrate the concept through examples such as the vertex of a right rectangular or square pyramid is vertically above the intersecting point of diagonals of the base, or the vertex of a right circular cone is vertically above the centre of the base circle. In this Learning Unit, students are only required to recognise intuitively the concept of a line segment being perpendicular to a plane. The

more formal mathematical concepts may be introduced later in Learning Unit 18 "Mensuration". Other than triangles, the mathematical definition of the centroid of a general figure is **not required**. Students are only required to recognise the above concepts in an intuitive way. For example, for a given regular polygon, a circle can be drawn through all the vertices of the regular polygon, and the centre of the circle is also the centroid of the regular polygon.

After recognising the concepts of right prisms and right pyramids, students are required to recognise that right prisms and right pyramids with regular polygons as their bases are called the regular prisms and regular pyramids respectively. Students are also required to recognise the concept of polyhedra, including a regular tetrahedron is a special case of a regular triangular pyramid, and its properties such as all its faces are equilateral triangles of the same size. Concepts of other regular polyhedra and the number of regular polyhedra are Enrichment Topics in Learning Objective 17.5.

Students are also required to recognise the concept of spheres, including all points on the sphere being equidistant from a fixed point, and this fixed point is called the centre of the sphere.

In Learning Objective 17.2, students are required to further recognise the sections (also known as cross sections) of different kinds of 3-D figures learnt in Learning Objective 17.1. These 3-D figures include prisms, circular cylinders, pyramids, circular cones, polyhedra and spheres. According to the recognition of the sections of prisms and circular cylinders in Key Stage 2, i.e., the sections parallel to the base of the prism/circular cylinder have the same shape and size as the base, students are required to recognise that all prisms and circular cylinders have the property of uniform cross sections, no matter they are right or not. Since the term "uniform cross sections" is **not required** in the primary Mathematics curriculum, teachers should introduce the term and its related concepts here. Teachers may introduce an intuitive concept of parallel planes: Two planes are parallel if they do not intersect with each other. For 3-D figures other than prisms and circular cylinders, students are only required to recognise that different sections of the same 3-D figure can have different sizes and shapes, thus establishing students' spatial sense.

In Learning Objective 17.3, students are required to sketch the 2-D representations of right prisms, right circular cylinders, right pyramids and right circular cones. Students are required to master the relevant drawing techniques, and identify the three-dimensional directions shown in the 2-D representations (such as above and below, left and right, front and back, etc.), but students are **not required** to learn sketching specifications such as in technical

drawings. Teachers may adopt common assistive tools for sketching 2-D representations, such as oblique grid and isometric grid, to help students learn to sketch the 2-D representations of 3-D figures on blank paper step-by-step. However, the use of oblique grid and isometric grid is not the focus of the curriculum, and students are **not required** to draw the representations according to specified dimensions. Students recognised intuitively the concept of right 3-D figures in Learning Objective 17.1, and in this Learning Objective, teachers should remind students to correctly mark the positions of vertices, right-angle symbols, etc. when sketching the 2-D representations of right prisms, right circular cylinders, right pyramids, and right circular cones to show that the figures being sketched are right 3-D figures (for example, as shown in Figure 17.1 and Figure 17.2). Students are **not required** to accurately draw the 2-D representations, but the plane figures they sketch should be visually aligned with the concept of right 3-D figures.

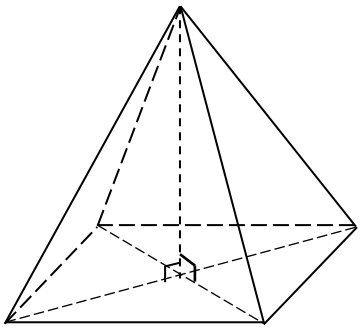


Figure 17.1

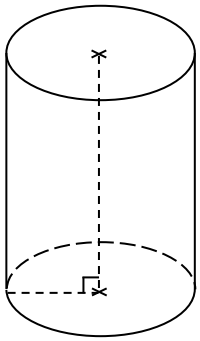


Figure 17.2

Recognition of the three orthographic views of 3-D figures belongs to the Enrichment Topics. Teachers may include appropriate content regarding three orthographic views of 3-D figures according to students' abilities and interests in Learning Objective 17.4, such as drawing the three orthographic views from a given 3-D figures, or guessing the possible corresponding 3-D figures from given three orthographic views.

Teachers may also include appropriate content regarding Euler's formula and regular polyhedra as Enrichment Topics in Learning Objective 17.5 according to students' abilities and interests. Teachers may introduce to students the simple regular polyhedra (also known as Platonic solids) other than regular tetrahedron and cube (regular hexahedron) as a continuation of Learning Objective 17.1. Teachers may let students explore the geometric properties of the polygons which may be used to construct Platonic solids, thereby the upper limit of the number of Platonic solids can be derived. Teachers may also introduce Euler's formula to students and let them explore what 3-D figures do not satisfy Euler's formula, and hence introduce the initial concepts of Euler characteristic which can be used as a classification of 3-D figures in topology.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
18. Mensuration	18.1 recognise the formulae for volumes of prisms, circular cylinders, pyramids, circular cones and spheres 18.2 find the surface areas of right prisms, right circular cylinders, right pyramids, right circular cones and spheres 18.3 recognise the relations among lengths, areas and volumes of similar figures and solve related problems 18.4 solve problems involving volumes and surface areas 18.5 **explore ways to form a container with the greatest capacity by folding an A4-sized paper with squares cutting from its four corners	15

Explanatory Notes:

This Learning Unit is the continuation of students' recognition of 3-D figures in Learning Unit 17 "3-D figures" and the recognition of 2-D representations of 3-D figures. Students are required to understand the 2-D representations of 3-D figures in this Learning Unit, extract information related to the measurement of the three-dimensional space and 3-D figures from the 2-D representations of 3-D figures, and apply them to solve problems involving volumes and surface areas. In Learning Unit 5M2 "Volume (I)" of the primary Mathematics curriculum, students recognised and applied the formulae for volumes of cubes and cuboids, and recognised the relation between capacity and volume in Learning Unit 6M2 "Volume (II)".

In Learning Objective 18.1, students are required to recognise the definition of a straight line being perpendicular to a plane. That is, if a straight line L is perpendicular to a plane, then L intersects the plane at a point P, and any straight lines on the plane passing through point P is perpendicular to L. Teachers should also introduce to students the conditions for determining whether a straight line is perpendicular to a plane. That is, if a straight line L intersects a plane at a point P, and there are two straight lines on the plane which pass through point P and are perpendicular to L, then L is perpendicular to the plane. Nevertheless, students are **not required** to prove that the condition is sufficient to ensure that the line is perpendicular to the plane. From the concept of a straight line being perpendicular to a plane,

students are required to recognise the concept of the orthogonal projection of a point on a plane. However, the term "orthogonal projection" is **not required** by the curriculum, students may use the term "projection" to describe the concept. Using the concept of the projection of a point on a plane, students are required to recognise that the distance from the point to its projection point defines the distance from the point to the plane. Students are then required to recognise the definitions of heights of prisms and pyramids from the aforementioned concept. Teachers may consider introducing the intuitive concept of the heights of general 3-D figures before the heights of prisms and pyramids, i.e. the maximum distance between all points and the specified base.

Students are required to recognise the formulae for volumes of pyramids and circular cones. Through dissecting and combining certain specific pyramids, students may discover that the volume of the pyramid is $\frac{1}{3}$ of the volume of the prism with the same base area and height as the pyramid. Students are **not required** to prove that such relation can be extended to general pyramids and circular cones, but students are required to apply the formulae for volumes of pyramids and circular cones to find the volumes and other related unknowns of general pyramids and circular cones. Students are also required to recognise and apply the formula for volumes of spheres to find the volumes of spheres or other unknowns. Students are **not required** to understand the proof of the formula. As an enrichment, teachers may provide appropriate demonstrations to explain the formula for volumes of spheres according to the abilities and interests of students.

In Learning Objective 18.2, students are required to find the surface areas of right prisms, right circular cylinders, right pyramids, right circular cones and spheres. In Learning Unit 5S2 "3-D figures (III)" of the primary Mathematics curriculum, students recognised the nets of cubes, cuboids and circular cylinders. On this basis, students are required to find the surface areas of cubes, cuboids and circular cylinders, and extend the related concepts to general right prisms and right pyramids. Students are required to recognise that the curved surfaces of right circular cones are sectors of circles, therefore the surface areas of right circular cones can be found by using the formula for areas of sectors of circles in Learning Unit 16 "Arc lengths and areas of sectors". Students are also required to recognise and apply the formula for surface areas of spheres to find surface areas of spheres or other unknowns. Students are **not required** to understand the proof of the formula. However, teachers may provide appropriate demonstrations to explain the formula for surface areas of spheres according to the abilities and interests of students as an enrichment.

In Learning Objective 18.3, students are required to build on the concept of similar plane

figures in Learning Unit 22 “Similar triangles” to recognise the concept of similar 3-D figures. Students should recognise that if the two 3-D figures are similar, all corresponding lengths of the figures (including the lengths of edges, the heights, the lengths of the diagonals, the perimeters of the corresponding faces, etc. of the 3-D figures) are proportional, and all the corresponding angles are equal. Students are **not required** to prove that two 3-D figures are similar, but teachers should give appropriate examples to illustrate that if two 3-D figures fail to meet the above conditions for similar 3-D figures (for example, not all the corresponding sides are proportional), then it can be determined that the two 3-D figures are not similar.

Students are required to recognise that for two given similar figures, the ratio of the square of any corresponding lengths is equal to the ratio of the areas of any corresponding surfaces; and the ratio of the cube of any corresponding lengths is equal to the ratio of the volumes of any corresponding parts of the figures. Students are required to recognise frustums from the relations between two similar pyramids or circular cones, and solve problems related to their surface areas and volumes.

In Learning Objective 18.4, students are required to integrate and apply the knowledge of Learning Objectives 18.1, 18.2 and 18.3 to solve problems involving volumes and surface areas of composite figures consisting of 3-D figures which have already been recognised in the above Learning Objectives. Students are also required to find the geometric quantities such as unknown lengths and areas from given volumes or surface areas.

Teachers may include appropriate enrichment topics in Learning Objective 18.5 according to the abilities and interests of students, such as exploring ways to form a container with the greatest capacity by folding an A4-sized paper with squares cutting from its four corners. Through cutting out squares of different sizes, students may discover that the change of the container’s capacity does not keep increasing or decreasing, thus establishing the concept of optimisation problems. Teachers may also enhance students’ understanding on graphs and variables by using graphs to show the change of the capacity.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
19. Angles and parallel lines	19.1 understand the concepts and properties of adjacent angles on a straight line, vertically opposite angles and angles at a point 19.2 understand the concepts of corresponding angles, alternate interior angles and interior angles 19.3 recognise the conditions for two straight lines being parallel 19.4 recognise the angle properties associated with parallel lines 19.5 understand the properties of the interior and exterior angles of triangles	11

Explanatory Notes:

From this Learning Unit to Learning Unit 25 "Pythagoras' theorem", students are required to gradually understand the theorems associated with various rectilinear figures in deductive geometry and the logical relationship between theorems, thereby establishing the basic knowledge and ability to learn plane geometry in a more formal way through logic and deduction.

In Learning Units 1S2 "2-D shapes" and 2S2 "Angles" of the primary Mathematics curriculum, students recognised preliminarily the intuitive concept of straight lines and angles respectively. In Learning Unit 6M1 "Angle (degree)", students also recognised the measures of angles, as well as reflex angles, straight angles and round angles. Learning Objective 19.1 summarises the above intuitive concepts, and lists out three properties related to straight lines, including:

- The sum of the adjacent angles on a straight line is equal to a straight angle,
- Vertically opposite angles are equal, and
- The sum of angles at a point is equal to a round angle.

Teachers may guide students to understand the logical relationship between premises and conclusions in arguments involving geometric theorems, including the use of the above properties in deducing the unknown angles from the figures formed by straight lines or line

segments. Students should also understand the logical relationship between the three properties. For example, the two properties "vertically opposite angles are equal" and "the sum of all the angles at a point is equal to a round angle" can be deduced from the property "the sum of the adjacent angles on a straight line is equal to a straight angle". According to students' interests and abilities, teachers may guide students to recognise that, on the other hand, just from one of the properties "vertically opposite angles are equal" or "the sum of all the angles at a point is equal to a round angle", one cannot conversely deduce the property "the sum of the adjacent angles on a straight line is equal to a straight angle". This helps students preliminarily recognise the logical relationship of geometric theorems.

Students are also required to recognise the concepts of complementary angle and supplementary angle in this Learning Objective. Teachers may consider organising inquiry activities for students to discover that when two angles are supplementary to each other, the two angles share a common vertex and lie on two sides of the common side, then the sides which are not common lie on the same straight line.

In Learning Objective 19.2, students are required to recognise that when a straight line L intersects with another two straight lines L_1 and L_2 respectively at different points, L is called a transversal of L_1 and L_2, where L_1 and L_2 are not limited to parallel lines. From this, students are required to understand the definitions of the respective pairs of corresponding angles, alternate interior angles and interior angles on the same side when L is the transversal, thereby linking to the content of Learning Objectives 19.3 and 19.4.

In Learning Unit 3S1 "Quadrilaterals (II)" of the primary Mathematics curriculum, students recognised preliminarily the concept of parallel lines. In Learning Objective 19.3, students are required to recognise further the conditions for determining two straight lines being parallel through the measure of particular angles. These conditions include, when there is a transversal L of the two straight lines,

- a pair of alternate interior angles are equal,
- a pair of corresponding angles are equal, or
- a pair of interior angles on the same side are supplementary.

Students are required to select a suitable condition to prove that the two straight lines are parallel according to the given measures of different angles. In addition, students are required to recognise that when the two straight lines are not parallel, each pair of alternate interior angles and corresponding angles are not equal, and both pairs of interior angles on the same side are not supplementary.

In Learning Objective 19.4, students are required to recognise the angle properties associated with parallel lines, including:

- Alternate interior angles are equal,
- Corresponding angles are equal, and
- Interior angles are supplementary.

Students are required to select appropriate parallel lines and transversals from the given rectilinear figures to find the measure of the unknown angles by choosing the appropriate properties described above. Students are also required to distinguish that the conditions mentioned in Learning Objective 19.3 and the properties mentioned in Learning Objective 19.4 are the converses of each other logically and in general they are not equivalent. Students are **not required** to recognise the logical terms such as "converse proposition" and "equivalent". Teachers may use appropriate examples to illustrate the logical relationships in everyday language. Teachers are advised to let students recognise that the properties mentioned in Learning Objective 19.4 can derive that two straight lines are not parallel if any one pair of the alternate interior angles or corresponding angles formed by the two straight lines and any one of the transversals are not equal, or any one pair of interior angles on the same side formed in the same way are not supplementary. Students can thus prove that two straight lines are not parallel from some of the given angles.

In Learning Objective 19.5, students are at first required to understand the definitions of interior and exterior angles of triangles. Through these definitions, the property of adjacent angles on a straight line in Learning Objective 19.1, and the angle properties associated with parallel lines in Learning Objective 19.4, students are required to prove that the sum of all the interior angles of any triangle is a straight angle, and that any exterior angle of a triangle is equal to the sum of its two interior opposite angles. After establishing the geometric theorems about the properties of the interior and exterior angles of triangles, students are required to use these properties to find the unknown angles in plane figures related with triangles. Students are also required to recognise the concepts of acute-angled triangles and obtuse-angled triangles.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
20. Polygons	20.1 understand the concept of regular polygons 20.2 understand the formula for the sum of the interior angles of a polygon 20.3 understand the formula for the sum of the exterior angles of a convex polygon 20.4 appreciate the triangles, quadrilaterals, and regular polygons that tessellate in the plane 20.5 construct equilateral triangles and regular hexagons with compasses and straightedge 20.6 **explore ways to construct regular pentagons with compasses and straightedge	8

Explanatory Notes:

In this Learning Unit, students are required to further understand the properties of sides, angles of different polygons based on the properties of the interior and exterior angles of triangles in Learning Objective 19.5.

Although students have recognised two kinds of regular polygons, namely squares and equilateral triangles, in Learning Objectives 3S1 "Quadrilaterals (II)" and Learning Objective 3S2 "Triangles" respectively in primary Mathematics curriculum, the term "regular polygons" is a new concept and mathematical term for them in Learning Objective 20.1. In this Learning Objective, students are required to understand the concept of regular polygons. They are required to understand that a polygon may not be a regular polygon if only all of its sides are equal or only all of its interior angles are equal. In order to deepen students' understanding of the concept of regular polygons, teachers may use models or information technology to provide students with examples and non-examples of different types of regular polygons, i.e. rectangles, rhombuses, equal-sided (or equilateral) pentagon, equal-angled (or equiangular) hexagon, etc.

In Learning Objective 20.2, students are required to understand the formula for the sum of the interior angles of an arbitrary polygon. Teachers may lead students to use an appropriate method to cut an arbitrary convex polygon into a finite number of triangles. Hence, students

can deduce the formula for the sum of the interior angles of convex polygons by using the relationship between the number of triangles formed and the number of sides of the polygons, and using the sum of interior angles of triangle as a straight angle. For concave polygons, students are only required to investigate the methods of dividing known-shaped concave polygons for discovering the formula for the sum of the interior angles of these polygons is the same as that for those convex polygons with the same number of sides.

Although students are required to understand the formula for the sum of the interior angles are valid for both convex polygons and concave polygons with same number of sides, students are **not required** to recognise the definition of convexity and concavity of general plane figures.

Students are required to find the measures of interior angles of different regular polygons by the formula for the sum of the interior angles and the definition of regular polygons.

In Learning Objective 20.3, students are required to understand the formula for the sum of the exterior angles of a convex polygon. Students may prove the sum of exterior angles of an arbitrary convex polygon is a round angle through the definition of exterior angles of convex polygons, the sum of adjacent angles on a straight line is equal to a straight angle, and the formula for the sum of interior angles.

In Learning Objective 20.4, teachers should introduce to students how to distinguish the polygons which can tessellate in the plane by considering the measures of interior angles of different types of polygons. Students may appreciate that all triangles and quadrilaterals can tessellate in the plane by itself through the sum of interior angles of triangles being a straight angle and the sum of interior angles of quadrilaterals being a round angle. By the same method, students are required to distinguish the kinds of regular polygons which can tessellate in the plane by itself. According to the abilities and interests of students, teachers may arrange exploratory activities for students to appreciate the methods of using more than one regular polygons to tessellate in the plane.

In Learning Objective 20.5, students are required to construct equilateral triangles and regular hexagons with compasses and straightedge. The straightedge mentioned in Learning Objective 20.5 is a straightedge without scale. Students are required to recognised the difference in features in constructing figures between using compasses and straightedge and other drawing tools such as protractor, set square and ruler: Geometric construction using compasses and straightedge applies geometric theorems to find construction procedures for geometric objects such as points, line segments, angles, arcs, etc. which mathematically fulfill

the requirements for construction. Teachers may let students recognise the basic knowledge about using compasses and straightedge for constructing the figures. Information technology may also be used to replace the physical compasses and straightedge for constructing figures. However, the focus of this Learning Objective should be placed at the geometric knowledge included in the procedures of constructing the equilateral triangles and regular hexagons.

After students learnt the methods and related geometric knowledge of using compasses and straightedge to construct equilateral triangles and regular hexagons, according to students' abilities and interests, teachers may add suitable enrichment topics in Learning Objective 20.6, such as continue to explore with students the methods of constructing a regular pentagon with compasses and straightedge.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
21. Congruent triangles	21.1 understand the concept of congruent triangles 21.2 recognise the conditions for congruent triangles 21.3 understand the property of isosceles triangles 21.4 understand the condition for isosceles triangles 21.5 construct angle bisectors, perpendicular bisectors, perpendicular lines, parallel lines, special angles and squares with compasses and straightedge 21.6 recognise the concept of congruent 2-D figures 21.7 **explore the angles that can be constructed with compasses and straightedg	14

Explanatory Notes:

In this Learning Unit, students are required to establish the concept of congruent triangles, and apply it as a basic knowledge to solve geometric problems.

In Learning Objective 21.1, students are required to understand the definition of congruent triangles and the geometric properties extended from its definition. Students are required to extend the intuitive concept of "coincidence" to understand that two triangles are congruent means that each of the three pairs of corresponding sides and each of the three pairs of corresponding angles are equal. Students are required to identify the corresponding sides and corresponding angles of given congruent triangles, and find the measures of sides and angles. Students are also required to understand from the definition that for a pair of triangles, if any pairs of corresponding sides or corresponding angles are not equal, then it can be determined that the pair of triangles are not congruent.

In Learning Objective 21.2, students are required to recognise the conditions for congruent triangles, including SAS, SSS, ASA, AAS and RHS. Students are required to recognise that it is sufficient to determine that a pair of triangles are congruent if they fulfill any one of the above conditions, and each of the remaining three pairs of corresponding sides and corresponding angles are consequently equal. Students are required to use the above conditions to prove the congruence of triangles. Students are also required to recognise through counterexamples that some conditions, such as SSA and AAA, cannot determine

whether a pair of triangles are congruent. According to the abilities and interests of students, teachers may consider exploring the logical relationship amongst the conditions of congruent triangles with students.

In Learning Unit 3S2 "Triangles" of the primary Mathematics curriculum, students recognised the concept of isosceles triangles. In Learning Objective 21.3, students are required to further understand and prove the property of isosceles triangles that its base angles are equal. Teachers may let students recognise the proof of base angles of isosceles triangles are equal by SAS. It should nevertheless be noted that, since this property of isosceles triangles (i.e. its base angles are equal) is necessary for the proof of SSS being a condition for congruent triangles, and using compasses and straightedge for constructing angle bisectors, perpendicular bisectors and perpendicular lines are based on the property of isosceles triangles that the base angles are equal, teachers should avoid using methods such as adding a line segment to join the vertex with the mid-point of the base to prove the above property of isosceles triangles. Otherwise, circular argument will result. Students are required to use the property that base angles of isosceles triangles are equal, as well as the properties of interior angles and exterior angles of triangles, to solve geometric problems related to finding unknown angles of triangles or proving the congruence of triangles.

In Learning Objective 21.4, students are required to understand if two angles of a triangle are equal, then the opposite sides of these two angles are equal, and hence this triangle is an isosceles triangle. Students are required to prove the above geometric property and apply it to solve geometric problems such as finding unknown length of sides of triangles and proving the congruence of triangles. In Learning Objectives 21.3 and 21.4, students may take equilateral triangles as a special kind of isosceles triangles to understand that equilateral triangles are also equiangular triangles, and vice versa. As such, when connecting with the concepts of regular polygons in Learning Objective 20.1, only in the discussion of triangles, regular triangles, equilateral triangles and equiangular triangles refer to the same kind of triangles.

In Learning Objective 21.5, students are required to construct angle bisectors, perpendicular bisectors, perpendicular lines, parallel lines, special angles and squares with compasses and straightedge without scale. Amongst these, special angles refer to those angles which can be constructed by the above construction skills and the methods of constructing equilateral triangles and squares. Students are required to manage these methods of construction. However, this Learning Objective focuses on the geometric reasoning behind the methods of construction. Hence, students are **not required** to construct figures which are over complicated or involve many repeated steps. Teachers are advised to demonstrate the

geometric meanings clearly in each step during teaching the above constructions, but students are only required to use the above construction techniques to complete the construction whilst explanation in words and proof of the steps for construction are **not required**. It is not necessary for students to stick to the use of physical compasses and straightedge, but they may use information technology with the same function as compasses and straightedge to do construction so as to focus on the geometric meaning of these geometric construction methods.

In Learning Objective 21.6, students are required to extend the concepts of congruent triangles to congruent 2-D figures. They are only required to recognise the congruent relationships of general 2-D figures (including closed figures which are not polygons) intuitively through the concept of "coincidence"; and if two 2-D figures are congruent, then each pair of the corresponding lengths (e.g. sides, diagonals, radii, perimeters, etc.) and each pair of the corresponding angles are equal. Students are **not required** to learn how to prove congruent 2-D figures except triangles, but are required to identify that two 2-D figures are not congruent.

After students learnt the construction methods in Learning Objective 21.5, according to the abilities and interests of students, teachers may arrange exploratory activities on using compasses and straightedge without scale as an enrichment topic as to enhance students' understanding in deductive geometry in Learning Objective 21.7. These activities may include using compasses and straightedge without scale to explore whether angles such as $\frac{360°}{2^n}, \frac{120°}{2^n}$ can be constructed, where n is any positive integers.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
22. Similar triangles	22.1 understand the concept of similar triangles 22.2 recognise the conditions for similar triangles 22.3 recognise the concept of similar 2-D figures 22.4 **explore shapes of fractals	9

Explanatory Notes:

In this Learning Unit, students are required to further understand the concept of similar triangles after understanding the concept of congruent triangles. Teachers are reminded to let students have some revisions of solving equations in the form such as $\frac{2}{x} = \frac{4}{3}$ as students may come across equations in similar form.

In Learning Objective 22.1, students are required to understand the definition of similar triangle, i.e. two triangles with all corresponding angles equal and all corresponding sides proportional. Teachers may discuss with students the similarity and difference between congruent relationship and similar relationship, so as to let them discover that congruent triangles are a special case of similar triangles. Teachers may also explain, through the intuitive geometric concept, that a triangle is similar to its original triangle by enlarging (reducing) the sides in scale. Students are required to find the measures of unknown sides and unknown angles in a given pair of similar triangles using the definition of similar triangles.

In Learning Objective 22.2, students are required to recognise the conditions for similar triangles, including AAA (or AA), corresponding sides proportional, and two corresponding sides proportional and their included angles equal. Students are **not required** to prove that any one of the above conditions is sufficient to prove that a pair of triangles satisfying the condition will consequently result in all corresponding angles equal and corresponding sides proportional. However, students are required to recognise the above results, and use these conditions to prove that two triangles are similar, and find the measures of the unknown sides and unknown angles in triangles.

According to the abilities and interests of students, after they have got hold of the mid-point theorem and the intercept theorem in Learning Objective 23.5, teachers may explore with students through some simple examples about why two triangles which fulfill any one of the

conditions for similar triangles will result in each pair of corresponding angles equal and corresponding sides proportional.

In Learning Objective 22.3, students are required to extend the concept of similar triangles to similar 2-D figures. Students are required to recognise intuitively that a 2-D figure is similar to its original figure by enlarging (reducing) the sides in scale. Students are also required to recognise the properties of two similar 2-D figures, namely all corresponding lengths (including sides, diagonals, radii, circumference, etc.) are proportional and all corresponding angles are equal. Except triangles, students are **not required** to prove any 2-D figures are similar figures. However, students are required to point out through examples that two figures are not similar considering the above properties. For example, although the corresponding sides of a square and a rhombus without right angle are proportional, their corresponding angles are not equal, and hence they are not similar; and although the corresponding angles of a square and a rectangle with different adjacent sides are equal, the corresponding sides are not proportional, and hence they are not similar. Students are also required to find the measures of unknown lengths and unknown angles in given similar 2-D figures. This Learning Objective should also connect with Learning Objective 18.3 "Recognise the relations amongst lengths, areas and volumes of similar figures and solve related problems" to consolidate students' concept on the similar relationships in geometry to apply to problems related to measurement in real-life scenarios.

According to the abilities and interests of students, teachers may introduce through enrichment activities the concept of self-similarity in Learning Objective 22.4 for students to explore shapes of fractals. Teachers may also introduce the methods of iteration to students and let them create simple shapes of fractals by using information technology.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
23. Quadrilaterals	23.1 understand the properties of parallelograms 23.2 understand the properties of rectangles, rhombuses and squares 23.3 understand the conditions for parallelograms 23.4 apply the above properties or conditions to perform simple geometric proofs 23.5 understand the mid-point theorem and the intercept theorem 23.6 **explore the conditions for congruent quadrilaterals	13

Explanatory Notes:

In Learning Units 2S4 “Quadrilaterals (I)”, 3S1 “Quadrilateral (II)” and 4S1 “Quadrilaterals (III)” of the primary Mathematics curriculum, students recognised the properties related to the length of sides and parallel properties of trapeziums, parallelograms, rectangles, rhombuses and squares. They also recognised the inclusion relationships between different kinds of quadrilaterals. However, teachers should pay attention that at the primary level students did not distinguish clearly which properties constitute the definitions and which properties can be deduced from the definitions of the quadrilaterals. Students recognise in Learning Unit 19 “Angles and parallel lines” the angle properties associated with parallel lines. They also recognise the concepts related to congruent triangles and similar triangles in Learning Unit 21 “Congruent triangles” and Learning Unit 22 “Similar triangles” respectively. In this Learning Unit, students are required to integrate and apply the above geometric knowledge to parallelograms, rectangles, rhombuses and squares, so as to further understand the logical relationships between the definitions and the geometric properties of these figures, and the applications of these geometric properties.

In Learning Objective 23.1, students are required to understand and prove the properties of parallelograms, including opposite sides equal, opposite angles equal, and two diagonals bisect each other, by the definition of parallelograms (that two pairs of opposite sides parallel), the concept of congruent triangles, and the angle properties associated with parallel lines.

In Learning Objective 23.2, students are required to understand the properties of rectangles, rhombuses, and squares. Students recognised that rectangles, rhombuses and squares are special types of parallelograms; and squares are a special type of both rectangles and rhombuses in Learning Unit 4S1 "Quadrilaterals (III)" of the primary Mathematics curriculum. In this Key Stage, students are further required to understand that as rectangles are parallelograms, they possess all geometric properties of parallelograms. Besides, students are required to understand that rectangles have the geometric properties of diagonals equal, and diagonals bisecting each other into four equal line segments, which general parallelograms do not have. Similarly, students are also required to understand that as rhombuses are parallelograms, and hence they possess all geometric properties of parallelograms. Besides, students are required to understand that rhombuses have the geometric properties of diagonals perpendicular to each other, and diagonals bisecting the opposite angles, which general parallelograms do not have. Lastly, students are required to understand that as squares are both rectangles and rhombuses, and hence they possess all geometric properties of rectangles and rhombuses, in addition to the property that the diagonals form angles of 45° with the sides of the squares.

In Learning Objective 23.3, students are required to understand if a quadrilateral has either: (1) opposite sides equal, (2) opposite angles equal, (3) diagonals bisecting each other, or (4) one pair of opposite sides equal and parallel, then this quadrilateral is a parallelogram. Students are required to prove the above four situations are the conditions for parallelograms by applying the geometric knowledge in congruent triangles, conditions for two straight lines being parallel, and so on.

In Learning Objective 23.4, students are required to integrate and apply the geometric definition, theorems, properties and conditions in Objectives 23.1 to 23.3 and other related Learning Objectives to perform simple geometric proofs related to quadrilaterals, such as to prove a pair of opposite sides being parallel in a quadrilateral, and a quadrilateral is a rectangle, rhombus, or square. Students are also required to identify the difference between geometric properties of and conditions for a specific type of quadrilaterals, as well as the logical relationships between the properties and conditions. For example, four sides equal is a property that squares possess, but it is insufficient to determine that a quadrilateral with four sides equal is a square; and all diagonals of rhombus are perpendicular to each other, but a quadrilateral with perpendicular diagonals may not be a rhombus.

In Learning Objective 23.5, students are required to understand the content, proof and applications of the mid-point theorem and the intercept theorem. Teachers should pay attention to avoid the possible circular argument in proving the mid-point theorem and the

intercept theorem, such as proving the mid-point theorem and the intercept theorem involving proving similar triangles. This is because in general, the generalized results of the mid-point theorem and the intercept theorem are more fundamental than the three conditions for similar triangles in the course of deduction. Although students are **not required** to understand the geometric proof of all three conditions for similar triangles, teachers may, according to the abilities and interests of students, use some simple examples to explore with students why two triangles fulfilling only one of the conditions for similar triangles will consequently result in all corresponding angles equal and all corresponding sides proportional as applications of mid-point theorem and intercept theorem. Teachers may guide students to prove the mid-point theorem by appropriate extension of the line segment connecting mid-points, the properties and the conditions for congruent triangles, and the conditions for two parallel lines and parallelograms. Similarly, teachers may also guide students to prove the intercept theorem by constructing an appropriate parallelogram, and using its properties, the conditions and the properties of congruent triangles. Teachers may provide opportunities for students to recognise the deductive methods in Euclid's *Elements* in this Learning Objective or other parts in this Learning Unit.

In Learning Objective 23.6, teachers may arrange enrichment activities for students to explore the conditions for congruent quadrilaterals according to the abilities and interests of students. Teachers may guide students to consider that besides the sides and the angles of quadrilaterals, the diagonals can also be used for the conditions for congruent quadrilaterals. Students may begin with exploring the conditions for congruent quadrilaterals by specific types. For example, the conditions for congruent squares can either be a pair of corresponding sides or diagonals equal; whilst the conditions for congruent rectangles can be two pairs of corresponding sides equal.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
24. Centres of triangles	24.1 understand the properties of angle bisectors and perpendicular bisectors 24.2 understand the concurrence of angle bisectors and the concurrence of perpendicular bisectors of a triangle 24.3 recognise the concurrence of medians and the concurrence of altitudes of a triangle	8

Explanatory Notes:

In this Learning Unit, students are required to understand the properties of angle bisectors and perpendicular bisectors, and hence the definitions and geometric properties of the incentre and the circumcentre of a triangle. Students are also required to recognise the definitions of the centroid and the orthocentre of a triangle. As prerequisite knowledge, students are first required to recognise the definitions of altitudes, medians, angle bisectors and perpendicular bisectors of a triangle, so as to further understand or recognise the geometric properties of these special line segments, and the relationships between these properties and the centres of triangles mentioned.

In Learning Objective 24.1, students are required to understand the geometric properties of angle bisectors and perpendicular bisectors, including (1) if a point lies on the angle bisector, then this point is equidistant from the two arms of the angle, and vice versa; and (2) if a point lies on the perpendicular bisector of a line segment, then this point is equidistant from the the two end points of the line segment, and vice versa. Students are required to prove the above properties by the definitions of angle bisectors and perpendicular bisectors, and the conditions for congruent triangles.

Based on the above properties in Learning Objective 24.2, students are required to prove that in any triangles, the three angle bisectors of a triangle and the three perpendicular bisectors of a triangle are respectively concurrent. Students may consider the properties of angle bisectors to understand that the intersection point of any two angle bisectors of a triangle is equidistant from the three sides of the triangle, and hence this point also lies on the last angle bisector. Students may use similar method to prove the concurrence of the three perpendicular bisectors of a triangle. Students are required to recognise the intersection points are the incentre and circumcentre of the triangle respectively. Students are required to recognise that

the incentre is equidistant from the three sides of the triangle, and hence with this distance as radius and the incentre as centre, a circle can be constructed and fit into the triangle. Students are also required to recognise that since the distances between the circumcentre and the three vertices of a triangle are equal, with this distance as radius and the circumcentre as centre, a circle passing through the three vertices of the triangle can be constructed. Teachers are **not required** to introduce the concepts like the inscribed circle, circumcircle or tangents to a circle in this Key Stage. These concepts belong to Learning Unit 11 “Basic properties of circles” in the Compulsory Part of senior secondary Mathematics curriculum.

In Learning Objective 24.3, students are required to recognise that the three medians of a triangle are concurrent and the three altitudes of a triangle are concurrent. They are required to recognise the definitions of the centroid and the orthocentre of a triangle as the intersection point of the medians and the intersection point of the altitudes respectively. Students are **not required** to prove the above geometric properties, but according to their abilities and interests, teachers may facilitate students to recognise the proofs of the concurrence of medians and concurrence of altitudes through the aids of information technology.

The learning content of the incentre, circumcentre, centroid and orthocentre of triangles in this Learning Unit connects with the Compulsory Part of senior secondary Mathematics curriculum in related Learning Units, such as Learning Unit 11 “Basic properties of circles”, where the learning content will further be integrated and applied.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
25. Pythagoras' theorem	25.1 understand the Pythagoras' theorem 25.2 recognise the converse of Pythagoras' theorem 25.3 solve problems related to Pythagoras' theorem and its converse 25.4 **explore Pythagorean triples	6

Explanatory Notes:

In Learning Objective 25.1, students are required to understand the Pythagoras' theorem, including its proof. There are numerous proofs of Pythagoras' theorem. Teachers may introduce different proofs of Pythagoras' theorem, for example, the proof by the ancient Chinese mathematician *Liu Hui* (劉徽), and the proof in Euclid's *Elements*. Teachers may also introduce the other names of Pythagoras' theorem with ancient Chinese origin, such as "*Gou Gu Dingli*" (勾股定理), "*Shang Gao Dingli*" (商高定理). Teachers may recall topics such as square numbers, square roots, etc. with students before teaching Pythagoras' theorem. This may help students manage the calculations when applying the Pythagoras theorem.

Students are required to understand at least one proof of Pythagoras' theorem in this Learning Objective. Teachers may choose one or more proofs to discuss with students according to their abilities and interests. Teachers may also let students appreciate and compare different proofs of Pythagoras' theorem through different classroom activities. Teachers may further introduce the contributions and philosophies of Pythagoras and the Pythagorean school in history of mathematics, such as introducing the first crisis in mathematics with the linkage with topics concerning rational and irrational numbers, or other related geometric topics. If students have higher abilities and greater interests, teachers may encourage them to find and compare different proofs of Pythagoras' theorem by themselves as an inquiry and investigation activity.

Teachers should remind students that the hypotenuse is the opposite side of the right angle in a right-angled triangle and is the longest side of a right-angled triangle. Students may first identify the hypotenuse when solving problems related to right-angled triangles for applying the Pythagoras' theorem correctly.

Students are required to solve problems related to one or more right-angled triangles by using Pythagoras' theorem. When handling more complicated situations, teachers may suggest students to draw the individual right-angled triangles required and considered, and solve them one by one.

In Learning Objective 25.2, students are required to recognise the converse of Pythagoras' theorem, and apply it to prove that a triangle is a right-angled triangle. Students are also required to identify the right angle by the lengths of this triangle. Teachers should explain clearly the difference between Pythagoras' theorem and its converse.

Students are required to learn Pythagoras' theorem and its proof, but they are only required to recognise the converse of Pythagoras' theorem and its applications while the proof is **not required**. However, teachers may, according to students' abilities and interests, introduce the proof of the converse of Pythagoras' theorem as an extension of learning.

After students learnt the content of Pythagoras' theorem and its converse, they are required to apply these to solve related application problems in Learning Objective 25.3, include finding the unknown sides in a right-angled triangle and identifying whether a triangle is a right-angled triangle.

The learning and teaching in this Learning Unit may connect with Learning Unit 4 "Rational and irrational numbers" and other geometric topics. For more able students, teachers may introduce Pythagoras and the Pythagorean school to them. Teachers may explain through the story of the first crisis in mathematics that the impact of the discovery of irrational numbers to the mathematical philosophies of the Pythagorean school, and its effect on the whole system of Euclidian Geometry, so that students may have chance to discuss the process in the development of mathematical knowledge. Teachers are advised to be aware that the discovery of irrational numbers originated from the ratio of lengths of the diagonal to the side in a regular pentagon instead of right-angled isosceles triangles.

According to students' abilities and interests, teachers may further arrange appropriate exploratory activities for students as an enrichment in Learning Objective 25.4. For example, students may explore what integral lengths may form a right-angled triangle to derive quadratic integral equations with roots being Pythagorean triples, so as to generate more Pythagorean triples.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
26. Rectangular coordinate system	26.1 recognise the rectangular coordinate system 26.2 find the distance between two points on a horizontal line and the distance between two points on a vertical line 26.3 find areas of polygons in the rectangular coordinate plane 26.4 recognise the effect of transformations on a point in the rectangular coordinate plane 26.5 understand the distance formula 26.6 understand the mid-point formula and the formula for the internal point of division 26.7 understand the slope formula and solve related problems 26.8 recognise the relation between the slopes of parallel lines and the relation between the slopes of perpendicular lines, and solve related problems 26.9 use coordinate geometry to perform simple geometric proofs 26.10 **explore the formula for the external point of division	19

Explanatory Notes:

This Learning Unit includes all learning content of the rectangular coordinate system in junior secondary Mathematics curriculum. Teachers may, according to students' learning progress and the coordination with other Learning Units, arrange the Learning Objectives by stages into the school-based curriculum at various year levels of junior secondary.

In Learning Objective 26.1, teachers may guide students to recognise that all points on the plane can be located by a pair of perpendicular axes and a pair of coordinates. Students are required to recognise the concepts of ordered pair (including that the two values in the ordered pair cannot be interchanged). Teachers may introduce the concepts of the rectangular

coordinate system through some common daily life examples, such as the seating plan in a classroom, the position of a piece on the checkerboard, and grid points on a map. Students are required to describe the position of a point by using the coordinates, and to mark the corresponding point with given coordinates on the rectangular coordinate plane.

In Learning Objective 26.2, students are required to find the distance between two points on a horizontal line and the distance between two points on a vertical line. By simple subtraction of directed numbers to calculate horizontal or vertical distances, students are required to establish a preliminary concept on combining numbers and shapes in coordinate geometry. According to students' abilities and interests, teachers may introduce the contributions made by René Descartes on the development of coordinate geometry and his influence on the development of geometry onwards.

In Learning Objective 26.3, students are required to integrate the knowledge of finding horizontal and vertical distance in Learning Objective 26.2 with the formulae for areas of rectilinear figures to find areas of polygons in the rectangular coordinate plane. The shapes of the polygons are limited to those could be dissected or filled as rectangles and triangles with its lengths and respective heights could be found by using the content in Learning Objective 26.2 as students' prerequisite knowledge.

In order to enhance students' knowledge on rectangular coordinates, in Learning Objective 26.4, students are required to recognise the effect of transformations on a point in the rectangular coordinate plane and describe it intuitively. These transformations include translation, reflection in a line parallel to the x-axis or y-axis, and clockwise or anti-clockwise rotation about the origin through 90° or a multiple of 90°. Students are required to find the position and coordinates of the image after the transformations of a point, or to find the original position and coordinates of a point by given the position and coordinates of the image after transformations. Teachers may conduct the related learning and teaching activities by using computer software or graph paper.

Students are required to extent the knowledge of horizontal distance and vertical distance in Learning Objective 26.2 to finding the distance of any two points in the rectangular coordinate plane by applying the knowledge in Learning Unit 25 "Pythagoras' theorem". Students are also required to understand how to use algebra to express the distance formula of any two points, so as to learn the feature of coordinate geometry which connects algebra with geometry.

In Learning Objective 26.6, students are required to understand the mid-point formula and

the formula for the internal point of division, including its proofs, by integrating the knowledge in Learning Unit 21 “Congruent triangle”, Learning Unit 22 “Similar triangle” and about horizontal and vertical distances in Learning Objective 26.2.

In Learning Objective 26.7, students are required to understand the concept of the slope of a straight line as a quantifier of the inclination of the straight line. By applying the knowledge in Learning Unit 6 “Rates, ratios and proportions” and Learning Unit 22 “Similar triangle”, students can understand that the vertical change and the horizontal change of any two points on the same straight line are proportional to each other, and hence the ratio of vertical change to horizontal change of any two points on a straight line is defined as the slope of the straight line. The formula for the slope of a straight line would hence be derived. Students should pay attention that the above analysis is not applicable to horizontal lines and vertical lines and their slope should be handled separately. Students are also required to understand the geometric meaning on positive, negative slope and different magnitudes of slope through observation. Besides finding slope, students are required to use the slope formula to find the x-coordinate or y-coordinate of points on straight lines, and the x-intercept or y-intercept of straight lines, from given conditions. For example, given the coordinates of two points on a straight line, find the x-intercept or y-intercept of the straight line. Students are required to recognise the concept of intercepts of a straight line in this Learning Objective.

In Learning Objective 26.8, through observation, students are required to recognise that the slopes of parallel lines are equal, and the product of the slopes of two perpendicular lines is -1. This property can also be regarded as the condition for two lines to be perpendicular (except for a pair of horizontal line and vertical line). Students are **not required** to prove these relations, but are required to apply the relations of the slopes of parallel lines and perpendicular lines to solve problems. While teaching the concept of slope, teachers may consider incorporating the content in Learning Unit 27 “Trigonometry” according to students’ abilities and interests. However, the relation between slope and inclination in rectangular coordinate plane is dealt with in the Compulsory Part of the senior secondary Mathematics curriculum.

In Learning Objective 26.9, students are required to integrate the knowledge in this Learning Unit to do simple geometric proofs, where the geometric properties and theorems concerned are limited to the Foundation Topics in the curriculum. This Learning Objective focuses on facilitating students to connect deductive geometry with coordinate geometry. Teachers may compare the similarity and difference on geometric proofs between deductive geometry and coordinate geometry through basic and simple examples to deepen students’ recognition on coordinate geometry.

According to students' abilities and interests, teachers may arrange exploratory activities for students to investigate the formula for the external point of division. As an extension of the formula for the internal point of division, students may first observe that there is an external point outside the given line segment which also satisfies the length ratio as required. A formula for the external point of division may then be derived. According to students' abilities and interests, teachers may discuss with students on how to apply the concepts of directed numbers on the length ratio of line segments to deduce a general formula for points of division.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
27. Trigonometry	27.1 understand sine, cosine and tangent of angles between 0° and 90° 27.2 understand the properties of trigonometric ratios 27.3 understand the exact values of trigonometric ratios of 30°, 45° and 60° 27.4 solve problems related to plane figures 27.5 solve problems involving gradients, angles of elevation, angles of depression and bearings	18

Explanatory Notes:

In this Learning Unit, students are required to learn the relationships between angles and sides of right-angled triangles. Through the concepts of similar triangles and ratios between sides, it is a new method for students to find the measures of angles in right-angled triangles with known sides, or to find the length of sides in right-angled triangles with the measure of an angle and the length of a side.

In this Key Stage, the learning content of trigonometry is confined to right-angled triangles. More comprehensive discussions on trigonometry in general triangles and trigonometric functions will be covered in the senior secondary Mathematics curriculum. Hence, in Learning Objective 27.1, teachers may introduce the significance of trigonometric ratios through simple real life examples related to right-angled triangles, such as estimating the height of a building. Students are required to understand the definitions of opposite side, adjacent side and hypotenuse with respect to a specific angle in a right-angled triangle. They are also required to understand through the knowledge of similar triangles that in all right-angled triangles with all corresponding angles equal, all corresponding sides of those right-angled triangles are proportional, so that the concepts of sine, cosine and tangent in a right-angled triangle are consequently understood. Since students understand the trigonometric ratios through right-angled triangles in this Key Stage, the trigonometric ratios are confined to 0° to 90°, excluding 0° and 90°.

In Learning Objective 27.2, students are required to understand the properties of trigonometric ratios including the geometric meanings of the changes in values of sine, cosine

and tangent when the size of the angle changes; the range of values to be taken by sine, cosine and tangent and the relationship amongst the three ratios based on the definitions of opposite side, adjacent side and hypotenuse; and some trigonometric identities derived from the interior angle sum of triangles and Pythagoras' theorem.

In Learning Objective 27.3, students are required to understand the exact values of trigonometric ratios of 30°, 45° and 60° through two kinds of special triangles, i.e. isosceles right-angled triangles (also known as right-angled isosceles triangles) and equilateral triangles. Students are required to express the exact values of trigonometric ratios of 30°, 45° and 60° in surd form by finding the ratios of the lengths of sides of isosceles right-angled triangles, the ratios of the lengths of sides and the altitudes of equilateral triangles, and using Pythagoras' theorem. Teachers may consider linking the exact values of trigonometric ratios of special angles to Learning Objective 4.3 according to students' abilities and interests.

In Learning Objective 27.4, students are required to solve problems related to plane figures by integrating the knowledge of trigonometric ratios and deductive geometry. Students are required to apply trigonometric ratios to solve problems by adding additional lines, dissecting or filling the plane figures into right-angled triangles.

In Learning Objective 27.5, students are required to solve problems involving gradients, angles of elevation, angles of depression and bearings. At primary level, students recognised the eight directions. In this Learning Objective, students are required to recognise the two kinds of bearing such as 010° and N10° E, and recognise the relation between gradients and inclinations. Students are required to link the relationship between gradients and tangents in the cases of positive gradients. However, the relationships between general gradients, inclination and tangent function will be handled in Learning Unit 14 "More about trigonometry" in the Compulsory Part of senior secondary Mathematics curriculum. Students are required to apply the knowledge in Learning Unit 19 "Angles and parallel lines" to solve problems involving gradients, angles of elevation, angles of depression and bearings. Teachers may emphasise on how to find parallel lines and right angles for applying the trigonometry ratios to solve problems. Teachers may note that students have chances to come across the topics such as bearings and gradients in Geography and other subjects. Hence, when using related terms such as reduced bearing (compass bearing), teachers may pay attention and compare the meanings of those terms in different subjects. Teachers may also take this opportunity to promote cross-KLA learning.

Learning Unit	Learning Objective	Time
Data Handling Stand		
28. Organisation of data	28.1 recognise the concepts of discrete data and continuous data 28.2 recognise organisation of data without grouping 28.3 recognise organisation of data in groups	4

Explanatory Notes:

Data handling is closely related to real life. Teachers may use data which are relatively more related to students' daily life as illustrations or objects of studies to enhance students' abilities and interests in applying related knowledge to solve real life problems.

At primary level, students learnt how to use different statistical charts to present discrete data, and have an intuitive recognition on statistics. In this Learning Unit, students are required to recognise further the concepts on discrete data and continuous data, as well as arranging and organising these data in different ways. Teachers may let students gain hands-on experience of recording, organising and presenting data using statistical charts through simple project learning activities in statistics.

In Learning Objective 28.1, students are required to recognise the concepts of discrete data and continuous data. They are required to distinguish examples of discrete data and continuous data, and link them up with Learning Objectives 28.2 and 28.3 on the similarity and difference on the methods of organising discrete data and continuous data.

In Learning Objectives 28.2 and 28.3, students are required to organise data without grouping and in groups respectively. Students are required to use the appropriate methods to organise raw data including both discrete data and continuous data. Students are required to recognise the organisation of data using frequency distribution tables. Teachers may use data with different types of distributions as examples to discuss with students the possibility of using different grouping methods to organise the same set of data in Learning Objective 28.3. Students are required to pay attention to the requirements that the groups (or classes) should cover all data, and the groups do not overlap with each other. Students should also pay attention that, in general, the width of each group (or class width) is equal, and the number of groups should be suitably set.

Learning Unit	Learning Objective	Time
Data Handling Stand		
29. Presentation of data	29.1 recognise stem-and-leaf diagrams and histograms 29.2 interpret stem-and-leaf diagrams and histograms 29.3 interpret statistical charts representing two different sets of data in daily life 29.4 recognise frequency polygons, frequency curves, cumulative frequency polygons and cumulative frequency curves 29.5 interpret frequency polygons, frequency curves, cumulative frequency polygons and cumulative frequency curves 29.6 choose appropriate statistical charts to present data 29.7 recognise the uses and abuses of statistical charts in daily life	17

Explanatory Notes:

In this Learning Unit, students are required to recognise how to construct different statistical charts to present data, and interpret different statistical charts to retrieve information from them. Teachers may flexibly arrange for the teaching sequence on constructing and interpreting the charts. They may first require students to interpret the statistical charts from different sources, then to learn how to construct statistical charts; or to construct statistical charts then conduct activities to interpret statistical charts. Teachers may also flexibly arrange the learning content of this Learning Unit for the teaching schedule at different year levels at junior secondary according to students' learning progress.

At primary level, students learnt how to construct bar charts and broken line graphs, as well as to intrepret simple pie charts. At junior secondary level, students are required to construct and interpret stem-and-leaf diagrams, histograms, frequency polygons, frequency curves, cumulative frequency polygons and cumulative frequency curves.

In Learning Objective 29.1, students are required to construct simple stem-and-leaf diagrams and histograms using paper and pen. When a larger amount of data is involved, it is

complicated for construction. Students may use information technology to save time for other learning and teaching activities. No matter which method is used, students are required to recognise the construction of statistical charts in appropriate scales for a clear presentation of data.

In Learning Objective 29.2, students are required to interpret stem-and-leaf diagrams and histograms. Students are required to read the information of data and data set. After students have learnt about mean, median and mode / modal class in Learning Unit 30 "Measures of central tendency", teachers may discuss with students how to find the mode or modal class of data from stem-and-leaf diagrams and histograms, and how to find the mean and median of data from stem-and-leaf diagrams. Nevertheless, students are **not required** to find the median directly from histograms. The content about reading the data to find the mean of data from histograms belongs to Learning Objective 30.3.

In Learning Objective 29.3, students are required to interpret statistical charts representing two different sets of data in daily life. The statistical charts are mixed types of statistical charts students have learnt, such as bar charts, broken line graphs, histograms, etc. These statistical charts feature dual parallel axes to present two different set of data. Teachers may use examples which students may encounter in daily life, such as the chart of temperature and rainfall by the Hong Kong Observatory, to provide opportunities for them to learn how to retrieve required information from these statistical charts having a greater amount of information, so as to facilitate their learning in Mathematics and other disciplines.

In Learning Objective 29.4, students are required to recognise frequency polygons, frequency curves, cumulative frequency polygons and cumulative frequency curves, including the construction of the above statistical charts from raw data, constructing cumulative frequency polygons from frequency polygons, and constructing frequency polygons from cumulative frequency polygons, etc. Teachers may discuss with students the similarity and difference between frequency polygons and frequency curves through some real life examples related to science, such as a survey on the height of the same kind of plants.

In Learning Objective 29.5, students are required to interpret frequency polygons, frequency curves, cumulative frequency polygons and cumulative frequency curves. Students are required to extract information related to frequency from the above statistical charts. Besides, they are also required to integrate their understandings of median in Learning Objective 30.1 to find the median, quartiles (upper quartile and lower quartile) and percentiles of the data, and the positions of individual datum in the populations from cumulative frequency polygons and cumulative frequency curves. However, the extension from quartiles to the concept of

inter-quartile range belongs to the content of Learning Objective 17.2 of the Compulsory Part of senior secondary Mathematics curriculum, and is **not required** to be introduced in this Key Stage.

In Learning Objective 29.6, students are required to choose appropriate statistical charts from stem-and-leaf diagrams, histograms, bar charts, pie charts and broken line graphs to present data. They are required to present the data with suitable statistical charts by considering the features of the charts to match the category of the data and the statistical objectives, or to point out why some specific kinds of statistical charts are not suitable for certain situations. Teachers may encourage students to explore different methods to present the same set of data, and comment on the suitability and validity of each method. To match the learning of applications of statistical charts in daily life in Learning Objective 29.7, teachers may use the statistical charts collected from different channels in daily life to guide students to pay attention to the advantages of various kinds of statistical charts in presenting data.

Teachers may require students to collect and interpret different statistical charts from various channels, such as newspapers, reports from Consumer Council, and encourage them to identify and describe the main features of the statistical charts through reading the charts. Beside learning to extract the correct information in real life examples of statistical charts, students are also required to learn to identify the potentially confusing or misleading elements which may be caused by improper constructions of statistical charts. Students are also required to judge whether interpretations of some statistical charts are reasonable through examples so as to recognise the abuses of statistical charts in daily life. Teachers are advised to encourage students to cultivate a rigorous attitude towards the handling and presentation of data, and to learn to present the data precisely and comprehensively to avoid the statistical charts being misleading. Teachers may consider using related learning elements in Science Education or Technology Education KLAs to enhance learning and teaching. For example, teachers may ask students to choose appropriate statistical charts to present and analyse the data collected from experiments so as to understand the related scientific principles.

Learning Unit	Learning Objective	Time
Data Handling Strand		
30. Measures of central tendency	30.1 understand the concepts of mean, median and mode/modal class 30.2 calculate mean, median and mode of ungrouped data 30.3 calculate mean, median and modal class of grouped data 30.4 recognise the uses and abuses of mean, median and mode/modal class in daily life 30.5 understand the effects of the following operations on the mean, median and mode: (i) adding a common constant to each item of the set of data (ii) multiplying each item of the set of data by a common constant 30.6 recognise the concept of weighted mean 30.7 solve problems involving weighted mean	10

Explanatory Notes:

Students learnt the concept of average in Learning Unit 6D1 "Averages" of primary Mathematics curriculum. In Key Stage 3, students are required to understand the concepts and applications of median and mode/modal class together with mean as the measures of central tendency of ungrouped data and grouped data. This Learning Unit emphasises the understanding of concepts, features and limitations for mean, median and mode/modal class, instead of over complicated calculations. Teachers may consider using calculators or computer software to explore with students the features of different measures of central tendency through modifying the data set.

In Learning Objective 30.1, students are required to understand the concepts of mean (i.e. arithmetic mean, but students are **not required** to recognise the term "arithmetic mean"), median and mode/modal class, include its definitions, features and limitations. Mean can also be called "average". Teachers may discuss with students the daily life applications of central tendency and to guide them to understand the features and limitations of each measure of central tendency. For example, a single extreme datum may have a great influence on the

mean, but not the median.

In Key Stage 2, students learnt the concept of average and calculated the average of some known values. In Learning Objectives 30.2 and 30.3, students are required to calculate mean, median and mode/modal class of ungrouped data and grouped data respectively. Students are required to understand that mean and median of grouped data are estimations only. Teachers may explore with students the possibility of obtaining different outcomes in calculating means and medians of grouped data when they are grouped differently.

In Learning Objective 30.4, students are required to recognise the uses and abuses of mean, median and mode/modal class in daily life through understanding the definitions, features and limitations of mean, median and mode/modal class. For example, students may comment whether it is appropriate to use certain kinds of measure of central tendency as the description and basis of analysis of data collected from some examples of real life statistical surveys. Teachers may design some examples to help students recognise that there may be a difference in describing the same set of data using different measures of central tendency because of the features and limitations of mean, median and mode/modal class. For example, mean can be easily affected by small amount of data with extreme values; median is normally less affected by extreme data; and mode/modal class is of little representative when the difference in frequencies of individual data or data class is not large. As such, teachers may discuss and analyse with students the statistical reasons, assumptions and limitations behind the conclusions and claims of surveys in real life to enhance students' information literacy and critical thinking skills.

In Learning Objective 30.5, students are required to understand the effects of the following operations to mean, median and mode based on their definitions:

(i) adding a common constant to each item of the set of data;

(ii) multiplying each item of the set of data by a common constant.

Students are required to analyse the above operations and understand that the mean, median and mode of the new set of data can be found directly from the mean, median and mode of the original set of data without repeated calculations from the new set of data again.

In Learning Objective 30.6, students are required to recognise the concept of weighted mean. Students may have a preliminary recognition on weighted mean intuitively in daily life. In this Learning Objective, students are required to recognise its mathematical definition and its real life applications, for example, the methods for calculating average scores for report cards and scores for university admission, and other examples related to students' daily life. Students are also required to solve problems related to weighted mean in Learning Objective

30.7. According to students' abilities and interests, teachers may discuss with students the real life applications of weighted mean. For more able students, teachers may use real life examples such as the Hang Sang Indexes of Hong Kong Stock Market, Consumer Price Index (A), Consumer Price Index (B), etc. to enhance students' interests and as an entry point for cross-KLA learning activities.

Learning Unit	Learning Objective	Time
Data Handling Stand		
31. Probability	31.1 recognise the concepts of certain events, impossible events and random events 31.2 recognise the concept of probability 31.3 calculate probabilities of events by listing the sample space and counting 31.4 solve problems involving probability 31.5 recognise the concept of expectation 31.6 solve problems involving expectation	12

Explanatory Notes:

In this Learning Unit, students are required to recognise and apply the concept of probability to mathematically describe some simple real life situations involving uncertainty. Students may have some general intuitive concept of probability through daily life, such as using daily terms of "chance", "possible" to describe and compare the chance of events to happen. In this Learning Unit, students are required further to handle some simple problems related to probability by means of quantitative method.

In Learning Objective 31.1, students are required to recognise the concepts of certain events, impossible events and random events. Through recognising the concept of probability in Learning Objective 31.2, students are required to recognise that the probability of a certain event is 1; the probability of an impossible event is 0; and the probability of a random event is a value between 0 and 1.

In Learning Objective 31.2, students are required to recognise that probability is a quantitative description for uncertainty. Students are required to recognise the concepts of sample space, possible outcomes, favourable outcomes and events, and the definition of probability in a finite sample space. Students are **not required** to recognise geometric probability and also **not required** to handle probability problems involving infinite sample space. Students are required to recognise the name and concept of relative frequency in this Learning Objective, and that relative frequency may be treated as an estimation of the probability for an event after statistical process or repeated trials. This would help students connect statistics with probability. However, the content about the accuracy of the above

estimation in probability, including the Law of Large Numbers, is **not required**.

In Learning Objective 31.3 and 31.4, students are required to calculate the probabilities of events and hence solve related problems. Students are required to calculate the probability of events by listing the sample space and counting. Students are required to use tables or tree diagrams to list sample spaces so that the work can be done effectively and systematically. In this Key Stage, students are only required to find the probability by counting and listing. The addition law and multiplication law of probability belong to Learning Unit 15 "More about probability" of the Compulsory Part of senior secondary Mathematics curriculum.

Teachers may use examples to let students recognise how tree diagrams and tables may list the sample space effectively so as to solve problems of probability. For example: When dealing with the case "there are three new born babies, assuming that the probability of a new born being a baby boy is the same as that of being a baby girl". Students may easily misunderstand that all the events in the sample space (i.e. 3 boys; 2 boys and 1 girl; 1 boy and 2 girls; and 3 girls) are of equal probability. However, a tree diagram can help list out all the possible outcomes with equal probability, and hence providing the sample space for finding the correct probabilities of different events.

Students are required to solve problems involving probability in Learning Objective 31.4. The methods used to calculate the probabilities of events are confined to the content covered in Learning Objective 31.3.

In Learning Objective 31.5, students are required to recognise the concept of expectation. Teachers should note that the concept of discrete random variables belongs to Learning Unit 11 "Discrete random variables" in Module 1 of the Extended Part of the senior secondary Mathematics curriculum which is **not required** in this Key Stage. In this Learning Objective, students are only required to list out the sample space (i.e. all the possible values) and find the corresponding probability for each value in the sample space, and hence find the expectation. Teachers may guide students to recognise that the concept of expectation may be interpreted as the weighted mean of the value of each event, with the probability of each event being its weight.

In Learning Objective 31.6, students are required to solve problems involving expectation. Further to Learning Objective 31.5, students are required to calculate and interpret the meaning of expectation under given scenarios. For example, further to the aforementioned scenario of three new born babies, students may use tree diagram to look into the scenario that, assuming the probability of a new born baby boy and that of a baby girl are equal, the

probabilities of 0, 1, 2, 3 baby boys are $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$ and $\frac{1}{8}$ respectively. Hence, the expectation of the number of baby boys is $\frac{1}{8}\times 0+\frac{3}{8}\times 1+\frac{3}{8}\times 2+\frac{1}{8}\times 3=1.5$. According to the above assumption, there are in average 1.5 baby boys per 3 new born babies. Teachers may also guide students to use the concept of expectation to analyse quantitatively on problems involving gain or loss, and hence let students recognise that decisions can be made based on expectation. As an entry point to include elements of value education, teachers may also discuss with students about the drawbacks of gambling by using expectation.

Learning Unit	Learning Objective	Time
Further Learning Unit		
32. Inquiry and investigation	Through various inquiry activities and mathematical modelling learning activities, construct and apply mathematical knowledge, further improve the ability to inquire, communicate, reason, conceptualise mathematical concepts and apply mathematics	20

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling learning activities. In other words, this is not an independent and isolated Learning Unit. Teachers may make use of the suggested lesson time to engage students in various learning activities, such as activities related to enrichment topics, cross-learning unit activities, and cross-Key Learning Area activities based on mathematical topics. Among the learning activities, teachers should plan about 2–3 hours of mathematical modelling learning activities for students in each school year, enabling them to analyse and make decisions with mathematical thinking when encountering problems in different real-world contexts, and strengthening their ability to apply mathematics to solve STEAM and other real-world problems. Teachers can make good use of Mathematics lessons, lessons designated for project learning or cross disciplinary learning, or other suitable lessons, together with other learning time, such as after-school or post-examination activity periods, to conduct mathematical modelling learning activities. These activities should be regarded as STEAM learning activities of Mathematics.

1. The Process of Mathematical Modelling

Mathematical modelling is the process of applying mathematical knowledge and tools to solve real-world problems. It emphasises on handling problems and analysing data in a mathematical way. Teachers should guide students to **analyse the real-world problem**, clarify key factors of the problem, identify related information/data and the underlying mathematical elements, simplify the problem by putting forward suitable assumptions, convey it into a mathematical problem, and determine relations between relevant quantities and variables. Students apply their prior mathematical knowledge and skills to **formulate a mathematical model** and **solve it for a mathematical solution**, and then **interpret the mathematical solution** by considering the meaning of the solution in the context of the real-

world problem to obtain **a real-world solution**. Subsequently, students **evaluate the mathematical model** through validation and reflection to suggest refinements of the model. Figure 1 and Table 1 show the five steps of the mathematical modelling process and the relevant elements of each step respectively:

Figure 1: The process of mathematical modelling

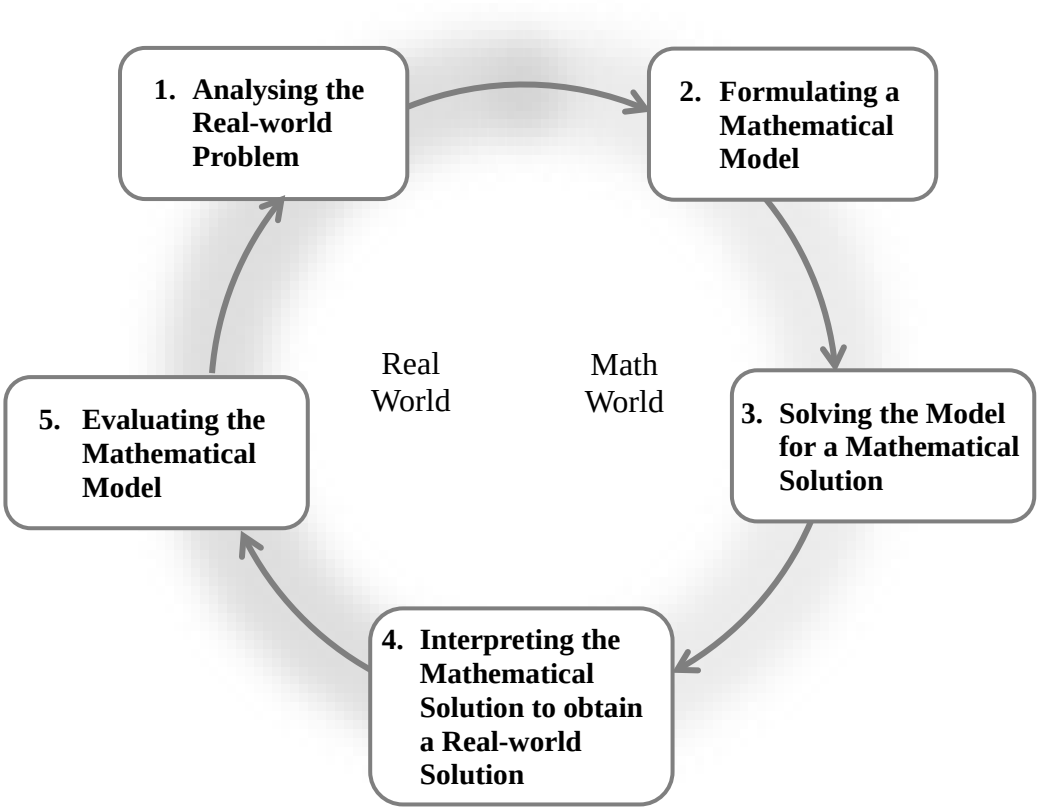


Table 1: Elements of the steps of mathematical modelling

Steps	Elements
1. Analysing the Real-world Problem	• Understand the real-world context • Clarify key factors of the problem • Identify related information/data and mathematical elements of the problem
2. Formulating a Mathematical Model	• Put forward assumptions to simplify the real-world problem • Convey the problem in a mathematical way • Determine relations between key quantities/variables
3. Solving the Model for a Mathematical Solution	• Apply mathematical knowledge, skills and different tools to solve the model
4. Interpreting the Mathematical Solution to obtain a Real-world Solution	• Consider the meaning of the mathematical solution with respect to the real-world problem
5. Evaluating the Mathematical Model	• Validate the model against the real-world context • Reflect on the strengths and limitations of the model • Compare different models • Suggest refinements of the model

The steps and elements of the mathematical modelling process shown on previous page aim to provide teachers with a general reference for designing mathematical modelling learning activities. Teachers may adjust individual steps or omit certain elements to suit the actual teaching circumstances and students' abilities. For example, the element "Comparing different models" in Step 5 "Evaluating the mathematical model" can be omitted if the activity does not involve multiple mathematical models.

In Key Stage 2, students have experienced a single cycle of the mathematical modelling process and realised that mathematics can be used to model real-world context for solving problems. In Key Stage 3, students are required to understand the five steps and their related elements of the mathematical modelling process through mathematical modelling learning activities. In addition, students should understand that upon completing Step 5 "Evaluating the mathematical model", the model can to be refined, leading to a new cycle of the mathematical modelling process. In Key Stage 4, students are expected to apply proficiently the modelling cycle comprising the five steps to solve real-world problems. For details of the learning of mathematical modelling at different key stages, please refer to Table 3 "Notes on the Learning and Teaching of Mathematical Modelling across Key Stages".

2. Design of Mathematical Modelling Learning Activities

Problems in mathematical modelling learning activities are different from conventional mathematical word problems. Contexts of word problems are usually simplified. The conditions and information needed for solving the problems are given, complete, and just sufficient. They are mainly designed to consolidate students' understanding of mathematical concepts and their application of specific skills. In contrast, problems in mathematical modelling learning activities are generally un-simplified real-world problems. They are open-ended and do not lead to a specified or explicit solving approach, nor a unique or optimal solution. Furthermore, the relevant conditions and information may not be fully provided. Students need to make suitable assumptions to simplify the problem, as well as collect and select the necessary information/data, to facilitate the formulation of a mathematical model that fits the context.

For students at Key Stage 3, teachers should take note of the following when designing mathematical modelling learning activities:

- The learning activities should be structed in alignment with the five steps of mathematical modelling, facilitating students' understanding of the mathematical modelling process;
- In designing mathematical modelling problems, teachers should adopt real-life contexts familiar to students (e.g. optimisation of packaging design, hand warmer scoring problem, or predicting tourist accommodation demand of Hong Kong) so that students can connect with their life experiences to comprehend the problems. The information or data provided for the problems may be excessive or insufficient;
- In selecting real-world problems, teachers should ensure they align with students' mathematical knowledge at this Key Stage, such as linear equations in one unknown, linear equations in two unknowns, polynomials, trigonometry, and probability;
- Teachers may incorporate the application of digital technology into mathematical modelling learning activities to foster students' ability to formulate and solve mathematical models using suitable digital tools (including spreadsheets, graphing software, dynamic geometry software, and artificial intelligence tools, etc.);
- Teachers may refer to the curriculum documents of other subjects, or collaborate with teachers of them, to design cross-subject mathematical modelling learning activities, thereby enhancing students' ability to integrate and apply multi-disciplinary knowledge to solve problems;
- Teachers may incorporate elements of values education into mathematical modelling learning activities based on the real-life situations involved, where appropriate;
- Teachers may consider students' abilities and learning needs to conduct mathematical modelling learning activities in diverse modes such as classroom activities or project learning.

3. Learning and Teaching of Mathematical Modelling

At this Key Stage, teachers should design modelling activities with reference to the five steps of the mathematical modelling process to facilitate students' understanding of the complete modelling process. While conducting the activities, teachers should offer adequate guidance and support to help students recognise each step and its relevant elements, and allow rooms for students to explore the problems. Regarding the learning and teaching of mathematical modelling, teachers should take note of the following:

<u>Step 1: Analysing the Real-world Problem</u>

- Teachers should guide students to understand the context of the real-life problem and identify its key factors. For instance, when solving a problem on selecting the

best route, key factors such as cost, time, comfort, and the number of attractions should be considered. Teachers should encourage students to reflect on which factors are mathematically relevant and more critical to solving the problem, thereby clarifying key factors;

- Students should be able to filter redundant information and collect necessary data where information is insufficient.

Step 2: Formulating a Mathematical Model

- Teachers should guide students to recognise that making appropriate assumptions is pivotal in formulating a mathematical model. As real-world problems are often complex and have intricate details, reasonable assumptions are necessary to simplify the situation for subsequent steps. Teachers should also explain that problems can be simplified in different ways by adopting varying assumptions;
- Teachers should guide students to convey problems in a mathematical way. For instance, converting the problem of purchasing hand warmers into a scoring model based on key factors, such as price, duration of warmth, and weight. As students at this key stage have developed an understanding of algebraic operations and graphs of basic equations, teachers should guide them to apply relevant mathematical knowledge to construct mathematical models, and to use algebraic expressions, graphs, or tabular forms to represent and understand the models. In model formulation, students should learn to represent the relationships between variables mathematically—for example, by assigning weightings to reflect the importance of variables, and normalising variables to a uniform scale for better comparability;
- Students may apply digital tools during model formulation. For example, students with a preliminary or intuitive understanding of the "line of best fit" may use spreadsheets or other tools incorporating the related function to generate trend lines for data prediction, without requiring knowledge of the underlying mathematical proofs.

Step 3: Solving the Model for a Mathematical Solution

- Teachers should guide students to apply their mathematical knowledge flexibly during the problem-solving process and make appropriate use of digital tools, such as spreadsheets, mathematical graphing software and dynamic geometry software, to obtain solutions to the models and reduce workload on repetitive calculations and graph plotting. For example, when a model involves quadratic functions, students can explore extreme values by sketching the graphs. However, students are not required to learn quadratic functions and the properties of their graphs at this stage;

- Students may also present their formulated mathematical models from multiple perspectives using diagrams, tables, algebraic expressions, and graphs.

Step 4: Interpreting the Mathematical Solution to Obtain a Real-world Solution

- Teachers should guide students through questioning or discussion to consider the meaning of model solutions in the context of the real-world problem, thereby solving the real-world problem. For example, when interpreting the output of a scoring model, students should assess whether a higher score necessarily represents a better outcome. Similarly, when a rainfall prediction model produces a probability of 60%, students should consider whether this implies that it will rain 60% of the time;
- Students should realise that if a certain solution of the model conflicts with the context of the real-world problem, such as values falling outside the defined range, it must be rejected or addressed through model evaluation and refinement.

Step 5: Evaluating the Mathematical Model

- Teachers should guide students to validate their models against the real-world context. For example, students may compare the predicted outcomes of the models with actual observations or experimental data, or by using one part of a dataset to construct the model and the remaining part to verify its accuracy;
- Teachers should guide students to reflect on the strengths and limitations of their models in light of the real-world context, and further prompt them to derive recommendations for model refinements;
- Teachers may guide students to compare different models and facilitate discussion of their respective strengths and limitations in terms of predictive accuracy, the reasonableness of assumptions, and computational complexity, to propose model refinements. Teachers may encourage students to express their views, fostering an open and rational atmosphere for discussion, for cultivating their critical thinking skills and creativity.

Depending on students' abilities, teachers may reengage students in the modelling process based on their proposed refinements, thereby constructing an optimised mathematical model. Students may revisit Step 1 "Analysing the Real-world Problem" to update key factors of the problem or collect additional data. Alternatively, they may bypass Step 1 and proceed directly to Step 2 "Formulating the Mathematical Model" to revisit their assumptions and update the mathematical model accordingly.

4．Assessment of Mathematical Modelling

Regarding the assessment of mathematical modelling, teachers should take note of the following:

- Teachers can assess students" performance in each modelling step by reviewing both the process and outcomes of the modelling process, such as what reflected in classroom discussions, oral presentations, worksheets, etc. Teachers can set appropriate assessment criteria to help understand students learning outcomes across different steps of the modelling process, for providing appropriate feedback.
- Teachers can refer to Table 2 as a reference to construct an assessment form for mathematical modelling learning activities, to assess students' performance in consideration of both modelling process and outcomes. An assessment form may be designed based on the modelling steps and related elements involved in the modelling activity. Teachers can then give comments and suggestions for improvement based on students' performance in each step and in the whole activity. For instance, when evaluating performance in Step 1 "Analysing the real-world problem", teachers may assess whether students can clarify key factors of the real-world problem and collect appropriate data. These criteria enable teachers to gain a more concrete understanding of students' mastery of the various modelling steps.
- Teachers can provide timely oral or written feedback at different stages of mathematical modelling learning activities. Peer assessment can also be conducted to help students understand their performance and improve their learning. Teachers can also use the assessment results to improve the delivery of mathematical modelling learning activities.

Table 2: Example of an Assessment Form

Step	**Element***	**Assessment / Feedback**
1. Analysing the Real-world Problem	- Able to understand the real-world context - Able to clarify key factors of the problem - Able to identify related information/data and mathematical elements of the problem	

Step	Element*	Assessment / Feedback
2. Formulating a Mathematical Model	- Able to put forward assumptions to simplify the real-world problem - Able to convey the problem in a mathematical way - Able to determine relations between key quantities/variables	Teachers may provide comments on student performance at different stages of the activity based on the modelling steps and elements involved. They can also assign marks or grades accompanied by corresponding descriptions.
3. Solving the Model for a Mathematical Solution	- Able to apply mathematical knowledge, skills and different tools to solve the model	
4. Interpreting the Mathematical Solution to obtain a Real-world Solution	- Able to consider the meaning of the mathematical solution with respect to the real-world problem	
5. Evaluating the Mathematical Model	- Able to validate the model against the real-world context - Able to reflect on the strengths and limitations of the model - Able to compare different models - Able to suggest refinements of the model	
Overall Feedback:		

*The elements in the table could be adapted to suit the mathematical modelling learning tasks by adding, removing, or further specifying individual elements. For example, if a task incorporates digital tools for computation, the element in Step 3 "Able to apply mathematical knowledge, skills, and tools to find model solutions" may be changed to "Able to utilise spreadsheets to organise data from different scenarios and calculate the required results".

5. Effective Use of Artificial Intelligence in the Learning and Teaching of Mathematical Modelling

Digital tools have served as important aids in enhancing effectiveness of learning and

teaching. Making effective use of artificial intelligence (AI) in the learning and teaching of mathematical modelling can further assist teachers and students in exploring modelling problems efficiently. Teachers may consider the following when incorporating AI elements into mathematical modelling learning activities:

- Teachers may guide students to utilise AI for facilitating them to understand the real-world problems. For instance, when evaluating the degree of comfort of a study room environment, students may first identify and list factors that may affect the comfort level, and then prompt the AI to provide feedback or generate questions that inspire them to explore other potential factors. This helps students deepen their thinking and clarify key factors of the problem;
- Teachers may guide students to utilise AI to facilitate data organisation and analysis. By reducing time spent on routine calculations and data processing, students can focus more on higher-order cognitive processes, such as problem comprehension, formulation of assumptions, exploration of variable relationships, and interpretation of results. For instance, in a decision-making activity on purchasing hand warmers, students may input data from different brands of hand warmers for AI to compute average values, rapidly generating comparison outputs or graphical representations;
- Teachers must be cautious about preventing AI from replacing student thinking. Teachers should guide students to think independently, leading them to view AI as a supporting tool for exploration and reflection. As AI-generated content may not always be accurate or appropriate, students need to examine AI outputs critically and cultivate a attitude towards use AI with care;
- Teachers should also guide students on how to use AI, preventing them from directly submitting AI-generated results and cultivating an awareness of responsible use of AI.

Teachers may utilise AI to assist in designing mathematical modelling learning activities, organising data, predicting students' learning difficulties, and producing learning resources. Teachers must carefully review the outputs of AI to ensure teaching quality and learning outcomes.

6. Learning and Teaching Resources

Regarding the examples of mathematical modelling learning activities, please refer to the learning and teaching resources on the website of the Mathematics Education Section of Education Bureau.

Table 3: Notes on the Learning and Teaching of Mathematical Modelling across Key Stages

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
Problem	1. Nature and complexity of problems	• Based on daily life contexts that students are familiar with • Related information/data are given • Guide students to formulate a simple model	• Based on real-life contexts that students are familiar with • The given information/data can be surplus or insufficient • Require students to formulate one or several models	• Based on real-life contexts that student may or may not be familiar with • The given information/data can be unstructured or scattered in diverse data sources • Require student to formulate several models or a series of models
Teacher	2. Teacher instruction	• Provide well-structured mathematical modelling learning activities along with clear and comprehensive instructions, as well as close guidance and support	• Provide sufficient guidance and support while allowing students to have the space to explore the problem themselves	• Facilitate students to conduct mathematical modelling activities themselves by providing timely guidance and support throughout the process
Student	3. Students' recognition of modelling process	• Be guided to experience the mathematical modelling process, with relatively less emphasis on model evaluation • Be aware that mathematics can be used to model real-world scenarios for solving problems, while not required to identify nor memorise modelling steps	• Be guided to recognise the modelling process according to the five steps of the modelling process	• Learn to conduct mathematical modelling activities themselves according to the five steps of the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
	4. Students' modelling capability	• Put forward or recognise the assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem and be able to provide an explanation on the relevance and reasonableness
		• Express real-world problems as simple mathematical problems and formulate related mathematical models after considering the key quantities/variables provided	• Express the real-life problem as a mathematical problem and formulate the related model(s) through identifying key quantities/variables and represent the relationship between them mathematically	• Express the more complicated real-life problem as a mathematical problem and formulate the related models through defining, identifying and organising key quantities/variables mathematically
		• Put forward or recognise possible strengths and limitations of the models	• Perform evaluation of model, such as comparing the results with real-world data, to identify the strengths and limitations of the model	• Understand the importance of model evaluation and perform evaluation of the model to identify the strengths and limitations of the model
			• Propose possible refinements to the model	• Make refinements to the model
	5. Students' application of mathematical knowledge and tools during the modelling process	• Apply mathematical knowledge and skills of lower and upper primary, such as those about arithmetic operations, areas, percentages, averages, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at junior secondary, such as linear equations, polynomial, trigonometry, probability, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at senior secondary, such as functions and graphs, variations, 3-D geometry, probabiliyt, differentiation*, matrices* and probability distribution* in the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
		• Use manipulatives, diagrams, tables and simple mathematical expressions to present the modelling results • Use calculators and simple digital tools, etc. as appropriate in the modelling process	• Use diagrams, tables, algebraic expressions and graphs to present the modelling results • Use digital tools, such as spreadsheet, graphing software, dynamic geometry software and artificial intelligence tools, etc. as appropriate in the modelling process	• Use diagrams, tables, functions and their graphical representations to present the modelling results • Use digital tools, such as spreadsheet, graphing software, dynamic geometry software, calculus calculator*, matrix calculator* and artificial intelligence tools, etc. as appropriate in the modelling process

* Applicable to students taking Module 1 or Module 2.

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CDC Committee on Mathematics Education

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Ad Hoc Committee on Secondary Mathematics Curriculum (Junior Secondary and Compulsory Part of Senior Secondary)

Ad Hoc Committee for Integrating Mathematical Modelling into Secondary Mathematics Curriculum

