Explanatory Notes to Senior Secondary Mathematics Curriculum :

Module 1 (Calculus and Statistics)

Mathematics Education Section
Curriculum Development Institute
Education Bureau
2009 (with updates in August 2018)

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Contents

Foreword

The *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)* (abbreviated as "C&A Guide" in this booklet) was updated in December 2017 to keep abreast of the ongoing renewal of the school curriculum. The Senior Secondary Mathematics Curriculum consists of a Compulsory Part and an Extended Part. The Extended Part has two optional modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus).

In the C&A Guide, the Learning Objectives of Module 1 are grouped under different learning units in the form of a table. The notes in the "Remarks" column of the table in the C&A Guide provide supplementary information about the Learning Objectives. The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of Module 1;
2. the strategies suggested for the teaching of Module 1;
3. the connections and structures among different learning units of Module 1; and
4. the curriculum articulation between the Compulsory Part and Module 1.

Teachers may refer to the "Remarks" column and the suggested lesson time of each Learning Unit in the C&A Guide, with the explanatory notes in this booklet being a supplementary reference, for planning the breadth and depth of treatment in learning and teaching. Teachers are advised to teach the contents of the Compulsory Part and Module 1 as a connected body of mathematical knowledge and develop in students the capability to use mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the C&A Guide does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequence that takes account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They should be sent to:

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Foundation Knowledge

Foundation Knowledge consists of two sections: Binomial Expansion, Exponential and Logarithmic functions. Binomial Expansion forms the basis of the binomial distribution. Many mathematical models related to natural phenomena involve the exponential function. The probability function of the normal distribution also involves the exponential function.

It should be noted that the content of Foundation Knowledge is considered as pre-requisite knowledge for Calculus and Statistics of Module One. Rigorous treatment of the topics in Foundation Knowledge should be avoided.

Learning Unit	Learning Objective	Time
Foundation Knowledge		
1. Binomial expansion	1.1 recognise the expansion of $(a + b)^n$, where n is a positive integer	3

Explanatory Notes:

At Key Stage 3, students understood the definition of a^0 and the laws of integral indices, and could perform addition, subtraction, multiplication as well as their mixed operations and factorisation of polynomials.

There are several ways to introduce binomial expansion, such as using multiplication to expand $(a+b)^n$ or using the concept of combination to explain the binomial expansion.

Students are required to recognise that

$(a+b)^n = C_0^n a^n + C_1^n a^{n-1}b + \cdots + C_{n-1}^n ab^{n-1} + C_n^n b^n = \sum_{r=0}^{n} C_r^n a^{n-r} b^r$, where n is a positive integer.

Notations such as "C_r^n", "${}_nC_r$", "${}^nC_r$", "$\binom{n}{r}$", etc. can be used.

To facilitate the expression of binomial expansion, students are required to recognise the summation notation "Σ", such as $\sum_{i=1}^{n} 2$, $\sum_{i=0}^{n} 4^i$ and $\sum_{k=1}^{7} k^3$. Students are also required to recognise the relations: $\sum_{i=1}^{n} a = na$ and $\sum_{r=1}^{n}(ax_r \pm by_r) = a\sum_{r=1}^{n} x_r \pm b\sum_{r=1}^{n} y_r$, where a, b are constants.

The following contents are not required:

- expansion of trinomials
- finding the greatest coefficient and the greatest term
- the properties of binomial coefficients
- applications to numerical approximation, e.g. finding the approximate value of $(1.01)^{10}$

Teachers may guide the students to expand the binomials by Pascal triangle and may introduce the following historical facts to the students: The arrangement of the binomial coefficients in a triangle is named after a mathematician, Blaise Pascal as he included this triangle with many of its application in his treatise, *Traité du triangle arithmétique* (1653). In fact, in 13th Century, Chinese mathematician Yang Hui (楊輝) presented the triangle in his book 《詳解九章算法》(1261) and pointed out that Jia Xian (賈憲) had used the triangle to solve problems. Thus, the triangle is also named "Yang Hui's Triangle" (楊輝三角) or "Jia Xian's Triangle" (賈憲三角).

Learning Unit	Learning Objective	Time
Foundation Knowledge		
2. Exponential and logarithmic functions	2.1 recognise the definition of the number e and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ 2.2 understand exponential functions and logarithmic functions 2.3 use exponential functions and logarithmic functions to solve problems 2.4 transform $y = ka^x$ and $y = k\left[f(x)\right]^n$ to linear relations, where a , n and k are real numbers, $a > 0$, $a \neq 1$, $f(x) > 0$ and $f(x) \neq 1$	8

Explanatory Notes:

Students are required to recognise that the value of $\left(1+\frac{1}{n}\right)^n$ tends to a fixed number e as n increases. When students have recognised the concept of limits, they are required to recognise that $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

Students may use calculators or computer software to find the approximate value of e.

Teachers may use real-life examples, such as population growth, radioactive decay, interest calculation to introduce the concept of e.

Students are required to recognise the exponential series: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$.

Students should be able to expand the exponential functions, such as e^{-x} , e^{kx} , e^{-x^2} and e^{x+k} into exponential series, where k is a constant.

Students learnt the properties of the exponential function $y = a^x$ and the logarithmic function $y = \log_a x$ ($a > 0$, $a \neq 1$) and their graphs in Learning Unit "Exponential and logarithmic functions" of the Compulsory Part. Students are required to understand that the exponential function $y = e^x$ and the natural logarithm function $y = \ln x$ ($x > 0$) are special cases of the exponential function $y = a^x$ and the logarithm function $y = \log_a x$ respectively.

Students are required to understand the relation between the graphs of exponential functions and that of logarithmic functions. To consolidate students' understanding of the properties of $y = e^x$ and $y = \ln x$, teachers may ask their students to explore the relations among the graphs of $y = 2^x$, $y = e^x$, $y = 3^x$, $y = 2^{-x}$, $y = e^{-x}$, $y = 3^{-x}$, $y = \log_2 x$, $y = \ln x$ and $y = \log_3 x$.

Students are required to understand the three equalities $e^{\ln x} = x$, $\ln e^x = x$ and $a^x = e^{x \ln a}$.

The exponential function can be used to model many natural phenomena, such as continuous growth, the bacteria growth, the rate of cooling of substances and the decay of radioactive elements, etc. Teachers may use the concept of compound interest that students learnt at Key Stage 3 to introduce the concept of continuous growth.

Students should be able to tackle the problems involving the following formulae:

Continuous growth : $f(t) = P_0 e^{rt}$, $r > 0$

Population growth： $f(t) = P_0 e^{kt}$, $k > 0$

Radioactive decay： $f(t) = P_0 e^{-kt}$, $k > 0$

Students should be able to transform $y = ka^x$ and $y = k[f(x)]^n$ to linear relations, where a, n and k are real numbers, $a > 0$, $a \neq 1$, $f(x) > 0$ and $f(x) \neq 1$. When experimental values of x and y are given, students should be able to plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercepts. The function $f(x)$ is not restricted to polynomial function.

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Calculus

Calculus consists of two sections: Differentiation with Its Applications and Integration with Its Applications. The concept of the derivative of a function involves the concept of the limit of a function. In the section "Differentiation with Its Applications", students are required to understand the definition of the derivative of a function, the fundamental formulae and the rules of differentiation. They are also required to use derivatives to find the equation of the tangent to a curve and to investigate the maximum and minimum values of a function.

Students also need to find the function $f(x)$ from its derivative $f'(x)$ in various situations related to science, technology and economics. This reverse process is the concept of the indefinite integral. Teachers need to introduce the idea of the definite integral as the limit of a sum of the areas of rectangles under a curve. Teacher can lead students to recognise that the Fundamental Theorem of Calculus can link the two apparently different concepts (the indefinite integral and the definite integral) together.

The approaches adopted should be intuitive but the concepts involved should be correct. In difficult topics such as the concept of limits, numerical approaches using calculators or computer software can help students understand the related concepts. Teachers may help students understand mathematics concepts by using interactive platform of dynamic mathematics software.

Learning Unit	Learning Objective	Time
Calculus		
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function 3.2 find the limits of algebraic functions, exponential functions and logarithmic functions 3.3 recognise the concept of the derivative of a function from first principles 3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$	5

Explanatory Notes:

The limit of a function is one of the basic concepts in calculus. Teachers may briefly review the concept and notation of a function before introducing the concept of the limit of a function.

Teachers may use table to show the small changes of the functional values close to $x = x_0$ to help students recognise the concept of the limit of $f(x)$ as x tends to x_0 .

Students are required to recognise that for a function $f(x)$, as x tends to x_0, the limit of $f(x)$ might not exist, such as $\lim_{x \to 0} \frac{1}{x}$.

Students are required to recognise the theorems involving the limits of sum, difference, product, quotient, scalar multiplication of functions, and the limits of composite functions. But the proofs are not required in the curriculum. By using these theorems, students should be able to find the limits of algebraic functions, exponential functions and logarithmic functions. The algebraic functions include polynomial functions, rational functions, power functions x^α, and functions derived from the above functions through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$. Students are also required to find the limits of functions as x tends to infinity, such as $\lim_{x\to\infty}\frac{2x+3}{x^3}$ and $\lim_{x\to-\infty}\frac{3e^x}{x}$, etc.

Students are required to recognise that for function $f(x)$, the derivative of a function $f(x)$ with respect to x can be defined as $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ if the limit exists. Teachers may demonstrate how to find the derivative of functions, such as x^2 and $\frac{1}{x-1}$ from first principles, but finding the derivatives of functions from first principles is not required in the curriculum.

Students are required to recognise the notations of derivative, e.g. y', $f'(x)$ and $\frac{dy}{dx}$. Students are required to recognise that $\frac{d}{dx}$ is an operator, and $\frac{dy}{dx}$ is not a fraction.

Students are required to recognise that the slope of the tangent of a curve $y=f(x)$ at a point $(x_0, f(x_0))$ is $f'(x_0)$ and its notation $\left.\frac{dy}{dx}\right|_{x=x_0}$. Students should be able to find the equation of tangent to the curve $y=f(x)$ at that point.

Learning Unit	Learning Objective	Time
Calculus		
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation 4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions	8

Explanatory Notes:

Students are required to understand the following rules of differentiation:

- addition rule: $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$
- multiplication rule: $\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$
- quotient rule: $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
- chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Students should also be able to find the derivatives of algebraic functions, exponential functions and logarithmic functions, such as $f(x)=\left(2x^2+5x+3\right)^8$, $f(x)=\sqrt{5x^2+2x-1}$, $f(x)=e^{x^2+1}$ and $f(x)=\ln\sqrt{x^3-1}$ etc., by integrated use of the following formulae:

- $(C)'=0$
- $(x^n)'=nx^{n-1}$
- $(e^x)'=e^x$
- $(\ln x)'=\frac{1}{x}$
- $(\log_a x)'=\frac{1}{x\ln a}$
- $(a^x)'=a^x\ln a$

Differentiation of inverse functions, differentiation of parametric equations, implicit differentiation, and logarithmic differentiation are not required in the curriculum.

Learning Unit	Learning Objective	Time
Calculus		
5. Second derivative	5.1 recognise the concept of the second derivative of a function 5.2 find the second derivative of an explicit function	2

Explanatory Notes:

Students are required to recognise the concept of second derivative of a function $f(x)$ and its notations: $f''(x)$ 、 y'' and $\frac{d^2y}{dx^2}$.

Finding the second derivative of an implicit function, and finding the third or higher order derivatives are not required in the curriculum.

Teachers may point out that, in general, $\frac{d^2y}{dx^2} \neq \left(\frac{dy}{dx}\right)^2$.

Students should be able to apply the second derivative of a function $f(x)$ to determine the concavity of its graph in $a \le x \le b$.

Learning Unit	Learning Objective	Time
Calculus		
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangents, rates of change, maxima and minima	10

Explanatory Notes:

Students should be able to use differentiation to solve problems involving tangent and rates of change, where the concepts of displacement, velocity and acceleration are required.

Students should be able to apply the first derivative or the second derivative to solve problems involving maximum and minimum. Local and global extrema are required.

Learning Unit	Learning Objective	Time
Calculus		
7. Indefinite integration and its applications	7.1 recognise the concept of indefinite integration 7.2 understand the basic properties of indefinite integrals and basic integration formulae 7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions 7.4 use integration by substitution to find indefinite integrals 7.5 use indefinite integration to solve problems	10

Explanatory Notes:

Students are required to recognise that indefinite integration is the reverse process of differentiation, and understand the meaning of the constant of integration C in the relation $\int f(x)\,dx = F(x) + C$.

Students are required to recognise the notation $\int f(x)\,dx$ and the terms "integrand", "constant of integration" and "primitive function".

Students are required to understand the basic properties of indefinite integrals and basic integration formulae:

The properties include:

- $\int kf(x)dx = k\int f(x)dx$, where k is a constant
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

The formulae include:

- $\int kdx = kx + C$, where k and C are constants
- $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$, where $n \neq -1$

- $\int \frac{1}{x} dx = \ln|x| + C$, $x \neq 0$ (students should recognise the concept of absolute value)
- $\int e^x dx = e^x + C$

Students should be able to use integration by substitution to find the indefinite integrals, such as $\int \left(3 + \frac{x}{2}\right)^3 dx$ and $\int 2x\sqrt{x^2+1}\, dx$, etc.

Using integration by parts to find indefinite integral is not required in the curriculum.

Learning Unit	Learning Objectives	Time
Calculus		
8. Definite integration and its applications	8.1 recognise the concept of definite integration 8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals 8.3 find the definite integrals of algebraic functions and exponential functions 8.4 use integration by substitution to find definite integrals 8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems	12

Explanatory Notes:

Students are required to recognise that definite integral is defined as the limit of a sum of the areas of rectangles under a curve, and should be able to distinguish its concept from that of the indefinite integral.

Students are required to recognise the notation: $\int_a^b f(x)\,dx$, and the concept of dummy variables, e.g. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$.

Students are required to recognise the Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$, and recognise the relationship between definite integral and indefinite integral by this theorem.

Students are required to understand the following properties of definite integrals:

- $\int_a^b f\left(x\right) dx = -\int_b^a f\left(x\right) dx$
- $\int_a^a f\left(x\right) dx = 0$

- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$, where k is a constant
- $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Teachers may help the students explore the geometrical meaning of the above properties.

Students should be able to find the definite integrals of algebraic functions and exponential functions and to use integration by substitution to find definite integrals. When the method of substitution is used to evaluate a definite integral, students should change the upper and lower limits of the definite integral accordingly.

Students should be able to use definite integration to find the area of the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$.

Using definite integration to find the area between a curve and the y-axis and the area between two curves are not required in the curriculum.

Students should be able to use definite integration to solve real-life problems, such as: rectilinear motion, growth model, etc.

Learning Unit	Learning Objective	Time
Calculus		
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4

Explanatory Notes:

It is sometimes hard or even impossible for students to evaluate the values of some definite integrals, such as $\int_1^2 e^{x^2} dx$. The trapezoidal rule is one of the methods to approximate the values of definite integrals.

In applying the trapezoidal rule, students are only required to use equal width of subintervals to approximate the values. Students are required to understand that a better approximation of the definite integral can be obtained by increasing the number of subintervals.

Error estimation in the application of the trapezoidal rule is not required in the curriculum.

Students should be able to judge whether an approximation is an under-estimate or over-estimate by the second derivative test and concavity, for example:

- If function $f(x)$ in $a \le x \le b$ is concave upwards, the trapezoidal rule will over-estimate the required integral $\int_a^b f(x) dx$.
- If function $f(x)$ in $a \le x \le b$ is concave downwards, the trapezoidal rule will under-estimate the required integral $\int_a^b f(x) dx$.

Statistics

Statistics consists of four sections: Conditional probability and Bayes' theorem, Discrete Probability Distributions and their Applications, The Normal Distribution and Its Applications, Point and Interval Estimation.

Probability in Statistics of Module One is basic and important. The concept of a random variable is new to students. The binomial, the Poisson and the normal distribution serve to widen students' knowledge on probability distributions. Discussions of statistical inference are also included in the curriculum.

A study of population parameters and sample statistics depicts the relationship between populations and samples. Point estimation and interval estimation are required.

Point estimation involves the use of sample data to calculate a statistic which is to serve as a guess for an unknown population parameter. A confidence interval is an interval estimate of a population parameter. The width of the confidence intervals is determined by the corresponding confidence level.

Learning Unit	Learning Objective	Time
Statistics		
10. Conditional probability and Bayes' theorem	10.1 understand the concept of conditional probability 10.2 use Bayes' theorem to solve simple problems	6

Explanatory Notes:

Students learnt the concept of independent events in Learning Unit "More about probability" of the Compulsory Part and were able to judge whether two events were independent or not. Students recognised the concept, the notation and the rule $P(A \cap B) = P(A) \times P(B|A)$ of conditional probability in the Compulsory Part. In this Learning Unit, students need to further understand the concept of conditional probability, and by integrating the laws in "More about probability", to understand that if A and B are independent events, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$, and vice versa.

Students are required to recognise that if A and B are independent events, then A' and B, B' and A, A' and B' are also independent events.

In handling problems involving a finite number of outcomes, drawing a tree diagram is an efficient way to consider all the outcomes. When using Bayes' theorem to solve simple problems, students may draw tree diagrams or other figures to list out the required conditional probability.

Teachers may explain the concepts of conditional probability and independent events by daily life examples.

Learning Unit	Learning Objective	Time
Statistics		
11. Discrete random variables	12.1 recognise the concept of discrete random variables	1

Explanatory Notes:

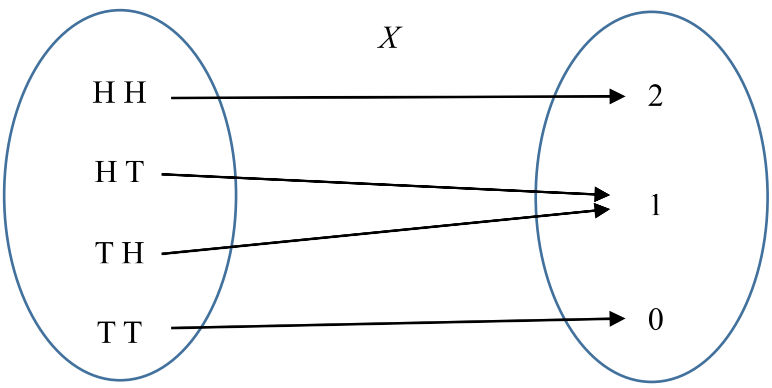
Teachers may introduce the concept of random variable by using daily life examples and the concept of function in the Compulsory Part, and may guide the students to recognise the differences between the concepts of random variables and variables in algebra.

Learning Unit	Learning Objective	Time
Statistics		
12. Probability distribution, expectation and variance	12.1 recognise the concept of discrete probability distribution and represent the distribution in the form of tables, graphs and mathematical formulae 12.2 recognise the concepts of expectation $E[X]$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems	7

Explanatory Notes:

Teachers may introduce the concept of discrete probability distribution by daily life examples, such as:

Suppose we toss two fair coins one time, we have four possible outcomes: HH, HT, TH and TT. Considering the number of heads that we get, the random variable X is as follows:



The probability distribution of the random variable X can also be tabulated as follows:

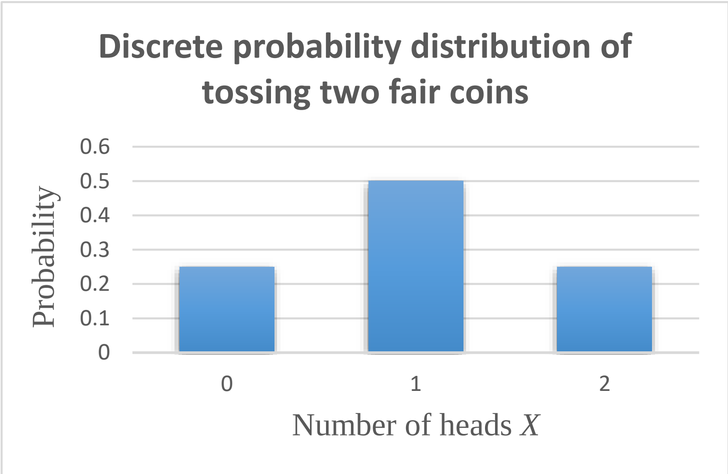
x	0	1	2
$P(X=x)$	0.25	0.5	0.25

where $P(X=x)$ represents the probability of the random variable X getting a value x, $0 \le P(X=x) \le 1$ and $\sum_x P(X=x) = 1$.

The probability distribution of the random variable X can also be represented by a formula:

$$f(x) = P(X = x) = C_x^2 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{2-x}, \text{ where } x = 0, 1, 2$$

The probability distribution of a random variable X can also be represented by a bar chart.



Students learnt the concepts and applications of the mean and standard deviation in the Compulsory Part. Teachers may briefly review these concepts before introducing the expectation and variance of a discrete random variable.

Students should be able to use the following formulae to solve simple problems:

- $E[X] = \sum xP(X = x)$
- $\text{Var}(X) = E\left[(X - \mu)^2\right]$
- $E[g(X)] = \sum g(x)P(X = x)$
- $E[aX + b] = aE[X] + b$
- $\text{Var}(X) = E\left[X^2\right] - (E[X])^2$
- $\text{Var}(aX + b) = a^2\text{Var}(X)$

The notation $E(X)$ can also be used.

The sums and differences of random variables, and joint probability distributions are not required in the curriculum.

Learning Unit	Learning Objective	Time
Statistics		
13. The binomial distribution	13.1 recognise the concept and properties of the binomial distribution 13.2 calculate probabilities involving the binomial distribution	5

Explanatory Notes:

Before learning the binomial distribution, students need to recognise Bernoulli distribution.

Students are required to recognise that a binomial experiment has the following properties:

- There are n identical trials.
- There are only 2 possible outcomes for each trial: success and failure.
- The probability of success is p and the probability of failure is $1 - p$ in each trial. The probability p will not change in the experiment.
- The trials are independent.

A binomial random variable X is the number of successes in n trials. Students are required to recognise that $E[X] = np$ and $\operatorname{Var}(X) = np(1-p)$, but the proofs of these two formulae are not required in the curriculum.

Besides, using the binomial distribution table to find corresponding probabilities is also not required in the curriculum.

The binomial formula requires time-consuming computation. Teachers may introduce free online calculator or using the built-in function in the spreadsheet software, such as BINOM.DIST(r, n, p, T), to find the individual and cumulative binomial probabilities.

Learning Unit	Learning Objective	Time
Statistics		
14. The Poisson distribution	14.1 recognise the concept and properties of the Poisson distribution 14.2 calculate probabilities involving the Poisson distribution	5

Explanatory Notes:

Teachers may use real-life examples to introduce the Poisson distribution, and may discuss with students that the Poisson distribution is in fact a binomial distribution under the limiting condition, and when n is large and p is small a binomial distribution can be approximated by a Poisson distribution.

Students are required to recognise that a Poisson experiment has the following properties :

- The occurrence of every event in an interval is independent of the occurrences of the events in other non-overlapping intervals.
- In any interval, the mean number of occurrences of events in an interval is proportional to the size of the interval.
- The probability of more than one event occurs in a very small interval is negligible.

Students are required to recognise that if X follows a Poisson distribution with λ as the mean number of occurrences of events in the interval, then $E\left[X\right]=\lambda$ and $\mathrm{Var}(X) = \lambda$. But the proofs of these formulae are not required in the curriculum.

Besides, using the Poisson distribution table to find the corresponding probabilities is not required in the curriculum.

The Poisson formula requires time-consuming computation. Teachers may introduce free online calculator or using the built-in function in the spreadsheet software, such as POISSON.DIST(x, n, T), to find the individual and cumulative Poisson probabilities.

Learning Unit	Learning Objective	Time
Statistics		
15. Applications of the binomial and the Poisson distributions	15.1 use the binomial and the Poisson distributions to solve problems	5

Explanatory Notes:

To identify the discrete probability distribution followed by a random variable, students are required to recognise the characteristics of discrete probability distributions. In the binomial distribution, the variance is less than the mean, whereas in the Poisson distribution, the variance is equal to the mean. These facts provide clues for students in the identification of the two distributions. If several random samples are collected, an appropriate probability distribution may be chosen by comparing the mean and variance of each sample.

Learning Unit	Learning Objective	Time
Statistics		
16. Basic definition and properties of the normal distribution	16.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution 16.2 recognise the concept and properties of the normal distribution	3

Explanatory Notes:

Students have already grasped the concepts of discrete random variable and discrete probability distribution. Teachers should extend those concepts to continuous random variables and continuous probability distributions.

Students are required to recognise the differences between the probability distributions of a discrete random variable and a continuous random variable.

Teachers may introduce the concept and properties of probability density function (p.d.f.) $f(x)$.

Finding the expectation and variance of a continuous probability distribution, and derivations of the mean and variance of the normal distribution are not required in the curriculum.

Students are required to recognise that the formulae in Learning Objective 12.2 are also applicable to continuous random variables.

Students are required to recognise the concept of the normal distribution, where $X \sim N(\mu, \sigma^2)$ represents that X follows a normal distribution with mean μ and standard deviation σ.

Teacher may introduce the probability density function of normal distribution $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, where $x \in (-\infty, \infty)$ with mean μ and standard deviation σ.

Students are required to recognise the properties of the normal distribution:

- the normal curve is bell-shaped and symmetrical about the mean
- the mean, mode and median are all equal
- the flatness of the normal curve is determined by the value of σ
- the area under the normal curve is 1

Learning Unit	Learning Objective	Time
Statistics		
17. Standardisation of a normal variable and use of the standard normal table	17.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution.	2

Explanatory Notes:

Students are required to recognise $Z \sim N(0,1)$ represents that Z follows the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. $N(0,1)$ is called "the standard normal distribution".

For $Z \sim N(0,1)$, students should be able to use the standard normal table to find the values of $P(Z > a)$, $P(Z \le b)$, and $P(a \le Z \le b)$, etc. Students should also be able to transform those distributions in $N(\mu, \sigma^2)$ to $N(0,1)$ before referring to the table.

Students are required to recognise that if $X \sim N(\mu, \sigma^2)$ and $Z = \dfrac{X-\mu}{\sigma}$, then

- $Z \sim N(0,1)$
- $E[Z] = 0$ and $\mathrm{Var}(Z) = 1$
- $P(a \le X \le b) = P(\dfrac{a-\mu}{\sigma} \le \dfrac{X-\mu}{\sigma} \le \dfrac{b-\mu}{\sigma}) = P(z_1 \le Z \le z_2)$

To find the standard score, the individual and cumulative probabilities of a normal random variable, teachers may introduce free online calculator or using some built-in functions in the spreadsheet software, such as:

- **NORM.DIST (*x*, *μ*, *σ*, *T*):** For $X \sim N(\mu, \sigma^2)$, when $T = 1$, we get $P(X \le x)$
- **NORM.INV(*p*, *μ*, *σ*):** For $X \sim N(\mu, \sigma^2)$, we get the value of x such that $P(X \le x) = p$
- **NORMSDIST(*z*):** For $Z \sim N(0,1)$, we get $P(Z \le z)$

- **NORMSINV(*p*):** For $Z \sim N(0,1)$, we get z such that $P(Z< z) = p$

- **STANDARDIZE (*x*, *μ*, *σ*):** We get $Z = \frac{x-\mu}{\sigma}$

Learning Unit	Learning Objective	Time
Statistics		
18. Applications of the normal distribution	18.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N\left(\mu,\sigma^2\right)$ 18.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N\left(\mu,\sigma^2\right)$ 18.3 use the normal distribution to solve problems	7

Explanatory Notes:

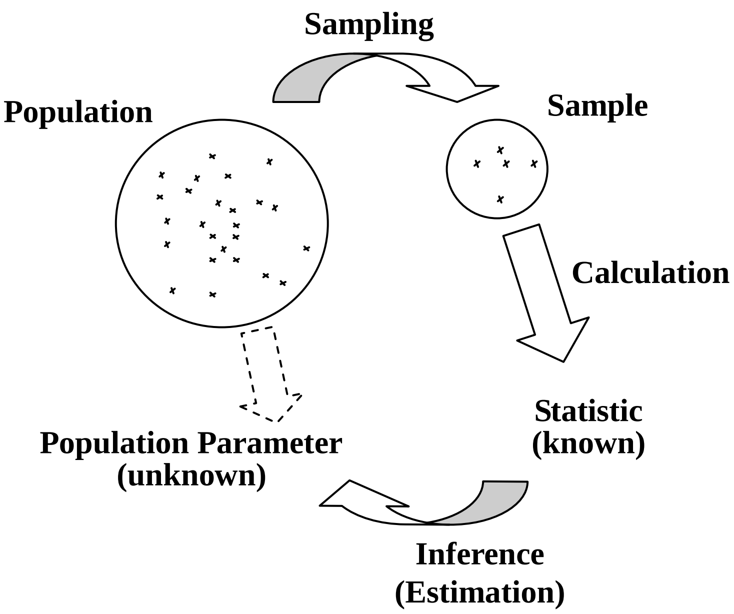
In finding the corresponding probabilities, students are required to recognise:

- $P(X > x_1) = P(X \geq x_1)$
- $P(X < x_2) = P(X \leq x_2)$
- $P\left(x_1 \leq X \leq x_2\right) = P\left(x_1 \leq X < x_2\right) = P\left(x_1 < X \leq x_2\right) = P\left(x_1 < X < x_2\right)$
- $P\left(X = x_1\right) = 0$

Learning Unit	Learning Objective	Time
Statistics		
19. Sampling distribution and point estimates	19.1 recognise the concepts of sample statistics and population parameters 19.2 recognise the sampling distribution of the sample mean $\overline{X}$ from a random sample of size n 19.3 use the Central Limit Theorem to treat $\overline{X}$ as being normally distributed when the sample size n is sufficiently large 19.4 recognise the concept of point estimates including the sample mean and sample variance	9

Explanatory Notes:

Students learnt the concepts of population and sample in the Compulsory Part. Students in this Learning Unit should recognise the concepts of sample statistics and population parameters and their relationships:



Students are required to recognise that:

- A sample statistic is not necessarily the same as the corresponding population parameter, but it can provide good information about that parameter.
- Most sample statistics are close to the population parameters. Few are extremely larger or extremely smaller than the corresponding population parameter.
- In general, samples that are larger produce statistics that vary less from the population parameter.

Students are required to recognise the following formulae about population mean μ and population variance σ^2 :

- $\mu = \frac{\sum_{i=1}^{N} x_i}{N}$
- $\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$

Teachers may conduct some sampling activities to help students recognise the concept of the sampling distribution of the sample mean $\bar{X}$, and that if the population mean is μ and the population variance is σ^2 , then $E\left[\bar{X}\right] = \mu$ and $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$.

Students are required to recognise that if $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, but the proof is not required in the curriculum.

Teachers may use some special case, such as n = 1 or n = N, or using computer simulation programmes to help students recognise that:

- no matter what the shape of the original distribution is, the sampling distribution of the mean approaches a normal distribution as the sample size increases
- most distributions approach a normal distribution very quickly as the sample size increases
- the number of samples is assumed to be infinite in a sampling distribution
- the spread of the distributions decreases as the sample size increases

Point estimation is one of the methods of parameter estimation. Students are required to recognise the concept of the estimation of an unknown population parameter from a sample statistic, e.g., to estimate a population mean μ by using a sample mean $\bar{x}$. Teachers may point out that sample median and sample mode can also be used as a point estimator.

In the process of sampling, different estimates may be obtained from different samples. It is difficult to determine which estimator is the most suitable one. We may use the unbiased estimator to estimate the unknown parameter. It is expected that, in the long run, the average value of our estimates taken over a large number of samples should equal the population value: E(sample estimator) = population parameter. Students are required to recognise that the unbiased estimator of a population parameter is not unique.

Students are required to recognise that the concept of unbiased estimator, e.g. the sample mean $\overline{x}$ is an unbiased estimator of the population mean μ, and the sample variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ is an unbiased estimator of the population variance σ^2.

Learning Unit	Learning Objective	Time
Statistics		
20. Confidence interval for a population mean	20.1 recognise the concept of confidence interval 20.2 find the confidence interval for a population mean	6

Explanatory Notes:

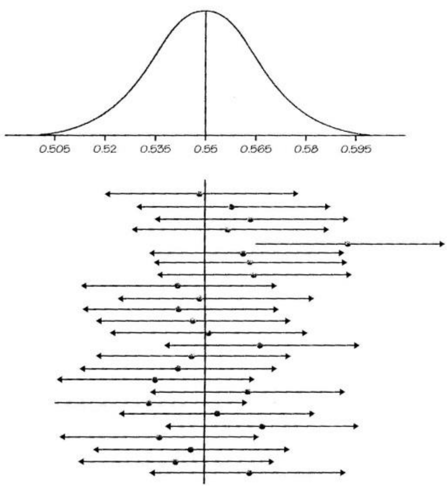
Students are required to recognise the concept of confidence interval, and that the 95% confidence interval and the 99% confidence interval are most commonly used.

Before constructing a confidence interval for μ, students should ask the following questions:

- Are the random samples taken from a normal population?
- Is the population variance known?
- Is the sample size large enough?

Teachers may use computer simulation programmes to create the diagram of confidence interval, such as the following, to help students recognise that:

- If random samples are taken from a population and 95% confidence intervals constructed for each sample, students may expect about 5% of the intervals do not include the population parameter. When students calculate a confidence interval, they will not know whether the parameter is included in the interval or not.



- The width of the confidence interval can be reduced by increasing the sample size or decreasing the confidence level.

- In constructing a confidence interval, it is desirable to have a narrow width (for a more precise estimate) with a high confidence level, but in most cases, we cannot attain these two conditions at the same time.

Students should be able to evaluate the confidence interval for the population mean μ under the following conditions:

Conditions	95% confidence interval for μ	99% confidence interval for μ
Normal population ▪ with known variance σ^2 ▪ large or small sample size n ▪ sample mean $\overline{x}$	$\left(\overline{x}-1.96\frac{\sigma}{\sqrt{n}},\ \overline{x}+1.96\frac{\sigma}{\sqrt{n}}\right)$	$\left(\overline{x}-2.575\frac{\sigma}{\sqrt{n}},\ \overline{x}+2.575\frac{\sigma}{\sqrt{n}}\right)$
Non-normal population ▪ with known variance σ^2 ▪ large sample size n ($n \geq 30$) ▪ sample mean $\overline{x}$	$\left(\overline{x}-1.96\frac{\sigma}{\sqrt{n}},\ \overline{x}+1.96\frac{\sigma}{\sqrt{n}}\right)$	$\left(\overline{x}-2.575\frac{\sigma}{\sqrt{n}},\ \overline{x}+2.575\frac{\sigma}{\sqrt{n}}\right)$
Non-normal population ▪ with unknown variance σ^2 ▪ large sample size n ($n \geq 30$) ▪ sample mean $\overline{x}$ ▪ sample variance s^2	$\left(\overline{x}-1.96\frac{s}{\sqrt{n}},\ \overline{x}+1.96\frac{s}{\sqrt{n}}\right)$	$\left(\overline{x}-2.575\frac{s}{\sqrt{n}},\ \overline{x}+2.575\frac{s}{\sqrt{n}}\right)$

Learning Unit	Learning Objective	Time
Further Learning Unit		
21. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units. In other words, this is not an independent and isolated Learning Unit and the activities may be conducted in different stages of a lesson, such as motivation, development, consolidation or assessment.

Acknowledgements

We would like to thank the members of the following Committees and Working Group for their invaluable comments and suggestions in the compilation of this booklet.

CDC Committee on Mathematics Education

CDC-HKEAA Committee on Mathematics Education

Ad Hoc Committee on Secondary Mathematics Curriculum (Extended Part/Elective of Senior Secondary)

CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (Module 1)