Explanatory Notes to Senior Secondary Mathematics Curriculum :

Module 2 (Algebra and Calculus)

Mathematics Education Section
Curriculum Development Institute
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Foreword

The *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)* (abbreviated as "C&A Guide" in this booklet) was updated in December 2017 to keep abreast of the ongoing renewal of the school curriculum. The Senior Secondary Mathematics Curriculum consists of a Compulsory Part and an Extended Part. The Extended Part has two optional modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus).

In the C&A Guide, the Learning Objectives of Module 2 are grouped under different learning units in the form of a table. The notes in the "Remarks" column of the table in the C&A Guide provide supplementary information about the Learning Objectives. The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of Module 2;
2. the strategies suggested for the teaching of Module 2;
3. the connections and structures among different learning units of Module 2; and
4. the curriculum articulation between the Compulsory Part and Module 2.

Teachers may refer to the "Remarks" column and the suggested lesson time of each Learning Unit in the C&A Guide, with the explanatory notes in this booklet being a supplementary reference, for planning the breadth and depth of treatment in learning and teaching. Teachers are advised to teach the contents of the Compulsory Part and Module 2 as a connected body of mathematical knowledge and develop in students the capability to use mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the C&A Guide does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequence that takes account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They should be sent to:

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Foundation Knowledge

The content of Foundation Knowledge comprises five Learning Units and is considered as the pre-requisite knowledge for Calculus and Algebra of Module 2. These Learning Units serve to bridge the gap between the Compulsory Part and Module 2. Therefore, it should be noted that complicated treatment of topics in Foundation Knowledge is not the objective of the Curriculum.

Learning Unit "Odd and even functions" provides the background knowledge to help students understand the properties of definite integrals involving odd and even functions. Learning Unit "The binomial theorem" forms the basis of the proofs of some rules in Learning Unit "Differentiation". Students should be able to prove propositions by applying Mathematical induction. Learning Unit "More about trigonometric functions" introduces the radian measure of angles, the three new trigonometric functions and some trigonometric formulae commonly used in the learning of Calculus. Students are required to understand the importance of the radian measure in Calculus. Learning Unit "Introduction to e" helps students understand that e and the natural logarithm are important concepts in mathematics and play crucial roles in differentiation and integration in Calculus.

As there is a strong connection between Foundation Knowledge, Calculus and Algebra, teachers should arrange suitable teaching sequences to suit their students' needs. For example, teachers may integrate Learning Unit "Introduction to e " into Learning Unit "Limits " when teaching the definition of e to form a coherent set of learning contents.

Learning Unit	Learning Objective	Time
Foundation Knowledge		
1 Odd and even functions	1.1 recongise odd and even functions and their graphs	2

Explanatory Notes:

In Learning Unit “Functions and graphs” of the Compulsory Part, students learnt the concept of functions. In this Learning Unit, students should recognise the definitions of odd function and even function and their graphs. In Learning Objective 10.2, some properties of definite integrals involve the concepts of odd and even functions. However, the properties of odd and even functions, for example, ‘odd function + odd function = odd function’ and ‘even function + even function = even function’, etc. are not required in the curriculum.

Students are required to recognise the definition of absolute value function $y=|x|$ and its graph, and that it is an example of even functions. Besides, the formula $\int \frac{1}{x}dx=\ln|x|+C$ and its proof in Learning Objective 9.2 also involve the concept of the absolute value function.

Learning Unit	Learning Objective	Time
Foundation Knowledge		
2. Mathematical induction	2.1 understand the principle of mathematical induction	3

Explanatory Notes:

Mathematical induction is an important tool in proving mathematical propositions. In this Learning Unit, students should be able to use mathematical induction to prove propositions related to the summation of a finite sequence. Students are required to understand the principle of mathematical induction, follows the procedures of mathematical induction and use mathematical induction to solve problems. Using mathematical induction to prove propositions involving inequalities and divisibility are not required in the curriculum.

Teachers may guide students to guess the formulae for some summations of finite sequences and ask them to examine their guesses. Teachers should point out that even though we know that a proposition is true for some positive integers, it was still not sufficient to guarantee that the proposition is true for all positive integers, e.g

$1+3+5+...+(2n-1)=n^2+(n-1)(n-2)(n-3)(n-4)(n-5)$ holds when n = 1, 2, 3, 4 and 5

but it is not true for other positive integers.

In order to prove mathematical propositions $P(n)$ that are true for all positive integers n by mathematical induction, students are required to notice that the following two steps in the principle of mathematical induction are crucial:

(1) Prove that $P(1)$ is true.

(2) Prove that for any positive integer k, if $P(k)$ is true, then $P(k + 1)$ is also true.

Teachers may use counter-examples to illustrate that if one of the two above steps is incomplete, we cannot prove that $P(n)$ is true for all positive integers n, for example:

(a) For any positive integer n, $1+2+3+\ldots+n=\dfrac{1+n}{2}$.

(b) For any positive integer n, $1+2+3+\ldots+n=\dfrac{n(n+1)}{2}+2$.

Example (a) shows that only step 1 but not step 2 can be completed for the statement. As a result, the statement " $P(n)$ is true for all positive integers n" cannot be proved by mathematical induction.

Example (b) shows that only step 2 but not step 1 can be completed for the statement. As a result, the statement "$P(n)$ is true for all positive integers n" cannot be proved by mathematical induction.

In Learning Unit "Arithmetic and geometric sequences and their summations" of the Compulsory Part, students learnt the formulae for the summation of the arithmetic sequence and that of the geometric sequence. Students may try to prove the related formulae by mathematical induction.

Students should be able to apply mathematical induction to prove the binomial theorem in Learning Unit 3.

Learning Unit	Learning Objective	Time
Foundation Knowledge		
3. The binomial theorem	3.1 expand binomials with positive integral indices using the binomial theorem	3

Explanatory Notes:

At Key Stage 3, students learnt the laws of integral indices, the operations of polynomials and the identity, $(a+b)^2 = a^2 + 2ab + b^2$. To introduce the binomial theorem in this Learning Unit, teachers may let students recognise that using the methods they have learnt at Key Stage 3 to expand $(a+b)^n$ will become very complicated when *n* is very large.

Students should be able to prove the binomial theorem by mathematical induction learnt in Learning Unit 2.

To present the binomial expansion in a more concise form, students are required to recognise the summation notation ($\sum$):

$(a + b)^n = C_0^n a^n + C_1^n a^{n-1} b + C_2^n a^{n-2} b^2 + \ldots + C_{n-1}^n ab^{n-1} + C_n^n b^n = \sum_{r=0}^{n} C_r^n a^{n-r} b^r$ where *n* is a positive integer.

Students are also required to recognise the relationships: $\sum_{i=1}^{n} a = na$ and $\sum_{r=1}^{n}(ax_r \pm by_r) = a\sum_{r=1}^{n} x_r \pm b\sum_{r=1}^{n} y_r$, where *a*, *b* are constants.

As the binomial theorem belongs to a learning unit in Foundation Knowledge, the problems and examples involved should be simple and straightforward. In this connection, the following contents are not required in the curriculum:

- expansion of trinomials
- the greatest coefficient, the greatest term, and the properties of binomial coefficients
- applications to numerical approximation

In Learning Unit 7, students should be able to use the binomial theorem to prove the formula $\frac{d}{dx}(x^n) = nx^{n-1}$, where n is a positive integer, from first principles.

Besides, teachers may introduce the following historical facts to students:

The arrangement of the binomial coefficients in a triangle is named after Blaise Pascal as he included this triangle with many of its application in his treatise, Traité du triangle arithmétique (1654). In fact, in the 13th century, Chinese mathematician Yang Hui (楊輝) presented the triangle in his book 《詳解九章算法》(1261) and pointed out that Jia Xian (賈憲) had used the triangle to solve problems. Thus, the triangle is also named Yang Hui's Triangle (楊輝三角) or Jia Xian's Triangle (賈憲三角).

Learning Unit	Learning Objective	Time
Foundation Knowledge		
4. More about trigonometric functions	4.1 understand the concept of radian measure 4.2 understand the functions cosecant, secant and cotangent 4.3 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine	15

Explanatory Notes:

In Module 2, students should be able to use radian to express the magnitude of an angle and perform the conversion between radian and degree. In Learning Unit 6 and 7, teachers may explain the significance of learning radian measure in deriving the formula $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (θ is in radians) and finding the derivatives of trigonometric functions.

In Learning Unit "More about trigonometry" of the Compulsory Part, students learnt the trigonometric functions sine, cosine and tangent, and their graphs and properties (including maximum and minimum values and periodicity). In this Learning Unit, students are required to understand the other three trigonometric functions cosecant, secant and cotangent, including their definitions and the two related identities: $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$. Students should also be able to use these identities to simplify others trigonometric expressions.

Students are required to understand the following formulae:

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

- $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$
- $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$
- $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
- $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
- $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
- $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
- $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

Besides, teachers may introduce some connections between the compound angle formulae and the Chord table constructed by Claudius Ptolemy (around AD 100 – 170) of Alexandria and the theorem mainly used in the Chord table, called “Ptolemy’s Theorem”. This theorem can be a topic for investigation in Further Learning Unit of the Compulsory Part.

Students will find that $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ and $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ together with the product-to-sum and sum-to-product formulae are important tools in finding integrals.

In Learning Unit “More about trigonometry” of the Compulsory Part, students learnt to solve simple trigonometric equations with solutions from 0° to 360° only. In this regard, students should also be able to solve trigonometric equations with solutions from 0 to 2π only and this content can be applied to solve optimisation problems in Learning Objective 8.4.

Subsidiary angle form is not required in the curriculum.

Learning Unit	Learning Objective	Time
Foundation Knowledge		
5. Introduction to e	5.1 recognise the definitions and notations of e and the natural logarithm	2

Explanatory Notes:

Students will find that e and the natural logarithm that are learnt in this Learning Unit will have a significance for the study of Calculus. In Learning Unit "Exponential and logarithmic functions" of the Compulsory Part, students learnt the exponential and logarithmic functions and their graphs. In this Learning Unit, students are required to understand the exponential function e^x and the natural logarithmic function $\ln x$.

Teachers may use different methods to introduce e. For example,

(1) $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$

(2) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

Students are required to recognise that $\left(1+\frac{1}{n}\right)^n$ will tend to a number, that is e if the value of n increases. After learning the concept of limits, students should recognise that $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$. However, the proof of the existence of $\lim_{n\to\infty}(1+\frac{1}{n})^n$ is not required in the curriculum.

Teachers may ask students to use calculators or spreadsheets to get the approximate value of $\lim_{n\to\infty}(1+\frac{1}{n})^n$. Besides, teachers may use dynamic mathematics software to plot the graph of $y = (1+\frac{1}{n})^n$ to help students observe the trend of $\left(1+\frac{1}{n}\right)^n$ as n increases, and estimate the value of $\lim_{n\to\infty}(1+\frac{1}{n})^n$.

Students are required to recognise that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, and can find the approximate value of e by putting $x = 1$ into this expression.

Students are required to recognise that the natural logarithmic function possesses all the properties of logarithm functions in Learning Unit “Exponential and logarithmic functions” of the Compulsory Part. In differentiation of Calculus, the formula for the change of base is important in finding derivatives of logarithmic functions of different bases .

Since this Learning Unit may involve the concept of limits, the teaching of this Learning Unit may be arranged before that of Learning Objective 6.1.

Calculus

The content of Calculus comprises six Learning Units related to limits, differentiation, integration and their applications.

Students are required to master the concepts of functions, their graphs and properties before studying limits and differentiation. The limit of a function is an important component of Calculus. With the knowledge of the limit of a function, students could understand the concept of the derivative of a function and the related rules of differentiation. In the applications of differentiation, students should be able to solve problems related to rate of change, maximum and minimum.

The indefinite integral and differentiation are related as reverse processes to each other. The Fundamental Theorem of Calculus links up the two apparently different concepts. At this stage, the applications of the definite integral focus on finding the areas of plane figures and the volumes of solids of revolution. Students can also appreciate how to apply the definite integral to calculate the areas of non-rectilinear figures, for example, the area of a circle, etc.

Learning Unit	Learning Objective	Time
Calculus		
6. Limits	6.1 understand the intuitive concept of the limit of a function 6.2 find the limit of a function	3

Explanatory Notes:

Students learnt the concepts of various functions and their graphs in Learning Units "Functions and graphs" and "More about graphs of functions" of the Compulsory Part. Dynamic mathematics software is very useful for the exploration of the graphs of functions. The limit of a function is an important component of Calculus. Students are required to use algebraic method and graphs of functions to understand the intuitive concept of the limit of a function. Teachers may use dynamic mathematics software to assist students to grasp the related concept. It should be noted that the rigorous definition of the limit of a function is not required in the curriculum.

Students are required to recognise that the limit of $f(x)$ does not exist for some functions $f(x)$ when x tends to a, such as the limit of the function $f(x)=\frac{1}{x}$ does not exit when x tends to 0.

To distinguish between continuous functions and discontinuous functions from their graphs is not required in the curriculum.

Students are required to recognise the theorems related to the limits of the sum, difference, product, quotient and scalar multiple of functions, and the limits of composite functions, but the proofs are not required in the curriculum. Students should recognise the required conditions for these theorems. For example, students are required to recognise that the truth of the theorem $\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$ has assumed that both $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exist. On the other hand, teachers may ask students to give examples in which $\lim_{x\to a}[f(x)\pm g(x)]$ $=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$ does not hold.

In this Learning Unit, teachers should introduce the concept of composite functions.

Students should be able to perform the conversion between the expressions such as, $\frac{1}{\sqrt{x+2}-\sqrt{x}}$ and $\frac{1}{2}\left(\sqrt{x+2}+\sqrt{x}\right)$. In this Learning unit, teachers may introduce the above method of conversion when computing the limits such as $\lim_{x\to 0}\frac{x}{\sqrt{x+2}-\sqrt{2}}$, $\lim_{x\to 0}\frac{x}{\sqrt[3]{x+5}-\sqrt[3]{5}}$ and $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}$, etc.

Students should be able to use two important formulae $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ (θ is in radians) and $\lim_{x\to 0}\frac{e^x-1}{x}=1$ to find the derivatives of trigonometric functions and the derivatives of exponential functions.

Teachers may use different methods, for example, using diagrams or dynamic mathematics software to explain the reasons why the above two formulae hold.

Besides, students should be able to find the limit of a rational function at infinity.

Learning Unit	Learning Objective	Time
Calculus		
7. Differentiation	7.1 understand the concept of the derivative of a function 7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation 7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions 7.4 find derivatives by implicit differentiation 7.5 find the second derivative of an explicit function	13

Explanatory Notes:

In Learning Unit 6 , students learnt the concept of the limit of a function. In this Learning Unit, students are required to understand: Given a function $y = f(x)$, if $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ exists, this limit is defined as the derivative of a function $y = f(x)$ at x . In addition, students should be able to use the graph of the function $y = f(x)$ to explain the derivative of the function $y = f(x)$ at x as the limit of the slope of the secant line passing through $(x, f(x))$ and $(x+\Delta x, f(x+\Delta x))$ when Δx tends to 0. Teachers should also introduce the concept of the tangent to a curve.

Students should be able to find, from first principles, the derivatives of elementary functions, e.g. constant functions, x^n (n is a positive integer), $\sqrt{x}$, sin x, cos x, e^x and ln x. They should also be able to apply method such as the conversion between the expressions $\sqrt{x} - \sqrt{x+\Delta x}$ and $\dfrac{-\Delta x}{\sqrt{x} + \sqrt{x+\Delta x}}$ to find the derivative of the function $\dfrac{1}{\sqrt{x}}$ from first principles. Students should be able to use the binomial theorem to prove $\dfrac{d}{dx}(x^n) = nx^{n-1}$, where n is a positive integer, from first principles. Students may also prove this formula by mathematical induction.

Students are required to recognise the notations: y', $f'(x)$ and $\dfrac{dy}{dx}$ for derivatives.

Testing differentiability of functions is not required in the curriculum.

Students are required to understand the addition rule, product rule, quotient rule and chain rule, and should be able to use these rules to find the derivatives of functions.

The rules include:

- Addition rule: $\dfrac{d}{dx}(u+v)=\dfrac{du}{dx}+\dfrac{dv}{dx}$
- Product rule: $\dfrac{d}{dx}(uv)=v\dfrac{du}{dx}+u\dfrac{dv}{dx}$
- Quotient rule: $\dfrac{d}{dx}(\dfrac{u}{v})=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$
- Chain rule: $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$

Teachers may choose suitable examples such as $\dfrac{d}{dx}(\sin^2 x)=\dfrac{d(\sin^2 x)}{d(\sin x)}\cdot\dfrac{d(\sin x)}{dx}=2\sin x\cos x$

to help students understand the chain rule.

Students are required to understand the following formulae:

- $(C)'=0$
- $(x^n)'=nx^{n-1}$
- $(\sin x)'=\cos x$
- $(\cos x)'=-\sin x$
- $(\tan x)'=\sec^2 x$
- $(e^x)'=e^x$
- $(\ln x)'=\dfrac{1}{x}$

Students should be able to use the above rules and formulae to find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions, and logarithmic functions. The following algebraic functions are required:

- polynomial functions
- rational functions
- power functions x^{α}
- functions formed from the above functions through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$

To find the derivatives of logarithmic functions with base not equal to *e* such as $y = \log_2 x$, students should be able to use the formula for the change of base learnt in Learning Unit "Exponential and logarithmic functions" of the Compulsory Part:

$$\frac{dy}{dx} = \frac{d}{dx}(\log_2 x) = \frac{d}{dx}\left(\frac{\ln x}{\ln 2}\right) = \frac{1}{\ln 2}\frac{d}{dx}(\ln x) = \frac{1}{x\ln 2}.$$

Students should be able to use implicit differentiation in finding derivatives of functions. Equations such as $x^3 - 3xy + y^3 = 3$ and $x = y + y^2$ are examples for illustrating the use of implicit differentiation to find $\frac{dy}{dx}$. It is not easy or impossible to express *y* in terms of *x* for some equations. If the purpose is to find the derivative only, it is not necessary for students to express *y* in terms of *x*.

Students should be able to use the technique of logarithmic differentiation to find the derivatives of functions such as $y = (x^2+2)(3x-2)^2(4x+5)^6$ and $y = \left(\frac{2x+1}{2x-1}\right)^4$, etc.

Students should be able to find the second derivatives of explicit functions and recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Students should be able to apply the second derivative of a function $f(x)$ to judge the concavity of its graph in $a \le x \le b$. The second derivatives are useful in determining the concavity of functions and finding the extrema of functions in Learning Objective 8.2.

Third and higher order derivatives are not required in the curriculum.

Learning Unit	Learning Objective	Time
Calculus		
8. Applications of differentiation	8.1 find the equations of tangents to a curve 8.2 find the maximum and minimum values of a function 8.3 sketch curves of polynomial functions and rational functions 8.4 solve the problems relating to rate of change, maximum and minimum	14

Explanatory Notes:

Students learnt to find the equations of straight lines in the Compulsory Part. In Learning Objective 8.1, students should be able to find not only the equations of tangents passing through a given point on a given curve, but also the equations of the tangents passing through an external point.

In Learning Unit "Functions and graphs" of the Compulsory Part, students learnt to use the graphical method and the algebraic method to find the maximum and the minimum value of a quadratic function. In this Learning Unit, students should be able to apply differentiation to find the maximum and the minimum values of other functions.

Students are required to understand the concepts of increasing, decreasing and concavity of functions, and should be able to apply the related concepts to find the maximum and the minimum values of functions.

Students should be able to use the first derivative and the second derivative to determine whether a turning point of a function is a maximum point or a minimum point, and to find local extrema (local maximum and minimum values) and global extrema (global maximum and minimum values) of functions. If $f''(x_0)=0$, the second derivative is not applicable to determine the extremum at $x=x_0$. In this case, students have to use the first derivative to find the extrema of the function.

Students should be able to sketch curves of polynomial functions and rational functions.

Students are required to consider the following points in curve sketching:

- symmetry of the curve
- limitations on the values of x and y
- intercepts with the axes
- maximum and minimum points
- points of inflexion
- vertical, horizontal and oblique asymptotes to the curve

Students should be able to use the second derivative to determine the concavity of a function and use these properties to find the points of inflexion of the curve. Students may use dynamic mathematics software to explore whether the tangent to a curve at a point of inflexion may be horizontal or oblique. For more able students, teachers may further discuss whether the tangent to a curve at a point of inflexion may be vertical.

Students should note that it is not necessary for them to consider all of these features when sketching the curve of a function.

Finding the equation of the oblique asymptote of a rational function may involve long division. Before teaching this Learning Objective, teachers may consolidate students' knowledge related to the division of polynomials in Learning Unit "More about polynomials" of the Compulsory Part should be consolidated in this Learning Unit.

Teachers should note that students are required to solve the problems relating to rate of change, maximum and minimum in Learning Objective 8.4, and the problems involving displacement, velocity and acceleration are required.

If the problems involve terms from other disciplines, the definitions of these terms should be provided in the problems unless they are "displacement", "velocity" and "acceleration" in Learning Objective 8.4.

Learning Unit	Learning Objective	Time
Calculus		
9. Indefinite integration and its applications	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals 9.3 understand the applications of indefinite integrals in mathematical contexts 9.4 use integration by substitution to find indefinite integrals 9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2-x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$ or $\frac{1}{x^2+a^2}$ 9.6 use integration by parts to find indefinite integrals	15

Explanatory Notes:

Students are required to recognise that indefinite integration is the reverse process of differentiation.

Students are required to recognise the notation of indefinite integral: $\int f(x)dx$ and the relation $\int f(x)dx = F(x)+C$ and understand the meaning of the constant of integration C in this relation. Students are required to recognise the terms "integrand", "primitive function" and "constant of integration", etc. Students are required to recognise that different methods of indefinite integration may lead to answers which look different, such as

$\int (x+1)^2 dx = \int (x^2+2x+1)dx = \frac{1}{3}x^3+x^2+x+C_1$ and

$\int (x+1)^2 dx = \int (x+1)^2 d(x+1) = \frac{1}{3}(x+1)^3+C_2$.

Students are required to understand the following properties of indefinite integrals:

- $\int kf(x)dx = k\int f(x)dx$ where k is a constant
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Students are required to understand and should be able to use the following formulae to find indefinite integrals:

- $\int k\ dx = kx + C$ where k and C are constants.
- $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$ where $n \neq -1$.
- $\int \dfrac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \sin x\, dx = -\cos x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sec^2 x\, dx = \tan x + C$

Students are required to understand the applications of indefinite integrals in mathematical contexts such as geometry. If the problems involve terms from other disciplines, the definitions of these terms should be provided in the problems unless they are "displacement", "velocity" and "acceleration" in Learning Objective 8.4.

Students should be able to use integration by substitutions for finding the indefinite integrals.

Students should be able to use trigonometric substitutions to find the indefinite integrals involving the forms $\sqrt{a^2 - x^2}$, $\dfrac{1}{\sqrt{a^2 - x^2}}$ or $\dfrac{1}{x^2 + a^2}$, and are required to recognise the notations: $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and the concepts of their principal values. The integrands containing $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ are not required in the curriculum.

Students should be able to use integration by parts to find indefinite integrals. Teachers can use $\int \ln x \, dx$ as an example to illustrate the method of integration by parts. It should be noted that in Module 2, the use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time
Calculus		
10. Definite integration	10.1 recognise the concept of definite integration 10.2 understand the properties of definite integrals 10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions 10.4 use integration by substitution to find definite integrals 10.5 use integration by parts to find definite integrals	10

Explanatory Notes:

Students are required to recognise the definite integral as the limit of a sum and find a definite integral from the definition. Students are required to recognise the notation: $\int_a^b f(x)\,dx$ and the concept of dummy variables, e.g. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. Using definite integration to find the sum to infinity of a sequence is not required in the curriculum.

Students are required to understand the following properties of definite integrals:

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ where k is a constant.
- $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_{-a}^a f(x)\,dx = 0$ if $f(x)$ is an odd function.

- $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ if $f(x)$ is an even function.

Teachers may discuss with students the geometric meanings of the above properties of definite integrals.

For the definite integral $\int_{-a}^{a} f(x)dx$, where $f(x)$ involves absolute values and is an odd or even function, students should be able to apply the properties of definite integrals of odd and even functions to have the result such as:

$\int_{-3}^{3} x|x|\,dx = 0$ for $y = x|x|$ is an odd function.

Finding other definite integrals whose integrands involve absolute values is not required in the curriculum.

When using integration by substitution to find definite integrals, students should be able to change the upper limit and the lower limit of the definite integral correspondingly.

Students are required to recognise the Fundamental Theorem of Calculus and through the theorem to recognise the relationship between definite integral and indefinite integral:

$$\int_{a}^{b} f(x)\,dx = F(b) - F(a), \text{ where } \frac{d}{dx}F(x) = f(x).$$

Teachers may also introduce the proof of the Fundamental Theorem of Calculus.

Students should be able to use integration by parts to find definite integrals but the use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time
Calculus		
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure 11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	4

Explanatory Notes:

In this Learning Unit, the applications of definite integration only confine to the calculations of areas of plane figures and volumes of solids of revolution. Teachers may give geometric demonstration on the relationship between the definite integral and the area of a plane figure.

Students are required to understand and should be able to use disc method to find the volumes of solids of revolution such as finding the volume by revolving the region about the *y*-axis if the region is bounded by the curves $y=\frac{x^2}{2}$ and $y=e^{-x^2}$, where $1 \le x \le 2$. The shell method is not required in the curriculum.

For the appreciation of the applications of definite integration, teachers may guide students to derive the formulae of the area of circle, the volume of right circular cone and the volume of sphere by using definite integration.

Algebra

Algebra consists of Determinants, Matrices, Systems of Linear Equations, and Vectors.

Students are required to understand the concepts, operations and properties of matrices, the existence of inverse matrices and the determinants. Determinants are important tools to investigate the properties of matrices.

Students learnt to solve the simultaneous linear equations in two unknowns by algebraic and graphical method at Key Stage 3. In Module 2, students are required to recognise the concepts of consistency and inconsistency and to further explore the conditions of consistency or inconsistency in a system of linear equations. They should be able to use Cramer's rule, inverse matrices and Gaussian elimination to solve systems of linear equations. Teachers should guide students to know the strengths and weaknesses of each method and how to choose appropriate methods to solve problems.

In order to extend students' knowledge in Algebra, the concepts, operations and properties of vectors should be included. The scalar product and the vector product are two useful tools to investigate the geometric properties of vectors including parallelism and orthogonality. In addition, students should be able to use the vector method to find the angle between two vectors and the area of a triangle or a parallelogram, etc.

Learning Unit	Learning Objective	Time
Algebra		
12. Determinants	12.1 recognise the concept of determinants of order 2 and order 3	2

Explanatory Notes:

Determinant is the important pre-requisite knowledge for the learning of matrices and systems of linear equations in the subsequent two Learning Units.

Teachers should emphasise that in Module 2 determinant is mainly used to find the inverse of a square matrix and to solve system of linear equations.

Students are required to recognise the definitions of determinants of order 2 and order 3, such as:

- $$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

- $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

- $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

- Use $\begin{array}{cccccc} & & & - & - & - \\ a_{11} & a_{12} & a_{13} & a_{11} & a_{12} & \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} & \\ & & & + & + & + \end{array}$ to denote the determinant of order 3 as follows:

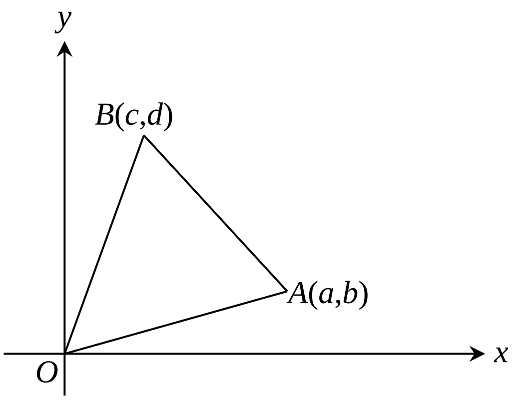
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

Teachers may explain to students that the above 3 definitions of a determinant of order 3 are the same.

Students are required to recognise that both $|A|$ and det A are two common notations of the determinant of the matrix A.

Teachers may introduce some geometric uses of determinants, for example:

In the figure, OAB is a triangle where O is the origin, $A=(a,b)$, $B=(c,d)$ and O, A and B are arranged in anticlockwise direction.



Area of the triangle $OAB = \frac{1}{2}\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

The properties of determinants are not required in the curriculum.

Learning Unit	Learning Objective	Time
Algebra		
13. Matrices	13.1 understand the concept, operations and properties of matrices 13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3	10

Explanatory Notes:

Students are required to understand the general form of a matrix with *m* rows and *n* columns, namely " $m \times n$ matrix". Students should be able to perform addition, subtraction, scalar multiplication and multiplication of matrices and understand the following properties:

- $A+B=B+A$
- $A+(B+C)=(A+B)+C$
- $(\lambda+\mu)A=\lambda A+\mu A$
- $\lambda(A+B)=\lambda A+\lambda B$
- $A(BC)=(AB)C$
- $A(B+C)=AB+AC$
- $(A+B)C=AC+BC$
- $(\lambda A)(\mu B)=(\lambda\mu)AB$
- $|AB|=|A||B|$

Students are required to understand that the commutative property does not hold for matrix multiplication , i.e. AB is not necessarily equal to BA.

The general proof of $|AB|=|A||B|$ where *A* and *B* are square matrices of order *n* is not required in the curriculum. However, teachers may have more in-depth discussions on this property with students for determinants of order 2 since the proofs are simpler.

Studens are required to recognise the terms, "zero matrix", "identity matrix", "transpose of a matrix" and "square matrix" and to understand the concepts, operations and the following properties of inverse of square matrices of order 2 and order 3:

- the inverse of A is unique
- $(A^{-1})^{-1} = A$
- $(\lambda A)^{-1} = \lambda^{-1} A^{-1}$
- $(A^{n})^{-1} = (A^{-1})^{n}$
- $(A^{T})^{-1} = (A^{-1})^{T}$
- $\left|A^{-1}\right| = \left|A\right|^{-1}$
- $(AB)^{-1} = B^{-1}A^{-1}$

where A and B are invertible matrices and λ is a non-zero scalar.

Students should be able to determine whether a square matrix is invertible and to find the inverse of an invertible matrix, e.g. by using the adjoint matrix and using elementary row operations. In addition, in some circumstances, students may need to use mathematical induction to prove propositions involving matrices.

In order to determine whether a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, students may consider to solve the matrix equation $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for unknowns x, y, z and w.

Learning Unit	Learning Objective	Time
Algebra		
14. Systems of linear equations	14.1 solve the systems of linear equations in two and three variables by Cramer's rule, inverse matrices and Gaussian elimination	6

Explanatory Notes:

At Key Stage 3, students learnt using the algebraic and graphical methods to solve linear equations in two unknowns. In this Learning Unit, students should be able to use Cramer's rule, inverse matrices and Gaussian elimination to solve systems of linear equations in two and three variables, and are required to recognise the terms "homogeneous", "non-homogeneous", "consistency" and "inconsistency".

Cramer's rule is an important topic of determinants. Students have to recognise that by Cramer's rule, for the system of linear equations $A\mathbf{x} = \mathbf{b}$, if Δ is the determinant of the coefficient matrix and $\Delta \neq 0$, the system has a unique solution. If $\Delta = 0$, Cramer's rule cannot be used. Teachers may discuss with students the logical relation between $A\mathbf{x} = \mathbf{b}$ and

$$\Delta \cdot x = \Delta_x ,\ \Delta \cdot y = \Delta_y \quad \text{and} \quad \Delta \cdot z = \Delta_z \ (*).$$

For example:
Should any solution in $A\mathbf{x} = \mathbf{b}$ be a solution in (*)?
Should any solution in (*) be a solution in $A\mathbf{x} = \mathbf{b}$?

Δ_x is the determinant obtained by replacing the first column of the coefficient matrix by the column vector $\mathbf{b}$, Δ_y is the determinant obtained by replacing the second column of the coefficient matrix by the column vector $\mathbf{b}$ and Δ_z is the determinant obtained by replacing the third column of the coefficient matrix by the column vector $\mathbf{b}$. Besides, students are required to recognise the following conclusions:

Case	Condition	Conclusion
1	$\Delta \neq 0$	The system has a unique solution.
2	$\Delta = 0$ and at least one of Δ_x , Δ_y or Δ_z $\neq 0$	The system has no solutions.
3	$\Delta = 0$ and $\Delta_x = \Delta_y = \Delta_z = 0$	The system has no solutions or infinitely many solutions.

In Case 1, the system has a unique solution and $x=\dfrac{\Delta_x}{\Delta}$, $y=\dfrac{\Delta_y}{\Delta}$, $z=\dfrac{\Delta_z}{\Delta}$.

In Case 2, as the given condition contradicts (*), the system has no solutions.
In Case 3, teachers may use the following examples to illustrate that the systems have no solutions or infinitely many solutions.

$$\begin{cases} x+y+z=1 \\ x+y+z=2 \\ x+y+z=3 \end{cases} \quad \text{(no solutions)} \qquad \begin{cases} x+y+z=3 \\ 2x+2y+2z=6 \\ 3x+3y+3z=9 \end{cases} \quad \text{(infinitely many solutions)}$$

Matrix is another important tool for solving systems of linear equations. With the knowledge of Learning Unit 13, students should be able to rewrite a system of linear equations in matrix form. If the inverse of the coefficient matrix exists, the system can be solved by using the inverse matrix. Students are required to recognise that this method becomes invalid if the inverse matrix does not exist.

Students should also be able to solve systems of linear equations by using Gaussian elimination. By setting up the augmented matrix, elementary row operations can be applied to solve systems of linear equations.

Teachers may demonstrate the linkage between matrices, determinants and elementary row operations in solving systems of linear equations.

Students are required to understand the theorem: a system of homogeneous linear equations has nontrivial solutions if and only if the coefficient matrix is singular. Teachers can use some simple systems of homogeneous linear equations in two variables to guide students to discover this theorem. Students are also required to understand that the systems of homogeneous linear equations in two and three variables are always consistent and know the way to find their nontrivial solution if the coefficient matrices are singular.

Learning Unit	Learning Objective	Time
Algebra		
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars 15.2 understand the operations and properties of vectors 15.3 understand the representation of a vector in the rectangular coordinate system	5

Explanatory Notes:

In this Learning Unit, teachers should emphasise that the magnitude and direction are two key concepts of vectors. Teachers should explain to students the difference between vectors and scalars. All vectors are restricted to $\mathbf{R}^2$ or $\mathbf{R}^3$ in the discussion of the vector properties. Students are required to understand the concepts of zero vector, unit vectors, equal vectors and negative vectors.

Students are required to recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$) ; and some notations for magnitude (including $|\mathbf{a}|$ and $|\vec{a}|$).

Students are required to understand the concepts of addition, subtraction and scalar multiplication of vectors, and the following properties of vectors:

- $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
- $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$
- $\mathbf{a}+\mathbf{0}=\mathbf{a}$
- $0\mathbf{a}=\mathbf{0}$
- $\lambda(\mu\mathbf{a})=(\lambda\mu)\mathbf{a}$
- $(\lambda+\mu)\mathbf{a}=\lambda\mathbf{a}+\mu\mathbf{a}$
- $\lambda(\mathbf{a}+\mathbf{b})=\lambda\mathbf{a}+\lambda\mathbf{b}$
- If $\alpha\mathbf{a}+\beta\mathbf{b}=\alpha_1\mathbf{a}+\beta_1\mathbf{b}$ (**a** and **b** are non-zero and are not parallel to each other), then $\alpha=\alpha_1$ and $\beta=\beta_1$.

Teachers may use the representation of vectors in the rectangular coordinate system to discuss the above properties of vectors with students.

Teachers should introduce the vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ representing the unit vectors in the directions of the positive x-, y- and z-axis respectively. Student should be able to use the form $x\mathbf{i}+y\mathbf{j}$ and $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ to express any vector in $\mathbf{R}^2$ and $\mathbf{R}^3$ respectively.
Students are required to understand the following formulae:

1. $|\overrightarrow{OP}|=\sqrt{x^2+y^2}$ when $\overrightarrow{OP}=x\mathbf{i}+y\mathbf{j}$ in $\mathbf{R}^2$.

2. $\sin\theta=\dfrac{y}{\sqrt{x^2+y^2}}$ and $\cos\theta=\dfrac{x}{\sqrt{x^2+y^2}}$ when θ is the angle that a non-zero vector $\overrightarrow{OP}$ makes with the positive x-axis and $\overrightarrow{OP}=x\mathbf{i}+y\mathbf{j}$.

3. $|\overrightarrow{OP}|=\sqrt{x^2+y^2+z^2}$ when $\overrightarrow{OP}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ in $\mathbf{R}^3$.

The concept of direction cosines is not required in the curriculum.

Learning Unit	Learning Objective	Time
Algebra		
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors 16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$	5

Explanatory Notes:

Students are required to understand that the definition of scalar product of two vectors $\mathbf{a}$ and $\mathbf{b}$ and its properties:

- $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$
- $\mathbf{a}\cdot(\lambda\mathbf{b})=\lambda(\mathbf{a}\cdot\mathbf{b})$
- $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$
- $\mathbf{a}\cdot\mathbf{a}=|\mathbf{a}|^2\geq 0$
- $\mathbf{a}\cdot\mathbf{a}=0$ if and only if $\mathbf{a}=\mathbf{0}$
- $|\mathbf{a}||\mathbf{b}|\geq|\mathbf{a}\cdot\mathbf{b}|$
- $|\mathbf{a}-\mathbf{b}|^2=|\mathbf{a}|^2+|\mathbf{b}|^2-2(\mathbf{a}\cdot\mathbf{b})$

Teachers may adopt one of the following definitions to introduce the vector product:

(1) For any non-zero and non-parallel vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbf{R}^3$,

$\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ ($0°<\theta<180°$), $\hat{\mathbf{n}}$ is the unit vector orthogonal (perpendicular) to both $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{a}$, $\mathbf{b}$, $\mathbf{a}\times\mathbf{b}$ follow the right-hand rule.

Otherwise, $\mathbf{a}\times\mathbf{b}=\mathbf{0}$.

(2) For vectors $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$,

$$\mathbf{a}\times\mathbf{b}=(a_2b_3-a_3b_2)\mathbf{i}+(a_3b_1-a_1b_3)\mathbf{j}+(a_1b_2-a_2b_1)\mathbf{k}.$$

Students are required to understand the determinant form of the vector product:

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Students are required to understand the following properties of the vector product:

- $\mathbf{a}\times\mathbf{a} = \mathbf{0}$
- $\mathbf{b}\times\mathbf{a} = -(\mathbf{a}\times\mathbf{b})$
- $(\mathbf{a}+\mathbf{b})\times\mathbf{c} = \mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$
- $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$
- $(\lambda\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(\lambda\mathbf{b}) = \lambda(\mathbf{a}\times\mathbf{b})$
- $|\mathbf{a}\times\mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2$

Teachers should remind students that in Learning Objective 16.2 all vectors concerned should be in $\mathbf{R}^3$.

Teachers should discuss the geometric meanings of the scalar product and the vector product with students in this Learning Unit and should emphasise the geometric applications of the scalar product and the vector product in Learning Unit 17.

The definition and properties of scalar triple product and the term “parallelepiped” are not required in the curriculum.

Learning Unit	Learning Objective	Time
Algebra		
17. Applications of vectors	17.1 understand the applications of vectors	6

Explanatory Notes:

At Key Stage 3, students learnt the conditions for two lines to be parallel and perpendicular in the rectangular coordinate system. In this Learning Unit, students should be able to use the properties of vectors to solve problems related to parallelism and orthogonality.

For examples, if $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors,

1. $\mathbf{a} = \lambda\mathbf{b}$ where λ is a real number if and only if $\mathbf{a}$ and $\mathbf{b}$ are parallel.
2. $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.
3. $\mathbf{a} \times \mathbf{b} = 0$ if and only if $\mathbf{a}$ and $\mathbf{b}$ are parallel.

Students should also be able to apply concepts of vectors to solve the problems related to the division of a line segment and the projection of a vector onto another vector. In addition, students should be able to find the angle between two vectors and the area of a triangle or a parallelogram by means of scalar product and vector product respectively.

Learning Unit	Learning Objective	Time
Further Learning Unit		
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units. In other words, this is not an independent and isolated Learning Unit and the activities may be conducted in different stages of a lesson, such as motivation, development, consolidation or assessment.

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CDC Committee on Mathematics Education

CDC-HKEAA Committee on Mathematics Education

Ad Hoc Committee on Secondary Mathematics Curriculum (Extended Part/Elective of Senior Secondary)

CDC-HKEAA Working Group on Senior Secondary Mathematics Curriculum (Module 2)

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