Exemplar 5: Constructing Squares

Learning Dimension: Measures, Shape and Space

Learning Unit: Quadrilaterals

Learning Stage: 3

Objective: To construct a square using information technology by applying different properties of a square

Prerequisite Knowledge:
(i) Properties of special quadrilaterals such as parallelograms, rectangles, squares, rhombuses, etc.
(ii) Basic concepts of transformation such as translation, reflection and rotation
(iii) Basic construction skills in using dynamic geometry software

Teaching Resources: Dynamic Geometry Software such as *Geometer's Sketchpad* (later referred as *Sketchpad*)

Description of the Activity:

1. The teacher introduces the activity to students and revises some basic construction skills in using dynamic geometric software such as constructing parallel lines, perpendicular lines, circles, etc.

2. The teacher discusses with students the method below to construct a square (See Fig. 1).

   Method 1:
   (i) Construct a line segment *AB*.
   (ii) Construct two perpendicular lines L_1 and L_2 through its end points *A* and *B*.
   (iii) Mark a point *P* on L_1.
   (iv) Construct the angle bisector of angle *PAB*. This angle bisector will intersect L_2 at point *C*.
   (v) Construct a line through *C* and perpendicular to L_2. This line intersects L_1 at point *D*. A square *ABCD* is then obtained.

3. The teacher then asks students whether the figure constructed is a square. Students are required to justify their assertion with reasons.

4. Students are guided to observe that not all the properties of a square are needed in the construction and to recall the idea of minimal condition in fixing a square.

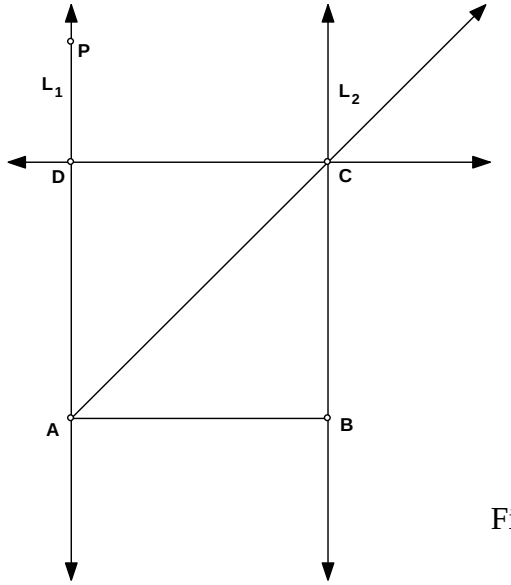

Fig. 1

5. Students are asked to construct their own squares using the properties of a square. For less able students, the teacher may give the hints below (Fig. 2 and Fig. 3):

Method 2:

(i) Construct a line segment *AB*.
(ii) Construct a circle centred at *A* with radius *AB*.
(iii) Construct a line through *A* and perpendicular to *AB*.
(iv) Mark the intersection of the circle and this line as point *C*.
(v) Construct a line through *C* and parallel to *AB*.
(vi) Construct a line through *B* and parallel to *AC*.
(vii) The point of intersection of the lines drawn in steps (v) and (vi) is labeled as *D*. Then *ABDC* is a square.

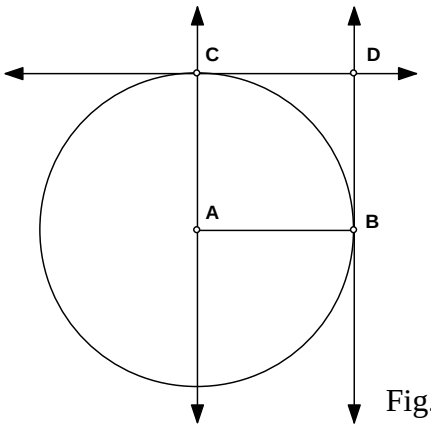

Fig. 2

Method 3:

(i) Construct a line segment *AB*.

(ii) Rotate the line segment by 90° in the anti-clockwise direction about point *A*.

(iii) Mark the end point of the resulting segment as *C*.

(iv) Rotate *AC* by 90° in the anti-clockwise direction about point *C*.

(v) Mark the end point of the resulting segment as *D*.

(vi) Rotate *CD* by 90° in the anti-clockwise direction about point *D*. Then *ABDC* is a square.

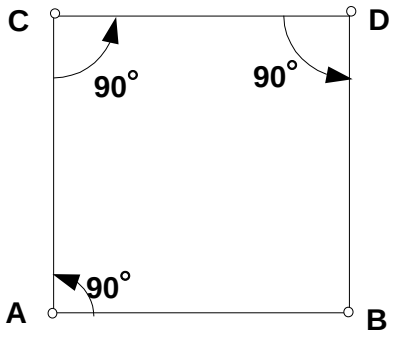

Fig. 3

6. Students are invited to demonstrate the steps in their constructions and give the geometrical explanations lying behind the construction of their squares. Other students may comment on the methods of construction and discuss which one is most efficient (i.e. which one involves the least number of steps).

7. The teacher then discusses with students the properties used in the construction of each square. Students are requested to write down the properties adopted in the constructions as homework.

Notes for Teachers:

1. This problem is a very interesting open-ended problem though the problem is embedded in the computer environment. Students may use different methods to construct a simple figure — a square. The computer also provides the environment for students to play freely with the properties of a square. More time should be allowed for students to explore the construction with the software. If time is not sufficient, the teacher can let students try the constructions at home or use the computer facilities in school after lessons, and discuss in the lessons.
2. Properties of a square used in each suggested construction procedure are listed in the table.

Method	Properties of square	Remarks
1	Two consecutive angles are right angles and the diagonals bisect opposite angles	These 2 methods are similar to the strategies of using straight edges and compasses in the construction of a square.
2	Opposite sides are parallel; adjacent sides are equal and perpendicular to each other	
3	All four angles are right angles and all four sides are equal	This method illustrates the application of transformation in the construction process by using the reflectional and rotational symmetries of a square.

This exemplar mainly involves the following generic skills:

1. Information Technology Skills
   - Use the dynamic geometry software to explore methods in the construction of a square

2. Numeracy Skills
   - Use the properties of a square in constructing the shape
   - Identify whether the shape constructed is a square or other form of quadrilateral

3. Creativity
   - Create one's own method in constructing squares

4. Problem-solving Skills
   - Plan and adjust their own strategy to construct squares, for example, using the transformation skill or other properties
   - Make use of a given tool to finish a task
   - Choose relevant information such as the minimal conditions of a square to construct the required shape