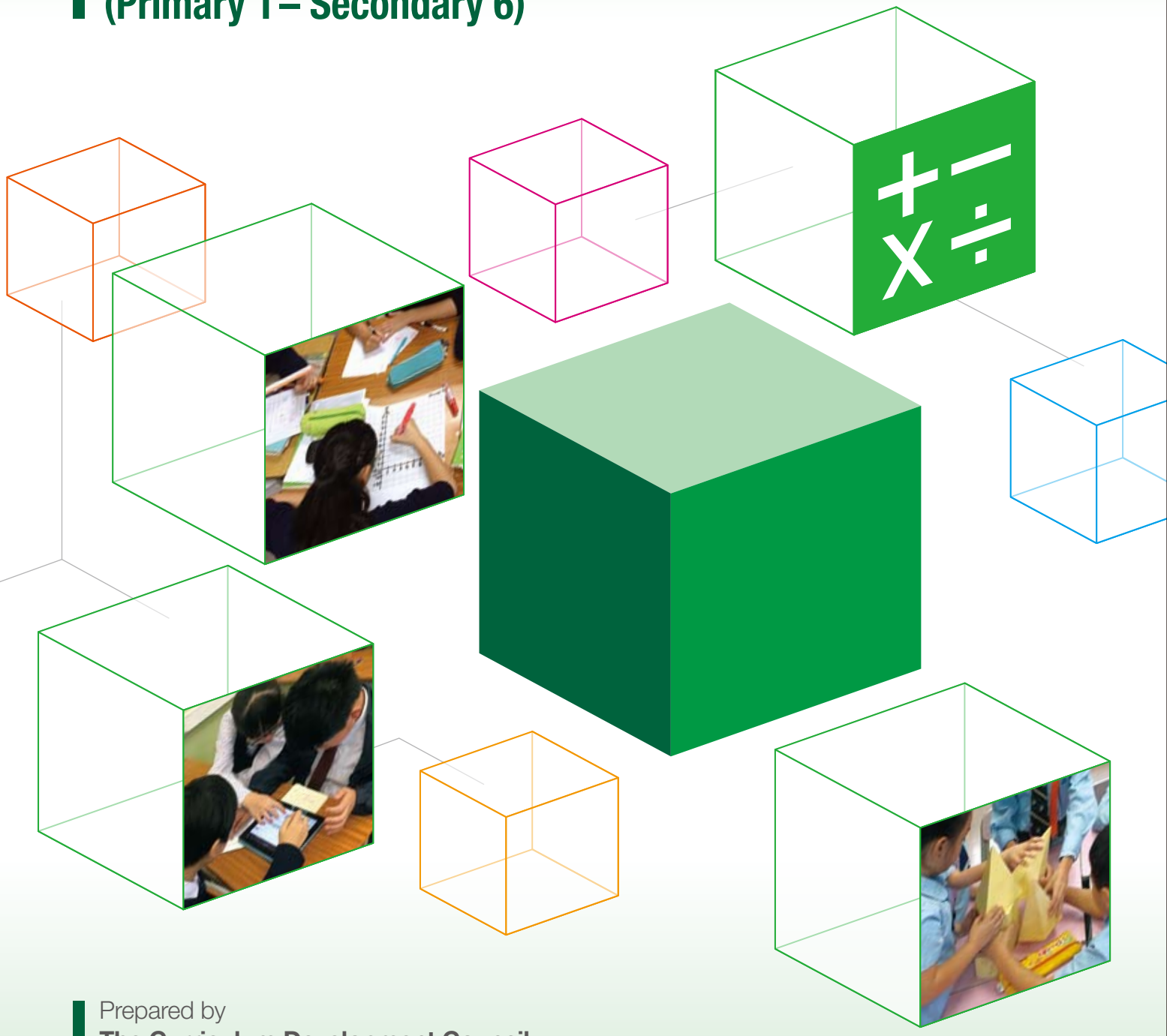
Mathematics Education

Key Learning Area Curriculum Guide (Primary 1 – Secondary 6)



Prepared by
The Curriculum Development Council

Recommended for use in schools by
The Education Bureau
HKSARG
2017

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(Primary 1 – Secondary 6)

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Preambles

The development of the Hong Kong school curriculum has advanced into a new phase of ongoing renewal and updating. It ushers in a new era for curriculum development to keep abreast of the macro and dynamic changes in various aspects in the local, regional and global landscapes in maintaining the competitiveness of Hong Kong. For the ultimate benefits of our students, schools are encouraged to sustain and deepen the accomplishments achieved since the Learning to Learn curriculum reform started in 2001, and to place new emphases on future needs in curriculum development for achieving the overall aims and learning goals of the school curriculum.

The eight Key Learning Area (KLA) Curriculum Guides (Primary 1 - Secondary 6) have been updated and recommended by the Curriculum Development Council (CDC)[1] to support the ongoing renewal of the school curriculum at the primary and secondary levels.

In updating the KLA Curriculum Guides, the respective KLA committees under the CDC have taken into consideration the concerns, needs and suggestions of various key stakeholders including schools, principals, teachers, students and the public at large. A series of school briefing cum feedback collection sessions coupled with a territory-wide school survey were conducted in 2015 to gauge schools' views on the major updates of the respective Curriculum Guides.

The eight KLA Curriculum Guides (2017) supersede the 2002 versions. Each KLA Curriculum Guide presents the updated curriculum framework which specifies the KLA's curriculum aims, learning targets and objectives, delineates the direction of ongoing curriculum development at the KLA level, and provides suggestions on curriculum planning, learning and teaching strategies, assessment, as well as useful learning and teaching resources. In addition, updated examples of effective learning, teaching and assessment practices are provided for schools' reference. Supplements to some KLA Curriculum Guides and subject curriculum guides are also available to provide further suggestions on their implementation at specific key stages. Schools are encouraged to adopt the recommendations in the KLA Curriculum Guides, taking into account the school contexts, teachers' readiness and learning needs of their students.

[1] The CDC is an advisory body offering recommendations to the Government on all matters relating to school curriculum development from kindergarten to secondary levels. Its membership includes heads of schools, teachers, parents, employers, academics from tertiary institutions, professionals from related fields or related bodies, representatives from the Hong Kong Examinations and Assessment Authority (HKEAA), and officers from the Education Bureau.

For a better understanding of the interface between various key stages and connections of different learning areas, and how effective learning, teaching and assessment can be achieved, schools should make reference to all related curriculum documents recommended by the CDC and the latest versions of the Curriculum and Assessment Guides jointly prepared by the CDC and the HKEAA for the senior secondary curriculum to ensure coherence in curriculum planning at the school, KLA and subject levels.

As curriculum development is a collaborative and ongoing process, the KLA Curriculum Guides will be under regular review and updating in light of schools' implementation experiences as well as the changing needs of students and society.

Views and suggestions on the development of the Mathematics Education KLA Curriculum are always welcome. These may be sent to:

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Key Messages

Mathematics Education KLA

Mathematics is essential in the school curriculum as it is a crucial mode of thinking that helps students acquire the ability to explore, conjecture and reason logically, a powerful means of communication, a foundation for the study of other disciplines, and an intellectual endeavor. Mathematics therefore plays an important role in helping students develop necessary skills for lifelong learning.

The Direction of Curriculum Development in Mathematics

In response to the changing needs of society, the rapid development of science and technology, the results of international studies on our education system, as well as views of stakeholders, the Mathematics Education KLA curriculum is developed in a direction to extend the existing strengths, to enhance students' learning progression and to align with the focal points of ongoing renewal of school curriculum. The focal points that connect with the development of the Mathematics Education KLA include STEM education, information technology in education, Language across the Curriculum, etc.

Aims of Mathematics Curriculum

To develop students' ability to conceptualise inquire, reason, communicate, formulate and solve problems mathematically; and their capability of appreciating the aesthetic nature and cultural aspects of mathematics.

The Central Curriculum of Mathematics: An Open and Flexible Framework

The central curriculum, in the form of an open and flexible framework, sets out what schools are encouraged to help students develop:

- Subject knowledge and skills as embodied in the learning units under different strands or areas;
- Generic Skills; and
- Positive values and attitudes.

Planning School-based Mathematics Curriculum

- Taking curriculum documents of the Mathematics Education KLA as major references

- Taking into account the school contexts, the overall aims of the Mathematics curriculum and the focal points and major renewed emphases (MRE) of the ongoing curriculum renewal (such as STEM education and information technology in education)
- Making use of the flexibility provided by the curriculum framework to cater for learner diversity, to enhance learning progression, and to plan learning and teaching sequences that facilitate cross-KLA learning activities
- Adopting appropriate learning and teaching resources, such as textbooks, e-resources and community resources

Learning and teaching of Mathematics

- Arranging diversified learning activities at different levels, such as hands-on exploratory activities, project work, mathematics reading activities, and activities that based on a topic in Mathematics to integrate relevant learning elements from other KLAs
- Incorporating the use of information technology for interactive learning and self-directed learning
- Adopting different strategies to cater for learner diversity, such as adapting the Mathematics curriculum and using the curriculum space created flexibly for consolidation and enrichment
- Assigning quality homework to consolidate learning, and discouraging mechanical drilling

Assessment

- Arranging assessments to collect ongoing information about the progress of student learning to provide timely and quality feedback for students to improve learning, and for teachers to adjust their teaching strategies
- Providing diversified modes of assessment (such as classroom observation, questioning, open-ended questions, exploratory tasks and projects) for improving learning and teaching
- Making use of suitable assessment tools, such as Learning Progression Framework (LPF) and Student Assessment Repository (STAR) to facilities assessment for learning and assessment as learning

(For more information on various curriculum matters, please refer to *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6) (*2014) and *Secondary Education Curriculum Guide* (2017).)

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Chapter 1
Introduction

Chapter 1 Introduction

In response to the changing needs of society, the rapid development of science and technology, the views of stakeholders collected through various surveys and engagement activities as well as the need to align with the direction for the ongoing curriculum renewal of the school curriculum, the recommendations provided in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3)* (2002) have been reviewed. Building on the strengths of Hong Kong students in mathematics, the curriculum content of the Mathematics Education Key Learning Area (ME KLA) have been updated to enhance students' learning progression and to align with the focal points of the ongoing renewal of the school curriculum, such as Science, Technology, Engineering and Mathematics (STEM) education and IT in education, for further enhancing the quality and effectiveness of learning, hence enabling students to become more effective lifelong learners in the 21st century.

The ***Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6)*** (2017) (this Guide) is prepared by the Curriculum Development Council Committee (CDCC) on Mathematics Education. It is an updated version of the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3)* (2002) and has been extended to include the three-year senior secondary Mathematics Education curriculum to provide reference for schools in developing a coherent school Mathematics curriculum.

The direction of development in this Guide aligns with the Seven Learning Goals of Primary and Secondary Education (see Appendix 1) and the major recommendations in the ***Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)*** (2014) and the ***Secondary Education Curriculum Guide*** (2017).

This Guide provides the overall direction for the development of the Mathematics Education curriculum in the five to ten years to come. It reinforces the curriculum emphases provided in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3)* (2002) to further enhance learning and teaching and supports the focal points and major renewed emphases (MRE) of the ongoing renewal of school curriculum which take into account the significant development in our society and around the world in various fields, and for the ultimate benefits of student learning. This Guide includes examples relevant to different key stages of learning to illustrate the concepts and ideas introduced and to narrow the gap in curriculum implementation.

1.1 What is a Key Learning Area?

A Key Learning Area (KLA) is an important part of a curriculum. It is founded on **fundamental and connected concepts within major fields of knowledge** which should be acquired by all students. A KLA provides a context for the development and application of generic skills (e.g. communication, collaboration skills and creativity) and subject-specific skills as well as positive values and attitudes through appropriate use of learning and teaching activities and strategies. It serves as a context for the construction of new knowledge and the development of understanding. The studies offered in each KLA may have an academic, social or practical orientation or a combination of these, depending on their purpose(s). They can be organised into subjects, modules, units, tasks or other modes of learning.

The three interconnected components of the curriculum framework, i.e. Knowledge in Key Learning Areas, Generic Skills, and Values and Attitudes, can be represented in Figure 1.

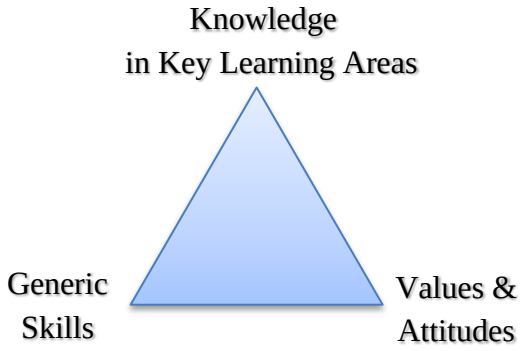

Figure 1

1.2 Position of the Mathematics Education KLA in the School Curriculum

Students require knowledge and skills that will help them meet the dynamic challenges in the 21st century, which is a knowledge-based information era driven by technology and creativity. Knowledge of mathematics is a necessity for every individual if they are to contribute towards the development and prosperity of their society. Mathematics and its applications pervade all aspects of life in the modern world. Many of the developments and decisions made in industry and commerce,

the provision of social and community services as well as government policy and planning etc., rely on the use of mathematics.

The Mathematics Education KLA is essential in the Hong Kong school curriculum as it is:

(a) *a powerful means for developing various abilities in a technology-oriented and information-rich society* – It helps students acquire the ability to communicate, explore, conjecture, reason logically and solve problems using a variety of methods.

(b) *a powerful means of communication* – It can be used to present information in many ways (e.g. figures, tables, charts, graphs and symbols) which can be processed to generate further information. The presentation skills help students lay a strong foundation for lifelong learning and acquire new knowledge in this rapidly changing world.

(c) *a tool for studying other disciplines* – It helps students enhance their understanding of the world and provides a basis as well as a foundation for studying other disciplines.

(d) *an intellectual endeavour and a mode of thinking* – It is a creative activity in which students can be fully involved and through which students can demonstrate their imagination, initiative and flexibility of mind.

(e) *a discipline, through which students can develop their ability to appreciate the beauty of nature, manage uncertainty and make sound judgements* – Mathematical experiences acquired in school enable students to become mathematically literate citizens and contribute towards social prosperity.

Being one of the KLAs that play a more active role in promoting STEM education, the Mathematics Education curriculum provides students with a solid knowledge base in mathematics. It also strengthens students' ability to integrate and apply the knowledge and skills of STEM-related subjects. As an integral part of general education, mathematics education supports the learning of other subjects. It contributes significantly to the whole-person development of students in primary and secondary schools, prepares them for multiple pathways to post-secondary education and future careers, and hence plays an important role in the Hong Kong school curriculum.

1.3 Rationale and Direction for Development

1.3.1 Rationale for the Development of the Mathematics Education KLA

Before the Learning to Learn Curriculum Reform

In July 1997, an ad hoc committee was set up by the Curriculum Development Council (CDC) to carry out a review of the Hong Kong Mathematics curriculum. Following two research studies[2] conducted in 1998, the ad hoc committee recommended in its final report (January 2000) that the Mathematics curriculum should be designed according to a set of content-based strands, the learning of abstract mathematical concepts should be backed up by adequate prior experience of manipulating concrete objects and an abundance of examples, and thinking skills should be developed through mathematical activities. These recommendations had been incorporated into the Mathematics curriculum as stipulated in the *Mathematics Curriculum Guide (P1 – P6)* (2000) and *Syllabuses for Secondary School – Mathematics (Secondary 1 – 5)* (1999).

Learning to Learn Curriculum Reform and Implementation of the New Academic Structure

In 2001, the Learning to Learn curriculum reform was launched to promote a curriculum and pedagogical change at the basic education level to help students become lifelong learners capable of meeting the challenges of a knowledge-based and changing society, globalisation and a competitive economy. In 2005, the report on *The New Academic Structure for Senior Secondary Education and Higher Education – Action Plan for Investing in the Future of Hong Kong* proposed a 3-year senior secondary and 4-year undergraduate academic system. A more flexible, coherent and diversified senior secondary curriculum was implemented at Secondary 4 in 2009. The curriculum and assessment reform at the senior secondary level under the New Academic Structure (NAS) was regarded as an extension of the curriculum reform at the basic education level. The *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)* (2007) provided details on the learning, teaching and assessment of the senior secondary Mathematics curriculum (SSMC) under the NAS.

[2] The two research studies were: (1) *Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries* conducted by The University of Hong Kong; and (2) *An Analysis of the Views of Various Sectors on the Mathematics Curriculum* conducted by The Chinese University of Hong Kong.

Review of the Senior Secondary Mathematics Curriculum under the New Academic Structure

There have been reviews of the SSMC in different aspects since its implementation in 2009. The first review was conducted in 2011 and in response to the views collected from the stakeholders, there was fine-tuning of the content and time allocation of the SSMC. Then a medium term review was conducted in the 2014/15 school year to solicit views of the Mathematics panel heads and teachers on the initial recommendations on the updating of the SSMC at the subject level. Teachers' views on the curriculum framework of the SSMC were also collected through a questionnaire survey and focus group interviews.

Ongoing Renewal of the School Curriculum

Alongside the implementation of the Learning to Learn curriculum reform, there have been a lot of changes and challenges in society and around the world, including those observed in economic, scientific, technological and social developments. To maintain Hong Kong's competitive edge and to prepare students well for the local and global changes taking place in various fields, it is necessary to enhance the Learning to Learn curriculum reform, to sustain and deepen the accomplishments achieved and to identify new focuses in the curriculum as we move to a new phase of curriculum renewal and updating.

In late 2015, a school survey was carried out to collect schools' views on the promotion of STEM education and the updating of the Mathematics Education KLA curriculum (P1 - S6). The results of the survey indicated schools' support for the promotion of STEM education as a key emphasis in curriculum development, the adoption of e-learning for effective learning and teaching, and the enhancement of the vertical continuity and lateral coherence of the Mathematics Education curriculum.

Stepping into the new phase of the ongoing curriculum renewal and following up the results of the NAS review in 2014/15, three ad hoc committees were set up under the CDCC on Mathematics Education in late 2015 to carry out a review of the Mathematics Education curriculum from P1 to S6 for updating purposes with due regard to the results of the previous school surveys. Following the completion of draft revised learning content of the Mathematics curriculum (P1 - S6) in late 2016, a multi-channel public consultation on the revised curriculum content and the curriculum framework of senior secondary Mathematics was conducted in 2016/17 school year. Views from different stakeholders, including school principals,

secondary career masters/mistresses, primary and secondary Mathematics panel heads and teachers, academics of universities and IVE, professional bodies and HKEAA were collected through focus group meetings, curriculum development visits, consultation forums and a school questionnaire survey. The revised Mathematics curriculum (P1 - S6) introduced in this Guide and its supplement was endorsed by CDC in 2017 after full consideration of the results of the public consultation.

The updated direction and strategies for the development of the Mathematics Education KLA are introduced in the following sections while the framework and content of the revised curriculum are introduced in the next chapter.

1.3.2 Direction for the Development of the Mathematics Education KLA

In face of the continual local and global changes in various fields, the rapid development of technology, the views of stakeholders, the results of international assessments (e.g. the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS)) which shed light on mathematics education in Hong Kong, as well as the direction for ongoing curriculum renewal, the recommendations provided in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 3)* (2002) on planning and implementing the school Mathematics curriculum are revisited. The following focal points of curriculum renewal are put forth for primary and secondary schools to incorporate into the school Mathematics curriculum to accommodate students' learning needs arising from the changing contexts and education trends.

(a) Strengthening students' ability to integrate and apply knowledge and skills through STEM education;

(b) Highlighting the importance of e-learning for enhancing learning and teaching effectiveness, facilitating self-directed learning and nurturing students' competence in applying information technology (IT) in learning mathematics;

(c) Highlighting Language across the Curriculum (LaC) in the school Mathematics curriculum, such as promoting reading in Mathematics to develop students' understanding of the connections between mathematics and

real life as well as other disciplines; and.

(d) Strengthening the development of generic skills and positive values and attitudes in an integrative manner through various Mathematics learning activities.

The above focal points are also some of the Major Renewed Emphases (MRE) introduced in Booklet 2 of the *Secondary Education Curriculum Guide* (2017). Besides, the development of the Mathematics curriculum is targeted at enhancing students' learning progressions through the updating of curriculum content. It is also important for the development to be based on the existing strengths.

1.4 Strategies for Development

In the ongoing renewal of the Mathematics curriculum, schools could build on their existing strengths, deepen and sustain the accomplishments achieved and identify new areas to focus on, to foster students' capabilities for whole-person development and lifelong learning. Schools are encouraged to take into account the suggestions and focal points set out in Sections 1.3.2 when planning the school Mathematics curriculum. Schools may select those relevant to their needs, set priorities and integrate them into the school curriculum. The following table summarises schools' existing strengths and suggested strategies for development to facilitate the ongoing curriculum renewal.

Existing Strengths	Suggested Strategies for Development
• Schools agree with the aims of the Mathematics Education KLA curriculum which cover the development of knowledge, generic skills, and positive values and attitudes. • Both students and parents show high regard for the Mathematics subject.	• Developing the school curriculum continuously by identifying areas for focusing, deepening and sustaining • Participating in research and development activities (such as "Seed" projects) to further develop the school Mathematics curriculum for enhancing students' whole-person development

Existing Strengths	Suggested Strategies for Development
	• Enhancing students' interest and confidence in learning mathematics through various means, such as hands-on activities, STEM activities, mathematics reading and effective use of IT
• As revealed in international assessments (e.g. PISA and TIMSS), Hong Kong students' performance in mathematics has been ranked among the top four of the participating countries/ regions in the past decade. • The proportion of the top performing students is increasing in general, as revealed in TIMSS 1995, 1999, 2003, 2007 and 2011.	• Sustaining the existing good practices in learning, teaching and assessment • Providing students with more opportunities and tools to apply mathematics in problem solving • Enhancing the design of learning and teaching materials and assessment tasks to cater for students' diverse abilities, e.g. providing diversified e-learning resources for students to work at their own pace and receive timely feedback.
• Most teachers support that STEM education is a focal point of the ongoing renewal of the school curriculum.	• Providing STEM learning activities in the best interest of students and within their abilities, e.g. through activities based on topics in the Mathematics curriculum and project work
• Most teachers support the incorporation of information technology for effective learning, teaching and assessment.	• Using IT in a well-integrated, pedagogically sound and effective way for the learning and teaching of mathematics • Applying IT to facilitate students' discussion and understanding of abstract concepts

Existing Strengths	Suggested Strategies for Development
	• Supporting students' self-directed learning by providing suitable e-resources and developing their e-learning strategies
• Mathematics teachers are usually professionally trained. • Teachers welcome the in-service training provided by the EDB and other professional bodies.	• Arranging teachers' professional development on the focal points of the ongoing curriculum renewal, such as promoting STEM education and IT in education • Encouraging teachers to participate in collaborative research and development projects or community of practice for sharing of good practices among schools • Extending teachers' understanding of the Mathematics curricula across different key stages for enhancing vertical continuity
• Teachers show high regard for assessing students' ability through formative and summative assessments. • Schools are making use of the internal and external assessment results to review and adjust their school curriculum and the learning and teaching strategies adopted.	• Adopting diversified modes of assessment, such as hands-on tasks, open-ended questions and problem-based tasks to assess students' diverse abilities • Deepening the use of more informative feedback to promote assessment as learning, in addition to assessment for learning, to help students set goals, and monitor, reflect on and evaluate their own learning

Existing Strengths	Suggested Strategies for Development
• Most schools have taken school-based measures to embrace learner diversity, e.g. organising remedial classes for students who are weak in mathematics. • There is flexibility in the curriculum design. Enrichment elements (e.g. Enrichment Topics in the primary and junior secondary Mathematics curricula) and Further Learning Units are available in the Mathematics curriculum.	• Offering enhancement/support measures to both ends of the student population • Planning a school curriculum which is adjusted and adapted to meet the needs of both less able students and more able students • Making use of the flexibility provided by the Mathematics Education curriculum to embrace learner diversity and adopt diversified learning and teaching strategies

Although the focal points of the ongoing curriculum renewal suggested in the previous section are not unfamiliar to schools, they are highlighted and supported through the following measures:

- Professional development programmes on the focal points under different categories including curriculum planning, learning and teaching and knowledge enrichment organised for curriculum leaders and teachers;
- Collaborative research and development projects and study groups organised for initialising, promoting, sustaining and improving the incorporation of the focal points in the learning, teaching and assessment of mathematics (e.g. STEM education and IT in education); and
- Others, such as provision of resource packages, newsletters and reading materials.

Suggestions on curriculum planning, learning and teaching, assessment and learning and teaching resources, and more elaboration on the focal points of ongoing curriculum renewal mentioned in this chapter are discussed in Chapters 3 to 6.

Chapter 2
Curriculum Framework

Learning to Learn 2+ — The Hong Kong School Curriculum

A broad and balanced curriculum with diversification and specialisations (choices) for academic, professional and vocational development according to students' needs

Nurturing lifelong & self-directed learning capabilities

Multiple pathways

Fostering whole-person development

SEVEN LEARNING GOALS

FIVE ESSENTIAL LEARNING EXPERIENCES

Moral and Civic Education | Intellectual Development | Community Service | Physical and Aesthetic Development | Career-related Experiences

Life-wide Learning

Secondary 4-6

SS

Core Subjects
Chinese Language
English Language
Mathematics
Liberal Studies

+

Electives
20 Elective Subjects
Applied Learning
Other Languages

+

Other Learning Experiences
Moral and Civic Education
Aesthetic Development
Physical Development
Community Service
Career-related Experiences

Secondary 1-3

JS

Four Key Tasks: Towards major renewed emphases (MRE) at the JS level and beyond
STEM education & ITE, Values education (incl. MCE & Basic Law education), Language across the Curriculum (incl. reading), etc.

Primary 1-6

P

Chinese Language Education	English Language Education	Mathematics Education	Science Education	Technology Education	Personal, Social & Humanities Education	Arts Education	Physical Education
Key Learning Area	Key Learning Area	Key Learning Area	Key Learning Area	Key Learning Area	Key Learning Area	Key Learning Area	Key Learning Area

General Studies

Kindergarten 1-3

KG

Values & attitudes, Skills and Knowledge

Language | Early Childhood Mathematics | Nature & Living | Self & Society | Arts & Creativity | Physical Fitness & Health

Values & attitudes

Seven priority values
- Perseverance
- Respect for Others
- Responsibility
- National Identity
- Commitment
- Integrity
- Care for Others

Generic skills

Basic Skills
- Communication Skills
- Mathematical Skills
- IT Skills

Thinking Skills
- Critical Thinking Skills
- Creativity
- Problem Solving Skills

Personal & Social Skills
- Self-management Skills
- Self-learning Skills
- Collaboration Skills

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Education Bureau
2017

Chapter 2 Curriculum Framework

2.1 Aims of the Mathematics Education KLA Curriculum

Students need mathematics to meet the dynamic challenges of their future studies, careers or daily life in an information-rich society with rapid development in technology. The overall aims of the Mathematics Education KLA curriculum are to develop in students:

(a) the ability to think critically and creatively, to conceptualise, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines;

(b) the ability to communicate with others, express their views clearly and logically in mathematical language;

(c) the ability to manipulate numbers, symbols and other mathematical objects;

(d) number sense, symbol sense, spatial sense, measurement sense and the capacity to appreciate structures and patterns; and

(e) a positive attitude towards mathematics learning and an appreciation of the aesthetic nature and cultural aspect of mathematics.

The focal points of curriculum renewal mentioned in the previous chapter, including the promotion of STEM education, Information Technology in Education and Language across the Curriculum, echo the aims of developing students' abilities to formulate and solve problems in daily life and other disciplines, ability to communicate with others clearly and logically. The development of Generic Skills and positive values and attitudes, being a continuous emphasis, is also a part of the curriculum aims.

2.2 Components of the Curriculum Framework

The curriculum framework for Mathematics Education KLA is the overall structure for organising the learning, teaching and assessment of Mathematics together with curriculum management, leadership and planning to achieve the overall aims and

learning targets of the Mathematics Education KLA.

Figure 2 shows a diagrammatic representation of the Mathematics Education curriculum framework. The central part of the framework comprises a set of interlocking components including subject **knowledge** organised under strands, **generic skills**, and **values and attitudes**, which sets out what students should learn and develop in the Mathematics Education KLA.

Curriculum management, leadership and planning, as well as effective learning, teaching and assessment of Mathematics involve not only the central part, but also the learning needs of students in the contemporary contexts, including the development of students' abilities in using language and information technology for learning. As the learning of Mathematics is also connected with other KLAs/subjects, one of the main concerns is integrating and applying knowledge and skills in different subjects, especially in promoting STEM education. Further, effective use of resources and partnerships between schools, the EDB and other organisations also lead to successful implementation of the Mathematics Education curriculum.

Further details on the knowledge, generic skills and values and attitudes of the Mathematics curriculum are given below.

2.2.1 Strands[3], Learning Targets and Objectives

Strands are categories of mathematical knowledge and concepts for organising the curriculum. Their main function is to organise mathematical contents for the purpose of developing knowledge, generic skills, and values and attitudes as a holistic process. There are basically three strands in the Mathematics Education curriculum, namely "Number and Algebra", "Measures, Shape and Space" and "Data Handling". At the primary level, these three strands are subdivided into five strands (see Figure 2).

To ensure meaningful and effective learning, there must be a coherent plan for students' learning at the primary and secondary levels. The learning targets and

[3] The term "Strands" has been refered to as "Dimensions" in earlier curriculum documents such as the *Mathematics Curriculum Guide (P1 – P6)* (2000) and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5)* (1999).

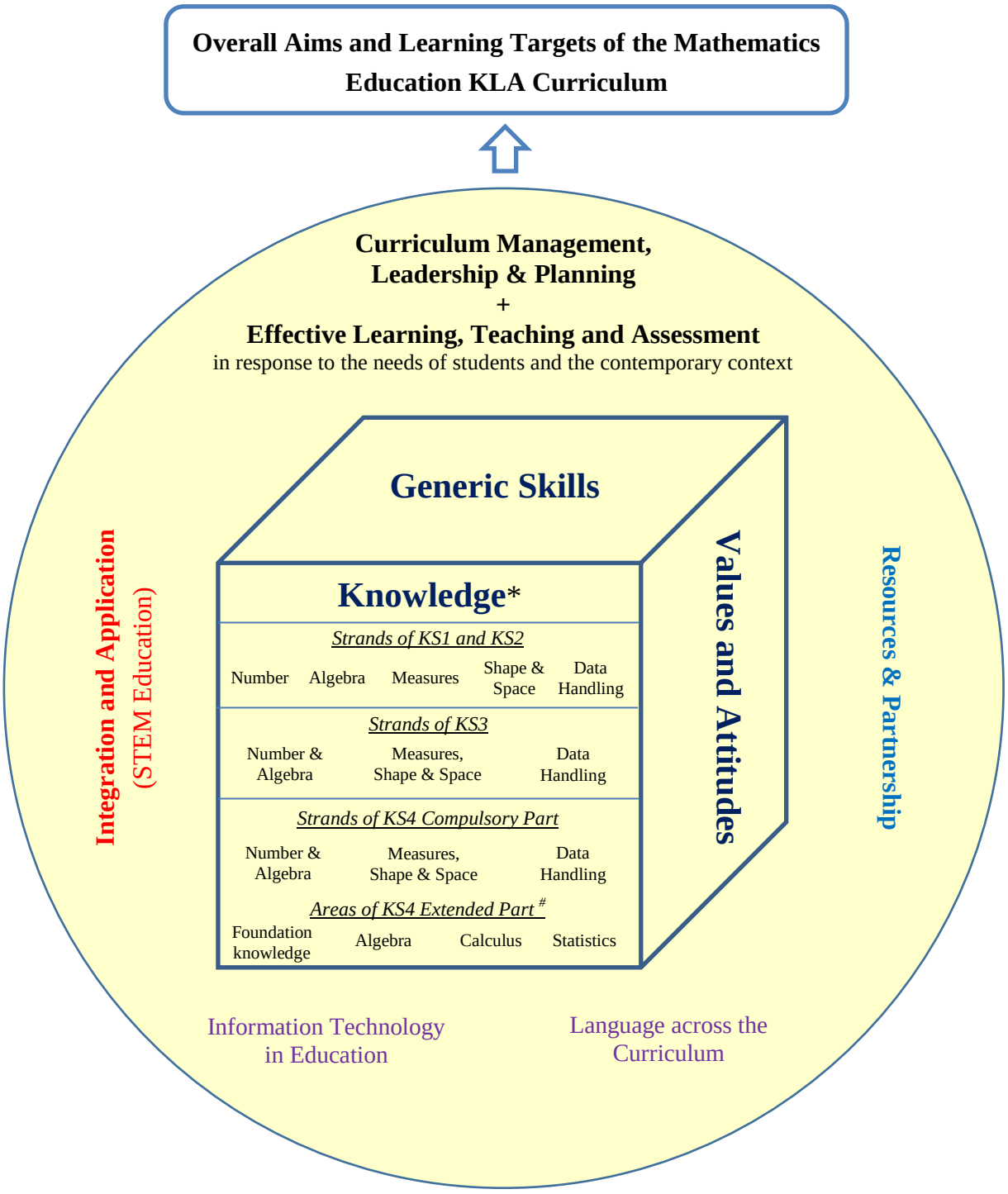

Figure 2 Diagrammatic Representation of the Mathematics Education Curriculum Framework

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* Flexibility in curriculum content is provided by the setting of foundation topics, non-foundation topics and enrichment topics (see Section 2.3 and 2.4 for details). A Further Learning Unit is also included in each key stage.

\# Module 1 of the Extended Part consists of "Foundation knowledge", "Calculus" and "Statistics" and Module 2 consists of "Foundation knowledge", "Algebra" and "Calculus".

objectives, which are geared towards the overall aims of the Mathematics Education curriculum, are organised progressively and systematically across Key Stage 1 (Primary 1 - 3), Key Stage 2 (Primary 4 - 6), Key Stage 3 (Secondary 1 - 3) and Key Stage 4 (Secondary 4 - 6).

As the content of the Extended Part of the senior secondary Mathematics curriculum are interwoven, they are not organised under strands but grouped under the areas of "Foundation knowledge", "Algebra", "Calculus" and "Statistics". Module 1 of the Extended Part consists of "Foundation knowledge", "Calculus" and "Statistics" while Module 2 consists of "Foundation knowledge", "Algebra" and "Calculus". At each key stage, in addition to the curriculum content in the strands and areas, a Further Learning Unit is designed to enhance students' ability to inquire, reason and conceptualise mathematical concepts and to allow students to integrate and apply knowledge and skills learned in different strands and areas.

The learning objectives of each strand or area are grouped and presented under different learning units which could, on one hand, reflect the relationship between learning content of similar nature, and on the other hand, enable teachers and students to relate the content in different units.

There are updates in the learning content of the Mathematics Education curriculum at different key stages in order to align with the direction of the development of mathematics education and the ongoing renewal of school curriculum as mentioned in Section 1.3.2 in Chapter 1 of this Guide. The main purposes of the updates include:

- to enhance the interface of the curriculum across key stages;
- to enhance the support to other subjects;
- to enhance the organisation of curriculum content for betterment of learning and teaching; and
- to provide more specific descriptions on the depth and breath of curriculum content.

The updated learning targets and learning units of the Mathematics curriculum for KS1 to KS4 are shown in the tables on the next pages. Readers may refer to the supplements of this guide for details of the learning content, including the learning objectives under each learning unit of the primary, junior secondary and senior secondary Mathematics curricula.

Overview of Learning Targets

Learning Targets of Primary Mathematics Curriculum (P1 – P3)			
Number Strand	**Measures Strand**	**Shape and Space Strand**	**Data Handling Strand**
Students are expected to:			
• recognise the concepts of whole numbers* and simple fractions; • recognise and use the commutative and associative properties of addition and multiplication; • perform four arithmetic operations of whole numbers and addition and subtraction of simple fractions, and check the reasonableness of results; and • use numbers to formulate and solve simple problems.	• recognise the concepts of length, distance, weight and capacity; • use different ways to compare the length, weight, capacity of objects and distance between objects, and record the results; • understand the need for using standard units of measurements; • choose and use appropriate measuring tools and standard units to compare the length, weight, capacity of objects and distance between objects, and record the results; • estimate the result of measurements; • recognise money, time and date, and their use in daily life; and • integrate the knowledge in the strands of Number, Measures, Shape and Space to solve simple problems.	• identify intuitively and describe 2-D and 3-D shapes; • recognise the properties of points and lines, and the concept of faces of 3-D shapes; • recognise the concepts of right angles, acute angles and obtuse angles; • recognise the concepts of perpendicular and parallel; • recognise the concepts and properties of squares, rectangles, parallelograms and trapeziums; • recognise the inclusion relations between parallelograms and squares, parallelograms and rectangles; • recognise the inclusion relations between different types of triangles; • make 2-D shapes and appreciate the beauty of geometric shapes; and • describe the relative position of different objects and recognise the four directions.	• recognise the importance of the organisation and representation of statistical data; • collect and group statistical data according to given criteria; • use appropriate scales to construct simple statistical charts and interpret them; and • formulate and solve simple problems arising from statistical data or statistical charts.

* In the primary Mathematics curriculum, “whole numbers” refers to non-negative integers.

Learning Targets of Primary Mathematics Curriculum (P4 – P6)				
Number Strand	**Algebra Strand**	**Measures Strand**	**Shape and Space Strand**	**Data Handling Strand**
Students are expected to:				
• recognise and use the distributive property of multiplication; • recognise the concepts of prime numbers and composite numbers; • understand the concepts of the highest common factors and the least common multiples; • understand the concepts of whole numbers, fractions, decimals, percentages and the relations among them; • perform four arithmetic operations of whole numbers, fractions and decimals, and check the reasonableness of results; and • use numbers to formulate and solve problems.	• use symbols to represent numbers; • use algebraic expressions to represent the operations of and relations between quantities that are described in words and involve unknown quantities; and • use algebra to formulate and solve simple problems and recognise how to check the reasonableness of results.	• recognise the concepts of perimeter, area, volume and speed; • use different ways to compare the perimeter and area of 2-D shapes, volume and speed of objects, and record the results; • choose appropriate standard units to measure and compare the perimeter and area of 2-D shapes, volume and speed of objects, and record the results; • use the measuring tool and the standard unit to measure, compare and draw angles of different sizes; • recognise the degree of accuracy of measurements; • estimate the result of measurements;	• recognise the concepts and properties of rhombuses and circles; • recognise the inclusion relations between different types of quadrilaterals; • recognise the concept of vertices and edges of 3-D shapes; • recognise the concept and property of sphere; • make 2-D shapes and 3-D shapes from given information and appreciate the beauty of geometric shapes; and • recognise the eight compass points.	• understand the criteria for organising and representing statistical data; • use approximate values and appropriate scales to construct statistical charts and interpret them; • recognise relations of data and patterns on the changes of data from statistical charts; • recognise the concept of average and solve problems; • formulate and solve problems arising from statistical data or statistical charts; • choose appropriate statistical charts to represent given data; and • judge the appropriateness of the representation of statistical charts.

Learning Targets of Primary Mathematics Curriculum (P4 – P6)				
Number Strand	**Algebra Strand**	**Measures Strand**	**Shape and Space Strand**	**Data Handling Strand**
Students are expected to:				
		• inquire and use measurements formulae of 2-D shapes and 3-D shapes; • recognise the relation between volume and capacity and solve problems; • perform the interconversion between units of time and solve problems related to time and speed；and • integrate the knowledge in the strands of Number, Measures, Shape and Space to formulate and solve problems.		

Learning Targets of Junior Secondary Mathematics Curriculum		
Number and Algebra Strand	**Measures, Shape and Space Strand**	**Data Handling Strand**
Students are expected to:		
• recognise the concepts of negative integers, negative rational numbers and irrational numbers; • further use numbers to formulate and solve problems; • investigate and describe relationships between quantities using algebraic symbols, including patterns of sequences of numbers; • interpret simple algebraic relations from numerical, symbolic and graphical perspectives; • manipulate simple algebraic expressions and relations; and apply the knowledge and skills to formulate and solve simple real-life problems and justify the validity of the results obtained; and • apply the knowledge and skills in the Number and Algebra strand to formulate and solve problems in other strands.	• recognise errors in measurement and apply the knowledge to solve problems; • extend concepts and formulae of measurements of 2-dimensional figures and 3-dimensional figures and apply the knowledge to solve problems; • explore and visualise the geometric properties of 2-dimensional figures and 3-dimensional figures; • use inductive and deductive approaches to study the properties of 2-dimensional rectilinear figures; • perform geometric proofs involving 2-dimensional rectilinear figures with appropriate symbols, terminology and reasons; • inquire and describe geometric knowledge in 2-dimensional space using algebraic relations and apply the knowledge to solve problems; • inquire and describe geometric knowledge in 2-dimensional space using trigonometric ratios and apply the knowledge to solve problems; and • apply the knowledge and skills in the Measures, Shape and Space strand to formulate and solve problems in other strands.	• recognise the methods of organising discrete and continuous statistical data; • further choose appropriate statistical charts to represent given data and interpret them; • understand the measures of central tendency; • select and use the measures of central tendency to describe and compare data sets; • investigate and judge the validity of arguments derived from data sets; • recognise the concept of probability and apply the knowledge to solve simple probability problems; and • integrate the knowledge in statistics and probability to solve simple real-life problems.

Learning Targets of the Compulsory Part of Senior Secondary Mathematics Curriculum		
Number and Algebra Strand	**Measures, Shape and Space Strand**	**Data Handling Strand**
Students are expected to:		
• extend the concepts of numbers to complex numbers; • further investigate and describe relationships between quantities using algebraic symbols; • generalise and describe patterns in sequences of numbers using algebraic symbols, and apply the results to solve problems; • interpret more complex algebraic relations from numerical, symbolic and graphical perspectives; • manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve more complex real-life problems and justify the validity of the results obtained; and • apply the knowledge and skills in the Number and Algebra strand to generalise, describe and communicate mathematical ideas and further solve problems in other strands.	• use inductive and deductive approaches to study the properties of 2-dimensional figures; • perform geometric proofs involving 2-dimensional figures with appropriate symbols, terminology and reasons; • further inquire and describe geometric knowledge in 2-dimensional space using algebraic relations and apply the knowledge to solve problems; • inquire and describe geometric knowledge in 2-dimensional space and 3-dimensional space using trigonometric functions and apply the knowledge to solve problems; and • apply the knowledge and skills in the Measures, Shape and Space strand to generalise, describe and communicate mathematical ideas and further solve problems in other strands.	• understand the measures of dispersion; • select and use the measures of central tendency and dispersion to describe and compare data sets; • further investigate and judge the validity of arguments derived from data sets; • acquire basic techniques in counting; • formulate and solve more complex probability problems by applying simple laws; and • integrate the knowledge in statistics and probability to solve more complex real-life problems.

Learning Targets of Module 1 (Calculus and Statistics) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	**Calculus**	**Statistics**
Students are expected to:		
• apply binomial expansion for the study of probability and statistics; • model, graph and apply exponential functions and logarithmic functions to solve problems; and • understand the relationships between exponential and logarithmic functions and apply the two functions to solve real-life problems.	• recognise the concept of limits as the basis of differential and integral calculus; • understand the idea of differentiation and integration through consideration of concrete phenomena; • find the derivatives, indefinite integrals and definite integrals of simple functions; and • apply the knowledge of calculus to solve real-life problems.	• understand the concepts of probability, random variables, and discrete and continuous probability distributions; • understand the fundamental ideas of statistical reasoning based on the binomial, Poisson and normal distributions; • use statistical reasoning and thinking to know when and how to apply statistical methods to make inferences and justify conclusions; and • develop the ability to think mathematically about uncertainty and then apply such knowledge and skills to solve problems.

Learning Targets of Module 2 (Algebra and Calculus) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	**Algebra**	**Calculus**
Students are expected to:		
• recognise odd and even functions and their graphs; • understand the principle of mathematical induction; • expand binomials using the binomial theorem; • understand simple trigonometric functions, important trigonometric identities and formulae involving compound angles; and • recognise *e*.	• understand the concepts, operations and properties of matrices and the inverses of square matrices up to order 3; • solve systems of linear equations; • understand the concept, operations and properties of vectors; and • apply the knowledge of vectors to solve problems in 2-dimensional space and 3-dimensional space.	• understand the concept of limits as the basis of differential and integral calculus; • understand the concepts and properties of derivatives, indefinite integrals and definite integrals of functions; • find the derivatives, indefinite integrals and definite integrals of simple functions; • find the second derivatives of functions; • apply the knowledge of calculus to sketch curves; and • apply the knowledge of calculus to solve real-life problems.

Overview of Learning Units

Learning Units of Primary Mathematics Curriculum (P1 – P3)			
Number Strand	**Measures Strand**	**Shape and Space Strand**	**Data Handling Strand**
1. Numbers to 20	16. Length and distance (I)	28. 3-D shapes (I)	37. Pictograms
2. Basic addition and subtraction	17. Money (I)	29. 2-D shapes	38. Bar charts (I)
3. Numbers to 100	18. Length and distance (II)	30. Directions and positions (I)	
4. Addition and subtraction (I)	19. Time (I)	31. Angles	
5. 3-digit numbers	20. Length and distance (III)	32. Directions and positions (II)	
6. Addition and subtraction (II)	21. Time (II)	33. Quadrilaterals (I)	
7. Basic multiplication	22. Money (II)	34. 3-D shapes (II)	
8. 4-digit numbers	23. Length and distance (IV)	35. Quadrilaterals (II)	
9. Addition and subtraction (III)	24. Time (III)	36. Triangles	
10. Basic division	25. Capacity		
11. 5-digit numbers	26. Time (IV)		
12. Multiplication (I)	27. Weight		
13. Division (I)			
14. Four arithmetic operations (I)			
15. Fractions (I)			
Further Learning Unit			
39. Inquiry and investigation			

Note: Learning units in the overview are not arranged in the order of teaching.

Learning Units of Primary Mathematics Curriculum (P4 – P6)				
Number Strand	**Algebra Strand**	**Measures Strand**	**Shape and Space Strand**	**Data Handling Strand**
1. Multiplication (II) 2. Division (II) 3. Multiples and factors 4. Common multiples and common factors 5. Four arithmetic operations (II) 6. Fractions (II) 7. Decimals (I) 8. Multi-digit numbers 9. Fractions (III) 10. Decimals (II) 11. Decimals (III) 12. Fractions (IV) 13. Fractions (V) 14. Decimals (IV) 15. Decimals (V) 16. Percentages (I) 17. Percentages (II)	18. Elementary algebra 19. Simple equations (I) 20. Simple equations (II)	21. Perimeter (I) 22. Area (I) 23. Area (II) 24. Volume (I) 25. Angle (degree) 26. Volume (II) 27. Perimeter (II) 28. Speed 29. Area (III)	30. Quadrilaterals (III) 31. Dissecting and forming shapes 32. Directions and positions (III) 33. Circles 34. 3-D shapes (III) 35. Symmetry	36. Bar charts (II) 37. Bar charts (III) 38. Averages 39. Broken line graphs 40. Pie charts 41. Uses and abuses of statistics
Further Learning Unit				
42. Inquiry and investigation				

Note: Learning units in the overview are not arranged in the order of teaching.

Learning Units of Junior Secondary Mathematics Curriculum		
Number and Algebra Strand	**Measures, Shape and Space Strand**	**Data Handling Strand**
1. Basic computation 2. Directed numbers 3. Approximate values and numerical estimation 4. Rational and irrational numbers 5. Using percentages 6. Rates, ratios and proportions 7. Algebraic expressions 8. Linear equations in one unknown 9. Linear equations in two unknowns 10. Laws of integral indices 11. Polynomials 12. Identities 13. Formulae 14. Linear inequalities in one unknown	15. Errors in measurement 16. Arc lengths and areas of sectors 17. 3-D figures 18. Mensuration 19. Angles and parallel lines 20. Polygons 21. Congruent triangles 22. Similar triangles 23. Quadrilaterals 24. Centres of triangles 25. Pythagoras' theorem 26. Rectangular coordinate system 27. Trigonometry	28. Organisation of data 29. Presentation of data 30. Measures of central tendency 31. Probability
Further Learning Unit		
32. Inquiry and investigation		

Note: Learning units in the overview are not arranged in the order of teaching.

Learning Units of the Compulsory Part of Senior Secondary Mathematics Curriculum		
Number and Algebra Strand	**Measures, Shape and Space Strand**	**Data Handling Strand**
1. Quadratic equations in one unknown 2. Functions and graphs 3. Exponential and logarithmic functions 4. More about polynomials 5. More about equations 6. Variations 7. Arithmetic and geometric sequences and their summations 8. Inequalities and linear programming 9. More about graphs of functions	10. Equations of straight lines 11. Basic properties of circles 12. Loci 13. Equations of circles 14. More about trigonometry	15. Permutations and combinations 16. More about probability 17. Measures of dispersion 18. Uses and abuses of statistics
Further Learning Unit		
19. Further applications 20. Inquiry and investigation		

Note: Learning units in the overview are not arranged in the order of teaching.

Learning Units of Module 1 (Calculus and Statistics) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	**Calculus**	**Statistics**
1. Binomial expansion 2. Exponential and logarithmic functions	3. Derivative of a function 4. Differentiation of a function 5. Second derivatives 6. Applications of differentiation 7. Indefinite integration and its applications 8. Definite integration and its applications 9. Approximation of definite integrals using the trapezoidal rule	10. Conditional probability and Bayes' theorem 11. Discrete random variables 12. Probability distribution, expectation and variance 13. The binomial distribution 14. The Poisson distribution 15. Applications of the binomial and the Poisson distributions 16. Basic definition and properties of the normal distribution 17. Standardisation of a normal variable and use of the standard normal table 18. Applications of the normal distribution 19. Sampling distribution and point estimates 20. Confidence interval for a population mean
Further Learning Unit		
21. Inquiry and investigation		

Note: Learning units in the overview are not arranged in the order of teaching.

Learning Units of Module 2 (Algebra and Calculus) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	**Calculus**	**Algebra**
1 Odd and even functions 2. Mathematical induction 3. The binomial theorem 4. More about trigonometric functions 5. Introduction to the number *e*	6. Limits 7. Differentiation 8. Applications of differentiation 9. Indefinite integration and its applications 10. Definite integration 11. Applications of definite integration	12. Determinants 13. Matrices 14. Systems of linear equations 15. Introduction to vectors 16. Scalar product and vector product 17. Applications of vectors
Further Learning Unit		
18. Inquiry and investigation		

Note: Learning units in the overview are not arranged in the order of teaching.

2.2.2 Generic Skills

Generic skills are fundamental in enabling students to learn how to learn. They can be seen as both process skills and learning outcomes in the Mathematics Education KLA. They are **developed through the learning and teaching of mathematics** and serve as a **means to help students acquire and master the mathematical knowledge and concepts.** It should be noted that generic skills are **not to be added to but embedded in the learning and teaching of mathematics.**

The following nine generic skills have been identified as essential for student learning for the 21st century in the school curriculum since 2001.

- Collaboration Skills
- Communication Skills
- Creativity
- Critical thinking Skills
- Information technology Skills
- Mathematical Skills
- Problem solving Skills
- Self-learning Skills
- Self-management Skills

"Mathematical skills" and "self-learning skills" have been referred to as "numeracy skills" and "study skills" respectively in earlier curriculum documents. "Mathematical Skills" replaces "Numeracy Skills" to represent a comprehensive set of revised skills in various areas such as measurement, logical thinking, describing patterns and data handling for applications in different disciplines. "Study Skills" is replaced by "Self-learning Skills" to align with the direction of nurturing self-directed lifelong learners.

Based on past experience of implementing the curriculum reform and in response to the dynamic changes in society and recent research, the nine generic skills are grouped in three clusters of related skills, namely Basic Skills, Thinking Skills and Personal and Social Skills, for better integrative understanding and application in a holistic manner (see the table below for details).

Basic Skills	Thinking Skills	Personal and Social Skills
Communication Skills	Critical Thinking Skills	Self-management Skills
Mathematical Skills	Creativity	Self-learning Skills
IT Skills	Problem Solving Skills	Collaboration Skills

Daily life applications and higher order thinking skills are emphasised in mathematics education at both the primary and secondary levels. The Mathematics Education curriculum provides numerous opportunities for the development of generic skills such as critical thinking skills, creativity and problem solving skills through problem solving activities in real life and in mathematical contexts.

Details of the nine generic skills and how the Mathematics Education KLA contributes to the development of these skills are available in Appendix 2 of this Guide.

When completing a more complicated learning task, the generic skills are often applied in an integrative manner rather than in isolation. Some generic skills are more likely to be used together. For example, collaboration skills are often used with communication skills and problem solving skills; critical thinking skills tend to be used with problem solving skills and creativity. The former cluster of generic skills can be referred to as “collaborative problem solving skills” and the latter as “holistic thinking skills”. Schools are encouraged to plan learning and teaching activities in a holistic manner whereby the clusters of skills can be effectively developed and applied through classroom learning experiences.

Further elaborations on “collaborative problem solving skills” and “holistic thinking skills” and examples on how the Mathematics Education KLA contributes to the development of these integrative uses of generic skills are provided in Appendix 3.

2.2.3 Values and Attitudes

Besides knowledge and generic skills, the development of positive values and attitudes is also important in mathematics education. According to the revised

Moral and Civic Education Curriculum Framework (2008), **seven priority values and attitudes** are identified to reflect the uniqueness of Hong Kong as an international city in which both Chinese and Western cultures and values co-exist and interact. They are **perseverance**, **respect for others**, **responsibility**, **national identity**, **commitment**, **integrity**, and **care for others**. They are of vital importance for students' whole-person development to meet their own needs as well as those of society.

In the Mathematics Education KLA, values education can be carried out through relevant topics and appropriate learning and teaching activities that help students apply and reflect on the priority values and attitudes, or other relevant ones, which permeate the curriculum in different key stages. The following objectives illustrate how mathematics learning is related to the development of positive values and attitudes and aim at facilitating the planning of relevant learning experiences in the Mathematics curriculum. These objectives, however, are neither exhaustive nor do they imply that the related values and attitudes should progress in the sequence presented. In fact, they can be realised at all key stages to different extents.

- Display **perseverance** in solving challenging mathematical problems.
- Show **respect for and acceptance to others** in seeking different solutions to a mathematical problem, or in comparing strategies for completing a mathematical project/task.
- Understand and take up one's **responsibilities** in group work and develop a sense of **commitment** by taking up different roles for completing group tasks.
- Foster a sense of **integrity** in discussing the misuse of statistics in different social contexts.
- **Think independently** in solving mathematical problems.
- Share ideas and experience, and work **co-operatively** with others in accomplishing mathematical tasks and solving mathematical problems.
- Be **open-minded**, willing to listen to others in the discussion of mathematical problems, respect others' opinions, and value and appreciate others' contributions.
- Develop **interest** in learning mathematics.
- Show **keenness** to participate in mathematical activities.
- Show **confidence** in applying mathematical knowledge in daily life, clarifying one's argument and challenging others' statements.

- **Appreciate** the preciseness, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

2.2.4 Developing Generic Skills, and Positive Values and Attitudes

Generic skills, values and attitudes are not developed in a vacuum. They are expected to be fostered through the learning of mathematical knowledge in the content areas. It is desirable for teachers to help students cultivate them through planned learning activities. Figure 3 illustrates how they intertwine to form a reference grid.

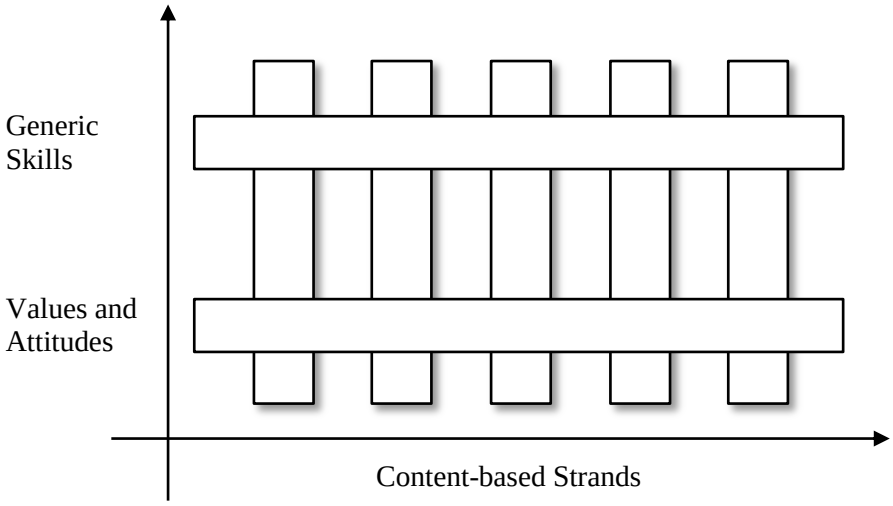

Figure 3

The Mathematics Education KLA provides meaningful contexts for the development of generic skills through appropriate learning and teaching activities and specific topics alongside the specific skills of mathematics. Schools should plan learning and teaching activities for the Mathematics curriculum in a holistic manner whereby individual or cluster of generic skills could be suitably and effectively applied and developed. Teachers are encouraged to

- embed elements of generic skills in the setting of the learning goals, as well as the design of the curriculum, learning activities and assessment plans for their schools;
- blend naturally the learning elements of generic skills with subject knowledge, values and attitudes;

- organise learning activities (such as projects, performance tasks and mini surveys) that require students to apply generic skills and reflect on their use of generic skills; and
- create authentic learning environments that bridge students' learning of these generic skills with real world issues.

Examples included in this Guide can illustrate how to link the learning targets and generic skills in learning and teaching. The following table summarises the main learning targets and generic skills from four examples (one from each key stage).

Examples	Objective(s)	Main generic skills applied / developed
Example 1 Hand in Hand (Key Stage 1)	• To understand even and odd numbers	• Communication • Critical thinking • Problem solving
Example 8 Knowing Your Community (Key Stage 2)	• To find the eight compass points with a compass • To become familiar with the district in the vicinity of the school • To sketch the plan of the district in the vicinity of the school; to be aware of the facilities and services offered in the community • To present the collected data with statistical charts	• Collaboration • Creativity • Critical thinking • Problem solving
Example 15 Slopes of Perpendicular Lines (Key Stage 3)	• To enhance the understanding of relation between slopes of perpendicular lines with the help of IT tools	• Information technology • Problem solving • Self-learning
Example 20 Modelling the Spread of a Disease (Key Stage 4)	• To let students recognise the mathematics in everyday life and apply information technology to solve problems • To let students recognise mathematics as a powerful tool for planning	• Critical thinking • Problem solving

Besides linking the learning targets and generic skills, positive values and attitudes can also be developed through different learning activities. Such integration comes naturally in the learning and teaching activities for the Mathematics Education KLA. Annotations on Example 8 are provided below for illustration.

In Example 8, the generic skills of collaboration skills, mathematical skills, critical thinking skills, problem solving skills and creativity are involved in the learning activities. While collecting data and constructing statistical charts in groups, students have to apply collaboration skills when they work out the roles of individual members and share the responsibilities. They also need to learn to respect others during discussion, be responsible and put effort into their part of the project and work cooperatively. Students formulate and solve the problems with the prudence of mathematics and using mathematical skills. Critical thinking and problem solving skills are involved when students are required to choose relevant information to solve the problems. Through the problem–solving journey, students' perseverance is enhanced. Their creativity is also developed and strengthened through the process of formulating solutions, presenting ideas in their own ways and suggesting ways of improving their solutions. Moreover, the activity enhances students' interest in learning mathematics and sensitivity towards the importance of mathematics in daily life.

2.3 Curriculum Organisation

Flexibility and diversification are provided in the Mathematics Education curriculum to help schools and teachers build on their strengths and adopt different modes of curriculum planning to meet different purposes of learning and teaching. At both primary and secondary levels, a "strand approach" is adopted with generic skills, values and attitudes incorporated into the content (see Figure 3 in Section 2.2.4) in the learning and teaching process.

Learning units of each strand are developed by key stages in both the primary and secondary curricula. Teachers are free to select the learning units among those suggested in the key stage concerned for planning their school Mathematics curriculum for each year level, provided that the units selected are organised logically and coherently. The same idea could be applied to the learning units for the areas in Module 1 and Module 2 of the senior secondary Mathematics curriculum.

Enrichment Topics are provided in the primary Mathematics curriculum and the junior secondary Mathematics curriculum, while Foundation Topics, Non-foundation Topics[4] are provided in the junior secondary Mathematics curriculum and the Compulsory Part of the senior secondary Mathematics curriculum to suit the different abilities of students. (see Section 2.4 for details)

Schools should make reference to the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and the *Secondary Education Curriculum Guide* (2017) for the suggested time allocation for the Mathematics Education KLA for different key stages, and exercise school-based lesson time planning. Sufficient lesson time should be allocated for Mathematics at all key stages to provide teachers with curriculum space to rearrange or to adapt the content and depth of the teaching materials, to incorporate e-learning strategies into the learning and teaching process, and to organise exploratory or cross-curricular activities based on topics in the Mathematics curriculum. There is lesson time reserved for the Further Learning Units in the curriculum for teachers to incorporate different learning activities in the lessons.

Teachers have the flexibility to design their school Mathematics curriculum to cater for the needs of their students. For example, teachers of Key Stage 3 are suggested to teach the topic *Linear Equations in One Unknown* at a lower level. After that, they are free to choose the teaching sequence for the following topics:

(a) *Linear Inequalities in One Unknown*

(b) *Formulae* followed by (c) *Linear Equations in Two Unknowns* or (d) *Identities*

The teaching sequence can be (a), (b), (c), (d) or (b), (a), (d), (c) or some other sequences that suits the needs and abilities of students (see Figure 4). In addition, teachers are giving a free hand to teach the combinations of topics anytime at the key stage. All the topics mentioned become the prerequisites for learning the related topics at Key Stage 4.

[4] The terms "Foundation Part" and "Non-foundation Part" have been used in the *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5)* (1999) instead.

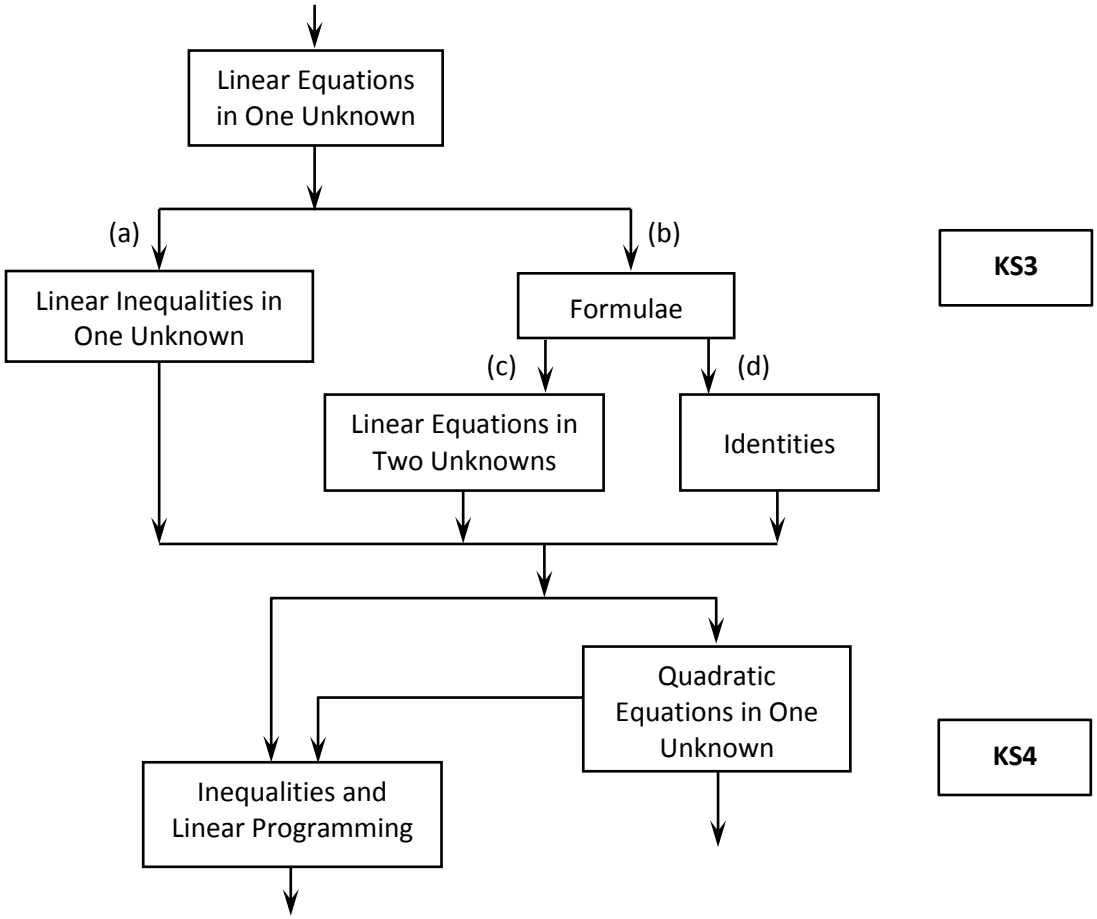

Figure 4

2.4 Core and Extension

Apart from meeting different purposes of learning and teaching, the issue of learner diversity has also been considered in the development of the Mathematics Education curriculum for primary and secondary schools.

The Foundation Topics of the junior secondary Mathematics curriculum and the Compulsory Part of the senior secondary Mathematics curriculum represent the topics that all students should strive to learn. They constitute a coherent set of essential concepts and knowledge, and are selected in accordance with the following principles in mind:

(a) to include basic concepts and knowledge necessary for the learning content in the Junior Secondary Mathematics Curriculum and the Compulsory Part, and for simple applications in real-life situations; and

(b) to cover topics from different areas to enable students to develop a coherent body of knowledge and to experience mathematics from an all-around perspective.

There are also contents beyond those in the Foundation Topics in terms of depth and breadth. They are identified as Non-foundation Topics and cover a wider range of contents. Teachers can judge for themselves the suitability and relevance of the Non-foundation Topics for their students at KS3 and KS4. On the other hand, no division between Foundation and Non-foundation Topics is made in the primary Mathematics curriculum as most of the concepts at the primary level are fundamental.

To cater for students who have different needs, interests and orientations, the senior secondary Mathematics curriculum includes an Extended Part in addition to the Compulsory Part. The Extended Part is designed for students who need more mathematical knowledge and skills for their future studies and careers. Two modules are offered for students to choose in the Extended Part. Module 1 emphasises applications rather than mathematical rigour and provides students with intuitive concepts of calculus and statistics. Module 2 emphasises understanding of mathematics for further progress in mathematically inclined disciplines and provides students with a concrete foundation in algebra and calculus. The contents of Module 1 and Module 2 are built upon the study of the Foundation and Non-foundation Topics in the Compulsory Part. It is advisable for students to study both the Foundation Topics and Non-foundation Topics in the Compulsory Part if they study the Extended Part.

For more able students at the primary and junior secondary levels, teachers can incorporate some of the suggested enrichment topics such as “curve stitching” at the primary level and “exploring the formula for external point of division” at the junior secondary level) into the curriculum at their discretion to extend students’ horizons and exposure in mathematics. Teachers may also adopt suitable enrichment topics other than those suggested in the curriculum to suit the interest and abilities of their students. At the senior secondary level, the learning unit “Further applications” is incorporated into the Compulsory Part to enable students to recognise and appreciate the interconnection between the different topics of the Mathematics curriculum they have learnt at both junior and senior secondary levels. It is also designed for students to integrate and apply knowledge and skills learned in the various strands to solve problems in real life as well as in mathematical

contexts. It allows teachers to plan different learning activities (such as cross-curricula activities based on mathematics topics) for students.

In addition, the further learning unit “Inquiry and investigation” is available in the Mathematics Education curriculum of all key stages. This provides students with learning space to engage in activities that facilitate the discovery and construction of knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying different Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling learning activities at Key Stages 2 to 4. This enables students to analyse and make decisions with mathematical thinking when encountering problems in different real-world contexts, and strengthens their ability to apply mathematics to solve STEAM and other real-world problems. For the arrangements regarding mathematical modelling learning activities and related recommendations on learning and teaching, please refer to the *Supplements to the Mathematics Education Key Learning Area Curriculum Guide* and the *Explanatory Notes to the Mathematics Curriculum* of the respective key stages.

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Chapter 3
Curriculum Planning

Chapter 3 Curriculum Planning

3.1 A Balanced Curriculum

A strand approach is adopted in designing the structure of the Mathematics Education curriculum. There are five strands at the primary level and three strands at the secondary (see Section 2.2.1 for details). The framework of strands from Key Stage 1 to Key Stage 4 enables the learning objectives and students' progress to be structured so that a balanced repertoire of mathematical knowledge and skills can be provided. Since students have different learning styles and abilities, a school Mathematics curriculum has to be designed to cater for their needs with reference to the school contexts. To ensure internal consistency and the provision of a balanced curriculum, opportunities should be provided to students to master all fundamental knowledge in each strand at each key stage. This is especially important at Key Stage 3 and Key Stage 4 when more flexibility is provided by the curriculum on the planning of topics under different strands for each year level. Care should also be taken to avoid over-emphasising a particular strand or allocating insufficient lesson time and resources to any strand.

Schools and teachers should consider the following in designing a school Mathematics curriculum: (a) overall aims of the central Mathematics Education curriculum; (b) focal points and major renewed emphases (MRE) of the ongoing curriculum renewal, e.g. STEM education and Information Technology in Education (ITE); (c) cognitive development of students; (d) learner diversity; (e) pedagogical strategies for deep learning; (f) interface between key stages; (g) lateral coherence with other subjects; (h) diversified modes of assessment; (i) provision of resources and support; and (j) students' multiple pathways for further studies and future careers. The following section provides some basic principles for planning and developing a school Mathematics curriculum.

3.2 Central Curriculum and School Curriculum Development

The central Mathematics Education curriculum is presented in the form of an open and flexible framework of learning content (including strands and learning units), generic skills, and values and attitudes, which all students are entitled to (see Sections 2.1 and 2.2 of this Guide for details). Schools may develop their school

Mathematics curriculum to suit their students' needs and abilities, and the school contexts through adapting the central Mathematics Education curriculum. Schools may consider varying the organisation of the following:

- content, contexts and examples;
- learning and teaching strategies;
- pace of learning and teaching; and
- modes of assessment.

Below are some basic principles for planning and developing a school Mathematics curriculum:

(a) Curriculum documents of the Mathematics Education KLA should be taken as major references on the curriculum aims, framework and content as well as allocation of lesson time.

(b) Schools are encouraged to set and work on clear and manageable aims and targets taking into account the school contexts, the overall aims of the Mathematics Education curriculum and the focal points/MRE of the ongoing curriculum renewal (such as STEM education and ITE).

(c) Schools could make use of the flexibility provided by the Mathematics curriculum framework to provide students (who may be streamed into different groups) with the curriculum content appropriate and relevant to their needs and abilities, for example,
   i) adapting the sequencing and depth of treatment of learning objectives;
   ii) planning the learning and teaching of Non-foundation Topics at the secondary level;
   iii) selecting some of the enrichment topics from the primary and junior secondary Mathematics curricula (or other suitable enrichment topics not in the curriculum) on the basis of students' interest and abilities, the time available to extend students' horizons and exposure to mathematics;
   iv) allowing suitable senior secondary students to take Module 1 or Module 2 in the Extended Part and arranging flexible lesson time across year levels for the two groups of students who are taking the Compulsory Part only or taking the Compulsory Part plus one module.

(d) Teachers could adopt different ways of sequencing the learning units for each year level or key stage, for example,

i) arranging the learning sequence so that students learn all strands spirally across year levels;

ii) reserving more Mathematics periods in S1 for students to revise and consolidate their learning in primary schools, and focusing only on the Foundation Topics of the curriculum in S2 and S3 to cater for students who learn at a slower pace (see Section 4.4.2 for more details on catering for learner diversity at the school level); and

iii) spending most of the lesson time allocated to Mathematics in S4 on teaching the Compulsory Part first until students are more mathematically mature and equipped with sufficient mathematical concepts and knowledge for the learning of the Extended Part.

(e) Before the start of a new school year, teachers could set and work on the learning activities to be carried out. Teachers are encouraged to provide more opportunities for primary and junior secondary students to engage in hands-on exploratory activities that help develop their abilities to inquire, communicate, reason and apply mathematical concepts. The inclusion of tasks or projects related to real-life problems would also provide opportunities for students to engage in deep learning.

(f) Teachers could make reference to the Learning Progression Framework (LPF) for planning their school Mathematics curriculum. The LPF provides a common language for teacher to describe students' learning progress in mathematics across primary and secondary levels. It consists of different levels of learning outcomes and descriptors about the growth of students on a continuum as they work towards the learning objective set out in the Mathematics curriculum (refer to Section 5.2.3 for more elaboration). The LPF for Mathematics is available at *http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/index.html* (under the links "Primary One to Six", "Secondary One to Three" and "Secondary Four to Six").

(g) Teachers could choose and adapt textbooks and other appropriate resources for the learning and teaching of Mathematics. Good management of different kinds of resources, including real models and e-resources, can facilitate the effective discharge of related learning and teaching strategies.

(h) Schools are encouraged to formulate the school assessment and homework policy that promotes assessment for learning and assessment as learning. Setting and working on the assessment strategies and modes for students at different key stages is essential for achieving the curriculum aims, including

nurturing students' capability for self-directed learning.

Curriculum development is an ongoing process. Schools should develop their own curriculum whenever appropriate and feasible. They should also encourage the professional development of teachers and collaboration with other stakeholders to achieve the curriculum aims, learning targets and objectives in the Mathematics Education curriculum framework.

School may refer to Chapter 2 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 2 of the *Secondary Education Curriculum Guide* (2017) for more ideas on whole-school curriculum planning.

3.3 Strengthening STEM Education

STEM is an acronym that refers collectively to the academic disciplines of Science, Technology, Engineering and Mathematics. The promotion of STEM education aligns with the worldwide education trend of equipping students to meet the changes and challenges brought by the rapid economic, scientific and technological developments in our society and around the world.

In the Hong Kong curriculum context, STEM education is promoted through the Science, Technology and Mathematics Education KLAs. The aims of promoting STEM education in schools are to strengthen the science, technology and mathematics education and to nurture versatile talents with different levels of knowledge and skills for enhancing the international competitiveness of Hong Kong. The objectives of promoting STEM education in schools include:

- to develop among students a solid knowledge base and to enhance their interests in science, technology and mathematics for further studies and careers in meeting the changes and challenges in the contemporary world; and
- to strengthen students' ability to integrate and apply knowledge and skills, and to nurture students' creativity, collaboration and problem solving skills, as well as to foster the innovation and entrepreneurial spirit as required in the 21st century

In STEM education, Mathematics serves as a discipline that equips students with knowledge and skills on algebra, geometry, data handling and logical reasoning that facilitate students to integrate and apply knowledge and skills across disciplines in solving real-life problems with practical solutions and innovative designs. Besides, mathematical modelling plays an important role in solving real-life problems, including those in science and technology contexts. STEM is not a new strand in the Mathematics Education curriculum, but it places more emphasis on applying mathematics in different contexts, with the integration of science and technology elements at different levels.

STEM education could be strengthened through creating opportunities for students to apply the mathematical knowledge and skills in analysing and modelling real-life problems that may or may not have a definite solution and in formulating solutions and eventually solving the problems.

On the other hand, real-life problems and the elements of science and technology can be useful examples and contexts for the learning of abstract mathematical concepts, and thus facilitate the learning of the subject knowledge for mathematics.

When designing a school Mathematics curriculum, it is desirable to provide all students with the opportunities to participate in STEM learning activities and let them experience integrating and applying their knowledge and skills of mathematics together with those of other STEM subjects. The following two approaches are recommended to organise STEM-related activities to serve this purpose:

- **Learning activities based on a topic in Mathematics for students to integrate relevant learning elements from other KLAs**

  In this approach, the design of the learning activity is based on a particular topic of the Mathematics curriculum. For example, in the topic "Laws of Integral Indices", students need to apply the index laws for handling numbers represented in the scientific notation. Related learning elements from other KLAs (such as representing and comparing sizes of cells using scientific notation) could be incorporated into the learning activities. Students could also be given problems with cross-KLA elements to assess their understanding of the related concepts and skills after the activities. There are a number of topics in the Mathematics curriculum under different strands which are more related to other STEM KLAs, e.g. "Ratios" under Number and Algebra, "Mensuration"

under Measures, Shape and Space, "Statistical Graphs" under Data Handling. In employing this approach to curriculum planning, teachers may consult colleagues of other STEM KLAs on students' prerequisite knowledge when they are designing the learning and teaching activities. More examples of topics and related STEM activities are available in Section 4.3 of Chapter 4.

- **Projects for students to integrate relevant learning elements from different KLAs**

  Projects on authentic problems provide opportunities for students to integrate the learning elements from different KLAs. In the course of conducting the project, students explore the issues in real-life contexts, and the relevant learning elements from the Science, Technology and Mathematics Education KLAs are brought in naturally by the students themselves. Teachers of different KLAs can jointly decide some themes of study for students at the curriculum planning stage. Students may start with these themes and try to tackle some related open problems. From the perspective of mathematics education, students have more opportunities to participate in mathematical modelling through identifying, formulating and solving the problem. Teachers of different KLAs will be the facilitators to support students in the different areas of knowledge and skills. A positive consequence of this approach is providing opportunities for teachers of these KLAs to collaborate and further enhance the effectiveness of learning and teaching. Some examples of implementing this approach are provided in Section 4.3 of Chapter 4.

Depending on the needs, interests and workload of students, schools can adopt one or both of the approaches, or other approaches as appropriate. In any cases, emphasis in learning and teaching should be shifted from routine calculations and drilling exercises to problem-based exploratory or mathematical modelling activities which encourage students to achieve deep learning.

The EDB will support schools in strengthening STEM education through: (i) renewing related curricula; (ii) strengthening the provision of quality learning experiences to students; (iii) developing KLA-based and cross-KLA resource materials; (iv) organising professional development programmes for curriculum leaders and teachers; (v) synchronising the contributions from different community key players; and (vi) reviewing the development of STEM education and consolidating good practices for dissemination.

3.4 Strengthening Information Technology in Education

With the rapid development of information technology (IT), the learning and teaching of mathematics can be greatly enhanced through careful planning and use of suitable strategies and e-resources. Major concerns for schools include: (i) the readiness of hardware infrastructure; (ii) the competencies of teachers and students in the basic operations of the commonly used mathematics education software packages; (iii) the necessity and advantages in using IT in the learning and teaching of different topics in the Mathematics curriculum; and (iv) the degree of integrating technology with pedagogy for enhancing the effectiveness of learning and teaching.

With the launching of the Fourth Strategy on Information Technology in Education (ITE4), it is anticipated that the IT infrastructure in schools will be updated and all classrooms will have smooth and stable Wi-Fi coverage to facilitate connection among electronic devices and with the Internet during lessons. Students and teachers can make use of appropriate resources such as cloud storage, cloud computing and e-platforms to promote interactive learning in Mathematics lessons. Some platforms provide interactive e-learning resources compatible with common dynamic geometry software and computer algebra systems. These learning and teaching resources and platforms can usually be used on both desktop computers and mobile computing devices. Due attention should be given to the system requirements of specific software packages for mathematics, especially those involve 3-dimensional simulations, in order to ensure a smooth flow of lessons.

Teachers may consider preparing learning and teaching materials for the Mathematics lessons through developing the resources themselves or customising some existing resources. In both cases, teachers need to have some understanding of the operations of the software packages to be used. Teachers can choose the appropriate software packages according to the nature of the learning activities, such as demonstration, simulation, mathematical experiment and exploration. With the aid of IT, the developed or customised e-resources could be shared amongst teachers within the school or even for inter-school collaboration. A collection of quality e-learning materials may form a resource bank for students' self-directed learning in mathematics, but it should be noted that learning and teaching instructions should be integrated with the e-learning materials.

Mathematics teachers are encouraged to consider the advantages of using e-learning materials in the learning and teaching of certain topics during the

curriculum planning stage. For example, for topics involving mathematical experiments or modelling, such as Probability and Permutation and Combination, well designed e-learning materials can allow students to conduct mathematical experiments that involve a large amount of repeated computations, which may not be handled smoothly by traditional computing equipment. Computer algebra system and dynamic geometry software can also shorten the computation time in exploration activities and provide good virtual models of complicated geometric figures, when physical models are hard to find in the market. Such appropriate use of IT could enhance learning effectiveness and stimulate deep learning. In order to use IT to maximise the efficiency in the learning and teaching of mathematics, teachers should consider the following on the design of e-learning materials:

- A clear focus towards specific learning objective should be identified. Modulised design in the same e-learning package should be employed if it serves more than one focus in the learning objectives.
- Instructions on how to use the e-learning materials should be provided so that students can have a clear target when they are working on exploratory or problem solving tasks.
- The limitations of the e-learning materials in aspects such as concept building, accumulation of computation experiences, and mathematical proofs should be figured out. Other learning and teaching activities should not be neglected, but could be used as complementary measures to ensure a comprehensive introduction of mathematical concepts.

3.5 Strengthening Values Education

Besides the learning of mathematical knowledge and skill, the development of positive values and attitudes in students is also important in mathematics education. Most values in the mathematics classroom are learnt and taught implicitly rather than explicitly. For example, in cases like Mathematics teachers advising students to check the reasonableness of results when they make estimations or to be careful when they read tables and graphs, values and attitudes like reasonableness and carefulness are implicitly taught. Therefore, values and attitudes such as carefulness, being systematic and analytic are nurtured in students through mathematics education. When applying mathematics to solve daily life problems, students have to analyse the situations or problems clearly. They decide which strategies or methods are more appropriate to use, and then apply them

systematically. In doing so, they can appreciate the beauty and importance of mathematics.

The following strategies can be implemented in the learning and teaching of mathematics to strengthen values education:

- **Routine problems:** The learning of mathematics should be both challenging and enjoyable for all students. The problems should be reasonably difficult and thus requiring efforts from students in attempting to solve them. Students will develop values such as willingness to try, persistence and perseverance when they take up such challenges. Likewise, the mathematical problems set must not be too difficult so that students are able to solve them without too much difficulty and with confidence. They should thus derive a sense of achievement and feel motivated. Nevertheless, the questions set must not be too easy or students will become bored and the process of learning will become meaningless and unenjoyable.

- **Non-routine problems (including open-ended questions):** Non-routine problems or open-ended questions can be introduced to reinforce or extend students' understanding of mathematical concepts, to help students develop problem solving strategies, and to encourage students to think logically. To solve these problems, students need to discriminate what knowledge is required for certain situations. Sometimes teachers may pose problems with too much or insufficient information, so students need to determine what is relevant and necessary. Openness, perseverance and creativity are therefore nurtured in students as they cannot just use the bookish knowledge which has been taught in schools. They need to apply general knowledge and common sense, make their own hypothesis or accept situations with undetermined solutions.

- **Designing problems:** Students can create their own problems to enhance their understanding about mathematical knowledge. This can encourage them to be flexible, and to realise that there can be more than one way of looking at a problem. Further, the teacher can set a theme (e.g. environmental protection) for the problems to be brought up to help students focus on the underlying values as well as the mathematical knowledge.

- **Develop an understanding of the history of mathematics:** Teachers can organise a one-off activity, or a collection of activities around a topic in history of mathematics to engage students' interest. For example, teachers may assign students to carry out a short research project about the historical

development of solving linear, quadratic and cubic equations in one variable. Students are also encouraged to investigate some aspects of Chinese mathematics, e.g. the use of abacus for arithmetic calculation, the use of counting board to solve systems of simultaneous linear equations, and the Chinese approach to proving the Pythagoras' Theorem. In ancient times, Chinese mathematics was very advanced and many important mathematical concepts, such as the study of circles and π, had been developed. The hardship and persistence of ancient Chinese mathematicians who had made great contributions to astronomy and calendar, algebra and geometry can also be introduced in the mathematics classroom.

- **Inquiry and Investigation:** The inquiry and investigation approach encourages critical and logical thinking, creativity and open-mindedness. The process of inquiry and investigation develops students' collaboration with and respect for others when group work is involved. Students are also expected to apply mathematical knowledge in a confident and self-motivated way. The use of an inquiry and investigation approach makes it difficult for students to behave like carrying out routine tasks without thinking about what they are doing. Teachers can provide students with some guiding questions, e.g. Does it always work? Why does that happen? How many cases are there? Is there any connection between this and that? Towards the end of an inquiry and investigation activities, students could be invited to compare their work with their classmates' and discuss who is right, especially if they have different solutions. This approach encourages students to be flexible, and develops their ability to respond to situations that are unexpected or do not have an immediate solution. It also helps students develop perseverance in the face of failure, construct their own ideas about solving mathematical problems and be responsible for their own learning.

3.6 Smooth Transition between Different Key Stages and Multiple Pathways

Students need to adapt to changes during the transition from Kindergarten to P1 as well as between different key stages of schooling. Their new learning needs arising from the promotion to a new school or the challenges of a new key stage should be addressed carefully. In planning a school Mathematics curriculum, vertical continuity in students' development of the subject knowledge and skills, generic skills, and values and attitudes have to be taken into account to ensure a smooth transition across key stages.

3.6.1 Smooth Transition between Kindergarten and Primary

Primary Mathematics teachers are advised to note the following concerning the transition between kindergarten and primary:

- Children at the pre-primary level are expected to acquire preliminary mathematics concepts (e.g. counting and comparing) through hands-on activities, games, storytelling and daily life experiences. Through a range of activities, children's communication skills, thinking skills and problem solving skills are nurtured. Their number sense and spatial sense are also fostered with the manipulation of real objects. At this stage, emphasis is put on developing children's interest in learning mathematics. Learning activities in different learning areas of kindergarten education are usually integrated using a thematic approach and through play.

- At the lower primary level, the approach which encourages integrating mathematics with children's experience and focusing on the application of mathematics in daily life is suggested to be continued. Diversified learning activities such as manipulation of real objects, making measurements and construction of lines and shapes are suggested in the curriculum to arouse students' interest and develop their basic mathematical knowledge and skills.

- Teachers teaching Primary 1 should consolidate students' mathematical concepts in the "Number" and "Shape and Space" strands, which are the foundation knowledge for further studies. The teaching should correspond to children's cognitive and thinking skills development. For example, when geometric shapes are taught, students should be allowed to manipulate and observe geometric shapes and to describe the shapes in mathematical language, such as using proper mathematical terms like "square" and "straight lines".

- At the primary level, homework could be used for consolidation of classroom learning and facilitation of self-learning but the quantity of assignments should be reasonable. Schools are encouraged to regularly review their homework policy in consultation with the key stakeholders including parents. Schools should also ensure that students have sufficient time for rest and leisure to develop an active and healthy lifestyle.

- At the upper primary level, mathematical concepts are further strengthened through diversified learning activities with more emphasis on students' thinking process and acquisition of concepts. As compared with lower primary,

more comprehensive learning activities and exploratory activities are designed for students to enhance their higher order thinking skills. At this stage, students should be encouraged to use appropriate mathematical language along with objects, pictures, diagrams, words and symbols to represent their work. Their ability to use abstract symbols should also be encouraged and gradually developed.

3.6.2 Smooth Transition between Primary and Secondary

The junior secondary Mathematics curriculum is built on the learning outcomes of the one at the primary level. It continues the development of the learning of primary Mathematics and equips students with the prerequisite knowledge and skills for their further learning at the senior secondary level.

Secondary Mathematics teachers should note the following concerning the transition between primary and junior secondary:

- It is essential for secondary Mathematics teachers to familiarise themselves with the curriculum as well as the learning and teaching approaches of the primary level to achieve a smooth progression in students' learning. Students coming from different primary schools may have different mathematics backgrounds, so revisiting the foundation knowledge that students are expected to have acquired at the primary level can effectively prepare them for the learning in KS3. Teachers can make use of assessment and consolidation activities to ensure that students have firm understanding of the prerequisite knowledge before proceeding to the learning and teaching of new content.
- At the primary level, students learn several areas of basic mathematical knowledge and the applications of the knowledge in simple situations. Junior secondary teachers can progressively introduce more complicated application problems, including some demanding a more integrative use of mathematical knowledge from different learning units.
- The junior secondary level plays the role of transition from the more concrete mathematics content in primary to the more abstract mathematical content in senior secondary. Learning and teaching approaches to achieve such a transition in consideration of students' cognitive development are encouraged. For example, teachers can provide hands-on activities (such as sketching the 2-D representation of simple solids, paper folding and geometric construction)

for the KS3 learning units in the "Measure, Shape and Space" strand to facilitate students' development of spatial sense and act as stepping stones for the learning of deductive geometry.

3.6.3 Smooth Transition between Junior Secondary and Senior Secondary

As far as content is concerned, the senior secondary Mathematics curriculum is built upon the solid foundation laid in the previous key stages. Similar to the layout of the curriculum in KS3, the content of the Compulsory Part is categorised into Foundation Topics and Non-foundation Topics. Students who learn only the Foundation Topics in KS3 should have sufficient background knowledge to study the Foundation Topics in the Compulsory Part of the KS4 curriculum.

Secondary Mathematics teachers should note the following concerning learning and teaching:

- In junior secondary, teachers usually explain the concepts in a concrete manner with simple language to facilitate students' learning. However, in senior secondary, teachers may present concepts in a more abstract and rigorous manner by using more advanced mathematical language such as set, logic, function etc. Students need to differentiate and integrate the concepts presented by symbols and words in a more abstract way.
- To enhance students' interest in learning mathematics in KS3, teachers usually use materials which are more related to daily life, more concrete and easier to follow, understand and resolve. But in KS4, teachers may use more advanced mathematical languages, more abstract and rigorous learning and teaching materials, create a more mathematically rich learning environment and delve into deeper understanding of mathematical concepts and knowledge. All these may lead to a higher demand on students' knowledge and skills and reduce their interest in learning mathematics. Some students may find it difficult to adapt to the new learning environment. Teachers are therefore advised to closely observe students' learning behaviour and provide timely support and assistance.
- Most topics at the senior secondary level require students to integrate the knowledge and skills acquired in KS3, e.g. solving 3-D problems in trigonometry involves the integration of both trigonometry and 3-D figures learnt in KS3. Integrated learning should be designed to encourage students to

make connections between mathematics topics. However, it is crucial for teachers to consolidate and reify students' previous knowledge so as to pave way for them to build up new knowledge and connect the knowledge with that acquired in KS3. In particular, teachers have to help students clarify misconceptions obtained in KS3.

- To develop students' ability to explore different problem solving strategies and to integrate various parts of mathematics, the learning unit "Further Applications" is incorporated into the Compulsory Part of the senior secondary Mathematics curriculum. Teachers can make good use of this learning unit to plan different learning activities and guide students to integrate what they have learnt in various areas of mathematics, and thus recognise and appreciate the interconnectedness between the concrete objects in junior forms and the abstract notions in senior forms, and strengthen their abilities to apply mathematics in problem solving.

3.6.4 Supporting Students in Educational and Vocational Pathways

While the primary Mathematics curriculum focusses on developing students' understanding of basic mathematical concepts and acquisition of basic computation skills, the secondary Mathematics curriculum aims to prepare students for a range of post-secondary pathways including tertiary education, vocational training, employment, etc. The senior secondary Mathematics curriculum, as introduced in Sections 2.3 and 2.4, consists a Compulsory Part to provide the knowledge and skills necessary for students' different career pathways, and an optional Extended Part for students who need more mathematical knowledge and skills for their future studies and careers.

The Mathematics curriculum equips students with mathematical knowledge and skills for vocational and professional education and training (VPET), and allowing them to explore and develop their potential talents and career interests. For example, students can develop essential knowledge and skills in computation, measurement and data handling for Applied Learning courses in the area of "Engineering and Production".

3.7 Cross-KLA Linkage

Mathematics is the foundation of many disciplines and provides supporting knowledge for them. It is linked to the other KLAs by providing a basis for making investigations as well as a tool for analysing data, representing findings and models with symbols, graphs and charts, and for theorising. On the other hand, other KLAs provide opportunities for students to apply mathematics in real-life situations.

It is important to relate students' daily life experience to the learning of mathematics. For some topics (like "Using percentages" and "Uses and abuses of statistics"), integration with other KLAs (like Science Education or Personal, Social and Humanities Education) is one of the ways of organising students' learning experiences mathematically. Integrated learning removes the subject boundaries and reflects the interdependent nature of reality and the complexities of life. It provides students with a holistic context for learning and enables students to make connections between what they are learning in class and real world problems. An example showing how this is done can be found in Example 8 of this Guide.

<u>Linkage with other KLAs</u>

Linkage between the Mathematics Education KLA and other KLAs are exemplified in the following paragraphs.

In the Chinese Language Education and English Language Education KLAs, mathematical concepts are essential if students are to understand essays with mathematical and statistical content. When reading information texts, students often have to extract relevant information from tables, charts and graphs, etc. to explain facts, infer results and draw logical conclusions.

In the Arts Education KLA, lines and shapes are important elements to create pictures, models and buildings, and patterns and symmetry are often explored in creative dance. Students can also apply tessellation in designing different patterns of tiles.

In the Physical Education KLA, mathematical knowledge can help students analyse sports data and design an appropriate strategy for improving sporting performance.

In the Personal, Social and Humanities Education KLA, a variety of mathematical tools and procedures are used for making rational and responsible social decisions,

such as identifying patterns and trends in statistical data and assessing validity in personal and social issues. Mathematical models are also used for theorising in social sciences, and particularly in economics.

In the Science Education KLA, laws and formulae are represented in mathematical language. Mathematical methods are employed to solve problems, organise data and generalize experimental findings, and mathematical models are used to represent physical phenomena.

In the Technology Education KLA, mathematical knowledge supports the writing of programmes. Mathematical models are used in computer simulations to explore the feasibility of realising some design ideas, and tables and charts are important tools in the representation of technical information.

In General Studies, the subject for primary students to develop and integrate knowledge and skills from the Science Education, Personal, Social and Humanities Education and Technology Education KLAs, mathematical knowledge supports students to participate in design-and-make activities and project work.

The role of Mathematics in STEM education is discussed in Section 3.4.

Linkage with Liberal Studies

Independent Enquiry Study (IES) is an integral component in the curriculum of Liberal Studies. It contributes to the curriculum aims by helping students develop higher-order thinking skills and communication skills through investigative exploration of issues. Students have to collect data, evidence and information through an appropriate means for their enquiry. They also have to analyse and evaluation information in order to construct knowledge, propose solutions to the issues and make decisions.

In conducting IES, students should know how to use a number of methods of data collection and analysis to suit different topics and contexts. The Mathematics curriculum can provide a solid background to the methods used in quantitative researches (such as sampling and questionnaire survey). Students should be aware of the limitations in data collection in terms of questionnaire design, sampling methods and sample sizes. As students have learnt a variety of statistical charts in Mathematics (such as pictogram, bar chart, pie chart, stem-and-leaf diagram, histogram, and broken line graph), they can choose an appropriate statistical graph

to present a set of data in their IES. They should be able to understand that different statistical graphs may lead to interpretations of the same set of data from different perspectives.

3.8 Time Allocation

The suggested time allocations for the implementation of the Mathematics curriculum at the primary and secondary levels, which is provided in Chapter 2 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014), and Booklet 2 of the *Secondary Education Curriculum Guide* (2017) respectively, is summarised in the table below:

Key Stage (Year Level)		**Suggested time allocation (%)**	**No. of hours (over 3 years)**
KS1 (P1-P3)		12 – 15%	285-356 hours
KS2 (P4-P6)		12 – 15%	285-356 hours
KS3 (S1-S3)		12 – 15%	331-413 hours
KS4 (S4-6)	**Compulsory Part**	10 – 12.5%	250-313 hours
	Compulsory Part with a module	15%	375 hours

The suggested time allocations are provided to assist schools and teachers in planning their school Mathematics curriculum for each key stage. Schools may vary the percentage of time allocation required for different year levels of a key stage as long as the total lesson time falls within the recommended range.

Schools are also encouraged to use the "flexible time" reserved in the central school curriculum to conduct cross-curricular activities, such as those for values education, Reading across the Curriculum and STEM education for students' whole-person development. The flexible time for the primary and junior secondary

levels is 19% and 8% respectively. As for the senior secondary level, the 10-15% time allocation is set aside for Other Learning Experiences (OLE). Schools should give due consideration to the overall planning and co-ordination among different KLAs and subjects with flexibility to organise OLE during and outside school hours.

Chapter 4
Learning and Teaching

Chapter 4 Learning and Teaching

4.1 Guiding Principles

In selecting or devising approaches and strategies for effective learning and teaching of Mathematics, the following principles should be noted:

- The main concern is the development of lifelong and self-directed learning capabilities, with emphases on equipping students with the necessary knowledge and skills to face the challenges of the contemporary world.
- All students are able to learn, but they may learn at a different pace and have different learning styles.
- The student-centered approach should continue to be adopted, with emphasis on harnessing IT for effective learning and teaching
- There should be a balance between what students are expected to learn as stated in the central Mathematics Education curriculum and aspects of the school Mathematics curriculum development based on the needs of the students and school contexts.

The Mathematics Education curriculum has been developed in line with the above principles. The main focus is not only on what mathematical topics should be learnt but also on how mathematics can be effectively learnt. The acquisition of generic skills and fostering of positive values and attitudes should be embedded within the learning of the mathematical content. The teachers' role is to help students learn how to learn mathematics and engage in self-directed learning.

In response to the economic, scientific and technological developments in the contemporary world, the learning and teaching of Mathematics should include elements related to the development of students' ability to integrate and apply the knowledge and skills of the Science, Technology and Mathematics Education KLAs. The use of IT for effective, interactive and self-directed learning should also be promoted.

As students have different abilities and learning styles, teachers have to understand their characteristics in order to enhance the effectiveness of learning and teaching.

The flexibility in the Mathematics Education curriculum, such as the provision of Foundation and Non-foundation Topics in the junior secondary Mathematics curriculum, enables teachers to adapt the curriculum to suit the needs of students with different abilities (refer to Section 4.3 of this Guide for strategies on embracing learner diversity).

A student-centered approach is recommended for the learning and teaching of Mathematics. The integrative use of IT to facilitate understanding of the mathematical concepts, timely assessments and students' self-directed learning is also encouraged. Due consideration should be given to students' abilities, experience, interests and learning styles in the design of diversified learning and teaching activities. To align with the worldwide education trend of equipping students to meet the challenges in our society and around the world with rapid economic, scientific and technological developments, greater emphasis is placed on motivating students to apply mathematical concepts to solve problems of different contexts. This can be realized in STEM education which is promoted through solving daily life problems with practical solutions and innovative designs.

Apart from the formal Mathematics curriculum, extra-curricular activities also play an important role in mathematics learning. It is generally agreed that well-chosen and organised mathematics extra-curricular activities (e.g. mathematical games, puzzles, mathematics trails and projects) help to arouse students' interest in learning the Mathematics subject and nurture their ability to apply mathematical knowledge and skills to solve problems.

4.2 Approaches to Learning and Teaching

The adoption of appropriate learning and teaching approaches and strategies is essential to address the needs of our students in face of the challenges of the 21st Century and to help them develop the capability for learning to learn. Approaches such as direct instruction, enquiry learning and co-construction could be adopted flexibly to facilitate students' acquisition, connection and construction of mathematical concepts and knowledge, as well as their development of generic skills and positive values and attitudes.

Direct instruction: This approach involves explanation, demonstration or modelling to enable students to gain knowledge and understanding of particular

concepts. Teachers introduce new concepts and new skills, and provide information and explanation on the learning content. Teachers also present factual materials in a logical manner and break complex ideas into simpler pieces or steps. Although direct instruction is more teacher-centered, it can be interactive and enhanced with the aid of IT to visualise abstract concepts.

Enquiry learning: This approach places emphasis on the process and action undertaken by students. It focuses on students' thinking and inquiring process, which promotes critical thinking and problem solving skills. Students are invited to make connections between facts, make predictions and debate on the alternatives. The approach essentially relies on students' interactions in the group or whole class. The enquiry activities involve open-ended questions, group discussion, investigations, experiments, hands-on exercises, and application of software for exploration.

Co-construction: This approach requires the whole class to form a learning community, and to learn through student-student and student-teacher interactions. A problem solving environment is created and students are allowed to discuss and exchange their ideas freely. During the knowledge construction process, teachers provide demonstrations, prompts, feedback, and opportunities for students to explain their arguments, while students develop social skills, organise thinking and produce rational arguments.

Teachers' flexible and integrative use of approaches will facilitate effective achievement of the different learning objectives of the school curriculum. No matter which approach is adopted, teachers are more than a transmitter of facts or knowledge. They have the roles of a resource person, a collaborator and a facilitator of learning. Teachers have the responsibility of delivering clear explanation, designing and conducting exploratory activities in lessons, providing suitable hints and feedback, creating an open atmosphere for discussion and showing concern for students' progress through suitable assessment strategies.

It is important to develop students' competencies in connecting learning to real-life contexts in a proactive manner, which would results in deep learning. Teachers may adopt the approaches and strategies mentioned above to design or identify challenging tasks to enhance students' intrinsic motivation in learning and to help them apply their learning in real-life contexts. The use of the Four Key Tasks and Life-wide Learning (LWL), as well as the role of e-learning and homework in the

learning and teaching of Mathematics, are discussed below.

4.2.1 The Four Key Tasks

The Four Key Tasks, namely Moral and Civic Education, Reading to Learn, Project Learning and Information Technology for Interactive Learning have been recommended for schools' implementation of the curriculum reform since 2001. Over the past decade, schools have been incorporating the Four Key Tasks into their school curriculum to help students develop independent learning capabilities. In the ongoing curriculum renewal, these tasks are updated as **Moral and Civic Education: Towards Values Education**, **Reading to Learn: Towards Reading across the Curriculum**, **Project Learning: Towards Integrating and Applying Knowledge and Skills across Disciplines**, and **Information Technology (IT) for Interactive Learning: Towards Self-directed Learning**. The updated key tasks are applicable in the Mathematics Education KLA to enliven learning and teaching, and to help students achieve whole-person development and become self-directed learners.

Moral and Civic Education: Towards Values Education

- The Mathematics Education KLA is an integral part of the school curriculum. It provides relevant learning experiences, and a conducive learning environment to implement values education and nurture students' positive values and attitudes from different perspectives.
    - Open-ended questions can be incorporated in the learning and teaching of Mathematics to develop students' respect for others. In tackling mathematical problems which may not have a definite solution, students evaluate not only their own solutions, but also alternative solutions proposed by others. Through discussing or debating on the different solutions proposed by various people, students are led to understand and accept different points of view and have respect for others. This also applies in the context of working out strategies for completing a mathematical project.
    - The use of daily life examples in the mathematics classroom may enhance students' awareness of the importance of mathematics in real-life contexts. For example, for the learning and teaching of statistics, mathematics teachers could make use of the environmental statistics on air, waste,

noise and water provided by the Hong Kong Environmental Protection Department for classroom activities. Teachers may use statistical diagrams to illustrate the wastes dumped in landfill to raise students' awareness of the importance of environmental protection. Students also develop appreciation of role of mathematics in everyday life.

- In project work, students are provided with opportunities to share out the responsibilities among group members and develop a sense of commitment by taking up different roles for completing the group tasks. Their leadership, social skills and sense of collaboration can also be fostered in authentic contexts.

Reading to Learn: Towards Reading across the Curriculum

- In a knowledge-based society, schools play an essential role to equip students with skills for reading to learn. Schools have, in general, established a reading culture among students at different year levels. With the implementation of a mathematics reading scheme, or systematic provision of mathematics reading materials, students' interest and knowledge in mathematics is enhanced. At the same time, students are provided with opportunities to apply and consolidate their reading skills and strategies developed in the language lessons.

- Schools are encouraged to build on their achievements in promoting "Reading to Learn" and further motivate students to read texts with themes related to mathematics to broaden their knowledge base and enhance their language competence and ability in handling reading materials made up of not only words, but diagrams, mathematical symbols and multimodal features (i.e. sounds, images, videos). To strive for improvement and enhancement of "Reading to Learn" and to promote "Reading across the Curriculum" in the mathematics context, schools could refer to the following suggestions and review the current strategies adopted.

  - The effective use of library resources facilitates the promotion of "Reading to Learn". Schools could borrow from the public libraries, or purchase for their school library mathematics reading materials that are appropriate to their students' age, language competence and learning interest.

  - Appropriate e-reading materials, including articles from mathematical journals or magazines, could be provided for students to read online for

the development of self-directed learning. Students can have access to these materials by using different devices including mobile phones, tablets and desk-top computers. Reading materials related to the Mathematics lessons can serve as pre-lesson preparation or further studies materials for students.

- Teachers could categorise reading materials with reference to students' diverse interest and abilities. Reading materials of diversified themes (e.g. applications of mathematics in daily life, history of mathematics, and stories of famous mathematicians) could help students connect their learning experiences of mathematics with those of other disciplines, mathematics in daily life and the cultural aspects of mathematics.

- Besides reading records, teachers may design suitable post-reading activities for students to share their reading experience. This can also reinforce students' understanding of the applications of mathematics in different disciplines and the cultural aspects of mathematics. To cater for students' different learning styles, teacher may allow students to make their presentations in varied ways, such as verbal reports, drawings and role-play.

- Mathematics teachers could collaborate with the English and Chinese teachers to help students transfer their reading skills and strategies learnt in the language lessons to read mathematics books or texts and enhance their reading skills development.

- Mathematics teachers could become role models of their students and share with them book reviews for some recreational mathematics books. It encourages students to develop reading habits and eventually a good reading culture can be established in the school.

(Please refer to Example 22 of this Guide for suggestions on organising and implementing a mathematics reading scheme.)

Project Learning: Towards Integrating and Applying Knowledge and Skills across Disciplines

- Apart from regular learning and teaching of Mathematics in the classroom, project learning is an effective means to arouse and enrich students' learning interest in mathematics. It provides an alternative learning experience and is a powerful learning and teaching strategy to promote self-directed, active and

self-reflective learning. It enables students to incorporate generic skills and values and attitudes in the process of constructing mathematical knowledge.

- The theme of project learning in mathematics can be a specific mathematical topic such as prime numbers or a study on using mathematics to solve a real-life problem that connects mathematics with the real world and other disciplines. Project learning is an effective approach to promoting cross-KLA learning and STEM education (see Section 3.4 and Section 4.3 of this Guide for details). When themes on STEM education are involved, collaboration among KLAs is strongly recommended to help students connect and integrate their learning experiences and outcomes from different disciplines.

- Project learning provides substantial opportunities for students' development of the nine generic skills. It enhances students' mathematical skills as charts and graphs are often used in data analysis and data presentation. Other elements of mathematical skills such as performing calculations and describing patterns might also be applied, depending on the themes of project learning. Besides mathematical skills, other generic skills can also be nurtured through project learning. Collaboration skills, self-learning skills and self-management skills are applied when students are required to plan and manage their individual and group work systematically. While creativity, critical thinking skills and problem solving skills are nurtured when students study the project questions, investigate them from different perspectives and explore different approaches to tackle them.

- Some issues only arise in group projects and while handling them, students learnt to listen and respond to alternative ideas and perspectives, respect others, delegate responsibilities and resolve conflicts. Positive values and attitudes are nurtured in these processes.

- In project learning, the roles of teachers change from a knowledge transmitter to a tutor, an advisor and a facilitator. Teachers need to monitor students' progress and provide continuous support, advice and regular feedback to students to develop their research skills in project learning. Once the students have become more independent in the learning process, teachers can merely act as a facilitator. Well established guideline and clear learning objective can help students conduct their projects effectively. Teacher may use samples of projects done by other students to exemplify the key elements of good projects

and guide students to improve the quality of their projects.

- Examples 3, 9, 12 and 13 of this Guide are provided to illustrate how project learning can be adopted for the learning and teaching of mathematics. Indeed, there are many other topics and types of projects, such as surveys, investigations on applications of mathematics, studies on the history of mathematics or famous mathematics problems, and biographies of mathematicians. The areas of study in project learning should be carefully chosen to suit students' interests and abilities. The depth of project work can also vary according to students' abilities. Similarly, different levels of guidance can be provided to cater for students' abilities, and with reference to the focus, difficulty and nature of the projects. Projects can be done individually or in groups, depending on their nature. Collaboration and communication skills, however, will be more effectively developed in projects carried out in groups. Project work can also be considered as an effective tool for promoting assessment for learning. Some pre-determined criteria could be set to assess the holistic performance of students (see Section 5.2 for details).

Information Technology for Interactive Learning: Towards Self-directed Learning

- The rapid advancement in IT in the past decades has led to its wide application in education. The use of mobile computing devices and application software during lessons has become increasingly popular and made a considerable impact on the learning and teaching of Mathematics. As discussed in Section 3.4, the use of IT in mathematics education is to harness its advantage to enhance student learning. Instead of merely application, IT should be used with effective pedagogies and meaningful learning tasks for enhancing effectiveness in student learning. The pedagogies and technology chosen should fit the characteristics of the topics to be delivered. Schools could refer to the TPACK[5] framework, which describes the integration of technological knowledge, pedagogical knowledge and content knowledge, for more ideas on the integration of IT with suitable pedagogy for the effective learning of a certain topic.

[5] TPACK stands for Technological Pedagogical Content Knowledge. It is a framework that incorporates the kinds of knowledge required by a teacher to develope effective pedagogical practice in a technology enhanced learning environment. More information about TPACK is available at *http://www.tpack.org*

- With the launching of ITE4, all classrooms in Hong Kong are Internet-connected to facilitate e-learning. With the help of Wi-Fi network and mobile computing devices, Mathematics lessons are provided with multimedia resources, authentic data, application software packages, communication/sharing platforms and other e-resources for interactive learning and exploratory activities. With the help of application software packages, there could be multiple representations of abstract mathematical concepts to enhance students' understanding. Moreover, with the increasing use of IT, there is a paradigm shift from a teacher-centered approach to a more interactive student-centered approach to teaching mathematics. During the lessons, students could also make use of the Wi-Fi network to share their ideas with their teachers and classmates.

- Nurturing students to apply IT tools and resources in learning mathematics is important as it also paves ways for students' self-directed learning. When students are learning mathematical concepts, they can use application software for simulations, analysis and modelling. For example, the dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum. With the help of IT tools, students can easily plot graphs for visualising mathematical relations, propose conjectures and make connections between different areas of mathematics. Appropriate use of IT tools can save students' effort in routine computation or presentation and create time and space for deeper understanding and thinking. Examples 15 and 19 of this Guide are provided to illustrate how IT can be utilised in the learning and teaching of mathematics.

In many cases, more than one of the updated Four Key Tasks can be connected to achieve a number of learning objectives. For example, a project on investigating the work of ancient mathematicians may require students to use IT skills to search for and evaluate relevant information. Students may also need to read books on the history of mathematics. Past endeavours of ancient mathematicians and collaboration among students during project learning are conducive to the development of perseverance, self-management and other qualities promoted through values education. The updated Four Key Tasks should be flexibly used in the learning and teaching of Mathematics to enhance students' capabilities for constructing knowledge, to promote the development and applications of generic skills in an integrative manner and to nurture positive values and attitudes.

School may refer to Chapter 3 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 6 of the *Secondary Education Curriculum Guide* (2017) for more ideas on the implementation of the Four Key Tasks as strategies for deepening, sustaining and focusing on the ongoing renewal of the school curriculum.

4.2.2 Life-Wide Learning

Learning may not be confined to the classroom. The provision of life-wide learning (LWL) opportunities enables students to learn mathematics in authentic contexts and gain learning experience which are more difficult to acquire in ordinary classroom settings. The choice of life-wide learning activities should be based on students' needs and school contexts.

At the senior secondary level, LWL can be effectively carried out through various kinds of experiential learning of the curriculum component of Other Learning Experience (OLE) organised by the school. Reflection on one's LWL experience and OLE can enhance students' deep learning and their reflective habit of mind as self-directed learners. These are essential for fostering students' whole-person development and lifelong learning.

There are many opportunities for students to learn mathematics through experiential learning outside the classroom, e.g.

- Conducting a real statistical survey for the student union election in the school;
- Inviting guest speakers or experts to deliver talks or run workshops on some interesting mathematics topics or real-life applications of mathematics, such as cryptography, history of mathematics, paper folding of polyhedron and mathematics games;
- Setting up a mathematics corner or a mathematics room for students to play mathematics games, investigate mathematics puzzles or construct 3-D figures; and
- Encouraging students to participate in a variety of mathematics competitions such as the following:

Competitions	Organising bodies
Mathematics Project Competition for Secondary Schools	Mathematics Education Section, EDB
Mathematics Book Report Competition for Secondary Schools	Mathematics Education Section, EDB
Statistical Project Competition for Secondary School Students	Hong Kong Statistical Society and EDB
Statistics Creative-Writing Competition for Secondary School Students	Hong Kong Statistical Society and EDB
Hong Kong Mathematics Olympiad	Mathematics Education Section, EDB and Department of Mathematics and Information Technology, EdUHK
Hong Kong Mathematics Creative Problem Solving Competition for Primary Schools Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools	Gifted Education Section, EDB and Hong Kong Federation of Education Workers in collaboration with Hong Kong Aided Primary School Heads Association

School may also refer to Chapter 6 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 7 of the *Secondary Education Curriculum Guide* (2017) for more details on LWL and OLE.

4.2.3 e-Learning

e-Learning refers to an open and flexible learning mode involving the use of electronic media (including digital resources and communication tools) to achieve the target learning objectives. In the context of the Mathematics Education curriculum, e-learning can be integrated for enhancing learning and teaching through:

- *Teachers' use of IT for presentation of abstract concepts*
  Teachers could apply different software packages (e.g. graphing tools, virtual 3-D manipulatives and dynamic geometry software) for multiple representations to enhance students' understanding of abstract mathematical concepts.

- *Students' application of software for exploration*
  Students could apply their IT skills for inquiry and investigation using suitable application software. With the help of graphing software, spreadsheet or computer algebra system, students could concentrate more on analysing the problem or formulating the solution. (see Example 15 of this Guide for details)

- *Interaction and collaboration on e-platform*
  The school e-platform enhances students' interaction and collaboration both inside and outside the classroom. Students can extend their learning by participating in mathematical activities, discussions or collaborative projects on the e-platform. Teachers may also use the e-platform to upload learning and teaching materials, notes, past papers and videos for students' revision. Students may be encouraged to leave messages (e.g. views and reflection) on the e-platform after using these materials.

- *e-Platform for learning outside classroom*
  With the e-learning environment, students are equipped with powerful tools for independent learning outside the classroom. For example, students could be assigned to study a new mathematical concept before the lessons through reading related notes/articles or watching short videos uploaded to the school e-platform so that the lesson time could be spent for further learning through discussion, collaboration, problem solving or exploration. In such a way, students are expected to have a greater engagement in self-directed learning.

The increasing use of IT has brought about changing mode of learning and teaching mathematics. Students have to learn to search for information efficiently and evaluate it critically. They also need to learn to organise data, analyse the results and make a presentation systematically.

Although IT is a useful means to facilitate learning and teaching, the importance of teachers' professionalism in stimulating the learning of mathematics through IT should also be highlighted. The "EDB One-stop Portal for Learning and Teaching Resources" (*http://www.hkedcity.net/edbsp*) has been set up to provide Mathematics teachers with web-based resources to support the learning and teaching of Mathematics and the implementation of STEM education (see Section 6.1 for details).

4.2.4 Meaningful Homework

The aim of homework assignments is extending and consolidating student learning after lessons. Quality homework also encourages independent learning. The main purposes of mathematics homework are:

- to further develop and consolidate what students have discovered or learned during lessons;
- to reinforce and maintain the mathematical concepts and skills developed during lessons;
- to facilitate students' self-learning after lessons;
- to encourage students to think independently, formulate and solve problems, assess situations, analyse and make decisions;
- to lead students to appreciate that mathematics is meaningful, enjoyable and useful, and to develop genuine interest in learning mathematics;
- to help teachers evaluate students' learning performance and progress; and
- to help teachers identify students' learning difficulties, provide feedback and adjust their teaching plans and strategies.

Apart from written assignments, students' homework may include discussions through e-platforms, group work, reading mathematics books, project work and other activities. When setting mathematics assignments, teachers should take note of the following:

- The quantity of each assignment should be appropriate in order to motivate students to learn. It is undesirable to assign too much homework at one time. The frequency of assigning homework should be regulated so that students will not be unduly overburdened. Co-ordination among different subject teachers is desirable and necessary.
- There should be a clear learning objective for each mathematics assignment. Questions selected should be geared to the objective. Too hard or too easy questions should be avoided.
- The exercises from a good textbook could form the main component of the assignment. Those designed by teachers themselves or derived from other

sources could be used as complementary or supplementary materials, e.g. fundamental questions for less able students and more demanding questions for more able students.

- It is essential to include a variety of question types in each assignment so as to maintain students' interest and provide different ways of consolidating their learning.
- The selection of questions should address students' diverse abilities. Core questions should be first identified and geared at students of general ability. A few more questions should be set on the two extremes for the more and less able students.
- Exercises requiring application of mathematics should be closely related to daily-life situation and designed with emphasis on practicality.

(School may refer to Chapter 8 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 4 of the *Secondary Education Curriculum Guide* (2017) for more details on meaningful homework.)

4.3 Learning and Teaching for STEM Education

As discussed in Section 3.4 of this Guide, promoting STEM education in the Mathematics Education KLA is one of the MRE/focal points of the ongoing renewal of school curriculum to enable students to become lifelong learners of science, technology and mathematics and to meet the changes and challenges in the era with rapid advancement in technology. When incorporating STEM learning activities in the school Mathematics curriculum, schools can adopt the following two approaches:

Approach One: *Learning activities based on topics of the Mathematics Education KLA for students to integrate relevant learning elements from the Science Education and Technology Education KLAs*

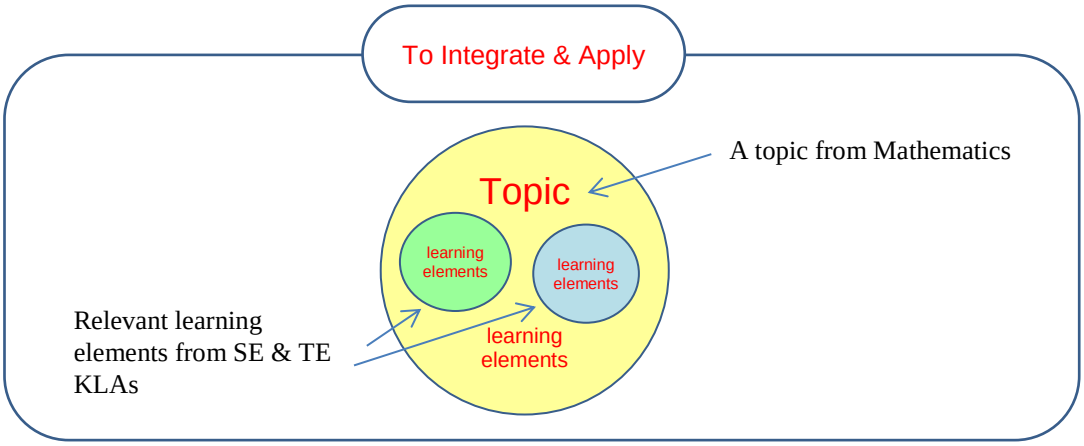

Figure 5

In this approach, the design and development of the learning activity is based on a particular topic of the Mathematics curriculum (see Figure 5). For example, P6 students could be requested to design and build rubber band powered toy cars in the learning unit "Speed". Students recognise and apply the related concepts through the attempts to improve the speed of the toy cars. Relevant learning elements are drawn from other KLAs such as recognizing energy conversion. Students also need to integrate and apply the knowledge and skills they have learned in Mathematics and General Studies (see Example 6 of this Guide for details).

At the secondary level, knowledge and skills of the subjects of the Science or Technology Education KLAs can also be integrated with different topics in the Mathematics curriculum. For example, teachers could use designing a measuring spoon as a real-life application of the mathematical knowledge related to the volume of spheres. The technological knowledge in the design of the product, such as how the thickness of the material affects the volume and design can also be embedded (see Example 18 of this Guide for details).

Below is a table showing some examples of STEM activities and the related topics under different strands in the junior secondary Mathematics curriculum.

Topics	Examples of STEM activities
Number and Algebra Strand	
Scientific notation	The applications of scientific notation in science or technology subjects can be discussed in lessons to illustrate the advantages of using this notation to present and compare values. For example, students could be asked to compare the sizes of different cells in our body, and the computational speeds of different computer processing units.
Ratios	Students could be asked to estimate the nutrition value of packed food product by applying knowledge on ratios and by referring to the nutrition information on food product labels.
Binary numbers	The application of binary numbers in computer systems could be used as an example of applications of mathematics. In a computer, the binary system is adopted for the expression of numbers and characters, and for performing logical and mathematical operations. Interested students could be asked to investigate the amount of data that an 8-digit binary number holds.
Measures, Shape and Space Strand	
Errors in measurement	Students could be asked to design a method to investigate the errors in measurement by some common GPS tracking apps. Students may compare the measurement results of the apps with those of other measuring tools, and discuss the reliability of the apps and ways to reduce errors in measurement.
Volumes of 3-D figures	Students could be asked to design a funnel or a box with specified volume and shape that would use the least amount of material to manufacture. Students could use spreadsheet software and apply the formulas for the volume and surface area of a circular cone or a cuboid for exploration. Students could also use 3D

Topics	Examples of STEM activities
	printers to realise their design for verification of their calculations.
Trigonometric ratios	Students could apply knowledge of trigonometric ratios to authentic problems. For example, students could find the distance between the school campus and a nearby building by measuring the angles of elevation of the building from two different levels using angle-measuring apps on a tablet computer or a mobile phone.
Data Handling Strand	
Organising data and constructing statistical charts	Students can conduct a cross-disciplinary project on healthy diet menu for the school lunch box supplier. They could analyse and estimate the nutritional values of different food ingredients and conduct a survey to collect information about the food preference of their fellow students. They can then design a diet menu meeting students' nutritional needs and their food preference.
Uses and misuses of statistical charts	When doing project work for the Science, Technology, Mathematics KLAs or on cross-KLA topics, students can collect data from secondary sources (e.g. the average annual rainfall in Hong Kong and the energy consumption in Hong Kong) to assist their exploration. Students need to interpret the data and present the results appropriately. Sometimes if the data come from graphs through the printed or electronic media, students need to evaluate whether the graphs is misleading to avoid misinterpretation.

Approach Two: *Projects for students to integrate relevant learning elements from different KLAs*

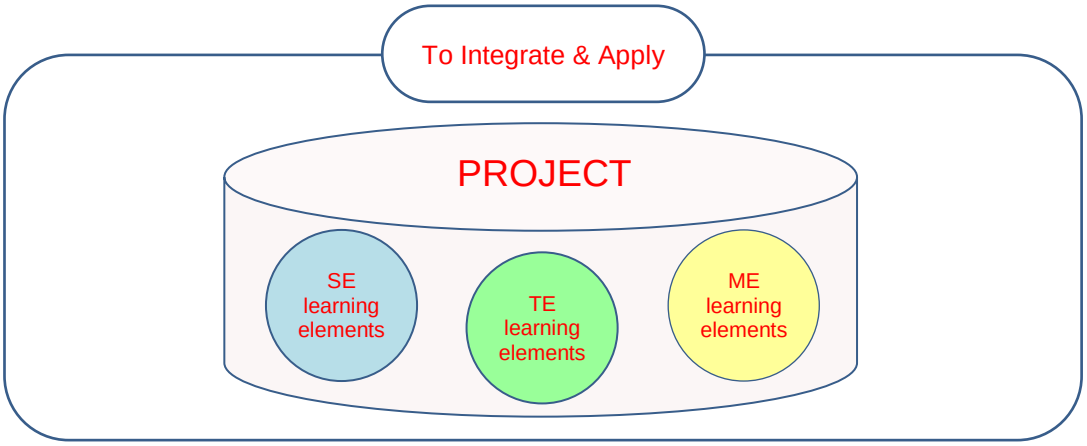

Figure 6

In this approach, a project learning activity is adopted to integrate all related learning elements from the Science, Technology and Mathematics Education KLAs (see Figure 6). Students start with an authentic object of studies, for example, designing a healthy diet menu for the school lunch box supplier. If the project is carried out by upper primary students, they could design the menu by applying their mathematical knowledge on fractions and decimals, their computation skills, together with the knowledge on food pyramid and nutrition in General Studies. Primary teachers could guide students to apply the concepts of percentages on studying the nutrients in different food and to construct statistical charts for presentation and analysis. For junior secondary students, their investigation could go further by applying their knowledge in Science about the contents of food, nutrient science and a balanced and healthy diet to formulate a theoretical framework in designing the lunch menu. They could apply suitable statistical methods learnt in Mathematics to collect and present data, such as students' food preference, and use an appropriate choice of measures of central tendency. They could apply the concepts of ratio and errors in calculating food ingredients. For senior secondary students who have been equipped with knowledge of linear programming, they could be guided to extend the use of these mathematical concepts to formulate a mathematical model for this real-life scenario, and with the aids of IT, the mathematical model could be solved for a menu with optimised nutrition. Students could also make use of their knowledge on food preparation in the Technology Education curriculum to actualise a practical menu. Teachers of

different subjects could be the facilitators in their own subject perspectives from time to time to support students, and students could endeavour to integrate and apply their knowledge and skills as the cross-disciplinary project is in progress.

It is important to note that projects in STEM-related topics are deep learning tasks that encourage students' collaborative explorations and integration of knowledge for solving real-life problems.

Teachers may refer to Appendix 4 of this Guide for some e-resources for organising learning and teaching activities related to STEM education.

4.4 Embracing Learner Diversity

Every student is unique. They are different in terms of level of maturity, gender, personality, ability, aspiration, interest, learning motivation, cultural and socioeconomic background. Their intelligence, thinking modes and learning styles influence their learning characteristics. To embrace the diversity, it requires the use of appropriate learning and teaching strategies, and the provision of different learning experiences and pathways for students to realise their potential. Embracing learner diversity is not intended to narrow the gap between individuals or even out their abilities and performances, but to enable all students to learn and perform to the best of their abilities. The ultimate goal is to help all students, whether they are gifted[6], with special educational needs (SEN[7]), more able or less able, to achieve maximum growth as learners. There could be three aspects of planning strategies to embrace learner diversity, namely the central curriculum aspect, the school aspect and the classroom aspect.

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[6] The current gifted education policy of the HKSAR Government encourages mainstream schools to nourish gifted students through school-based gifted programmes. Support services are provided by Gifted Education Section, Curriculum Development Institute, Education Bureau and The Hong Kong Academy for Gifted Education (HKAGE) together with tertiary institutes and educational organisations/bodies. Teachers are encouraged to nominate their gifted students with outstanding performance/great potential in Mathematics to join the HKAGE programmes.

[7] On the principle of "one curriculum framework for all", students with special educational needs (SEN) follow the mainstream school curriculum and are offered essential life-long learning experiences. Materials and teaching packages on curriculum for students with SEN have been uploaded to the website: *http://www.edb.gov.hk/en/curriculum-development/major-level-of-edu/special-educational-needs/index.html* for teachers' reference.

4.4.1 Central Curriculum Aspect

In designing the central Mathematics Education curriculum, the needs of students at both ends of the ability scale are equally important. All students' opportunities to learn should be maximised so the attention should not be placed only on low academic achievers but also the average and more able students.

The design of the Mathematics Education curriculum allows flexibility in the organisation of a school Mathematics curriculum at different key stages to embrace learner diversity. Enrichment Topics are provided in the primary and junior secondary Mathematics curricula, while Foundation Topics and Non-foundation Topics are provided in the junior secondary Mathematics curriculum and the Compulsory Part of the senior secondary Mathematics curriculum (refer to Section 2.4 of this Guide for more details).

4.4.2 School Aspect

Nowadays, both primary and secondary schools develop their school Mathematics curriculum with reference to the central Mathematics Education curriculum. It is a general practice that panel heads work in collaboration with other panelists to diagnose students' strengths and weaknesses as well as their learning needs in mathematics to provide a basis for the planning of the school Mathematics curriculum. The following are some strategies to embrace learner diversity at the school level.

(a) *Adopting school-level arrangements such as providing additional lessons to students in need and introducing ability grouping strategies like streaming, split class, withdrawal and cross-level subject setting*

For more able or gifted students, teachers can group them for attending mathematics pull-out programme during or after regular school hours to allow structured learning.

(b) *Adjusting the inclusion of Non-foundation Topics of the secondary Mathematics curriculum or the depth of treatment of topics in the primary Mathematics curriculum while selecting topics as the common core learning contents for all students*

(c) *Arranging the learning contents in a logical sequence for each year level*

taking into consideration the cognitive development, the mathematical abilities, learning styles and interests of students; the learning objectives of each learning unit; the inter-relation of learning units; and the inter-relation of mathematical learning at different year levels

For less able students, it is necessary to review the essential topics regularly. The spiral approach assists students to refresh their knowledge. However, it should be noted that the teaching of too many topics in a specific year and fragmentation of learning (i.e. breaking learning down into unconnected bits of knowledge or skills) should be avoided. In addition, measures like organising bridging programmes can be introduced to ensure that students of different abilities can progress in the curriculum.

For more able or gifted students, teachers can employ curriculum enrichment, compacting and acceleration to address their learning needs. Curriculum compacting refers to the skipping of certain basic concepts or basic skills that students have mastered so that teachers could focus more on the harder part of the topics or introduce related enrichment topics. Teachers can also help students develop self-directed learning by selecting suitable basic contents for pre-lesson study. Nevertheless, teachers should secure students' mastery of basic skills through appropriate assessments. Curriculum acceleration refers to learning and teaching at a pace faster than the one suggested in the central curriculum.

(d) Choosing appropriate textbooks, selecting proper learning and teaching resources and adapting or producing instructional materials

Schools may either use different textbooks for students from different ability groups of the same year level, or use the same textbook but centrally produce different instructional materials to support students in different classes. Teachers can also provide learning and consolidation tasks to set scaffolding for students of different abilities in learning mathematics.

(e) Designing a wide variety of learning activities such as weekly questions posted on the mathematics bulletin boards, mathematics books reading schemes, poster design, and mathematics club

Students with different inclinations and abilities may participate in different activities that suit their needs or interest. Students could be encouraged to participate in activities outside of school, e.g. Mathematics Creative Problem Solving Competition for Primary Schools, Hong Kong Mathematics Olympiad

(HKMO), Mathematics Project Competition for Secondary Schools, Mathematics Book Report Competition for Secondary Schools, Hong Kong Statistical Project Competition for Secondary School Students and Hong Kong Statistics Creative-Writing Competition for Secondary School Students.

(f) Formulating the school assessment policy and methods of recording and reporting to provide feedback for learning and teaching.

Schools may empower teachers to use diversified modes of assessment to suit the needs of individual classes and allow certain percentages (say 5% to 10%) of students' mathematics scores to teachers' own discretion. Teachers may design their own test papers, project work, daily marks, etc. which account for the percentage of the mathematics scores set.

Schools may also consider using different question papers for classes of different mathematical abilities. Such practice can provide flexibility for teachers to assess students' learning at their particular ability levels. Information provided would reflect the performance of students through suitable assessment criteria and serve the purpose of assessment for learning.

(g) Caring for non-Chinese speaking (NCS) students

In general, NCS students have a lower Chinese language competence and English is usually adopted as the medium of instruction for Mathematics. Some mathematics learning and teaching materials produced by the EDB have been translated into English and are available at the EDB website. Schools with NCS students have to note that besides language, there might also be differences between Chinese and NCS students in terms of their cultural backgrounds and learning experiences. For example, as NCS students might have a different counting method in their own culture, the learning content of the Mathematics curriculum might need to be rearranged to address the NCS students' needs. Using more graphical illustrations in instructional materials may also help NCS students to grasp the learning content of the Mathematics curriculum.

(h) Providing gifted students with information on off-site support programmes

A variety of enrichment programmes are offered by professional bodies, universities and non-governmental organisations including The Hong Kong Academy for Gifted Education (HKAGE) to provide opportunities for more advanced learning to enhance the performance of gifted students with

exceptional achievements or potential in specific aspects. The Gifted Education Section of Education Bureau and HKAGE also co-operate with or/and commission tertiary institutes (or educational organisations/bodies) to provide challenging off-site enrichment and extension learning opportunities for exceptionally gifted students nominated by schools.[8] Schools could visit the related websites and find out details about programmes and activities suitable for their students who are gifted in mathematics.

(i) *Adapting the school curriculum according to the needs of SEN students with due consideration given to their pace and style of learning*

Curriculum adaptation is not equivalent to trimming but about tailoring the learning objectives, content, materials, teaching strategies and learning environment to suit the learning needs of students with SEN and particularly their learning pace and style. Schools are encouraged to help students with SEN identify learning objectives, achievement targets and expected level of performance in mathematics according to their prior knowledge, abilities and learning needs.

4.4.3 Classroom Aspect

No matter how the school Mathematics curriculum is designed or how students are streamed or grouped in schools according to their abilities in learning mathematics, it is more important that the class teacher should be flexible enough to adjust his/her teaching plan to address the needs of students. Below are some suggested strategies for teachers in designing their classroom activities

<u>General Strategies</u>

- *Diagnosis of students' needs and differences*
  Teachers need to gather background information of students, including their interests, their strong and weak areas. Teachers' observation of students' performance in class and in assignments and tests is also a reliable basis for diagnosis.

[8] More information on off-site support could be found on the homepage of the Gifted Education Section: http://www.edb.gov.hk/en/curriculum-development/major-level-of-edu/gifted/index.html , and the homepage of the Hong Kong Academy for Gifted Education: http://ge.hkage.org.hk/en/ .

- *Variation in level of difficulties and contents covered*

  Based on the above findings, teachers can plan relevant mathematics learning activities for each lesson. For example, primary teachers could include hands-on activities, or other activity-based learning in their lessons that fit the needs of students. Teachers have to select, adapt or design materials to suit the range of abilities of their students. Too easy or too difficult tasks will not stimulate and sustain student's internal drive to learn. For less able students, tasks should be relatively more fundamental in nature as these activities can give students greater sense of satisfaction. On the other hand, for more able students, tasks assigned should be challenging enough to cultivate as well as to sustain their interest in mathematics learning. Teachers may prepare different sets of activity materials for students of different abilities in a class, or prepare one set of materials with a core part for all students, a part that reinforce the prerequisite knowledge for the less able students and a part that challenges the more able students. Students could then choose the parts that suits their ability and needs.

  For example, the planning of the learning content for classes with different abilities may vary for the KS3 learning unit on the rectangular coordinate system under the Measures, Shape and Space strand. For less able students, teachers can consider not including the Non-foundation Topic about the formula for internal points of division. For more able students, teachers can cover all the learning objectives and select the Enrichment Topic, which is about the formula for external point of division.

- *Variation in questioning strategies*

  Appropriate questioning can help students achieve the learning goals effectively and make improvement. Through providing students with different clues when questions are asked, teachers can enable students at the same year level but with different abilities to learn the same topic together. In general, teachers can pose simple and straightforward questions to less able students, and on the contrary, pose more challenging questions to more able ones. It should be noted that feedback from teachers is essential in facilitating students' learning. Teachers can request even the less able students to explain their strategies for solving the problems and to modify their answers instead of giving the solutions right after they have given the wrong answers.

  Teachers may decide what questions to ask, in terms of levels of thinking, to

address students' different needs. There are questions on memory and information recall, on interpretation, comparison and explanation, and also on new ideas and alternatives. Questions of thinking levels fitting students' progress in the development of concepts can greatly assist students learning.

- *Variation in clues provided in tasks*

  Teachers can also provide students with the same task or exercise but give additional support such as diagrams to aid comprehension and structuring long question for less able students. For more able students, teachers may provide fewer hints in the process of solving problems. Further, open-ended problems (such as Examples 5 and 14 of this Guide) and graded exercises can also be used to motivate students to solve the problems with strategies suited to their abilities and concerns.

- *Variation in approaches to introducing concepts*

  Teachers can introduce mathematical concepts with different approaches. Concrete examples may be used to illustrate the concepts for less able or younger students but symbolic language can be used for more able or more mature students. For example, a diagram of dots arranged in a triangular pattern can be used to illustrate the sequence of triangular numbers for less able students, whereas the idea of the sum of arithmetic sequence can be introduced to more able students.

  Teachers could adopt multiple means of presentation to cater for different learning styles. For example, in the learning and teaching of topics related to 3-D figures, diagrams, real models of 3-D figures for hands-on manipulation and virtual 3-D figures by software packages can be used to address the needs of visual, auditory and kinesthetic learners. Example 17 of this Guide shows a learning activity designed to cater for students' different learning styles.

- *Variation in peer learning*

  Besides whole-class teaching, teachers can also consider different grouping strategies to cater for the needs of different students. However, it should be noted that the way the groups are formed, the suitability of the tasks designed for the groups, the durability of the grouping and the ongoing assessment of the group dynamic are ingredients for successful collaborative learning. Further, it is very important to build up a collaborative instead of competitive atmosphere that is found undesirable for effective learning.

Teachers may consider grouping students with similar or different learning abilities, or changing the group size for collaborative learning. However, care must be taken to avoid labelling effect on students due to homogeneous grouping (i.e. grouping students of similar abilities), especially for a long duration. Heterogeneous grouping (i.e. grouping students of different abilities) may lead to both positive academic and remedial outcomes. The high-ability students may benefit from group interaction as much as the average or less able students. For maximum communication among members, group size should not be too large. Groups of three to four students work quite well in mathematics tasks.

- *Importance in arousing learning motivation*
  Motivation is probably one of the most important factors in relation to learning performance because a well-motivated student is usually more determined to achieve a higher standard and overcome a lot of learning difficulties. Motivation is not constant over time and may change according to the circumstances and disposition of the student. Teachers must be aware of the possibilities of such changes and be flexible enough to adjust their strategies when necessary. It is particularly crucial for teachers to plan learning activities to initiate their students' motivation.

- *Variation in using e-learning tools*
  e-Learning packages usually provide different levels of mathematics exercises or activities. Appropriate use of IT provides teachers with a way to embrace learner diversity as it allows students with different abilities to learn at different paces. The e-features to record students' performance also provide information for teachers to diagnose students' misconceptions or general weaknesses so that they can re-adjust the teaching pace and teaching strategies. e-Learning tools and resources can also engage different types of students, and to enrich their learning experiences.

<u>Strategies for Students with SEN</u>

- *Diagnosis of students' needs and differences*
  Teachers need to recognise the needs of students. For some SEN students, they may have problems in building up number concepts in their long-term memory, and in retrieving procedures or strategies when solving problems. Even when

SEN students give a correct answer, they may not show any confidence towards their work.

- *Variation in level of difficulties and contents covered*
  Students with SEN may show anxiety about mathematics learning even though they can give a correct answer. Teachers should recognise their learning abilities and adopt appropriate strategies in learning and teaching to provide successful learning experience for them. For example, for SEN students having a weaker working memory, teachers should break the instruction of learning activities into small steps or use teaching aids such as counting blocks and pictorial cards for helping them to follow the activities and understand the concepts. Solving one type of problems with many different methods and requesting for a prompt reply in mental calculation also make some SEN students feel overburdened and anxious. Teachers could concentrate on some strategies and encourage SEN students to use jottings to reduce their burden.

  Below is an example of the P3 learning unit "Multiplication (I)" under the "Number" strand of the primary Mathematics curriculum.

	1	2
×		3
	3	0
		6
	3	6

Students need to know that the values of "1" in 12 is 10 and the result of 10 × 3 is 30 before performing the multiplication. Connecting the new technique with their previous one by writing "30" instead of "3" in the calculation can effectively reduce the loading of their working memory.

When students are requested to perform multiplication with a 1-digit multiplier and a 2-digit multiplicand, teachers usually start the discussion with less able students by adopting calculations without carrying and together with counting blocks to help students understand the calculation in column form. Same strategies can also be adopted for SEN students with modifications to reduce their burden on memory.

Strategies for Gifted Students

- *Diagnosis of students' needs and differences*
  To get assistance in identifying more able or gifted students, teachers may refer to the information[9] provided by the Gifted Education Section of the EDB.

- *Variation on level of difficulties and contents covered*
  In the learning and teaching of conditions for congruent triangles, teachers can extend the concept of fixing a triangle to fixing a quadrilateral to challenge gifted students. Teachers may also ask students to guess the condition(s) sufficient for identifying congruent triangles.

  In the learning and teaching of the methods to construct angle bisectors, perpendicular bisectors and special angles by compasses and straight edges, teachers can prompt students to connect the geometric construction with the angle bisector theorem and the perpendicular bisector theorem.

- *Variation in clues provided in tasks*
  Teachers can adopt tiered assignments to embrace learner diversity. Tiered assignments are differentiated learning tasks developed by teachers based on students' abilities to meet their individual needs. It provides a better matching between students and their learning needs, and involves different levels of difficulty, complexity, abstractness, depth and creativity. According to Heacox (2002), there are six ways to structure graded assignments: (i) by challenge level; (ii) by complexity; (iii) by resources; (iv) by process; (v) by outcome; and (vi) by product. To prepare for a tiered assignment, teachers may first consider the instructional level of average students. Then the assignment can be modified to become more challenging by increasing the level of difficulty and complexity for the mathematically gifted students. More details can be found in the resources[10] produced by the Gifted Education Section of the EDB and the HKAGE.

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[9] *http://www.hkedcity.net/article/project/webcourses_gifted/doc/BehaviouralCharacteristicsChecklistOfMathematicallyGiftedStudentsAppI&II_2.pdf*

[10] *http://resources.edb.gov.hk/gifted/ge_resource_bank/files/TeachingLearning/C/course%20report%20web4_070921.pdf* and *http://www.hkage.org.hk/file/teaching_resources/1449/%E4%B8%ADA01%20Tiered%20Task%20Info%20Sheet.pdf*

- *Variation in approaches to introducing concepts*

  Teachers may consider using more abstract presentation method for mathematically gifted students. For example, in general teachers are advised to let students build and manipulate the concrete models by themselves before learning the 2-D representations of a 3-D solid. For more able or gifted students, teachers may skip the use of concrete models by using an appropriate computer program to demonstrate the effect of rotation of a 3-D solid on the 2-D representations.

- *Importance in arousing learning motivation*

  Teachers can use some mathematics paradoxes to provoke gifted students' curiosity. For example, teachers may demonstrate the proof of $0.999\ldots = 1$ to challenge students' intuitive view on rejecting the equality of $0.999\ldots$ and 1.

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Chapter 5
Assessment

Chapter 5 Assessment

5.1 Guiding Principles

Assessment is an integral part of the curriculum planning, pedagogy and assessment cycle. It involves collecting evidence about student learning, interpreting information and making judgement about students' performance with a view to providing feedback to students, teachers, schools, parents and other stakeholders. Assessment also plays a critical role in helping students become self-directed learners. The purposes of assessment are manifold, but the prime purpose should always be for facilitating and improving students' learning. Suitable assessment enables

- students to understand their abilities and hence improve their ways of learning;
- teachers to understand the performance of their students and the effectiveness of the learning and teaching strategies adopted so that suitable and timely measures can be provided; and
- parents to understand the performance of their children so that they can, in collaboration with teachers, provide suitable support to help the learning of their children.

A framework for organising school assessment is provided in Figure 7. The figure illustrates the interrelationship of formative and summative assessment, and the connection among learning and teaching, internal assessment and external assessment. Schools can refer to this figure in developing their assessment plans for the Mathematics Education curriculum.

As reflected in the Mathematics Education curriculum, the learning process and content are both important in learning mathematics. Assessments in mathematics should align with the aims and objectives of the curriculum to obtain a complete picture of students' performance. In general, the focus of assessment in mathematics should not be the answers provided by students only, but also how it reflects students' learning process, skills used in solving mathematical problems as well as their development of thinking abilities and positive values and attitudes.

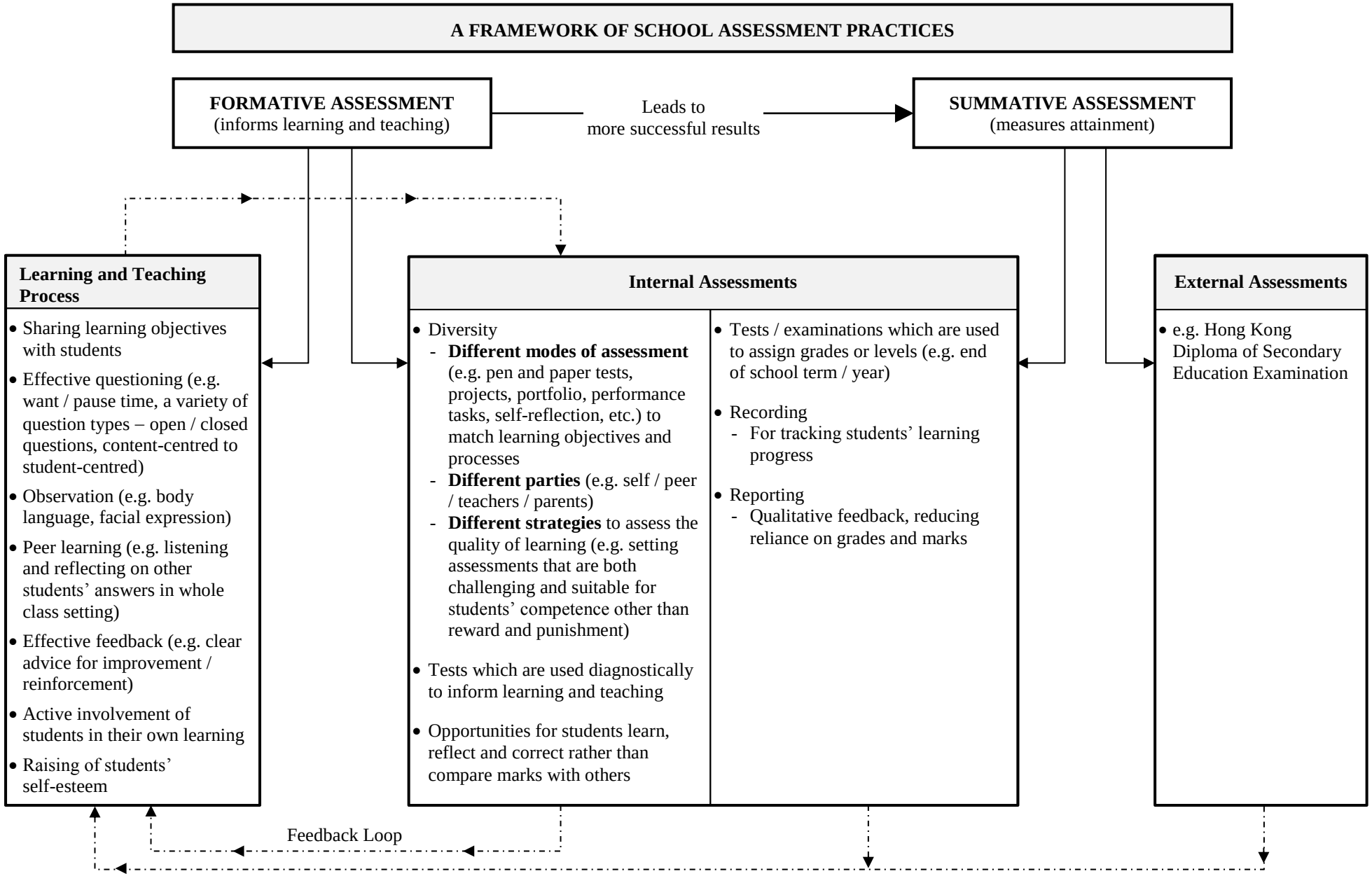

Figure 7

(Adapted from Shirley Clarke)

A collection of balanced and diversified assessment activities is definitely helpful to teachers in obtaining a comprehensive view of students' achievements and performance in learning. Teachers should note the following points in the design of suitable mathematics assessment activities to collect information about students' performance.

Suitable Assessment	Unsuitable Assessment
• Helps students develop confidence and interest in learning • Enables teachers to provide immediate feedback to enhance learning and teaching • Addresses different learning objectives	• Causes anxiety in students, puts undue pressure on them, and in extreme cases makes them lose confidence and interest in learning • Reduces lesson time for learning and teaching, increases teachers' workload and hence poses pressure on both students and teachers • Over-emphasises drilling

Schools' assessment policy should aims at achieving a good balance of different modes and purposes. Schools should consider the following aspects while formulating the assessment policy:

- What are the expected learning outcomes at the level concerned?
- What assessment activities will be adopted in assessing students' performance in the various aspects of mathematics and cross-KLA areas (e.g. STEM education)?
- Will the assessment activities include life-wide learning activities and activities other than the paper-and-pencil type (e.g. performance tasks)?
- What assessment criteria will be used in activities like projects, oral presentation?
- Have learner diversity been condisered in the design of assessment activities and setting of assessment criteria?
- Will e-assessment be considered for some topics to facilitate assessment for learning?

- What is the number of tests and examinations in the school year?
- What assessment data will teachers collect and how will teachers use it to inform learning and teaching and school curriculum planning?
- What kind of feedback will be provided to students and is it comprehensive enough for studetns' self-directed learning?

An appropriate frequency of assessment is useful to inform students' learning. Over-assessment should be avoided as it disrupts learning and teaching and overburdens students and teachers. In planning the school assessment policy, enough space has to be reserved for students' learning and teachers' teaching.

Schools may refer to Chapter 5 of the *Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 4 of the *Secondary Education Curriculum Guide* (2017) for more details on developing a school assessment policy.

5.2 Formative and Summative Assessment

Assessment in mathematics can be both summative and formative. Summative assessment focuses more on providing a comprehensive and summary description of students' performance and progress in learning. Formative assessment focuses more on diagnosing students' strengths and weaknesses in learning, providing feedback and reviewing learning and teaching strategies.

5.2.1 Purposes of Assessment

In general, assessment can serve the following three purposes:

Assessment *of* Learning	• Providing evidence to make judgements on student achievement against learning targets, objectives or standards at a certain point of time, e.g. the end of a teaching unit, a school term and a key stage • Being summative in nature and resulting in marks, grades or certification

Assessment *for* Learning	• Collecting ongoing information about the progress of student learning to provide timely and quality feedback for students to improve learning, and for teachers to adjust their curriculum planning and teaching strategies. • Being formative in nature and integrating assessment into learning and teaching
Assessment *as* Learning	• Using learning tasks and feedback collected by students to enhance own learning • Being formative in nature and encouraging students' understanding of their learning, evaluation of learning effectiveness, adjusting learning strategies, planning for follow-up actions and setting future learning objectives and strategies

"Assessment for Learning" and "Assessment as Learning" can be used when formulating appropriate modes of assessment for Mathematics to collect evidence of students' learning process, provide feedback to enhance learning and establish students' roles and responsibilities in their learning. "Assessment of Learning" can be used when comparing different groups of students or different systems and should not be used as the only assessment strategy. Students should be encouraged to make use of feedback from both formative ands summative assessment for self-reflection and teachers should carefully analyse the results of formative and summative assessment to revise their curriculum planning and class instructions.

5.2.2 Modes of Assessment

Different modes of assessment serve different purposes. Teachers may use them deliberately to focus on the process, progress and product of learning mathematics. Various assessment activities are required to help teachers collect, judge and interpret information about students' performance in their development of mathematical knowledge and skills, generic skills, and positive values and attitudes. Designing appropriate and diversified assessment activities according to the learning targets and students' learning needs is essential in implementing assessment as learning. Below are some common mathematics assessment activities.

- *Class Discussion*

  In the learning and teaching of mathematics, there are often discussions, asking and answering questions between teachers and students or among students. A discussion in class not only enables teachers to discover what students understand about a particular topic, but also provides opportunities for students to present their views and foster their communication skills. Teachers could also understand students' attitudes and abilities in applying thinking skills through class discussions. Assessment criteria for a class discussion may include: Can students explain how they have got the solutions put forward and what strategies they have employed? Do students know how to raise questions? Do students raise questions actively? Constructive feedback from teachers allows students to understand the correctness of their response and achieve an in-depth analysis of the topic. Topics suitable for class discussions include: *Why* $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$ *? Why* $(x + y)^2 \neq x^2 + y^2$ *? How can the height of the school building be estimated? Are the areas of figures with the same perimeter equal? How can a right-angled triangle be constructed by using a straight rule and a pair of compasses? How statistics be applied in real life?*

- *Classwork and Homework*

  It is a common practice to assign classwork and homework to help students consolidate their concepts in mathematics and help teachers assess students' performance. To embrace learner diversity, teachers may consider assigning different classwork to students of different abilities. In assigning homework, it is important for teachers to understand that it is about quality and not quantity. Moreover, each assignment should be set at a suitable level of difficulty and appropriately related to specific learning objectives. It is inappropriate, for example, to give students an assignment on the drilling of addition and subtraction while the aim is to assess their application of addition and subtraction. Finally, classwork and homework for mathematics should not be confined to routine mathematical problems. There could be open-ended questions, reading assignments, hands-on assignments and preparatory work for class discussions. For example, collecting newspaper cuttings on the use of statistics in daily life for a class discussion on the misuse of statistics; collecting containers with the volumes in litre written on them for measuring activities in school.

- *Project Work*

  Project work is a useful activity to assess students' performance. It is also an effective learning and teaching strategy to promote self-directed learning and enables students to connect their mathematical knowledge, generic skills, values and attitudes. It can also foster students' critical thinking skills, creativity and problem solving skills. Teachers should note that collaboration and communication skills will be more effectively developed through group projects than individual projects. Teachers may use the following criteria to assess students' performance in project work:

  - comprehension of the project;
  - use of strategy and approach;
  - coverage, depth and accuracy of content;
  - presentation and communication; and
  - attitude.

  Students' reflections and peer assessment can be an integral part of the assessment for project work and parents can also contribute their views. Using assessment rubrics is a good practice for self-reflection and peer assessment for achieving self-directed learning. IT may be employed to facilitate assessment of project learning, e.g. submitting students' project work on schools' e-platform and sharing of comments.

  Typical examples of project work for mathematics include conducting a survey on students' favourite extra-curricular activities, comparing the heights of boys and girls in the school, investigating the relationship between students' eyesight and time spent watching TV or using display screen equipment, investigating the story of π, designing containers with the optimal use of materials, investigating the application of mathematics in sport, and conducting a statistical survey on students' time management. Cross-KLA topics or topics on real-life problems can also be assigned for promoting STEM education (refer to Examples 10 and 12 of this Guide for details).

- *Exploratory Task* and *Performance Task*

  Exploratory tasks and performance tasks are classroom activities that require students to apply various skills to perform mathematical investigations or solve problems in a hands-on way. Through getting students to complete the

tasks in groups, teachers can look at students' problem solving skills and collaboration skills. The assessment criteria can also include comprehension of the problem, use of strategy and approach, degree of participation and attitude, etc. Typical examples of exploratory tasks and performance task include constructing parallel lines, measuring volumes of irregular objects, geometric constructions by compasses and straightedge (or by dynamic geometry software), constructing special sequences by spreadsheet, making 3-D models of polyhedrons, constructing loci by dynamic geometry software, etc.

- *Tests and Examinations*

  Generally, there are both tests and examinations in schools. Nevertheless, teachers should note the following when setting tests or examination papers.

  - The coverage in the paper should be complete and the item format should be diversified.
  - Each item should have clear assessment objective(s).
  - The levels of difficulty of items should reflect and cater for students' diverse abilities.
  - The number of items in each paper should be reasonable.
  - The language used in the paper should be simple, clear and straightforward.

  Before setting a test or examination paper, teachers should prepare a specification table with allocation of marks to different learning units/objectives clearly shown. The allocation of marks should reflect the aims and focus of the paper and ensure adequate coverage of topics. The paper should embrace various types of items like explanatory, fill in the blanks, and multiple choice to assess students' knowledge in various aspects of mathematics. Open-ended questions should also be included to assess students' critical thinking skills, creativity and communication skills (refer to Examples 2, 5 and 14 of this Guide for details).

The incorporation of peer assessment and self-assessment in various assessment activities is also beneficial as they allow students to learn from peers and understand their own learning. These assessments also direct students to consider the main learning objectives, skills involved and perform self-reflection on the

learning process and attitude. Learner diversity should also be considered in designing assessment activities. For example, there should be a balance among the number of elementary, intermediate and advanced problems in tests and examinations.

The use of e-assessment may enhance learning and teaching by providing instant feedback to both students and teachers. Students can base on the feedback to perform self-reflections when they are still working on the tasks, while teachers can understand students' performance when their instruction is still in progress. e-Assessment can also help adjust the levels of difficulty of assessment items to embrace learner diversity. Besides, students' work and assessment data can be stored for future analysis.

On the whole, the most appropriate assessment activities for gathering information about students' performance should be decided according to how the required information will be used and the kind of performance that will provide the information required. No matter which assessment activities are adopted in collecting information about students' learning, the main purpose of assessment should always be for facilitating and improving students' learning.

5.2.3 Promoting Formative Assessment

Assessment of learning has long been a usual practice in schools. Many schools have now incorporated assessment for learning and assessment as learning into their assessment policy to enhance learning and teaching effectiveness. Besides, schools have put effort in using diversified modes of assessment to get a full picture of student learning and at the same time embrace learner diversity and enhance self-directed learning. In recent years, the Learning Progression Framework (LPF) and Student Assessment Repository (STAR) have been developed by the EDB to promote assessment for learning and assessment as learning in schools.

Learning Progression Framework

In support of the curriculum reform and the recommendations detailed in the report *Learning to Learn - The Way Forward in Curriculum Developmen*t (CDC, 2001), schools are encouraged to put emphasis on assessment for learning as an integral part of the learning-teaching-assessment cycle. The LPF for Chinese Language,

English Language and Mathematics have been developed to articulate students' performance and help teachers plan strategically to enhance learning and teaching in the three subjects from primary one to secondary six.

The LPF for Mathematics provides a common language for teachers to describe students' learning progress in mathematics across primary and secondary levels. It consists of different levels of Learning Outcomes (LOs) and descriptors about the growth of students on a continuum as they work towards the learning objectives set out in the Mathematics Education curriculum. The continuum is divided into eight attainment milestones for each of the three strands. They are expressed in the form of outcome statements which give a general description of students' performance. There are pointers as specific examples of what students are able to do in demonstrating the LOs, which support the outcome statements. It should be noted that the pointers are meant to elucidate the LOs. They are neither prescriptive nor exhaustive, and must not be regarded as a checklist for mathematical development. The LPF for Mathematics is available at the website *http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/index.html* (under the links "Primary One to Six", "Secondary One to Three" and "Secondary Four to Six").

Student Assessment Repository

The Student Assessment Repository (STAR) is an online assessment bank currently under development for the three subjects of Chinese Language, English Language and Mathematics with a view to enhancing teachers' assessment literacy and improving students' learning through promoting assessment for learning and assessment as learning with the use of technology. The STAR platform will be made accessible to both students and teachers.

The STAR provides diversified assessment items on mathematical knowledge and skills, which are developed with reference to the learning outcomes and pointers of the LPF for Mathematics and designed to suit the online environment. Students can take the initiative or be assigned by their teachers to work on assessment tasks on the STAR platform, and reports for diagnosing students' performance will be generated on completion of the tasks attempted. Through the STAR, teachers can understand students' attainment and plan for remediation or progression in teaching. Students can also be guided to use the STAR for self-directed learning.

5.3 Internal and External Assessment

5.3.1 Internal Assessment

Internal assessment refers to the assessments of different modes and scales that schools employ as an integral part of learning and teaching. Teachers are encouraged to incorporate diversified modes of formative and summative assessment tasks as suggested in Section 5.2.2 to provide timely feedback for enhancing learning and teaching.

As discussed in Section 5.1 and illustrated in Figure 7, feedback is a crucial component of the learning, teaching and assessment cycle. Without proper feedback, assessment becomes meaningless. However, feedback does not mean “praise" or “blame" and is not identical to “guidance". Effective feedback is able to help students know what they can and cannot do and where their strengths and weaknesses lie, and it also tells them what to do to make improvement. Mathematics teachers should note the following points about feedback:

- Feedback from assessment can be in verbal or written form. It should be related to particular quality of students’ work and give advice on what they should do to make improvement. Comparisons among students should be avoided.
- Timely feedback on the basis of formative assessment is effective and should be given wherever possible. For example, homework should be marked and promptly returned to students with appropriate comments whenever necessary.
- The STAR or school-based e-assessment platform can help teachers get a quick analysis of students’ learning and provide timely feedback to students. Through careful selection of suitable assessment items for students, teachers can provide more constructive feedbacks to them according to the data provided by the e-assessment platform. Students can then acquire a clearer picture of their own performance and learn to adjust their learning objectives and strategies.
- Feedback from summative assessment activities can provide information for students to plan their subsequent study, and for teachers to plan the next teaching sequence, and adjust the breadth and depth of the school Mathematics curriculum for the subsequent term or year.

- Communication between teachers and parents is essential for establishing a partnership between schools and families in supporting students' learning. Based on the evidence collected from assessment activities, parents could be provided with more information on how to improve their children's learning through different channels. Through the feedback from teachers, parents can better understand the learning objectives and collaborate with teachers to support their children's learning.

In sum, schools need to formulate their policy on internal assessment according to their context, teachers' experience and students' needs and interests. The assessment policy should be balanced in terms of assessment activities and reporting of the information collected. Learning objectives, learning activities and assessment tasks should be aligned to ensure that what is intended to teach and be learnt will be properly achieved.

5.3.2 External Assessment

External assessments, including public examinations, are assessments conducted by external organisations (such as the Hong Kong Examinations and Assessment Authority) for evaluating the learning progressions or outcomes of students by the end of a specific stage of education. For the Mathematics Education curriculum, students have to take various external assessments after their completion of their learning at several key stages for different purposes.

- Primary and junior secondary: Mathematics assessments of Basic Competency Assessment Research Study at P3 and the Territory-wide System Assessment (TSA) at P6[11] and S3
- Senior secondary: Hong Kong Diploma of Secondary Education (HKDSE) Examinations for Mathematics (Compulsory Part, Extended Part - Module 1 and Extended Part - Module 2)

Feedback from external assessment also plays an important role to inform learning and teaching. The results of the HKDSE Examination provide feedback to students on their performance and to teachers and schools on the quality of the teaching provided. Regarding Basic Competency Assessment Research Study and TSA, each participating school will receive a school report containing information on

[11] TSA at P6 is implemented in alternate year starting from 2011.

students' performance in different strands of the curriculum. With the report, schools can identify their students' strengths and weaknesses and adjust the teaching time, sequence and strategies accordingly. For example, if the report of a school shows that the performance of S3 students in using the conditions for congruent triangles to perform simple proofs is unsatisfactory, the school may adjust the learning and teaching strategies of this topic in S3 and related topics at the senior secondary level. However, when designing related classwork and homework, schools are advised to help students consolidate and extend their learning based on their established foundation and to avoid mechanical drilling. The same principal applies to primary schools.

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Chapter 6
Learning and Teaching Resources

Chapter 6 Learning and Teaching Resources

6.1 Quality Textbooks and Other Learning and Teaching Resources

Learning and teaching resources provide students with a basis for enriching their learning experiences. The effective use of learning and teaching resources helps students extend and construct knowledge and consolidate what they have learned, and is therefore an important factor for the successful implementation of the school Mathematics curriculum. Schools need to select, adapt and, where appropriate, develop relevant resources to support student learning.

Among the variety of learning and teaching resources, textbooks are usually regarded as the most common type. However, textbooks are not the curriculum but tools to facilitate the learning and teaching of the curriculum. Before teaching, teachers are encouraged to refer to this guide and its supplement for learning content, curriculum planning, and learning and teaching of the primary, junior secondary and senior secondary Mathematics curriculum[12].

Other than textbooks, teachers should develop or select relevant learning and teaching resources to help students achieve the target learning objectives.

6.1.1 Textbooks

Nowadays students learn in a dynamic learning environment and from a variety of learning resources, resulting in broadened learning experiences. However, textbooks still play an important role in mathematics education in such an environment, and have great influence on the learning and teaching that takes place in the mathematics classroom. Textbooks are also materials for students' self-directed learning, preparation and revision. Therefore, well-written Mathematics textbooks should align with the aims of the Mathematics Education curriculum and facilitate effective learning.

[12] Before the implementation of the revised Mathematics curriculum (P1 – S6), teachers may refer to the *Mathematics Curriculum Guide (P1-P6)* (2000), *Syllabuses for Secondary Schools – Mathematics (Secondary 1 - 5)* (1999) and *Mathematics Curriculum and Assessment Guide (Secondary 4-6)* (with updates in November 2015).

A set of guiding principles for quality textbooks has been developed by the EDB to provide criteria for schools and teachers in choosing quality textbooks for their students. It is accessible at *http://www.edb.gov.hk/en/curriculum-development/resource-support/textbook-info/GuidingPrinciples/index.html*.

Schools can make reference to the Recommended Textbook List (RTL) and the Recommended e-Textbook List (eRTL) at *www.edb.gov.hk/rtl* and *www.edb.gov.hk/ertl* respectively when selecting textbooks. All textbooks placed on the RTL or eRTL are written in line with the curriculum documents issued by the CDC and deemed acceptable in terms of the content, learning and teaching, structure and organisation, and language used.

In the selection of Mathematics textbooks, schools and teachers have to consider:

- Whether the sequencing of topics is compactable with the school Mathematics curriculum and meets students' abilities for effective development of mathematical concept, generic skills, values and attitudes;
- Whether different learning activities are included for students to investigate, discuss, make conjectures, and test hypotheses;
- Whether the examples and illustrations of mathematical concepts and skills meet the needs and abilities of students;
- Whether the learning activities provided by the textbooks allow teachers to engage students in applying their mathematical knowledge to solve problems, including real-life problems; and
- Whether peer and self-assessments are incorporated in learning activities to help students assess their own learning and reflect on what and how they have learned for further improvement.

In choosing Mathematics textbooks for use by students, schools should set up mathematics textbook selection committees to evaluate the relevance and suitability of the textbook in meeting the educational needs and abilities of their students. The extent to which teachers can exercise professionalism in adapting the contents to meet the learning and teaching objectives should also be taken into consideration.

6.1.2 Learning and Teaching Resources Other than Textbooks

Textbook is only one of the many learning and teaching resources. Other resources for the learning and teaching of mathematics may include:

- physical models and teaching aids (such as counters, blocks and geometric models);
- audio-visual materials (such as ETV programmes);
- printed resources (such as library books, magazines, journals, and reference materials developed by the EDB);
- e-resources (such as commonly used application software for desktop computers or mobile computing devices and resources available on the school e-platform), including Internet resources (such as web-based graphing tools and short videos on mathematical topics); and
- community resources (such as statistical reports and graphs in the newspaper and resources provided by professional bodies).

Using textbooks as the only resource is not sufficient to achieve the aims and objectives of the Mathematics Education curriculum. Adaptation of textbooks and other learning and teaching resources may be necessary to meet the needs and abilities of different students. When new mathematical concepts are to be constructed, manipulative materials can be provided and suitable apps can be used to help students visualise abstract concepts. When mathematical skills are to be sharpened or facts to be memorised, consolidation activities are needed. When independent work is assigned, activity cards, projects and application software can be used. Providing quality learning and teaching resources to support the implementation of the Mathematics curriculum is therefore basic to effective teaching. At the same time, teachers could develop their learning and teaching resources to suit their students.

A list of examples of websites and application software for the learning and teaching of mathematics is provided in Appendix 4 of this Guide for teachers' reference.

6.1.3 Resources in Support of Curriculum Development

Education Bureau Resources

The EDB has developed a variety of resource materials for primary and secondary teachers to support the implementation of the Mathematics Education curriculum and enhance the effectiveness of the learning, teaching and assessment of mathematics. Some of the resource materials are listed below as examples:

- EDB One-stop Portal for Learning and Teaching Resources (*http://www.hkedcity.net/edbosp/*)
  It provides learning and teaching materials for the learning units of different key stages.
- Learning and teaching resource packages, such as Addenda Series for Primary Mathematics
- Printed materials, such as Mathematics Cabinet Series and School Mathematics Newsletter
- Website of the Mathematics Education KLA (*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/index.html*)
- The Assessment Tasks Reference (*www.hkedcity.net/edbatr*)
  It provides assessment tasks on various topics
- Web-based Learning and Teaching Support (WLTS) (*http://wlts.edb.hkedcity.net/*)
- Educational Television (ETV) programmes (*https://www.hkedcity.net/etv/en/?currId=71*)

For more details of the EDB resources, please refer to Appendix 5 of this Guide or visit the homepage of the Mathematics Education KLA at: *http://www.edb.gov.hk/cd/maths/*

Collaborative Projects

To facilitate the implementation of the primary and secondary Mathematics curricula and support teachers further, the EDB has engaged in a series of collaborative research and development ("Seed") projects with schools since September 2001. These projects are geared towards promoting the learning

capabilities of students, generating good practices, suggesting useful teaching methodologies and developing a critical mass in order to achieve the aims of the school Mathematics curriculum.

Since 2001, more than 30 "Seed" projects have been initiated by the Mathematics Education Section and more than a hundred primary and secondary schools have participated in them. The key concerns of these projects include:

- curriculum planning and learning and teaching strategies;
- development of critical thinking and creativity through learning activities;
- assessment for learning; and
- focal points of ongoing curriculum renewal, e.g. STEM education.

Based on the experience gained in these projects, learning and teaching packages are produced and disseminated to schools and mathematics teachers in the professional development programmes. For the list of "Seed" projects conducted, please refer to Appendix 6 of this Guide or visit: *https://cd.edb.gov.hk/seed/chi/seed.asp*.

Community Resources

Authentic materials such as advertisement leaflets, statistical reports presented on the Internet and graphs printed in the newspaper could provide up-to-date information that could easily arouse students' interest in learning. Different organisations in the community also organise seminars and forums to familiarise teachers with the current development in mathematics education. Some of the community resources are listed in Appendix 5 of this Guide for teachers' reference.

6.2 Effective use of Learning and Teaching Resource

Appropriate use of learning and teaching resources helps create meaningful learning experiences for students. Not only can it facilitate the interaction between teachers and students, it can also broaden students' learning experiences and embrace learner diversity. Apart from textbooks and reference books, resources such as real models of 3-D figures, online learning materials, computer software packages, mobile application software, e-learning platforms, and community resources, do play a key role in student's learning. Therefore an effective way of

using these learning and teaching resources is important. In preparing Mathematics lessons, teachers need to

- refer to the learning objectives as stated in the curriculum documents;
- consider whether the learning and teaching resources:
  - provide a means for students to acquire the mathematical concepts or master necessary skills;
  - promote students' interest and active involvement in learning mathematics;
  - provide students with the opportunity to explore, discuss and co-construct knowledge to nurture their mathematical thinking and logical reasoning;
  - provide materials at different levels of difficulty and related learning experiences to cater for students' diverse needs and abilities;
  - can complement or extend the content of the Mathematics textbooks, enable students to make connections and achieve a wider understanding of concepts and skills; and
  - help students develop their positive values and attitudes as well as generic skills and self-directed learning strategies;
- adapt the curriculum contents, textbooks and learning and teaching resources to meet the needs and abilities of different students; and
- choose on-line resources from reliable sources, such as getting statistical data from websites of government departments or official websites of relevant organisations.

Nowadays, the adoption of e-resources has become a common practice in the mathematics classroom. Teachers make use of the interactive and real-time functions of e-resources in their lessons to cater for the diversified needs of students and to build up an effective learning community in class so as to enhance students' learning effectiveness. Besides, teachers are empowered to play a better role of facilitators and students are provided with more supports for their self-directed learning. However, students should be reminded to use non-electronic resources whenever appropriate so that they can understand the mathematical concepts and skills more comprehensively. Teachers are encouraged to bring real-life scenarios to the classroom to allow students to apply what they have learnt in real-life situations.

6.3 Resource Management in Schools

Proper use of resource materials in teaching mathematics can enhance students' learning effectiveness and make learning and teaching process more interesting. However, it is important that available resources for the learning and teaching of mathematics are well managed. Below are some suggestions on resources management:

- Schools should make good and flexible use of grants to build up a suitable stock of both electronic and non-electronic resource materials for the learning and teaching of mathematics.
- An up-to-date list of resources, with resources grouped under proper categories, is suggested to be uploaded to the school intranet for teachers' easy reference. Teachers are encouraged to make flexible use of different kinds of learning and teaching materials.
- If space is available, new resources and learning packages should be displayed for teachers' information and perusal.
- Small workshops or experience-sharing sessions can be conducted to provide teachers with a better understanding of the resources available.
- The stock of library books suitable for teachers and students on the academic and recreational aspects of mathematics could be increased. It is also advisable to compile or update the list of available mathematics journals and reference books in the school library for teachers' perusal.
- Regarding e-resources, teachers are encouraged to
    - check that webpages or on-line tools are still readily available on the Internet;
    - compare software of similar functions and choose an appropriate one for their students;
    - choose appropriate e-platforms to facilitate sharing and communication among students and teachers, such as sharing learning and teaching resources with appropriate measures to protect intellectual property rights; and
    - form communities of practice among teachers within and outside schools to investigate, reflect on and share experiences in the use of different types of e-resources.

School may refer to Chapter 7 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 10 of the *Secondary Education Curriculum Guide* (2017) for more ideas on learning and teaching resources.)

Examples

This Curriculum Guide includes a number of examples, which illustrate how the spirit of the Mathematics curriculum can be realised through activities, how the focal points of the ongoing curriculum renewal can be promoted, how generic skills can be fostered and how learner diversity can be catered for. The table below shows the main concerns of the examples.

Title	Key Stage and Strand	Main Concerns	Major Generic Skills Fostered*	Page
1. Hand in Hand	KS1 Number	Interface	◆ Communication ◆ Critical thinking ◆ Problem solving	p.112
2. Sharing Game	KS1 Number	Diversified Learning activities	◆ Communication ◆ Problem solving	p.117
3. Making Your Own Measuring Cup	KS1 Measures	Project learning	◆ Collaboration ◆ Critical thinking ◆ Problem solving	p.122
4. Discovering Symmetry	KS1 Shape and Space	Learner diversity	◆ Communication ◆ Creativity ◆ Critical thinking	p.126
5. Finding Area	KS2 Measures	Diversified Learning activities	◆ Communication ◆ Creativity ◆ Problem solving	p.130
6. Rubber Band Powered Car	KS2 Measures	STEM education	◆ Collaboration ◆ Creativity ◆ Problem solving	p.133
7. Cross Sections of 3-D Shapes	KS2 Shape and Space	e-Learning	◆ Information technology ◆ Self-learning	p.138
8. Knowing Your Community	KS2 Shape and Space, Data Handling	Project learning	◆ Collaboration ◆ Creativity ◆ Critical thinking ◆ Problem solving	p.141

Title	Key Stage and Strand	Main Concerns	Major Generic Skills Fostered*	Page
9. Extra-Curricular Activities	KS2 Data Handling	Project learning	◆ Collaboration ◆ Communication ◆ Critical thinking	p.145
10. Four Seasons	KS2 Data Handling	STEM education	◆ Creativity ◆ Critical thinking ◆ Information technology	p.148
11. Investigating Errors of Measurements by GPS Tracking Apps	KS3 Number and Algebra	STEM education	◆ Communication ◆ Critical thinking ◆ Information technology ◆ Problem solving	p.153
12. Design a Healthy Diet Menu	KS3 Number and Algebra, Data Handling	STEM education	◆ Collaboration ◆ Creativity ◆ Problem solving	p.156
13. Mathematics Magic	KS3 Number and Algebra	Project learning	◆ Collaboration ◆ Communication ◆ Critical thinking ◆ Problem solving	p.158
14. Open-ended Geometric Problem	KS3 Measures, Shape and Space	Diversified modes of assessment	◆ Communication ◆ Critical thinking ◆ Problem solving	p.162
15. Slopes of Perpendicular Lines	KS3 Measures, Shape and Space	e-Learning	◆ Information technology ◆ Problem solving ◆ Self-learning	p.164
16. Volume of Frustums	KS3 Measures, Shape and Space	History of mathematics	◆ Communication ◆ Creativity ◆ Problem solving	p.167

Title	Key Stage and Strand	Main Concerns	Major Generic Skills Fostered*	Page
17. Surface Areas of Prisms	KS3 Measures, Shape and Space	Learner diversity	◆ Communication ◆ Critical thinking ◆ Problem solving	p.172
18. Flipping Measure Spoons	KS3 Measures, Shape and Space	STEM education	◆ Critical thinking ◆ Self-learning	p.178
19. Translations of Functions	KS4 Number and Algebra	e-learning	◆ Creativity ◆ Information technology	p.182
20. Modelling the Spread of a Disease	KS4 Number and Algebra	STEM education	◆ Critical thinking ◆ Problem solving	p.189
21. Return and Risk	KS4 Data Handling	Entrepreneurial spirit	◆ Critical thinking ◆ Problem solving	p.195
22. Mathematics Reading Scheme	KS1-4	Language across the Curriculum	N.A.	p.202

*Note: "Mathematical Skills" is not included in the list as all examples involve development and applications of Mathematics Skills in different aspects.

Example 1
Hand in Hand

Key Stage: 1

Strand: Number

Learning Unit: Numbers to 20

Objective: To understand odd and even numbers

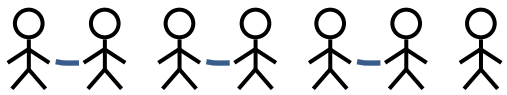
Prerequisite Knowledge: Understand numbers 1-10

Resources Required:
(i) Beads
(ii) Worksheets

Description of the Activity:

Activity 1

1. The teacher chooses a group of not more than 10 (e.g. 7) students. Every student in the group tries to find a partner and stands hand in hand with him/her. Students record the results on the worksheet "Hand in Hand", e.g.

No. of Students	Draw lines joining hands to indicate how the classmates stand hand in hand	Does everyone get a partner? (✓ or ×)
7		

2. Repeat Step 1 with different numbers of students in a group and record the results on the worksheet.

Questions for discussion:

1. Every two students in a group will hold hands together. Will everyone get a partner?
2. If there are 7 students in a group, how many students could be added so that each student gets a partner? At least how many students should be added?
3. In what situation can each student get a partner? In what situation is a student left without a partner?
4. The teacher chooses a group of not more than 10 students to enter the "three-legged

race" competition. The rules of the competition require teams of 2 students. How many students can be chosen? (Students can provide more than one answer.)

Activity 2

1. The teacher gives each student 10 beads. When the teacher says a number (e.g. 5), each student takes out the corresponding number of beads and arranges them in pairs.
2. Students record the results in Part A of the worksheet "Odd and Even Numbers".
3. Students observe and compare the results, and divide the numbers into two types. Then the teacher introduces the concept of "odd numbers" and "even numbers".

Questions for discussion:

1. Observe the arrangements of beads, what are the similarities and differences?
2. Based on the above observation, how many types of numbers can 1 to 10 be divided into?
3. Which numbers are of the same type? What is the common characteristic of these numbers?

Activity 3

1. Students generalise a conclusion based on the discussion in Activity 2, and complete Part B of the worksheet "Odd and Even Numbers".
2. Students observe the results in Part B of the worksheet. Then they discuss and find out the relationship between odd numbers and even numbers.

Questions for discussion:

1. Which numbers are odd numbers? Which numbers are even numbers?
2. What is the relationship between the order of appearing of odd numbers and even numbers?
3. You have odd number of beads. If you want to make it even, how many more beads should be added? What is the least number of beads that should be added? How many beads should be taken away? What is the least number of beads that should be taken away?
4. You have even number of beads. If you want to make it odd, how many more beads should be added? What is the least number of beads that should be added? How many beads should be taken away? What is the least number of beads that should be taken away?

This example mainly involves the following generic skills:

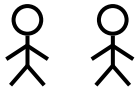
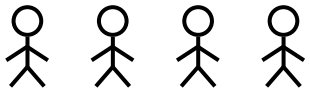
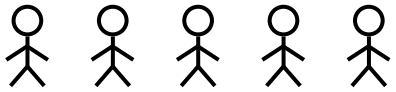
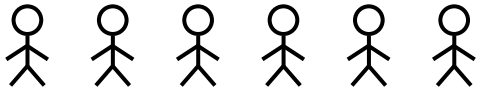
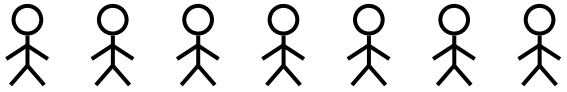
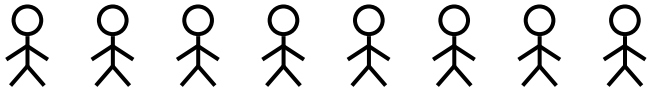
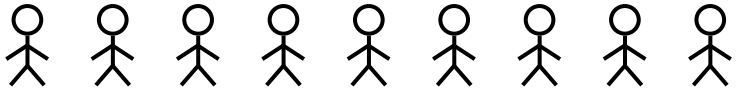
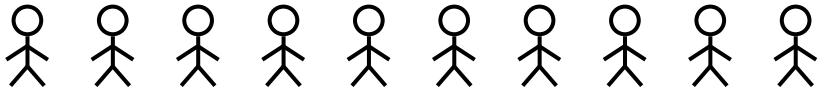
1. Communication Skills
    - Comprehend and act appropriately on spoken instructions, for example, by following the teacher's instructions to draw and record the findings on the worksheet
    - Present results of tasks with drawings and symbols
    - Discuss with others in accomplishing tasks, for example, by finding out the differences between odd and even numbers

2. Critical Thinking Skills
    - Reason inductively when exploring the characteristics of numbers
    - Draw logical conclusions based on adequate data and evidence

3. Problem Solving Skills
    - Adopt various ways of solving problems, for example, by finding ways to make an odd number even or make an even number odd

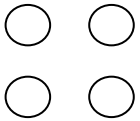
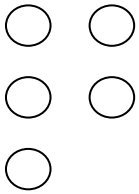
Hand in Hand

Can you record how the classmates stand hand in hand?

No. of students	Draw lines joining hands to indicate how the classmates stand hand in hand	Does everyone get a partner? (✓or ×)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Odd and Even Numbers

(A) Draw the arrangements of beads in the table below.

1	2	3	4	5	6	7	8	9	10
									

(B) Colour the boxes with **odd numbers** in **red** and **even numbers** in **blue**.

1	2	3	4	5	6	7	8	9	10

Example 2
Sharing Game

Key Stage: 1

Strand: Number

Learning Unit: Division (I)

Objective:
(i) To understand the meaning of equal sharing
(ii) To perform equal sharing of objects

Prerequisite Knowledge: Basic multiplication

Resources Required:
(i) Pencils, beads
(ii) Worksheet

Description of the Activity:

Activity 1

1. The teacher shows students 6 pictures of an apple and asks a student to share the apples evenly between two classmates. Then the student repeats the sharing using 7 apples.
2. The teacher asks students to pay attention to the sharing process and helps students to recognise the concept of equal sharing.
3. Students repeat the sharing of apples among three and four classmates. By comparing the sharing processes and the respective results, students further recognise the concept of sharing.

Questions for discussion:

1. If we do not know the total number of objects, how could we share the objects evenly between 2 people?
2. What are the similarities and differences in the processes and results between the cases of sharing 6 apples and 7 apples between two people?
3. When 6 apples are shared between two people, will each one get the same number of apples? Can all the apples be shared evenly? How about sharing among three or four people?
4. What are the similarities and differences in the processes and results when 6 apples are shared between two, three or four people?

5. If all the apples have been shared, but the amount of apples each person get is not the same, can this still be considered as equal sharing?

Activity 2

1. The teacher shows students 12 pencils. Students guess how many persons can share the pencils evenly and the number of pencils each person can get.
2. Students perform the real sharing and see if their guesses are correct or not.
3. The teacher reminds students to pay attention to the sharing processes and results. The teacher guides students to find out the relationship between the amounts of objects being divided and the numbers of people involved in the sharing.

Questions for discussion:

1. Can you guess how many people can share the pencils evenly? How many pencils will each person get?
2. Are your answers correct? How do you get the answers?
3. What other method(s) can be used to speed up the sharing process apart from sharing the pencils one by one?
4. How is the change in the number of pencils each person gets if more people share the pencils?
5. What is the relationship between the number of pencils, the number of people sharing the pencils and the number of pencils each person gets under the situation of equal sharing?

Activity 3

1. Students take the worksheet "Sharing Game" and 18 beads.
2. Students complete Questions 1 to 3.
3. The teacher guides students to find out the multiplication relation among the quantities.

Questions for discussion:

1. Will each person get the same number of pencils when 15 pencils are all shared among four people?
2. How many beads will each person get when 18 beads are shared evenly between two people?
3. If each person gets 9 beads, then how many beads do two people get in total? How is it calculated?
4. What is the relationship between the total number of beads, the number of people and the number of beads each person gets under equal sharing? Does this relationship still

hold when the beads cannot be shared evenly?

Activity 4

1. After students have returned the beads to the teacher, they complete Question 4 of the worksheet. The teacher encourages students to write down their ideas.
2. Students share their answers of Question 4e.
3. The teacher guides students to observe the answers in the worksheet and try to find out the characteristic or pattern of the sharing process. Students share their findings with others.

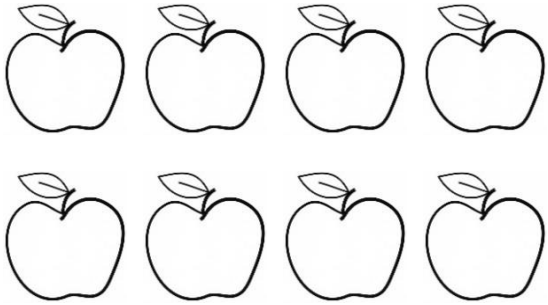
Questions for discussion:

1. In each activity of sharing objects, can the objects be shared evenly every time?
2. How to tell if the objects are shared evenly?
3. When the objects are shared evenly, how can we find out the number of objects that each person gets?
4. If a certain number of objects can be shared evenly into two portions, what is the characteristic of the amount?

This example mainly involves the following generic skills:

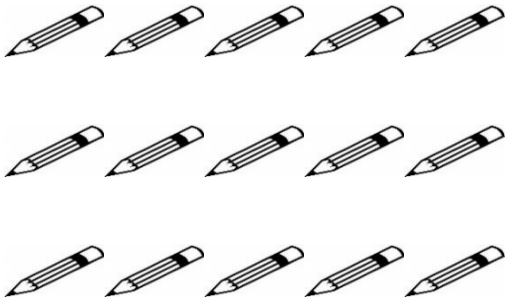
1. Communication Skills
   - Explain how do they know that their answers are correct
   - Explain how the problems are solved
   - Explain the differences between the cases
2. Problem Solving Skills
   - Transfer knowledge of basic multiplication and solve the problems
   - Use real objects and diagrams to assist the thinking process
   - Record results systematically

Sharing Game

1. 

a) Can the apples be shared evenly between two people? (Yes / No)
If yes, each one can get _____ apples.

b) Can the apples be shared evenly among four people? (Yes / No)
If yes, each one can get _____ apples.

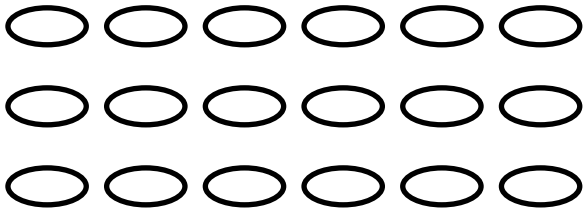
2. 

a) Can the pencils be shared evenly between two people? (Yes / No)
If yes, each one can get _____ pencils.

b) Can the pencils be shared evenly among three people? (Yes / No)
If yes, each one can get _____ pencils.

c) Can the pencils be shared evenly among five people? (Yes / No)
If yes, each one can get _____ pencils.

d) Can the pencils be shared evenly among four people? (Yes / No)
If yes, each one can get _____ pencils.

3. 

a) Can the beads be shared evenly between two people? (Yes / No)
If yes, each one can get _____ beads.

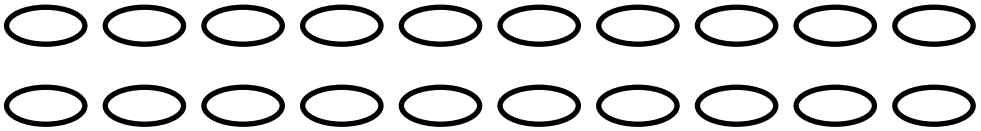
b) Can the beads be shared evenly among three people? (Yes / No)
If yes, each one can get _____ beads.

c) Share among six people, each one can get _____ beads.

d) Share among nine people, each one can get _____ beads.

e) Can the beans be shared evenly among four people? (Yes / No)
If yes, each one can get _____ beads.

f) 18 beads also cannot be shared evenly among _____ people.

4. 

a) Evenly shared between two people, each one can get _____ beads.

b) Evenly shared among four people, each one can get _____ beads.

c) Evenly shared among five people, each one can get _____ beads.

d) Evenly shared among ten people, each one can get _____ beads.

e) How do you get these answers? Try to explain.

Example 3
Making Your Own Measuring Cup

Key Stage: 1

Strand: Measures

Learning Unit: Capacity

Objective: To consolidate students' concept on measurement and capacity through making a measuring cup

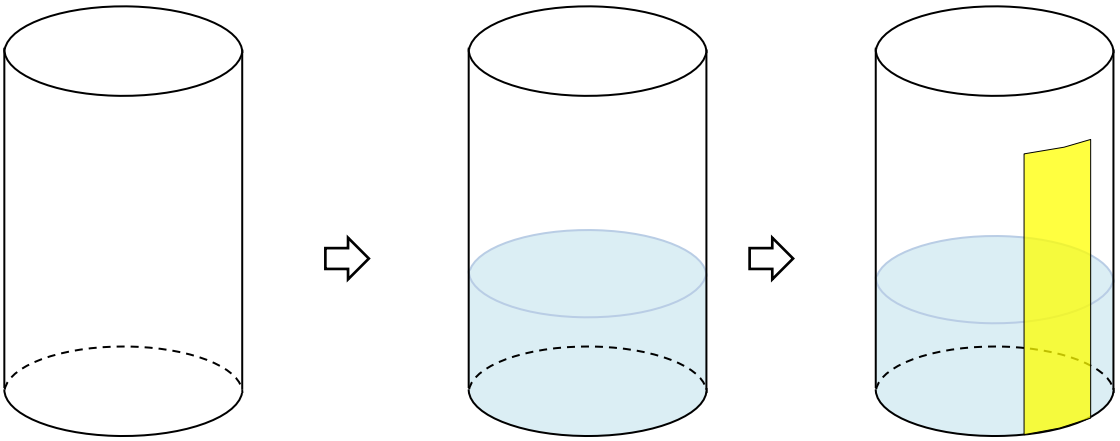
Prerequisite Knowledge:
(i) Compare the capacity of containers directly
(ii) Recognise "litre" and "millilitre" (or "L" and "mL") as standard units of capacity

Resources Required: Transparent containers with capacity for more than 1L, 500mL measuring cups, some daily life containers of known capacities, papers strips and adhesive tape

Description of the Activity:

Activity 1:

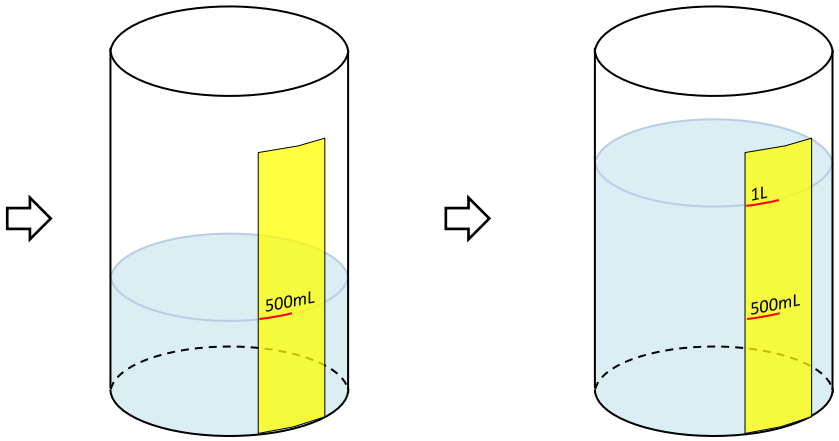
1. The teacher shows some transparent containers with capacity for more than 1L, which are commonly found in our daily life.
2. Students form groups of four or five. Each group is given a 500mL measuring cup and a transparent container.
3. The teacher asks the students to fill the measuring cup with 500mL of water and then pour the water into the transparent container.
4. Students stick a paper strip vertically outside the transparent container, and mark "500mL" on the paper strip at the height of the water level.
5. Students pour another 500 mL water into the transparent container and mark "1L" on the paper strip at the height of the water level (as shown in the figure on the next page).



Transparent container

Fill in 500mL of water

Stick the paper strip



Mark “500mL” at the water level on the paper strip

Mark “1L” at the water level on the paper strip

Questions for discussion:

1. Can the transparent container be used as a measuring cup to measure the capacity of other containers?
2. What are the drawbacks of this self-made measuring cup? (can measure 500mL and 1L only)
3. How will you improve it?

Notes for Teachers:

1. The teacher should make sure that students have a clear direction to improve their self-made measuring cup.
2. The teacher may use containers with a uniform cross-section or other containers for the activities.
3. If the cross sections of the container are not uniform, the marking of "500mL" may not lie exactly in the middle between the marking of "1L" and the lower end of the paper strip.

Activity 2:

1. Students are given a period of time (e.g. two weeks) to improve the self-made measuring cup.
2. Then each group presents how they improve the measuring cup and the problems they encountered.
3. The teacher provides some containers (with capacity between 500mL and 1L preferred) for each group to test their measuring cup.

Notes for Teachers:

1. The teacher may allow students to conduct the first part of Activity 2 at home.
2. The teacher can decide whether to give hints to students in accordance with students' abilities.
3. The teacher can guide students to repeat the method in Activity 1 (i.e. to mark "500mL" and "1L") to locate markings like "200mL", "400mL", etc. using small measuring cups at home.
4. Students should be given adequate time to do the project.
5. Suggested criteria for assessing students' performance:

Criteria	Description of the Criteria
Understanding the theme of the project	• Do students mark "500mL" and "1L" at the appropriate positions? • Do students provide appropriate new markings?
Use of strategies and methods	• Do students use appropriate methods to locate the marking of "1L" instead of doubling the length of the water level of 500mL on the strips? • Do students develop a strategy to add some more markings on the paper strip?

Criteria	Description of the Criteria
Accuracy	• Do student give markings accurately with respect to the water level? • Do students demonstrate a strategy (e.g. repeat the measure for several times) to locate the markings in a more reliable manner?
Presentation and communication	• Can students elaborate their ideas on the design clearly? • Can students present their process on measurement clearly?
Attitude	• Do students work collaboratively? • Do students participate actively in the task?

The criteria suggested are by no means exhaustive and each criterion may have different weightings.

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Participate actively in discussing and selecting a suitable strategy for locating the markings for different capacities
   - Listen to others with patience, and explain own ideas in response to group members' views on selecting a suitable design for the measuring cup

2. Critical Thinking Skills
   - Find reasons to account for the non-uniform spreading of markings.
   - Identify advantages and limitation of their measuring cup and suggest some ways to improve it

3. Problem Solving skills
   - Develop a strategy to put new markings at appropriate positions

Example 4
Discovering Symmetry

Key Stage: 2

Strand: Shape & Space

Learning Unit: Symmetry

Objectives:
(i) To recognise symmetrical shapes
(ii) To identify symmetrical shapes
(iii) To make symmetrical shapes

Prerequisite Knowledge: Recognise regular and irregular 2-D shapes

Resources Required: Handicraft paper, scissors and worksheets

Description of the Activity:

Activity1

1. Each student is given a pair of scissors and a piece of handicraft paper. They cut out some simple shapes such as a heart or a butterfly in a simple way.
2. The teacher selects some of the students' shapes that are more or less symmetrical and posts them on the blackboard for discussion.

Questions for discussion:

1. What is the fastest way to cut the shapes on the blackboard?
2. Is there any difference between the shapes cut and not cut by folding the paper?
3. For the shapes cut by folding the paper, what are the features of the two halves along the folding line?

Activity 2

1. The teacher introduces the concepts of symmetrical shapes and lines of symmetry.
2. Each student is given some handicraft paper. They are asked to create symmetrical shapes by folding the handicraft paper.
3. The teacher selects some of students' shapes and posts them on the blackboard.

Questions for discussion:

1. Which of the shapes are symmetrical? What are their common properties?
2. How do we identify symmetrical shapes?
3. Are there any other methods to identify symmetrical shapes?
4. What is the difference between the shapes made by folding the paper once and twice?

Notes for Teachers:

1. Teacher can prompt students to fold the paper more than once when they have finished their first shape.
2. Besides folding, a mirror can also be used for identifying symmetrical shapes. Students can also be guided to observe that the line segment joining "overlapping points in folding" is perpendicular to the line of symmetry and also cut into two equal line segments by the line of symmetry.

Activity 3

1. The teacher puts students in groups of three or four.
2. Each group is given the worksheet "Symmetrical Shapes". (The teacher could also provide students with enlarged copies of the 2-D shapes in the worksheet to assist students in identifying symmetrical shapes.)
3. The teacher asks students to identify the symmetrical shapes and add lines of symmetry on those symmetrical shapes.

Questions for discussion:

1. Which shapes in the worksheet are symmetrical?
2. Where are the lines of symmetry for each symmetrical shape? (The teacher can post the shapes on the blackboard and ask students to draw the lines of symmetry.)
3. Which of the symmetrical shapes have more than one line of symmetry?
4. Can you find any symmetrical shapes in your daily life? (After the discussion, the teacher can ask students to collect pictures of symmetrical shapes at home.)

Notes for Teachers:

1. According to students' abilities, the teacher can decide if it is necessary to provide students with the enlarged symmetrical shapes in the worksheet to verify if the shapes are symmetrical by folding, measuring or using other methods.

This example mainly involves the following generic skills:

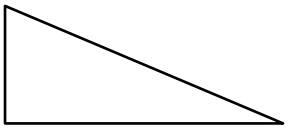
1. Communication Skills
   - Describe the features of symmetrical shapes
   - Discuss the advantages and disadvantages of the methods of identifying symmetrical shapes

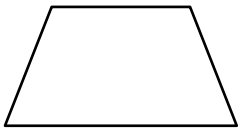
2. Creativity
   - Create symmetrical shapes by folding the handicraft paper

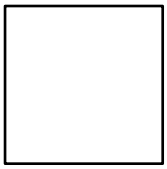
3. Critical Thinking Skills
   - Justify if the shapes are symmetrical according to the concept and properties of symmetrical shapes

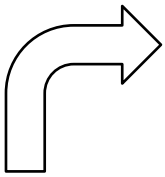
Symmetrical Shapes

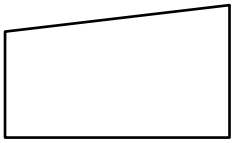
For the shapes below, identify the symmetrical shapes and add lines of symmetry.

1. 

2. 

3. 

4. 

5. 

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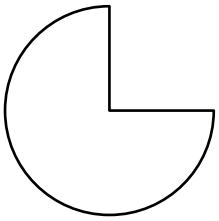
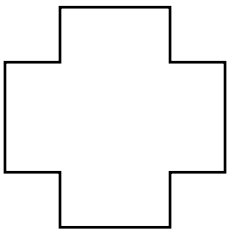
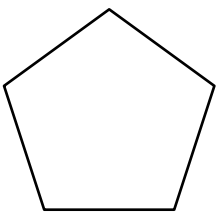
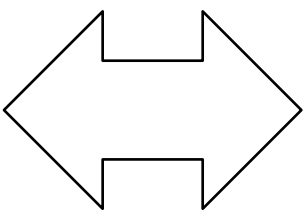
7. 

8. 

9. 

10. 

Example 5
Finding Area

Key **Stage**: 2

Strand: Measures

Learning Unit: Area (I)

Objective: To consolidate the concept of area

Prerequisite Knowledge: Finding the area of figures using the standard unit: square centimeter (cm^2)

Resources Required: Worksheet (grid paper and dotted paper)

Description of the Activity:

1. The teacher distributes the grip paper to students and guides them to answer the following questions:
   (a) Can you draw three figures that are of the same area as the shaded region but of different shapes?
   (b) How do you know that each figure drawn is of the same area as the shaded region?
2. When students have drawn the figures, the teacher could invite them to share their drawing strategies.
3. (Extension) The teacher distributes the dotted paper to students and guides them to answer the following questions:
   (a) Is the area of the shaped region on the dotted paper the same as that on the grid paper?
   (b) Can you create some figures on the dotted paper that are different from the shaded region in terms of both shape and area?
   (c) What are the areas of the figures created? (in cm^2)

Notes for Teachers:

1. Sufficient time should be allowed for students to draw the figures.
2. Figures drawn could be common ones. They could also be irregular polygons.

3. Students' understanding of the concept of area and creativity in drawing the figures are the foci of this activity.
4. Teachers can use the following criteria to assess students' performance:
   (a) Can students understand the concept of area? (Are the figures drawn closed figures?)
   (b) Can students express areas using square centimetres (cm^2) correctly?
   (c) Can students reach the requirements on the number and area of figures to be drawn?
   (d) Can students conclude their strategies applied in drawing the figures?

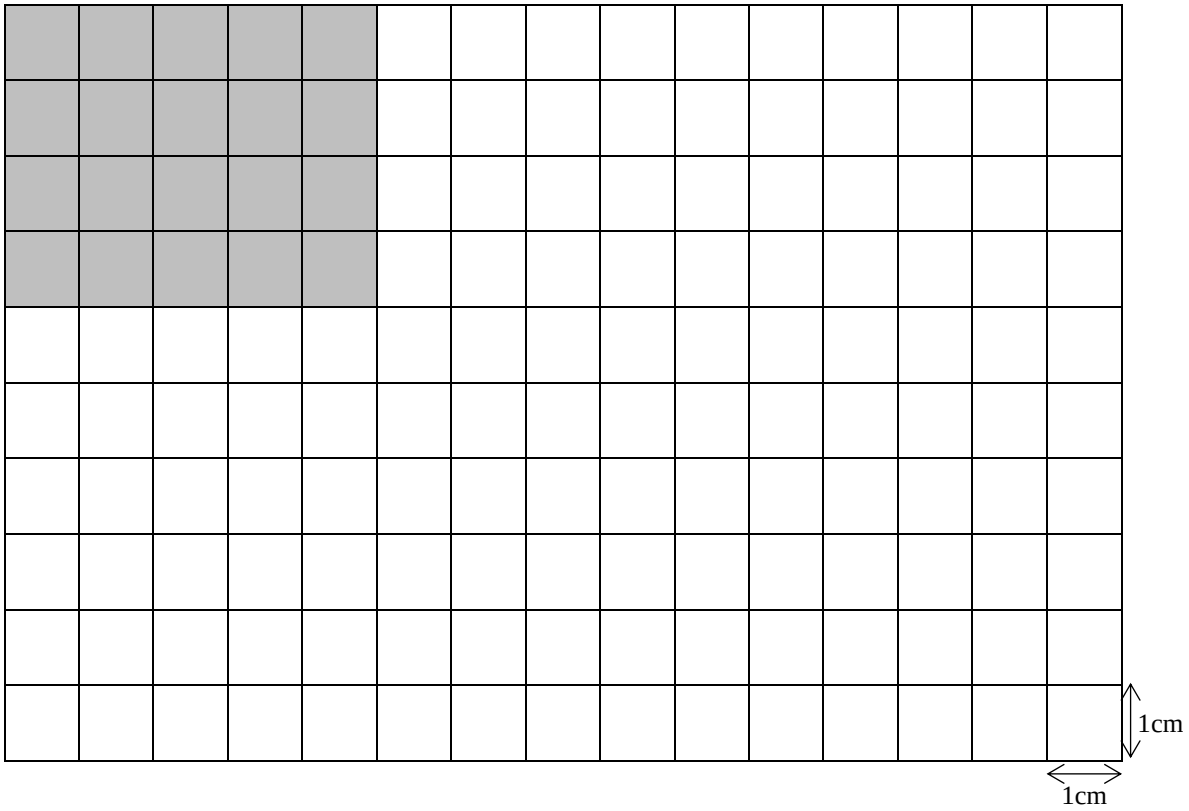
This example mainly involves the following generic skills:

1. Communication Skills
   - Explain, with the aid of diagrams, the reasons why the figures have the same area

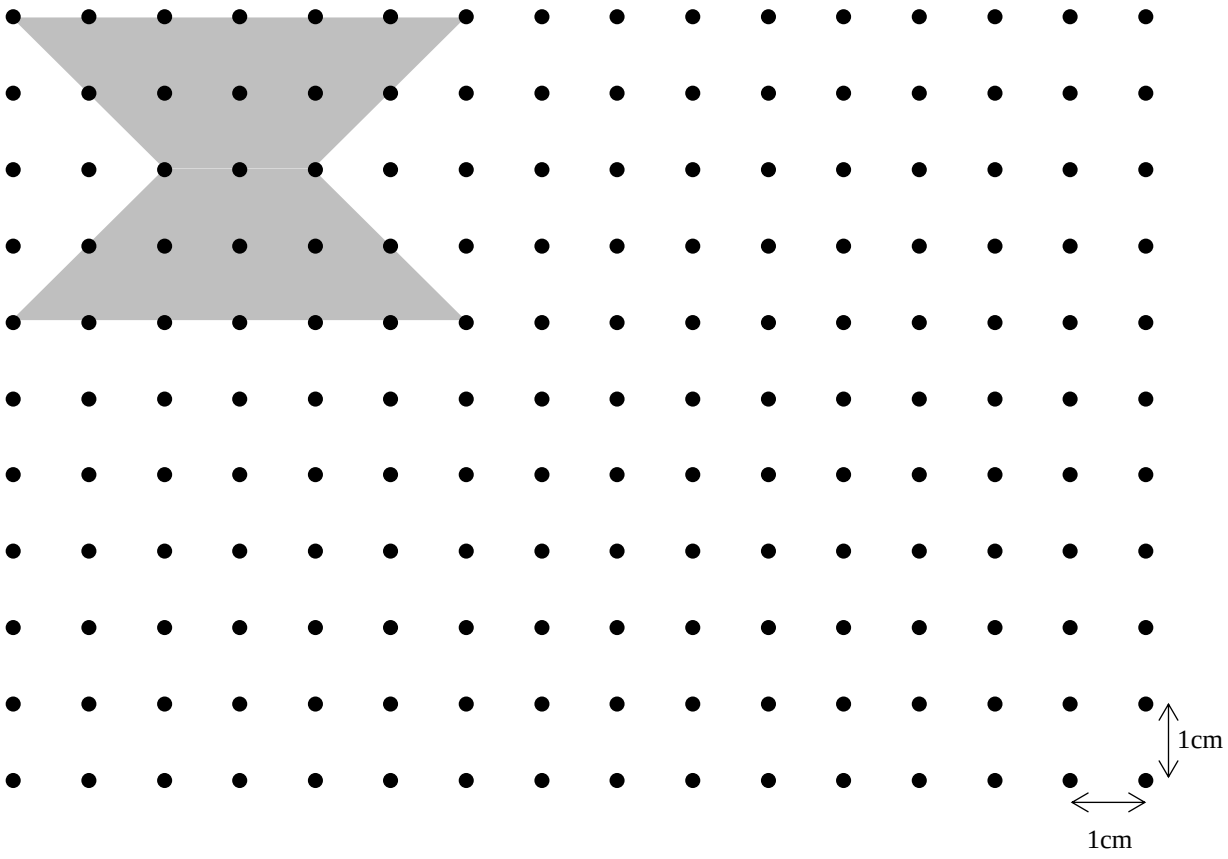
2. Creativity
   - Draw figures other than the common ones
   - Use different methods of finding the areas of figures

3. Problem Solving Skills
   - Use simple methods to find the areas of figures
   - Draw figures using different strategies
   - Apply previous knowledge to draw figures

Grid paper:



Dotted paper:



Example 6
Rubber Band Powered Car

Key Stage: 2

Strand:

Mathematics: Measures (Learning Unit: Speed)

General Studies: Science and Technology in Everyday Life
(Core Elements: Efficient transfer of energy and the interaction between energy and materials)

Objectives:

(i) To consolidate the concept of speed

(ii) To use "metres per second" (m/s) and "kilometres per hour" (km/h) as units of speed

(iii) To recognise efficient transfer of energy and the interaction between energy and materials

(iv) To design and build models by using different materials and to test selected functional characteristic of the model built with the chosen materials

(*In collaboration with General Studies*)

Prerequisite Knowledge: Understanding the relationship among speed, time and distance

Resources Required: Worksheets, stopwatches, measuring tapes and calculators

Related Links: Scientific Investigation Series - Potential Energy Car
http://www.hkedcity.net/edb/teachingresources/project/?p=science&path=/car
Science Magician
http://sites.google.com/site/sciencemagician/ (click the link "橡皮筋動力車")

Description of the Activity:

Activity 1

The teacher raises questions to consolidate students' concept of speed.

Questions for discussion:

1. If A and B run 100 metres in 20 seconds and 30 seconds respectively. Who runs faster on average?
2. If C and D run 50 metres and 60 metres in 10 seconds respectively. Who runs faster on average?
3. What is the relationship between time and speed if they run the same distance?

   What is the relationship between distance and speed if they run at the same time?
4. What information must be obtained to determine their running speed if they do not start in the same place?

Notes for Teachers:

Students are divided into groups and provided the related links. They could prepare and make the rubber band powered car prior to the lesson.





<u>Activity 2</u>

1. The teacher reads out Part 1 of the worksheet as an introduction. Students work in pairs and discuss the ways to compare two persons' speeds when the distances they run and the time they spend are different. The teacher guides students to work out the formula for finding average speed.
2. Each group is given the worksheet. Students complete Part 1 of the worksheet and present the results.
3. Students collect the data and complete Part 2 of worksheet by using their rubber band powered cars.

Questions for discussion:

1. If we obtain the distance an object moves and the time it spends to move, how can we calculate the average distance it moves in 1 second?

2. How can we calculate the average time it spends to move 1 metre?
3. Which method will you use to compare the speeds of A and B? Why?
4. The distances the cars of the two groups run and the time they spend are different. How can you compare their speeds?

Notes for Teachers:

1. The activities aim at measuring the speed of their rubber band powered cars, instead of the speed racing. There is no need to arrange the cars to start at the same place and the same time.
2. To save time, the teacher may allow students to complete Part 2 of the worksheet using a calculator.
3. The teacher can remind students of how to use the stopwatch and measuring tape for measurement.

Activity 3

The teacher discusses with students the energy conversion and the ways to improve the structure of the car.

Questions for discussion:

1. When a student is scrolling the rubber band, what kind of energy is stored?
2. When students put their cars on the ground and release the wheels, what kind of energy will be converted from the energy stored in the rubber band? What is the relationship between this kind of energy and the speed of the cars?
3. How can the speed and running time of the rubber band powered car be increased?

Notes for Teachers:

1. The teacher can remind students to refer to the design and materials used by the other groups in order to improve their own rubber band powered car.

Integration and Application:

Science Education: Conversion of energy

Technology Education: Model design and choosing appropriate materials

Mathematics Education: Concept of speed and measurement of speed

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Collaborate with classmates in group activities
   - Share responsibilities and understand the roles of individual members in the construction of the rubber band powered car

2. Creativity
   - Improve and refine the designs of the rubber band powered car

3. Problem Solving Skills
   - Propose different solutions to increase the speed and running time of the rubber band powered car
   - Try out and improve the solutions through different experiments

Worksheet

1. The running records of Student A and Student B are as follows:

	Distance	Time
A	100 m	20 s
B	150 m	25 s

How to compare the speed of A and B?

Method 1	Method 2
A has run _____ m in 1 second on average. B has run _____ m in 1 second on average. Student _____ runs faster, because on average the distance he/she runs is _____ in 1 second.	A has spent _____ s to run 1 m on average. B has spent _____ s to run 1 m on average. Student _____ runs faster, because on average he spends _____ time to run 1 m.
Which method do you like to use to compare their speeds? Why?	

2. Complete the following table and calculate the speed of rubber band powered car.

Distance (m)	Time (s)	Speed (m/s)

Remark: The speed calculated can be rounded off to 1 decimal place.

Example 7
Cross Sections of 3-D Shapes

Key Stage: 2

Strand: Shape and Space

Learning Units: 3-D Shapes (III)

Objectives:
(i) To recognise the concepts of the cross sections of a prism and a cylinder
(ii) To recognise that the sizes and shapes of the cross sections of a prism and a cylinder, which are parallel to the bases, are the same as that of the bases

Prerequisite Knowledge:
(i) Identify different 3-D shapes
(ii) Identify different 2-D shapes

Resources Required:
(i) Objects or plasticine shaped in 3-D figures
(ii) Physical models of 3-D figures (including triangular prisms, cuboids, cubes, triangular pyramids, rectangular pyramids, cylinders and cones)
(iii) Tablet computer
(iv) E-resources on cross sections of 3-D solids (such as https://sites.google.com/site/hkgeogebra/ks2/ss/3d_sections)

Description of the Activity:

Activity 1

1. The teacher shows some plasticine or objects shaped in prisms or circular cylinders (such as fruits and cakes.) to students, and cuts the objects or plasticine by a plastic knife or ruler in different directions. The teacher then introduces the cutting planes to students and calls the planes "cross sections".
2. Students are requested to guess the shapes of the cross sections of circular cylinders and circular cones, which are parallel to the base of the two 3-D shapes, by observing the physical models given by the teacher.
3. Students then use the tablet computers to access the e-resources to verify their guesses.

4. Students discuss the similarities and differences between the cross sections of circular cylinders which are parallel to their bases and those of circular cones, with the help of tablet computers.

Questions for discussion:

1. If a cylinder is cut in a direction parallel to the bases, are the shapes of cross sections obtained equal? How about cones?
2. If a cylinder is cut in a direction parallel to the bases, are the sizes of cross sections obtained equal? How about cones?
3. (Further discussion) If you want to get cross sections with the same shapes and sizes, how should you cut a cylinder?
4. (Further discussion) Can you get a rectangular section when cutting a cylinder/cone? How do you cut it?

Notes for Teachers:

1. If needed, the teacher may let students cut real objects to develop the concept of cross sections before using the software for exploration. The teacher should notice that cross sections which are parallel to the bases should be the major topic for discussion.
2. Students are requested to discuss the cross sections of cylinders first as it is a simpler case as compared with that of cones.
3. In order to facilitate explanation and discussion, students are reminded to capture the screen of the software when needed.

Activity 2

1. Students are requested to explore the cross sections of triangular prisms and triangular pyramids, which are parallel to their bases, with the help of tablet computers. The teacher provides assistance to students whenever necessary.

Questions for discussion:

1. What is the difference between the cross sections of a triangular prism, which are parallel to its bases, and those of a circular cylinder?
2. What is the difference between the cross sections of a triangular pyramid, which are parallel to the base, and those of a circular cone?
3. What are the similarities and differences between the cross sections of a triangular prism, which are parallel to the bases, and those of a triangular pyramid?
4. (Further discussion) Which types of polygonal cross sections can be obtained by cutting a triangular prism/pyramid?

Notes for Teachers:

1. When students have acquired enough experience in operating the software, teachers can let them explore the cross sections of cuboids and rectangular pyramids for self-learning.

This example mainly involves the following generic skills:

1. Critical Thinking Skills
   - Recognise the similarities and differences between cross sections of circular cylinders which are parallel to their bases and those of circular cones through observations, discussions, making guesses and verifications

2. Information Technology Skills
   - Use software packages for learning the concepts of cross sections

3. Self-learning Skills
   - Take the initiative in identifying and organising main points from sources, for example, record the figures of cross sections in different 3-D shapes for comparison

Example 8
Knowing Your Community

Key Stage: 2

Strands: Shape and Space, Data Handling

Learning Units: Directions and Positions (III) and Uses and Abuses of Statistics

Objectives:
(i) To find the eight compass points with a compass
(ii) To become familiar with the district in the vicinity of the school
(iii) To sketch the plan of the district in the vicinity of the school
(iv) To be aware of the facilities and services offered in the community
(v) To present the collected data with statistical charts

Prerequisite Knowledge:
(i) Finding the four directions with a compass
(ii) Constructing a simple bar chart

Resources Required: Compasses

Description of the Activity:

Activity 1:

1. Students use compasses to find the four directions and describe the community facilities in the vicinity of the school in the four directions.
2. Students use compasses to find the eight compass points and record the community facilities in the vicinity of the school in the eight directions.

Questions for discussion:

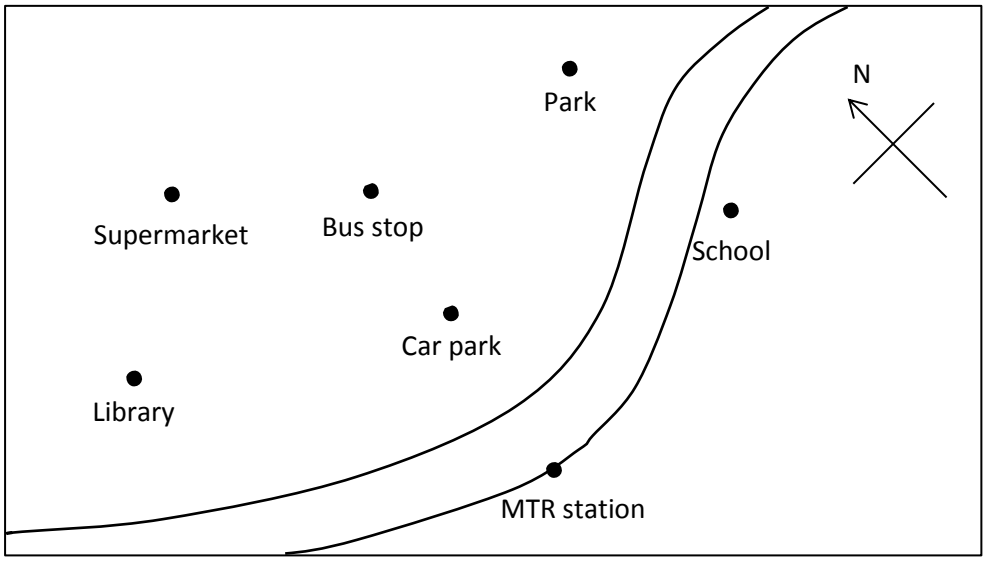
1. If you are in a fixed position of the school campus, will the directions of the community facilities determined by the compass change when you change the direction faced?
2. Will the directions of the community facilities determined by the compass change when you are in different positions of the school campus?

Activity 2:

Students are divided into groups to accomplish the following tasks:

1. Sketch the plan of the area in the vicinity of the school, with the help of compasses.

For example,



(Since distances between facilities are not the main concern of this activity, students can estimate them intuitively.)

2. Collect information on the community facilities and services available in the vicinity of the school, for example, the number of markets, parks and libraries. Choose one or two community facilities for investigation in detail, e.g. the scale of the facilities, the frequency with which the facilities are used by the residents of the community. (Students may collect information by conducting a small-scale survey.)

3. Organise the information collected and present the data with appropriate statistical charts, such as bar charts. The teacher can encourage students to make use of information technology in presenting their findings. For example, students can record data and draw statistical charts by spreadsheet software and take pictures of the facilities with mobile digital devices.

4. Prepare a report and present the findings orally.

5. (Extension) Suggest ways to improve the community facilities.

Questions for discussion:

1. What kinds of facilities are provided in the vicinity of the school?
2. Which types of facilities do you think are in excess? Why?
3. Which types of facilities do you think are insufficient? Why?
4. If you were a town planner, how would you redesign this district?

5. How can we protect and improve the facilities and environment of the community?

Notes for Teachers:

1. Students should be given adequate time to do the project.
2. Teachers should advise students not to use a map (including printed map, online map and apps) when they are sketching the plan of the vicinity of the school, but to apply the knowledge on compasses points and get to know the community through sketching the plan. However, students could use a map if the location and surrounding environment of the school make the sketching process difficult.
3. Teachers should give explicit instructions and suggest methods for the collection of data.

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Share responsibilities and understand the roles of individual members in collecting data and presenting projects results
   - Discuss and exchange ideas with others in sketching the plan, and decide the information to be collected and the strategies to be used
   - Participate actively in the project and listen to others patiently in discussions
   - Understand that there might be different views in ways of improving the community facilities among members with different cultural backgrounds

2. Creativity
   - Propose creative solutions to problems, present own views on the provision of community facilities and suggest ways of improving the facilities

3. Critical Thinking Skills
   - Extract, classify and organise information collected from the area in the vicinity of the school
   - Identify relevance and irrelevance information, for example, finding relevant information useful for supporting the suggested ways to improve the community facilities

4. Problem Solving Skills
   - Choose the relevant information to solve problems, for example, by identifying the statistical data necessary for giving suggestions for improving the community facilities

- Make use of various tools in solving problems, for example, by using a tally in counting the frequency with which the facilities are used, and spreadsheet software to record data and construct statistical charts.

Example 9
Extra-Curricular Activities

Key Stage: 2

Strand: Data Handling

Learning Units: Bar Charts (II)

Objectives :
(i) To use appropriate methods to collect and organise data
(ii) To use relevant statistical charts to represent collected data
(iii) To read and discuss charts constructed
(iv) To make recommendations based on the data collected

Prerequisite Knowledge: Collecting data and drawing simple bar charts

Resources Required: Grid paper

Description of the Activity:

1. The teacher helps students form groups of 4 students.
2. The teacher guides each group to fix a project topic related to extra-curricular activities, for example, time students spend on extra-curricular activities, more popular extra-curricular activities, and usage rates of venues for extra-curricular activities.
3. Students, working in groups, discuss the method to collect information about the topic they have chosen.
4. Students collect data and organise the data collected.
5. Students select suitable and relevant statistical charts to present the data.
6. Students read and discuss the charts with groupmates and make recommendations about extra-curricular activities based on the set of data collected, for example, which activities should be organised or which kinds of activity rooms should be increased in number.
7. Each group report to the class orally and prepare a report.

Notes for Teachers:

1. The teacher prepares information about the extra-curricular activities organised at school before the project.

2. Assessment of students' performance in the project should not focus only on the product (the written report). Their attitude towards the project, their participation, etc. should also be considered (see point 5 below for the suggested assessment criteria).
3. Part of the project can be done at home and students should be given adequate time for discussion and preparation of the project.
4. The teacher should give students suggestions on the time schedule for the project and tell them the deadline for completing their work.
5. Suggested criteria for assessing students' performances:

Criteria	Description of the Criteria
Understanding the theme of the project	• Do students demonstrate a thorough understanding of the theme of the project? • Do students know how to carry out the project and what information they are expected to collect?
Use of strategies and methods	• Are proper methods of data collection designed and relevant information collected? • Do students use appropriate statistical charts to represent the data collected? • Do students analyse and compare data?
Accuracy	• Do students construct statistical charts accurately? • Do students analyse the statistical chart accurately? • Do students make feasible recommendations based on their analysis?
Presentation and communication	• Can students present their work in a systematic and logical way? • Do students report their work accurately?
Attitude	• Do students exercise a good team spirit? • Are students actively involved in the activities? • Have students reviewed their work in a timely manner?

The criteria suggested are by no way exhaustive and each criterion above may have different weightings.

This example mainly involves the following generic skills:

1. Collaboration Skills
    - Collaborate in groups and share responsibilities, understand the roles of individual members in collecting data, preparing statistical charts and presenting the project results
    - Share ideas freely with others in conducting the project
    - Discuss, negotiate and compromise with others in designing methods for collecting data and presenting the results of the project
    - Share ideas in making recommendations on extra-curricular activities at school

2. Communication Skills
    - Use suitable means to collect information
    - Discuss with group members during the group activities
    - Report to the class and discuss with classmates and teacher during reporting

3. Critical Thinking Skills
    - Read and discuss charts, and find out the important or critical parts
    - Draw logical conclusions and make feasible and logical recommendations based on the information collected

Example 10
Four Seasons

Key Stage: 2

Strand:

Mathematics: Data Handling (Learning Unit: Broken line graphs)

General Studies: People and Environment (Learning Objective: to identify and describe climate and seasonal changes and their effects)

Objectives:

(i) To read and discuss broken line graphs

(ii) To construct broken line graphs

(iii) To recognise the temperature change in the four seasons

(*In collaborate with General Studies*)

Prerequisite Knowledge:

(i) Finding the average of a group of data

(ii) Interpreting and constructing bar charts of greater frequency counts

Resources Required: World maps, globes and computers

Related Links: Hong Kong Observatory: *http://www.hko.gov.hk*

Bureau of Meteorology, Australia: *http://www.bom.gov.au*

Description of the Activity:

Activity 1

The teacher asks students to record the daily temperature of 7 days in a week. It is not necessary to guide the students to record the data in any means or in any forms. Students may collect the data from various sources, e.g. thermometer and weather report. One week later, the teacher discusses with students the data collected.

Questions for discussion:

1. Why are the data collected by different students on each day different?
2. If the temperature of one day is selected to represent that of the week, which day should be selected?

Notes for Teachers:

1. The teacher may guide students to account for the difference. For example, the temperature varies within a day and the readings taken at different time of a day are not the same. Measuring the temperature at different places or by different instruments may also lead to inconsistent results. Students are guided to conclude that some conditions or criteria are necessary to be set in data collection.
2. By using the result in the discussion, the teacher may introduce that different regions use maximum temperature, minimum temperature or average temperature for describing the temperature of a particular period of time. The following links can be referred to:
   - Climatological Information Services (Hong Kong Observatory): *http://www.weather.gov.hk/cis/climat_e.htm*
   - Climate statistics for Australian location (Bureau of Meteorology, Australia): *http://www.bom.gov.au* (follow the path Bureau Home > Climate > Climate Data Online)
3. Through discussion, the teacher can introduce the use of the average of a group of data to represent the group in daily life situations.

Activity 2

According to the results in Activity 1, the teacher guides the students to agree with a scheme for measuring temperature. Students will measure the temperature of 7 days in another week in accordance with the scheme.

Questions for discussion:

1. Are the temperatures measured at one or some particular times each day? If so, which times are the most appropriate?
2. If the temperatures are measured at different time of a day, is the maximum temperature or minimum temperature selected to be the target data for recording? How can you collect the data for the maximum temperature or minimum temperature?

Notes for Teachers:

To arouse students' interest, the teacher can refer students to the following webpage and introduce conventional instruments installed at an Automatic Weather Station in Hong Kong by the Hong Kong Observatory:
http://www.hko.gov.hk/education/article_e.htm?title=ele_00454

Activity 3

1. The teacher introduces and discusses with students the temperature information available on the Internet, e.g. the monthly mean of maximum daily temperature of Hong Kong available on the webpage of the Hong Kong Observatory.
2. The students collect temperature information from the webpage of the Hong Kong Observatory and the Bureau of Meteorology, Australia.
3. The students construct broken line graphs using Excel under the teacher's guidance.
4. There can be a discussion on the points to note in the construction of broken line graphs using Excel.
5. The students read and discuss the characteristics of the broken line graphs.
6. The teacher shows the broken line graphs constructed with the use of the temperature information of the two places, and guides the students to compare and find the difference. Students can explore the reason to account for the phenomenon.

Questions for discussion:

1. When is summer and winter in Hong Kong?
2. When is summer and winter in Australia?
3. Ying Ying's grandma lives in Australia. If Ying Ying is going to visit her during Christmas holidays, should Ying Ying wear winter clothes or summer clothes?
4. Describe the changes of temperature in the two places. Why is there such a difference?

Notes for Teachers:

1. When students construct broken line graphs using Excel, the teacher should guide them to use an appropriate graph.
2. When students have finished their construction, they should be reminded to check if the graph has provided adequate information, such as the title and labels for the two axes, etc.
3. For the ease of teacher's preparation, two sets of temperature data of Hong Kong and Perth, Australia are included in the Information Sheet.
4. The teacher may use the monthly mean of daily maximum/minimum temperature of two places for comparison.
5. If necessary, data from places other than Hong Kong and Australia can be used to construct broken line graphs. They can be used as reference to support students' conclusion.

Integration and Application:

Science Education:	Revolution of the Earth around the Sun
Technology Education:	Construction of statistical graphs by IT
Mathematics Education:	Data collection and handling

This example mainly involves the following generic skills:

1. Creativity
   - Design their own scheme in recording the daily temperature.

2. Critical Thinking Skills
   - Explore reasons which account for the difference in daily temperatures recorded by different classmates and the difference in seasonal temperature variations between Hong Kong and Australia, and justify the reasons.

3. Information Technology Skills
   - Use a spreadsheet program to construct statistical charts.

Information Sheet

Monthly mean of daily maximum, mean and minimum temperature recorded at the Hong Kong Observatory and Bureau of Meteorology, Australia between 1981-2010*

Hong Kong	Mean Daily Maximum (℃)	Mean Daily Minimum (℃)
January	18.6	14.5
February	18.9	15.0
March	21.4	17.2
April	25.0	20.8
May	28.4	24.1
June	30.2	26.2
July	31.4	26.8
August	31.1	26.6
September	30.1	25.8
October	27.8	23.7
November	24.1	19.8
December	20.2	15.9

Perth, Australia#	Mean Daily Maximum (℃)	Mean Daily Minimum (℃)
January	31.9	17.3
February	32.1	17.6
March	29.9	16.2
April	26.1	13.4
May	22.2	10.9
June	19.2	9.0
July	18.1	8.1
August	18.7	8.2
September	20.4	9.3
October	22.9	10.5
November	26.5	13.3
December	29.2	15.2

Note:

The sources of two sets of data are the Hong Kong Observatory (*http://www.weather.gov.hk*) and the Bureau of Meteorology, Australia (*http://www.bom.gov.au*)

* To make the data comparable and more reliable, data of thirty years (1981 – 2010) of two places are extracted.

\# Perth is chosen from Australia as it lies in the same time zone as Hong Kong.

Example 11
Investigating Errors of Measurements by GPS Tracking Apps

Key Stage: 3

Mathematics Education

Strand: Measures, Shape and Space
(Learning Unit: Error in measurement)

Technology Education

Knowledge Contexts: Information and Communication Technology
(Module: Computer systems)

Objectives:

(i) To consolidate the concept of error in measurement

(ii) To understand and apply ICT as a prime tool for learning and in our daily life

Prerequisite Knowledge:

(i) Calculate different types of errors

(ii) Use rate, ratio and proportion to solve real-life problems

Resources Required: Scaled floor plans of the school playground, trundle wheels, and tablet computers with GPS tracking apps installed



Description of the Activity:

1. The teacher introduces the activity to students and revises the concepts of ratio and proportion.
2. The teacher gives each student a scaled floor plan of the school playground.
3. Students, working in groups, measure the dimensions of the playground on the floor plan and calculate the actual dimensions of the playground in consideration of the scale ratio of the floor plan.
4. Students discuss how to verify the answer found in (3).
5. Students use trundle wheels and GPS tracking apps at the same time to measure the dimensions of the playground.
6. Students compare the results obtained in (3) and (5) and by taking the measurements by trundle wheels as reference, compare the errors of the results obtained by measuring the floor plan of the school playground and by GPS tracking apps.
7. Students discuss which method (scaled floor plan or GPS) is more reliable and how to reduce the errors in measurement.

Notes for Teachers:

1. It is desirable for students to work in small groups.
2. The teacher should allow ample opportunities for students to discuss and draw conclusion by themselves instead of giving them straightforward hints.
3. The teacher should prepare prior information about the dimensions of the school playground.
4. The teacher should install the GPS tracking apps in the tablets before the lessons.
5. The teacher could also conduct the activity in a standard athletic ground and asks students to measure the lengths of tracks for studying the errors of measurements by GPS tacking apps.
6. GPS function is widely used nowadays. Teachers could encourage students to find out the principle and usage of GPS from the Internet.

This example mainly involves the following generic skills:

1. Communication Skills
   - Understand, analyse and respond to spoken instructions from the teacher and written instructions in worksheets
   - Use appropriate language and mathematical expressions to present the methods and results of calculations
   - Discuss and work with others to accomplish tasks, for example, determining the most appropriate way to take the measurement through discussion

2. Critical Thinking Skills
   - Understand the restrictions of real measurement
   - Evaluate the ways of finding the actual dimensions of the school playground
   - Draw logical conclusions based on adequate data and evidence, for example, by comparing different methods and related errors in measuring to conclude the most appropriate method for taking measurements

3. Information Technology Skills
   - Use the GPS tracking apps in the tablet to carry out the exploratory activities

4. Problem Solving Skills
   - Compare the results of different measurement methods and justify the method selected

Example 12
Design a Healthy Diet Menu

Key Stage: 3

Key Learning Areas: Science, Technology and Mathematics Education KLA

Learning Elements:

KLA	Learning Elements
Science Education	• Common food substance • Function of food substance • Food pyramids • Balanced diet • Healthy lifestyles
Technology Education	• Food groups, dietary goals and eating habits • Meal planning • Principles and skills, hygienic and safe practices in food preparation • Food product development – using a design cycle to create and develop food products to meet the design specifications of a task e.g. address the health concerns of teenagers and sensory requirements of the products • Health lifestyle / sedentary lifestyle / unhealthy lifestyle
Mathematics Education	• Approximation and estimation • Collect and organise data • Construction and interpretation of statistical graphs • Measures of central tendency

Objectives: To integrate and apply knowledge and skills in Science, Technology and Mathematics Education KLAs to solve real-life problems

Description of the Activity:

In this activity, teacher adopts a cross-disciplinary approach to integrating the learning of Science, Technology and Mathematics Education KLAs. The project itself is arranged as an independent activity. Learning elements from the three KLAs are drawn by the students

themselves or by the teachers in the project learning activities.

- The teacher first chooses an authentic problem which most students are concerned. Quite often, students complain about the taste, quality and quantity of the lunch boxes provided by the school lunch box provider. However, the nutritional values of the lunch boxes are also important to the health of teenage students. The teacher asks students to conduct a project on designing a healthy diet menu for the lunch box supplier to help students understand the importance of healthy diet.
- Starting with the enquiry questions, plenty of learning opportunities can be provided for the students to construct, integrate and apply knowledge and skills from the three KLAs. Students can use IT skills to search for information from the Internet about food and diet, including the functions of various food substances, nutritional values, recommended daily intake, etc. Students can apply mathematical skills to calculate and analyse the nutritional values of different types of food. Students could collect data about the food preference of their fellow students, and prepare food samples for tasting. After proper analysis, student can submit the healthy diet menu to the school lunch box provider for reference.
- During the progress of the project, teachers can provide proper guidance and feedback, resources and assistance to their students when needed.

This example mainly involves the following generic skills:

1. Collaboration Skills
    - Share responsibilities and understand the role of each member in the project
    - Agree on suitable strategies for carrying out the project through discussion

2. Creativity
    - Under the constraints in the aspects of nutrition and proportion of ingredients, students design their healthy diet menu.

3. Problem solving Skills
    - Identify the problems associated with the project
    - Make use of data collected to design the menu

Example 13
Mathematics Magic

Key Stage: 3

Strand: Number and Algebra

Learning Units: Algebraic Expressions
Laws of Integral Indices

Objectives: To apply algebraic language and concepts of place value to explore the principles behind some number games

Prerequisite Knowledge:
(i) Representing word phrases by algebraic expressions
(ii) Representing a number in the sum of products of each digit and place value of the corresponding place

Description of the Activity:

1. The teacher helps students form groups for the project work. Since the complexity of the project is not great, the group size could be less than four.

2. The teacher shows an example of "mathematics magic", such as an age guessing game, and explains to students the topic of the project.

   An example of a simple game:
   (a) Choose a two digit number.
   (b) Add 6 to it and multiple the sum by 2.
   (c) Do something "reverse", that is, subtract 6 from the product in (b) and divide the result by 2.
   (d) Subtract the original two-digit number from the result of (c).
   (e) I know what you get. It is 3.

   An example of a more complicated "magic":
   (a) You are asked by the "mathematics magician" to use the format yyyymm to form two numbers, one from the present year and month and one from your year and month of birth, and then subtract the second number from the first.
   (b) Then you find the difference between the result of (a) and the number formed

by reversing the digits of it, e.g. if the result of (a) is 12345, then calculate 54321 – 12345, the larger one minus the smaller one.

(c) You choose a digit from the result of (b) to delete and tell the magician the sum of the remaining digits.

(d) Although the magician does not know your year and month of birth, he can tell the digit that you choose to delete.

(Reference: *http://www.guokr.com* "數學魔術：瑪雅人告訴你哪個數字被偷了")

3. Students are required to
   (i) search for some "mathematics magic" or number guessing games, from books or the Internet, butt the principles behind is not provided;
   (ii) investigate the principles of the games or magic;
   (iii) make comparisons between different games or magic; and
   (iv) design their own mathematics magic.

4. The teacher should keep track of the progress of each group and provide guidance/assistance whenever necessary.

5. Students are required to write a brief report and present their findings to their classmates.

Notes for Teachers:

1. According to students' abilities, the teacher could choose to give further explanations to the examples provided to students. For example, for the first game mentioned above, the teacher could demonstrate the use of algebraic language to represent the process:

$$[(x+6)\times 2-6]\div 2-x=(2x+6)\div 2-x=3$$

2. There are different types of number games for the project, for example,

   Guessing numbers or dates

   http://www.cut-the-knot.org (search for "a matter of age")

   Using binary numbers

   https://scratch.mit.edu (search for "生日密碼與數學魔術")

3. Below are some suggested criteria for assessing students' performance in the project:

Criteria	Description of the Criteria
Understanding the theme of the project	• Do students demonstrate a thorough understanding of the topic of the project? • Do students know how to carry out the project and what information they are expected to collect?
Use of strategies and methods	• Do students use algebraic expressions to investigate the principles of the games/magic? • Do students adopt feasible strategies to investigate the principles of the games/magic? Are variables set systematically?
Accuracy	• Are the algebraic expressions formulated correctly? • Are the algebraic manipulations carried out correctly? • Can the principles suggested fully explain the games/magic?
Creativity	• Can the groups design their own game/magic?
Presentation and communication	• Can students present their exploration process in a systematic and logical way? • Is the report clear and concise, and can it match the theme?
Collaboration	• Is the division of duties among students in the group appropriate? • Is there good collaboration among members?

The criteria suggested are by no way exhaustive and each criterion above may have different weightings.

4. Each group, after completing the project, is invited to present their findings to their classmates. Groups that have designed a game can play the game with their classmates during their presentation. Time for each group to respond to questions raised by classmates and the teacher should also be reserved.

5. It should be emphasised that the process of the project is considered to be more important than the corresponding results/findings in this project. Encouragement should be rendered to the group members who participate in the project but end up with unsatisfactory/incorrect results.

6. Students should be discouraged from spending too much time on the art design of the report. The teacher should make it clear that sophisticated computer art-work in the report is not necessary. In addition, the teacher should give students suggestions on the time schedule and indicate the deadline for the completion of their project work.

7. Students should be given sufficient time to complete the project.

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Share responsibilities and understand the role of each member in the project
   - Select suitable strategies for investigating the principles of the games through discussion and by considering supporting reasons

2. Communication Skills
   - Discuss and exchange ideas openly with other members
   - Put forward ideas in a systematic and logical way during discussion
   - Prepare a concise and relevant report after the completion of the project
   - Respond to questions raised by the teacher and classmates during an oral presentation

3. Critical Thinking and Problem Solving Skills
   - Identify the main task of the project
   - Make use of appropriate knowledge and skills to find out the principles of the games

Example 14
Open-ended Geometric Problem

Key **Stage**: 3

Strand: Measures, Shape and Space

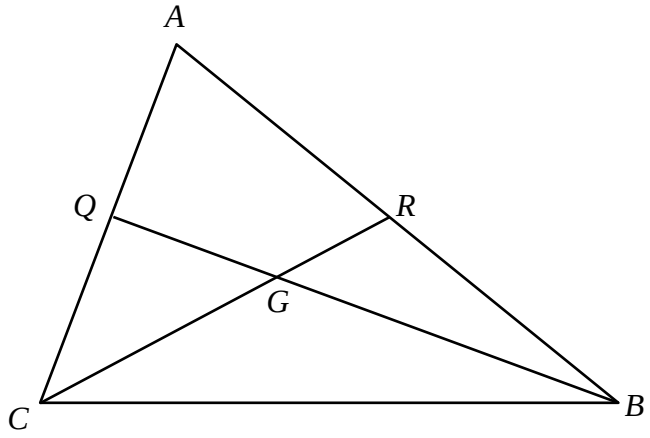
Learning Units: Congruent triangles
Centres of triangles

Objective: To explore relations between angles, line segments and areas in triangles

Prerequisite Knowledge:
(i) Conditions for congruent triangle
(ii) Properties of isosceles triangles
(iii) Centres of triangles

Description of the Activity:
Students are asked to solve the following problem.

Problem
The figure below shows an isosceles triangle ABC with $BA = BC$. G is the centroid of $\triangle ABC$. Q and R are points on AC and AB respectively such that BQ and CR are line segments passing through G. Write down as many as possible equalities related to the figure and justify your answers. Your equalities may involve angles, line segments and areas of geometric figures.



Notes for Teachers:

This is an open-ended question. Students' performance can be assessed according to their ability in listing equalities involving angles, line segments and areas with justifications. Teachers are free to adjust points, marks or grades assigned to students if they provide only some of the equalities.

This example mainly involves the following generic skills:

1. Communication Skills
    - Describe the findings and explain conjectures using mathematical language

2. Critical Thinking Skills
    - Categorise information using various basic geometric facts, for example, locating equal angles related to congruent triangles
    - Reason deductively in the process of obtaining triangles equal in area

3. Problem Solving Skills
    - Choose relevant information and geometric facts to solve problems
    - Understand the problem by noting the important information given in the question
    - Apply knowledge learned to solve new problems

Example 15
Slopes of Perpendicular Lines

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: The Rectangular Coordinate System

Objective: To enhance understanding of the relation between slopes of perpendicular lines with the help of IT tools

Prerequisite Knowledge:
(i) The changes to the coordinates of a point after a rotation about the origin through multiples of 90°
(ii) Finding the slope of a straight line
(iii) The relation between the slopes of parallel lines

Resources Required:
(i) Dynamic geometry apps for tablets (e.g. *GeoGebra*)
(ii) A video clip on the topic "Coordinates of Points rotate about the Origin"
(iii) Tablets computers

Description of the Activity:

Pre-lesson Preparatory Activity

The teacher asks students to watch the video clip "Coordinates of Points rotate about the Origin" and answer the question "What are the coordinates of a point $P(s, t)$ after a rotation of 90° about the origin?" before the lesson.

Notes for Teachers:

Activities 1 and 2 below are designed to guide students to discover and understand the relation between slopes of perpendicular lines in the lessons. Before the activities, the teacher may first give a brief review on the concepts introduced in the pre-lesson activity to see whether students have any questions.

Activity 1

1. Students are asked to use the dynamic geometry app *GeoGebra* to

   (a) construct a straight line L, which passes through the origin O, and the point $P(6, 5)$;

   (b) rotate the straight line L through 90° about the origin to form a new straight line L_1 ; and

   (c) mark the point Q (which is the image of P after a rotation of 90° about the origin) on L_1 by rotating P.

2. Students are then asked to

   (a) find the slopes of L and L_1 by considering the coordinates of P and Q respectively; and

   (b) observe the relationship between the slopes of L and L_1 and draw a conclusion.

3. Repeat steps (1) and (2) above a few times with different coordinates of P and verify the conclusion drawn in step 2(b).

4. During the construction process, the teacher may prompt students with the following questions:

   (a) [in step 1(b)] What is the relation between L and L_1?

   (b) [in step 1(c)] What have you got for the coordinates of Q ?

   (c) [in step 2(a)] How do the results relate to the coordinates of P ?

   (d) [in steps 2(b) and 3] What conclusion can you draw? Can you think of another way to present your conclusion?

5. Let students have sufficient time for discussion and exploration before drawing any conclusion.

Activity 2

1. Teacher may then repeat Activity 1, but this time the line L passes through the points $P(5, 0)$ and $Q(1, 6)$ and ask students to rotate the line about point $R(3, 3)$ and consider

the slopes of the two lines.

2. Students are required to have a group discussion on whether the conclusion in Activity 1 step 3 still holds for lines not intersecting at the origin. Students are required to provide a logical explanation on their conclusion.

Notes for Teachers:

1. It is desirable for students to work in small groups.
2. The teacher should allow ample opportunities for students to discuss and draw conclusions instead of giving them straightforward hints.

This example mainly involves the following generic skills:

1. Information Technology Skills
   - Use dynamic geometry apps and tablet computers to facilitate learning

2. Problem solving Skills
   - Identify the main focus of the problem by building connection between the prerequisite knowledge of point rotation and the perpendicularity of straight lines.

3. Self-learning Skills
   - Check the mastery of prerequisite knowledge for the learning of the new topic through the pre-lesson preparatory activity

Example 16
Volume of Frustums

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: Mensuration

Objective:
(i) To let students appreciate the mathematical achievements of ancient China through understanding the methods in finding the volume of frustums in *The 9 Chapters on the Mathematical Art* (九章算術)
(ii) To enhance students' understanding of the cultural aspect of mathematics.

Prerequisite Knowledge:
(i) Properties of similar triangles
(ii) Method of dissection in finding volumes
(iii) Finding the volume of rectangular pyramids

Resources Required: Cube blocks

Extended Reading Materials: Chapter 5 "*Shanggong*" (商功) of *The 9 Chapters on the Mathematical Art*

Description of the Activity:

1. Students are asked to find the volume of a square-based frustum, with lengths of upper square and lower square 40 cm and 50 cm respectively, and with height 50 cm. In this activity, students would try to derive the formula of the volume of a frustum of square bases, namely $V = \frac{1}{3}(a^2 + b^2 + ab)h$, where *V, a, b*and *h* being the volume, length of upper square, length of lower square and height of the frustum respectively.

2. The teacher introduces the method of dissection (棊驗術) used in the problem of "Square Pavilion" (方亭) in Chapter 5 of *The 9 Chapters on the Mathematical Art*:
   a. As the basic units in the discussion of the volume of a solid, the solids "*li-fang*" (立方), "*qian-du*" (壍堵), "*yang-ma*" (陽馬) and "*bie-nao*" (鱉臑) are

introduced to students, which may be done with the aid of computer software on cutting a *li-fang* gradually, with an example of illustration as Figures 1a and 1b below;

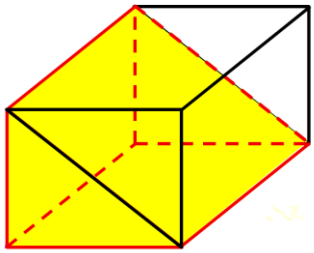


Figure 1a

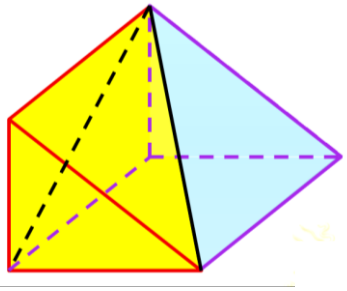


Figure 1b

b. Students are guided to explore the ratio of the volumes of *li-fang*, *qian-du* and *yang-ma* (or cuboid, right-angled triangular prism and rectangular pyramid with the vertex vertical above one of the vertices of the rectangular base) of the same dimension (i.e. *qian-du* and *yang-ma* being cut from the *li-fang* being considered);

c. Students are guided to explore how a square-based frustum could be vertically dissected into *li-fang*, *qian-du* and *yang-ma* and fill in a table as below, which corresponds to the top view of dissection.

yang-ma	*qian-du*	*yang-ma*
qian-du	*li-fang*	*qian-du*
yang-ma	*qian-du*	*yang-ma*

Figure 2 below shows an example of dissection. Students should note that the dimensions of *li-fang*, *qian-du* and *yang-ma* resulted from the dissection are not necessarily the same (i.e. not being cut from the same cuboid).

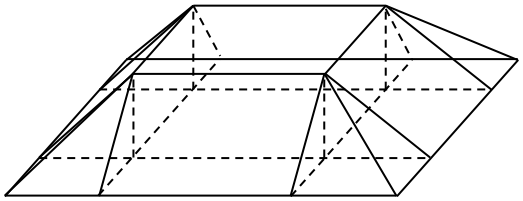


Figure 2

d. Students are guided to consider multiplying the number of solids in the table

with the same number so as to represent the total volume in each cell of the table as integral multiples of some *li-fangs*, i.e. after multiplying the table by 6;

2 *li-fang*	3 *li-fang*	2 *li-fang*
3 *li-fang*	6 *li-fang*	3 *li-fang*
2 *li-fang*	3 *li-fang*	2 *li-fang*

(*li-fang*s in the above table may have different dimensions)

e. Students are given some cube blocks to visualise the table as a model (as in Figure 3 below), and hence guided to explore how the formula of the volume of a frustum in Step 1 can be derived through the model.

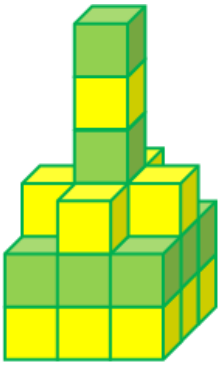


Figure 3

Remark**:** Students should be reminded that the dimensions of the solids represented by each cell in the tables of (c) and (d) above may **NOT** be the same, and that the term "li-fang" does not necessarily mean a cube as the modern terminology does. The cube blocks should only be understood as a tool to simulate the abstract ideas of the method introduced.

3. **(Enrichment)** Students discuss the generalisation of the formula of volume to general prismatoids (i.e. a polyhedron with all vertices lying on two parallel planes) of square or rectangular bases (such as wedges and truncated wedges) by using a similar method as in Step 2 on frustums of rectangular bases. Students are expected to appreciate that the method stated in *The 9 Chapters on the Mathematical Art* is indeed a more generalised and powerful method to find the volume of prismatoids as compared with applying the properties of similar triangles.

Notes for teachers:

1. The key feature of this method of dissection builds on the use of the basic units, known as *qi* (葦) in the original text. In this example, the basic units *li-fang*, *qian-du* and *yang-ma* are used. It should be emphasised that the ratio of the volumes of these basic units is fixed **if they are of the same dimensions**.

2. It should be noted that when comparing the basic units of two different cells in the table in Activity 2c or 2d, they may not be of the same dimensions, which means that there is no assumptions that the frustum is made from a right pyramid. Hence, instead of finding the volumes of each solid after dissection, the multiplication done in Activity 2d is essential to form the parts of solids of which the volumes can be found using the given dimensions in the question.

3. After a suitable multiplication to represent each cell of the table in *li-fang*, there is no need to know all the dimensions of each basic unit. Only the lengths and widths of the upper and lower bases together with the height of the prismatoids are sufficient for finding the volume. In other words, students could be guided to understand that the information about relative positions of the upper and lower bases is not necessary to find the volume of this of kind prismatoids. To students with higher ability and more interest in mathematics, this can serve as an entry point to the important concept of Cavallieri's Principle.

4. Teacher could use the 3-D printing technology to make a model for illustrating the dissection of the rectangular based frustum (as in Figure 3a and 3b below). This allows students to understand the concept of the method of dissection through hands-on manipulation of the model.



Figure 3a

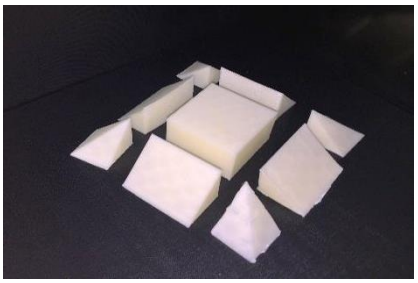


Figure 3b

5. Though the original Chinese terminologies used in *The Nine Chapters on the Mathematical Art* may arouse the interests of some students, the use of these terminologies is not essential if it would hinder students' understanding of the mathematical concepts. However, students more capable of understanding classical Chinese texts should be encouraged to read the original remarks by LIU Hui (劉徽)

to have a taste of the mathematical quality of ancient Chinese mathematicians. Samples of images of the book can be found in some free electronic library (Figure 4).

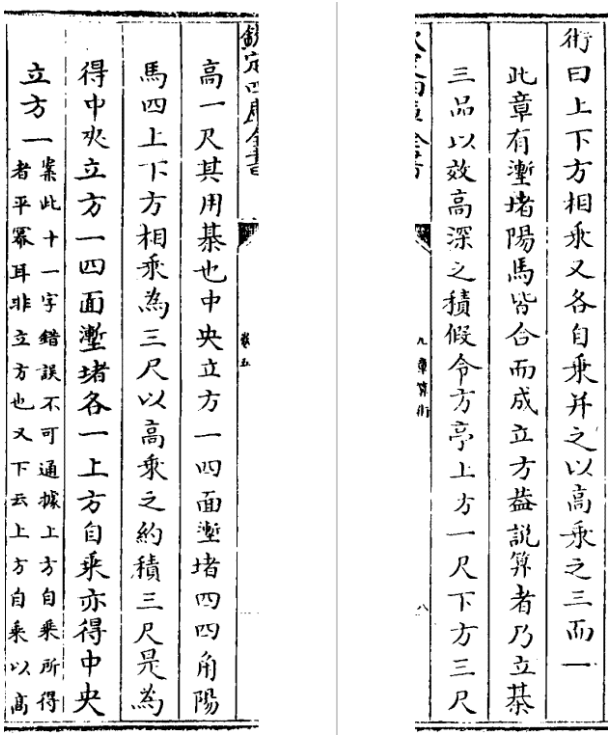


Figure 4

This example may serve the purpose of promoting students' understanding, appreciation and interest in mathematics in the cultural and historical aspects.

This example mainly involves the following generic skills:

1. Creativity
   - Appreciate and elaborate an alternative methods in finding the volume of a frustum

2. Communication skills
   - Understand and analyse the classical Chinese text and translate them into modern mathematical language and illustration

3. Problem Solving skills
   - Deriving the formula of volumes of frustums by referring to solids with known formulae of volume, through planned dissections and multiplications.

Example 17
Surface Areas of Prisms

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: Mensuration

Objectives:
(i) To recognise the concept of surface areas of prisms
(ii) To investigate the relation between the dimensions of a square-based prism and its surface area

Prerequisite Knowledge: Finding areas of simple polygons and volumes of prisms

Resources Required: Empty paper boxes (such as biscuit box and chocolate box), colour pens, scissors, a set of unit cubes, worksheets and computers

Description of the Activity:

Activity 1

This activity allows students to recognise the concept of surface areas of prisms.

1. Students are divided into groups of four. The teacher distributes to each group some empty paper boxes (such as biscuit box and chocolate box) and Worksheet 1.
2. The teachers then guides students to answer the following questions:
   (a) Regarding the surface of each box, how many faces are there?
   (b) What kinds of two-dimensional figures (e.g. rectangles and triangles) are the faces of the boxes?
   (c) Which edges of the boxes can determine the surface areas of the boxes?
   (d) Are there any congruent faces?
3. Each group is requested to cut the boxes along its edges to form plane nets.
4. Each group measures the lengths of suitable edges of the boxes for calculating the surface areas.
5. Students are required to complete Worksheet 1.

Notes for Teachers:

1. Teacher should guide students to recognise that the surface area of a prism is the sum of all the faces.
2. The main focus of the activity is to determine which edges or lengths are needed to be measured in order to calculate the surface area.

Activity 2

This activity allows students to explore the setting of dimensions of a square-based prism with a fixed volume, for the least surface area.

1. Students are divided into groups of four. Each group is given 64 unit cubes and Worksheet 2. Students have to use the cubes to make square-based prisms of different base lengths (a unit) and heights (h unit), and calculate the surface area (A square unit) of the prism for each case.
2. Students are required to record the data and complete Worksheet 2.
3. Students try to find the values of a and h such that the square-based prism has the least surface area.

Notes for Teachers:

In this activity, students should discover that the surface area of square-based prism will not always increase when the height of the prism decreases from 64 units to 1 unit.

Activity 3

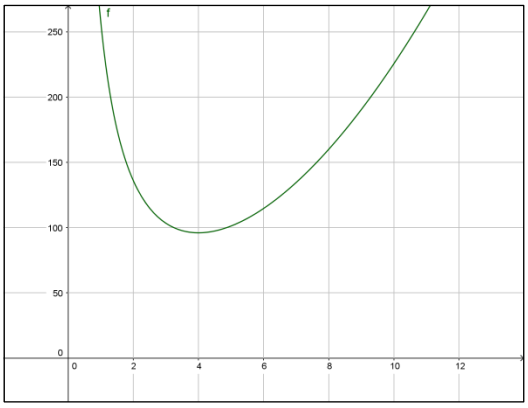
This activity allows students to investigate the surface areas of prisms as in Activity 2 using the graphical method. The dimensions of a and h will not be limited to only some values as in Activity 2. In this activity, a can take any positive value.

1. Students write down the formula for the volume (V cubic unit) and surface area (A square unit) of the square-based prism in terms of a and h.
2. From the two expressions in (1), students write A in terms of a and V.
3. Considering $V = 64$ and a takes the values 1, 2, 3, … , 12, students use a spreadsheet software to construct a table showing the corresponding values of h and A.

Volume (V)	Base length (a)	Height (h)	Surface area (A)
64	1	64.00	258.00
64	2	16.00	136.00
64	3	7.11	103.33
64	4	4.00	96.00
64	5	2.56	101.20
64	6	1.78	114.67
64	7	1.31	134.57
64	8	1.00	160.00
64	9	0.79	190.44
64	10	0.64	225.60
64	11	0.53	265.27
64	12	0.44	309.33

4. Students plot the graph of A against a with V = 64 using a spreadsheet software or any graphing software.





5. Students find out from the graph the values of a such that A is minimum. Students repeat using other values of V (such as 125, 216, and 1 000) and try to draw conclusion on the dimensions of a square-based prism of fixed volume that has the least surface area.

Notes for teachers:

1. For Activities 1 and 2, there is more use of concrete models. Such a teaching approach favours students who learn best through classroom activities and hands-on exploratory tasks.

2. In all the activities, teachers are recommended to adopt strategies like questioning, group discussion and group reporting to enhance interaction. Such a teaching approach favours students who learn best by oral lectures, group work and discussions.

3. Activity 2 could be skipped if students are comfortable with abstract algebraic manipulations as the table constructed in Activity 3 may serve a similar purpose.

4. The graphical method in Activity 3 is not a proof for the minimum surface area. Students could be encourage to learn calculus at the senior secondary level if they want to prove that A is minimum when $a = h$.

This example mainly involves the following generic skills:

1. Communication Skills
   - Describe the correspondence between the dimensions of square-based prism to their surface area verbally in simple and appropriate mathematical terms
   - Present the results logically with appropriate drawings, graphs and mathematical expressions

2. Critical Thinking Skills
   - Determine the dimensions of a square-based prism that lead to the least surface area from the results of calculations

3. Problem Solving Skills
   - Solve the problem by considering all the cases in Activity 2 and the data computed by the spreadsheet software
   - Compare different approaches to tackling the same problem

Worksheet 1

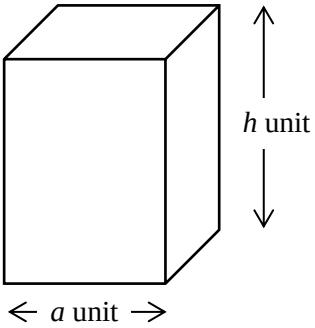
Surface Areas of Prisms

Sketch of the prism	Sketch of the net (mark the dimensions required to calculate the surface area in the net)	Surface area (correct to the nearest cm^2)

Worksheet 2

Suface Areas of Square-based Prisms

Use the 64 cubes provided to make square-based prisms of different base lengths (*a* unit) and heights (*h* unit) and to find the dimensions which result in the least surface area.



Record the base lenghts and heights of the suqare prisms that you have constructed in the table below. (Let the length of an edge of the cubes be 1 unit)

Sketch of the prism	Base length *a* unit	Height *h* unit	Surface area *A* sq. unit

Example 18
Flippable Measure Spoons

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: Mensuration

Objective: **To u**nderstand the applications of calculating volumes and lengths in industrial designs in consideration of appropriately chosen materials.

Prerequisite Knowledge:
(i) Basic concepts on calculating circumferences of circles and volumes of spheres
(ii) Understand and use the relationship between sides and volumes of similar figures
(iii) Imagine the 3-D objects from given 2-D representations

Relationship with other KLA(s) in STEM Education:
Related topics include "Particle model for the three states of matter", "Corrosive nature of acids" and "Materials of the modern world" from *Supplement to the Science Education KLA Curriculum Guide: Science (S1 – 3)* (2017).

Resources Required:
(i) Internet resources with information on flippable measuring spoons and the physical and chemical properties of silicone
(ii) Tablet computers with Internet connections

Scenario:
A measuring spoon is an important cooking utensil for measuring the ingredients correctly. Traditionally, one measuring spoon is used to measure one fixed amount (conventionally 1 tablespoon, 1/2 tablespoon, 1 teaspoon and 1/2 teaspoon). However, with the aid of extensible and flexible materials such as silicone rubber, a flippable measuring spoon can be designed so that two fixed amount can be measured using the same measuring spoon (reference link: *https://www.youtube.com/watch?v=yKTgaZnkaqc*)

This task is to discuss the industrial design of flippable measuring spoons by applying students' mathematical knowledge in finding various measurements and scientific

knowledge in explaining why a specific material is or is not suitable for making kitchen utensils.

Description of the Activity:

Preparatory Activity (Optional)

Students familiarise themselves with the steps in finding the radius of a hemisphere from a given volume. For example, if the measure of a tablespoon is 15 cm^3, and assuming the shape of a measuring spoon is a hemisphere, students are requested to find the radius of the measuring spoons that can be used to measure 1 tablespoon and 1/2 tablespoon.

Teachers may suggest students to use the knowledge of similar figures to find the ratio between the radii of the two measuring spoons by considering the ratio between their volumes.

Activity 1:

The teacher may play the video clip (link provided above) to introduce the ideas behind the industrial design of a flippable measuring spoon which can measure two fixed amounts.

After the video, the teacher may use the cross-section model of the flippable measuring spoon to further facilitate students' understanding of the design (see Figures 1a and 1b):

Figure 1a

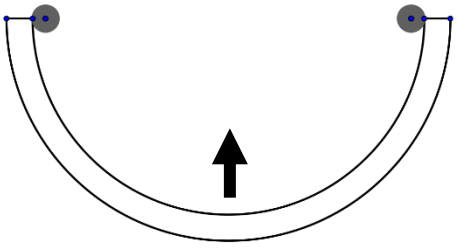
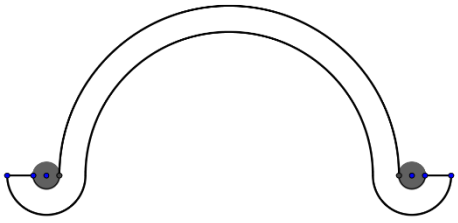


Figure 1b

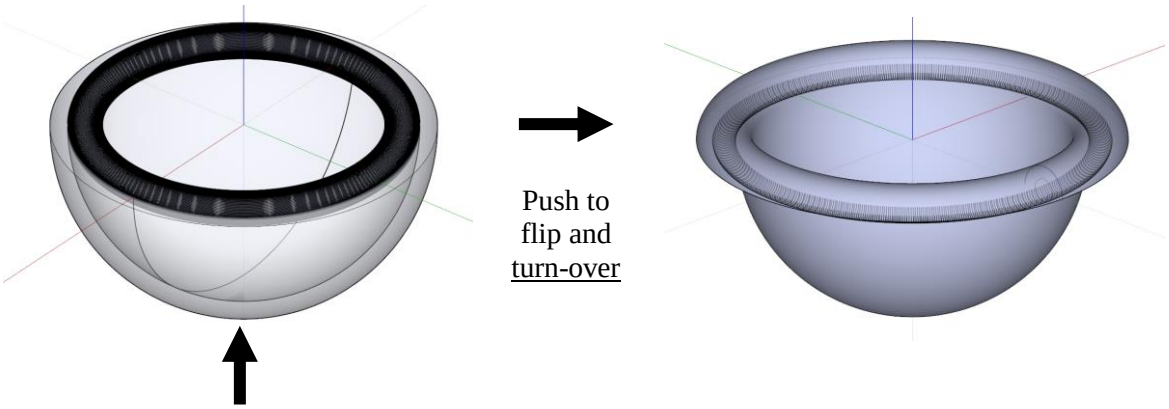


The grey circles are the mount for fixing the flappable part

Remarks: **The t**eacher can also use 3-D graphics software packages to illustrate the design of the measuring spoon (see Figures 2a and 2b):

Figure 2a

Figure 2b



Questions for discussion:

1. If a flippable measuring spoon is designed to measure 1 tablespoon and 1/2 tablespoon of ingredients respectively, and assuming that in both cases the interior is a hemisphere, what should the radii of the two hemispheres be?
2. Let t be the thickness of the material for making the flippable measuring spoon and r be the radius of the circular mount, can you suggest a formula to link up t and r such that the capacities of the two states of the measuring spoon are 1 tablespoon and 1/2 tablespoon respectively?
3. Would there be errors due to the assumption on hemispheres, and could they be reduced by varying the values of t and r ?

Activity 2:

Students are asked to search on the Internet for the physical and chemical properties of silicone, such as its melting point, durability, flexibility, conductivity, toxicity, chemical reactivity and acid resistance to critically analyse whether this material is good for making kitchen utensils.

Mathematics teachers are encouraged to collaborate with Science teachers to establish a stronger linkage of the lesson to the Science Education KLA. Related science topics at the junior secondary level include “Particle model for the three states of matter”, “Corrosive nature of acids” and “Materials of the modern world”.

Questions for discussion:

1. Why is it important to consider the melting point of a matter in choosing the material for making kitchen utensils? Besides the melting point, what other factors concerning temperature should be considered?
2. In order to make the design of the flippable measuring spoons possible, what are the physical properties of silicone featured? Are there other matters having the same

physical properties?

3. Compare silicone and plastic. Which materials is preferred for making kitchen utensils in consideration of its impact on health and environment?

This example mainly involves the following generic skills:

1. Critical Thinking Skills
   - Critically discuss the advantages and limitations of the design of and materials for making flippable measuring spoons
   - Employ the ideas of mathematical modelling as a powerful tool to quantitatively simulate and solve the problems of practicability in designing flippable measuring spoons

2. Self-learning Skills
   - Autonomously gather useful information on the materials for making flappable measuring spoon from various sources
   - Initiate an enquiry into modifying and refining the designs of flippable measure spoons

Example 19
Translations of Functions

Key Stage: 4

Strand: Number and Algebra

Learning Unit: More about graphs of functions

Objective: To understand the vertical and horizontal translation of a function $f(x)$ from graphical and symbolic perspectives

Prerequisite Knowledge:
(i) Basic concepts of translation in rectangular coordinates system
(ii) Using algebraic and graphical methods to represent functions
(iii) Basic operation skills of dynamic geometry software

Resources Required: Dynamic geometry software such as *GeoGebra*

Preparation:

1. In order to utilise the teaching resources in conducting learning activities, teachers may consider, with the aid of suitable software packages, developing or customising electronic worksheets according to students' needs.

2. The following questions should be addressed in developing or customising the learning and teaching materials and electronic worksheets:
   - What are the key elements students should observe?
   - How can functions of the software be used to highlight these key elements?
   - What are the key elements that may not be effectively demonstrated by functions of the software?

3. The following steps are the suggestions on creating the electronic worksheet for exploring the translations of graphs of functions using *GeoGebra*:

   3.1 Create two sliders, labelled with a and b.

   3.2 Create a function $f(x)$ by keying in "$f(x) = x$" in the input field.

   3.3 Create an input box which is linked to the function $f(x)$ so that students can

freely key in any functions in x to define $f(x)$.

3.4 Create two functions $g(x)$ and $h(x)$ such that

$$g(x) = f(x) + a \text{ and } h(x) = f(x + b).$$

3.5 Add a point P on the graph of $y = f(x)$.

3.6 Define another two points $P' = P + (0, a)$ and $P'' = P + (-b, 0)$.

3.7 Define another two points Q' and Q'' on the graphs of $y = g(x)$ and $y = h(x)$ respectively such that PQ' and PQ'' are a horizontal and a vertical line respectively.

Remarks:

a. It can be done by adding a horizontal and a vertical line at P , and locate Q' and Q'' by finding the correct intersections.

b. Q' and Q'' may not exist depending on the type of $f(x)$. Also, Q'' may not be unique. However, these drawbacks can be used to create the contrast of whether one can view a horizontal translation as a vertical translation, or vice versa.

3.8 Add the line segments PP', PP'', PQ' and PQ'', and measure the lengths.

3.9 Add a check box, labelled with "show distance", and link it with the objects PP', PP'', PQ' and PQ''. Right-click the check box and rename it as s.

3.10 Add two check boxes, labelled with "vertical translation" and "horizontal translation". Rename the check boxes as t and u. For the check box "vertical translation", link it with the objects $g(x)$, slider a, P', Q', PP' and PQ'. For the check box "horizontal translation", link it with the objects $h(x)$, slider b, P'', Q'', PP'' and PQ''.

3.11 Right click PP' and PQ'. In the "Advanced" page in "Object Properties", type "s&&t" in "Condition to Show Object", as illustrated Figure 1:

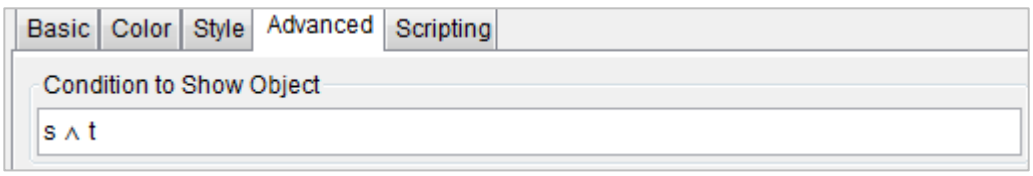


Figure 1

Similarly, set the condition to show PP'' and PQ'' as "s&&u".

3.12 Set the condition to show the check box "show distance" as "t||u", i.e. t or u.

Description of the Activity:

Activity 1a

1. The teacher revises some basic concepts of translation in the rectangular coordinate system, including the use of algebraic symbols such as $(x, y) \rightarrow (x + a, y + b)$ to describe a translation.

2. In the *GeoGebra* electronic worksheet, the teacher could ask students to enter any single-variable quadratic functions in x and check the "vertical translation" module (see Figure 2) using their tablet or notebook computers.

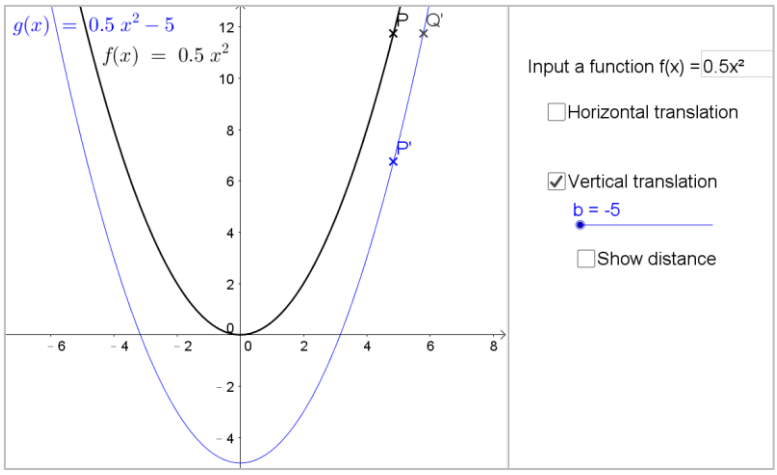


Figure 2

3. By dragging the slider, students are asked to observe and verbally describe the translation of the graphs. Students are guided to discuss the relationship between the sign of the slider and the direction of translation.

4. The teacher then asks students to check the "show distance" box and move the point on the graph of the original function (see Figure 3). Students are guided to observe the changes of horizontal and vertical distances between the graphs at different positions.

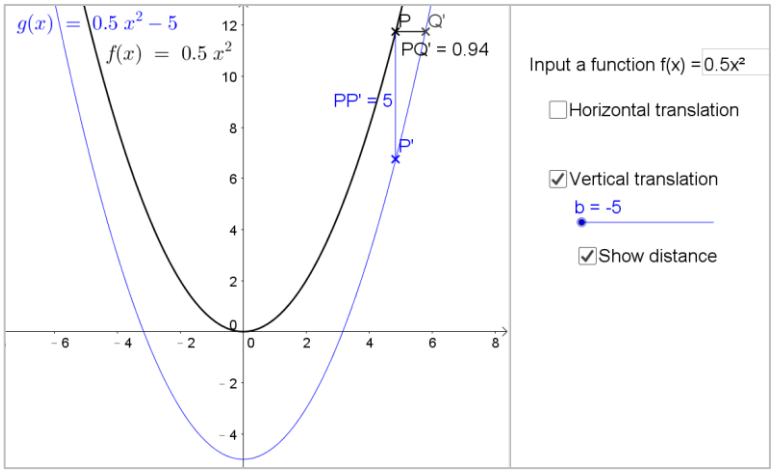


Figure 3

5. Students are guided to observe and discuss the relationship between the values of the slider, the horizontal and vertical distances between the graphs, the algebraic expression of the functions shown, and the displacement of the translation. Students are then asked to make hypothesis on describing the vertical translation of a function algebraically.

6. Students can then be asked to type in any other functions of different types to verify their hypothesis.

Activity 1b

1. The teacher provides a definition of vertical translation of a function.

2. Students are guided to consider the graph of a function f as a point set with points $(x, f(x))$ and hence discuss how to describe the graph of the translated function f' algebraically in terms of f.

3. The teacher then asks students to verify and hence prove that the vertical distances between the graphs of the functions $y = f(x)$ and $y = f(x) + a$ are always the same at different values of x.

4. The teacher guides students to draw the conclusion on the effect of transforming $f(x)$ through $f(x) + k$.

Activity 2a

1. In the *GeoGebra* electronic worksheet, the teacher asks students to uncheck the "show distance" box and "vertical translation" module, enter any single-variable quadratic functions in x and check the "horizontal translation" module (see Figure 4) using their tablet or notebook computers.

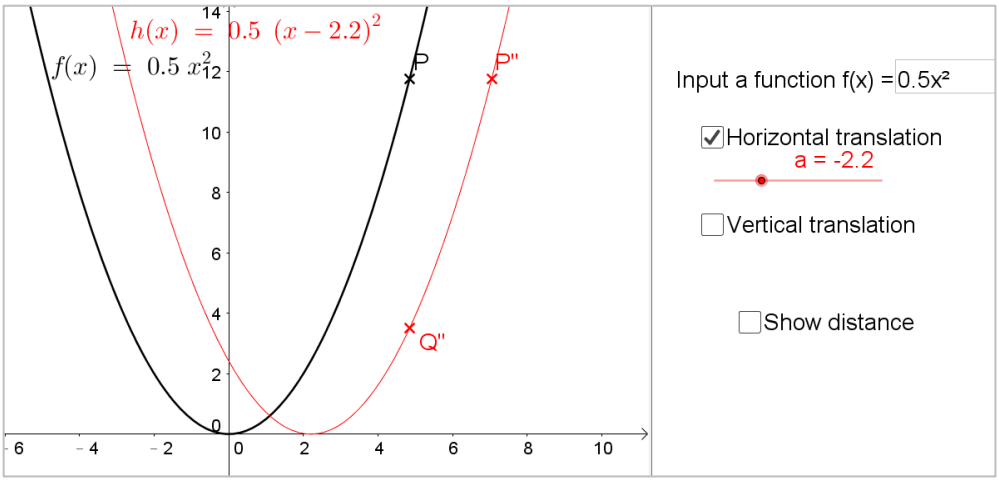


Figure 4

2. By dragging the slider, students are asked to observe and verbally describe the translation of the graphs. Students are guided to discuss the relationship between the sign of the slider and the direction of translation.

3. The teacher then asks students to check the "show distance" box and move the point on the graph of the original function (see Figure 5). Students are guided to observe the changes of horizontal and vertical distances between the graphs at different positions.

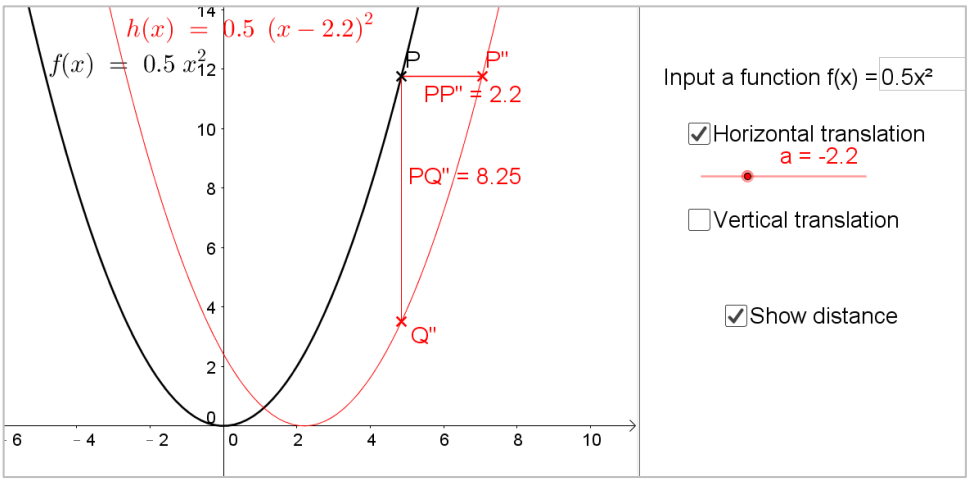


Figure 5

4. Students are guided to observe and discuss the relationship between the values of the slider, the horizontal and vertical distances between the graphs, the algebraic expression of the functions shown, and the displacement of the translation. Students are then asked to make hypothesis on describing the horizontal translation of a function algebraically.

5. The teacher asks students to type in any other functions of different types to verify their hypothesis.

Activity 2b

1. The teacher provides a definition of horizontal translation of a function.

2. Students are guided to consider the graph of a function f as a point set with points $(x, f(x))$ and hence discuss how to describe the graph of the translated function f' algebraically in terms of f.

3. The teacher then asks students to verify and hence prove that the horizontal distances between the graphs of the functions $y = f(x)$ and $y = f(x - b)$ are always the

same at different values of y.

Activity 2c (Enrichment learning activity for more able students)

1. The teacher asks students to observe the vertical translation and horizontal translation of the linear function $y = x$ on whether there is any difference between the two translations.

2. The teacher asks students to enter any single-variable linear function in x and check the "vertical translation" module using their tablet or notebook computers.

3. By checking the "show distance" box, the teacher asks students to drag the slider and move the point on the graph of the original function to observe the changes of horizontal and vertical distances between the graphs at different positions.

4. Students are guided to discuss whether the translation $(x, f(x)) \rightarrow (x, f(x) + a)$ is a vertical or horizontal translation when f is a linear function.

5. Similarly, students are asked to explore and discuss whether the translation $(x, f(x)) \rightarrow (x - b, f(x))$ is a vertical or horizontal translation when f is a linear function.

6. Students are guided to show that for all linear functions $f(x)$, $f(x - p)$ can always be written in the form of $f(x) + q$ for some q.

Notes for Teachers:

1. Teachers are encouraged to ask students to discuss the horizontal translation after the vertical translation. With appropriate design, it is also good for students to compare and contrast the different effects of a vertical translation and a horizontal translation to the same function.
2. While the use of information technology can significantly provide the graphs of all functions for the discussion, teachers should be aware that it may not directly help students conceptualise the transformation algebraically. Hence, traditional consolidation activities are essential.
3. Teachers can use Activity 2c as an enrichment activity to stretch the potential and interest of students who are more able in Mathematics.
4. To cater for the diverse learning style, teachers can rewrite the form of transformation of $y = f(x) + k$ to $y - k = f(x)$ and $y = f(x + k)$ to $y = f(x - k)$, and help students to recognise the effects of the translation by observing the position and sign of the constant k. (Remark: Such written forms, together with a similar treatment of other

transformations such as $y = f(kx)$, and $y = kf(x)$, can be explained to students with high mathematical ability and strong interest in mathematics using the concepts of transforming axis of the coordinate system as an enrichment activity.)

This example mainly involves the following generic skills:

1. Creativity
   - Making hypothesis based on one's own observations

2. Information Technology Skills
   - Using dynamic geometry apps to explore properties of transformations of functions

Example 20
Modelling the spread of a disease

Key Stage: 4 (Compulsory Part and Module 1/Module 2)

Learning Unit: Further applications
Inquiry and investigation

Objectives:
(i) To help students relate STEM education with the real life
(ii) To let students recognise the mathematics in everyday life and apply information technology to solve problems
(iv) To let students recognise mathematics as a powerful tool for planning

Prerequisite Knowledge:
(i) Content of the learning unit "Exponential and logarithmic functions" in Compulsory Part
(ii) Content of the learning unit "Probability" in junior secondary Mathematics
(iii) Topics on calculus in Extended Part Module 1 or Module 2

Background information

Bird flu, SARS and Ebola are examples of fatal epidemics that have emerged in a large scale in the past two decades. They badly threaten human lives in the world. One may ask in what pattern do diseases spread among people. Below are two mathematical models on the spread of diseases.

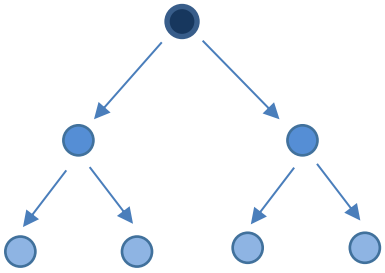
Basic assumptions for the following two models:

1. The population is in a closed area.
2. The total number of the population is fixed.
3. The disease is transmitted by direct individual contact.
4. The recovered people are not considered.

Model 1: A simple epidemic model

The spread of the disease starts from an infected person. The person spreads the disease directly to two susceptible persons. Each of the two newly infected persons then transmit

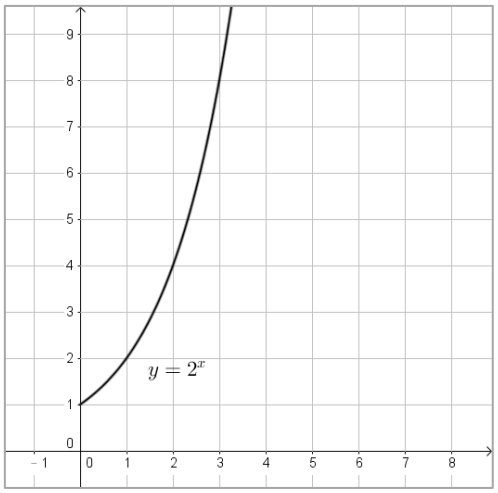
the disease to two more persons (as in the following figure), and so on. The process is continued until everyone is infected.



The number of persons infected at each stage is a power of 2. We can model the situation by the exponential function: $y = 2^x$. The teacher can take the chance to review the properties of exponential function with students.

The teacher may raises the following questions about the graph of the function:

- Does the graph of the exponential function get steeper or less steep as x increases?
- Does the graph have a y-intercept? If so, what is it? If not, explain the reason.
- Does the graph have an x-intercept? If so, what is it? If not, explain the reason.



Discussion questions on this model:

- How many steps will it take to infect all the people in the classroom? How about the whole school?
- If the spread starts from another infected person, will the result be the same?
- What is the difference if 3 persons are infected at each stage?
- What are the limitations of this model?

Note for Teachers:

This model is one of the simplest way to model the spread of disease. The model has many restrictions and seems unrealistic. It does not take into account people who recover and are then immune. It also does not consider isolated communities. Real epidemics do not really follow this model.

Model 2: Counter Plague model:

A dice is used to simulate the variability of the number of infected people in this model. The number of newly infected people caused by each infected people at each step is determined by the value of tossing a fair dice, so the infection rate is not fixed. Two scenarios are investigated for the model:

(i) infection rate greater than 1

(ii) infection rate less than 1

Case 1:

Number on dice	Number of newly infected people
1	0
2	0
3	1
4	2
5	2
6	3

The expected rate of infection = $\frac{1}{6}\times 0+\frac{1}{6}\times 0+\frac{1}{6}\times 1+\frac{1}{6}\times 2+\frac{1}{6}\times 2+\frac{1}{6}\times 3=\frac{4}{3}$ which is greater than 1. **Epidemics take off**.

Case 2:

Number on dice	Number of newly infected people
1	2
2	1
3	1
4	0
5	0
6	0

The expected rate of infection = $\frac{1}{6}\times 2+\frac{1}{6}\times 1+\frac{1}{6}\times 1+\frac{1}{6}\times 0+\frac{1}{6}\times 0+\frac{1}{6}\times 0=\frac{2}{3}$ which is less than 1. **Epidemics die out**.

Classroom Activity:

The teacher asks students to form small groups and provide each group with one dice and about 30 counters. For each group, students take the following steps:

1. Choose a case from Case 1 and Case 2.

2. Place one counter on the table top to represent one infected person, i.e. the originally infected person.
3. Throw the dice once. Suppose the dice shows 4. In Case 1, this means that two new persons are infected. Two more counters have to be placed on the desk. In case 2, no one is infected and the disease dies out.
4. Repeat for EACH newly infected persons and place the appropriate number of counters on the desk.
5. Repeat until either the epidemic dies out or the counters are run out.
6. Record the progress of the epidemic on a graph.
7. Run the simulation several times for the two cases and compare the graphs.

Discussion questions on this model:

- Does the epidemic take off or die out in each case?
- On average, how many steps does the epidemic run in each case?
- What aspects of the model help explain how epidemics spread?

Challenging Problem:

Suppose a population is threatened by an infectious disease. The population can be divided into two groups, i.e. the healthy and the infected groups. Let *p* be the probability that a healthy person gets the disease, *r* be the probability that an infected person recovers in each month. Initially there are 8 healthy people and 2 infected people.

Setting up the model

Students may try to model what happens to the 10 people over a period of 10 months by tossing a fair 6-sided dice. It is assumed that no one dies in 10 months and we roll the fair dice for each person in each month. If we get a 1 , 2 or 3 for a healthy person, then he becomes infected that month ($p = \frac{1}{2}$). If we get a 4 or 5 for an infected person, then he recovers that month ($r = \frac{1}{3}$). For other outcomes, their situations remain unchanged.

Theoretical suggested solution

Let *x* be the proportion of the population that is infected. We can present the rate of change of *x* by the following expression:

$$\frac{dx}{dt} = p(1-x) - rx$$

$$\frac{dx}{dt} = \frac{1}{2}(1-x) - \frac{1}{3}x$$

$$\boxed{\frac{dx}{dt} = \frac{3-5x}{6}}$$

Notes for Teachers:

(i) From the given condition, students are able to set up the expression for the rate of change of *x*. As it is not required to solve differential equation in our curriculum, teachers may demonstrate the technique of solving the problem for the more able students.

$$\frac{dx}{dt} = \frac{1}{2} - \frac{5}{6}x$$

$$\int \frac{6}{3-5x}\,dx = \int dt$$

$$-\frac{6}{5}\ln(3-5x) = t + k \qquad \text{where } k \text{ is a constant}$$

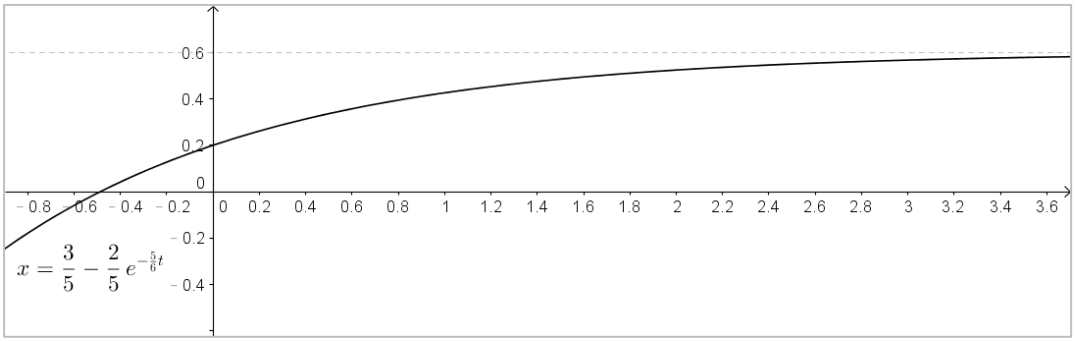
When $t = 0$, $x = 0.2$, so $k = -\frac{6}{5}\ln 2$, thus

$$-\frac{6}{5}\ln(3-5x) = t - \frac{6}{5}\ln 2$$

$$-t = \frac{6}{5}\ln\left(\frac{3-5x}{2}\right)$$

$$e^{-\frac{5}{6}t} = \frac{3-5x}{2}$$

So $x = \frac{3}{5} - \frac{2}{5}e^{-\frac{5}{6}t}$ and when t tends to infinity, x tends to $\frac{3}{5}$. The graph is:



What happens to the solution if the value of *p* and *r* is changed?

(ii) Another way of dealing with this problem is to give the suggested solution $x = \frac{3}{5} - \frac{2}{5}e^{-\frac{5}{6}t}$ to students and ask them to verify that this solution satisfies the differential equation.

Reference:

Epidemics: Modelling with mathematics: *https://motivate.maths.org* (following links *Maths and Our Health* and *Epidemics: Modelling with mathematics*)

A program for simulating Model 2, the Counter Plague model and a demonstration video about it can be accessed through the above webpage.

This example mainly involves the following generic skills:

1. Critical Thinking Skills
   - Compare mathematical model and real situation, and analyse the shortcomings of the model

2. Problem Solving Skills
   - Formulate a mathematical solution when tackling a real-life problem
   - Use real objects to simulate in an abstract mathematical context

Example 21
Return and Risk

Key Stage: 4

Strand: Data Handling

Learning Unit: More about probability
Measures of dispersion

Objective: To understand the use of expected value and standard deviation to describe return and risk

Pre-requisite Knowledge:
(i) Understand the concept of probability
(ii) Recognise the concept of expectation
(iii) Understand the concept of standard deviation for both grouped and ungrouped data sets

Description of the Activity:
Two games will be played to introduce the basic idea of return and risk, which are described by expectation and standard deviation of the given data. Players need to consider both return and risk in making their decisions.

Game 1

1. Students are required to form groups of about 2 to 4.
2. Before the game, the teacher gives each group 10 chocolates and a pack of 52 playing cards.
3. Each group shuffles the playing cards and randomly picks a card each time to determine the change to the number of chocolates. Students need to record the changes (the teacher needs not collect or give chocolates at this stage).
   - *Scenario 1*
     If the group picks a club, the reward is 10 chocolates; otherwise, they lose 5 chocolates.
   - *Scenario 2*
     If the group picks a red card, the reward is 7 chocolates; otherwise, they lose 3 chocolates.

- *Scenario 3*
  If the group picks a red card less than 4, the reward is 40 chocolates; otherwise, they lose 10 chocolates.

If the number of chocolates drops to zero or negative, or increases to 40 or above, the game is over.

4. All the groups need to play the game for at least three times within 10 minutes. Then they can stop or continue playing the game until the time is up. If a group gets a total of 40 chocolates or more at a particular game, then that group can keep the 10 chocolates in hand.

5. Before each game, each group needs to toss a fair six-sided die to decide which scenario to play in. If they get "1" or "6", they will play the game in Scenario 1. If they get "3" or "4", they will play the game in Scenario 2. If they get "2" or "5", they will play the game in Scenario 3.

6. Each group needs to calculate the expected value of the game under different scenarios according to the theoretical situation and decide in which scenarios to play if they are free to choose.

Questions for discussion:

1. Under the rule in step 5 above, does each group have equal chance of choosing the three scenarios?

2. In the long run, which scenario should each group choose to play the game to get a greater reward? Please explain.

Notes for Teachers:

1. Based on the theoretical probability, the expected gain of chocolates in each scenario is as follows:

   - *Scenario 1*

     The expected gain of chocolates = $\frac{13}{52}\times 10+\left(1-\frac{13}{52}\right)\times(-5)=-1.25$

   - *Scenario 2*

     The expected gain of chocolates = $\frac{26}{52}\times 7+\left(1-\frac{26}{52}\right)\times(-3)=2$

- *Scenario 3*

  The expected gain of chocolates = $\frac{4}{52}\times 40+\left(1-\frac{4}{52}\right)\times(-10)=-6.15$

From the result, it seems that we should choose Scenario 2 in the long run to get a greater reward and the case in Scenario 3 will bring us a great loss. The expected value reflects the risk behind the large reward in Scenario 3.

2. In practice, as students only play the game at limited number of times, they are expected to recognise that the actual gain of chocolates in a particular scenario may be different from what is reflected by the expected gain; and the meaning of each expected value in relation to the risk of losing chocolates under the corresponding scenario.

Game 2

Players need to concern about and analyse the variability of return in making their decision. Suppose we have two plans and the expected returns of them are the same. It is quite reasonable to choose the plan with lower fluctuation in return. Variance and standard deviation of the return could be used to measure the level of risk in relation to the fluctuation in return.

1. Students are required to form groups of about 2 to 4.

2. The teacher distributes 10 chocolates and two dice to each group. Then the teacher asks each group to play the game according to the following rules:

   Scenario 1

   (i) Throw two dice and subtract the smaller number from the larger one. We call the results "Dice Differences".

   (ii) At each throw, if the "Dice Difference" is 0 or 1, each group can get 2 chocolates. Otherwise, each group needs to return 1 chocolate to the teacher.

   (iii) The game will stop when all the chocolates of a group have been returned to the teacher after a throw.

   (iv) If a group still owns chocolates after a throw, it needs to continue until it has performed 20 throws.

(v) Each group needs to record their results of each throw and calculate their final number of chocolates for comparison.

Scenario 2

(i) Throw two dice and add the two numbers of the outcome. We call the results "Dice Sums".

(ii) At each throw, if the "Dice Sum" is 5 or below, each group can get 9 chocolates. Otherwise, each group needs to return 3 chocolates to the teacher.

(iii) The game will stop when all the chocolates of a group have been returned to the teacher after a throw.

(iv) If a group still owns chocolates after a throw, it needs to continue until it has performed 20 throws.

(v) Each group needs to record their results at each throw and calculate their final number of chocolates for comparison.

Each group gets the amount of chocolates as recorded at the end of the game. The group with the largest number of chocolates wins the game.

Questions for discussion:

1. According to the rules, what are the expected gains of chocolates in Scenario 1 and Scenario 2? Are the games fair?
2. Suppose you need to choose between Scenario 1 and Scenario 2, which scenario will you choose in order to win the game? Please give justification for your choice. You may consider the standard deviation of the gain in number of chocolates for each scenario.
3. Are there any drawbacks if we use standard deviation to describe the risk of playing a game?

Notes for Teachers:

The possible outcomes for throwing two dice are listed as follows:

2nd die \ 1st die	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1. Based on the theoretical probability, the expected gain of chocolates in each scenario under the given rules is as follows:

   Scenario 1

   The expected gain of chocolates = $\frac{16}{36}\times(+2)+\left(1-\frac{16}{36}\right)\times(-1)=\frac{1}{3}$

   Scenario 2

   The expected gain of chocolates = $\frac{10}{36}\times(+9)+\left(1-\frac{10}{36}\right)\times(-3)=\frac{1}{3}$

   It seems that in the long run, each group can gain chocolates. The expected return is the same. The game is favourable to the players.

2. Each group needs to record their results like the following tables. We may use an online simulator (such as *http://www.virtualdiceroll.com/*) to get the outcomes of virtual dice throws. The following tables list simulated results under the two scenarios:

   Scenario 1

1st set	Dice diff.	Gain	Number of chocolates	2nd set	Dice diff.	Gain	Number of chocolates
1. (6,2)	4	–1	9	1. (3, 5)	2	–1	9
2. (6,3)	3	–1	8	2. (1, 3)	2	–1	8
3. (1,1)	0	2	10	3. (3, 6)	3	–1	7
4. (1,5)	4	–1	9	4. (2, 4)	2	–1	6

1st set	Dice diff.	Gain	Number of chocolates	2nd set	Dice diff.	Gain	Number of chocolates
5. (3,5)	2	–1	8	5. (4, 1)	3	–1	5
6. (3,1)	2	–1	7	6. (2, 2)	0	2	7
7. (2,2)	0	2	9	7. (6, 4)	2	–1	6
8. (5,5)	0	2	11	8. (5, 6)	1	2	8
9. (6,2)	4	–1	10	9. (3, 5)	2	–1	7
10. (1,4)	3	–1	9	10. (2, 5)	3	–1	6
11. (1,5)	4	–1	8	11. (4, 5)	1	2	8
12. (3,6)	3	–1	7	12. (2, 5)	3	–1	7
13. (4,4)	0	2	9	13. (4, 5)	1	2	9
14. (6,5)	1	2	11	14. (5, 5)	0	2	11
15. (6,3)	3	–1	10	15. (6, 6)	0	2	13
16. (2,3)	1	2	12	16. (5, 4)	1	2	15
17. (3,6)	3	–1	11	17. (1, 5)	4	–1	14
18. (3,6)	3	–1	10	18. (6, 6)	0	2	16
19. (1,4)	3	–1	9	19. (1, 1)	0	2	18
20. (6,2)	4	–1	8	20. (5, 6)	1	2	20
Standard deviation of gain = 1.4				Standard deviation of gain = 1.5			

Scenario 2

1st set	Dice sum	Gain	Number of chocolates	2nd set	Dice sum	Gain	Number of chocolates
1. (6,2)	8	–3	7	1. (3, 5)	8	–3	7
2. (6,3)	9	–3	4	2. (1, 3)	4	9	16
3. (1,1)	2	9	13	3. (3, 6)	9	–3	13
4. (1,5)	6	–3	10	4. (2, 4)	6	–3	10
5. (3,5)	8	–3	7	5. (4, 1)	5	9	19
6. (3,1)	4	9	16	6. (2, 2)	4	9	28
7. (2,2)	4	9	25	7. (6, 4)	10	–3	25
8. (5,5)	10	–3	22	8. (5, 6)	11	–3	22
9. (6,2)	8	–3	19	9. (3, 5)	8	–3	19
10. (1,4)	5	9	28	10. (2, 5)	7	–3	16
11. (1,5)	6	–3	25	11. (4, 5)	9	–3	13
12. (3,6)	9	–3	22	12. (2, 5)	7	–3	10
13. (4,4)	8	–3	19	13. (4, 5)	9	–3	7
14. (6,5)	11	–3	16	14. (5, 5)	10	–3	4
15. (6,3)	9	–3	13	15. (6, 6)	12	–3	1
16. (2,3)	5	9	22	16. (5, 4)	9	–3	–2

1st set	Dice sum	Gain	Number of chocolates	2nd set	Dice sum	Gain	Number of chocolates
17. (3,6)	9	–3	19	17. /	/	/	/
18. (3,6)	9	–3	16	18. /	/	/	/
19. (1,4)	5	9	25	19. /	/	/	/
20. (6,2)	8	–3	22	20. /	/	/	/
Standard deviation of gain = 5.5				Standard deviation of gain = 4.7			

According to the simulations, it seems that players can gain more chocolates by choosing Scenario 2 (a maximum of 28 in both trials), but the risk arising from the fluctuation of returns is great. The group may lose all their chocolates.

3. For interested students, the teacher could provide further reading materials, for example: the booklet *Applications of Mathematics: Investment Portfolios and Market Efficiency* (downloadable from *http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/portfolio.pdf*)

This example mainly involves the following generic skills:

1. Critical Thinking Skills

   - Make logical judgements based on adequate data and evidence, for example, stop playing the game after three rounds

2. Problem Solving Skills

   - Identify the problems associated with the rules of the game
   - Choose relevant information and strategies to solve problems

Example 22
Mathematics Reading Scheme

Key Stages: 1-4

Dimension/Strand: All

"Reading to Learn" is one of the Four Key Tasks in the curriculum reform. It aims to enhance students' skills in learning to learn and self-directed learning. Schools can build on their achievements in "Reading to Learn" and further promote "Reading across the Curriculum". This example provides suggested reading programme with activities that can be used:

(i) to develop students' interest and habit in reading texts related to mathematics and enhance their literacy skills;

(ii) to develop students' generic skills (such as communication skills and self-learning skills);

(iii) to provide opportunities for students' to link their mathematical knowledge with learning in other KLAs/subjects; and

(iv) to broaden students' understanding of the applications of mathematics in real life and the cultural aspect of mathematics.

Planning of Reading Programme/Activities

Below are some suggestions to help schools set up a Mathematics reading programme:

- Schools could set long term and short term objectives of their own reading programme and implement it by stages
- Schools could design their Mathematics reading programme with reference to
  - ✧ schools' major concerns,
  - ✧ students' reading competency,
  - ✧ students' interest and ability in mathematics,
  - ✧ teachers' experiences in promoting reading, and
  - ✧ reading materials available for students.
- Schools could encourage collaboration between the Mathematics panel, other subject panels and the school library for organising reading activities for students
- Teachers could review and categorise mathematics reading materials which suit the interest and ability of the target students and plan in advance to increase the quantities of good mathematics reading materials available in the classrooms or school library. Teachers are also encouraged to make good use of resources of public libraries.

- Teachers could design suitable post-reading activities to reinforce students' learning and help students discover the mathematics concepts, application of mathematics in different disciplines and understand the cultural aspects of mathematics in the reading materials.
- Schools could choose appropriate methods and suitable success criteria to evaluate the effectiveness of the implementation plan regularly.

School examples:

School A (Primary)

Background: Reading to learn is a major concern of the school in the current school development cycle. Students of the school are keen on reading but they have very little experience in reading mathematics books. Parents are supportive to school's policies and the reading scheme. The school has provided more mathematics books for lower primary students in the school library.

Implementation Plan for the Reading Scheme

Target: Primary 2 Students

Objectives:

1. To create a positive atmosphere of learning Mathematics through reading
2. To arouse students' interest and develop reading habits in reading books or passages with themes related to mathematics
3. To enhance students' communication skills through sharing activities

Activities and Strategies:

- Introduction of the Reading Scheme
  To brief parents and target students on the objectives, activities and details of the reading scheme in a morning assembly in September.

- Reading with parents
  Students read with their parents four times in each term. Teachers deliver a book or a reading passage in a reading bag each time for students to read at home with their parents and fill-in the record book.

- School-wide Reading Programme
  Students join the School-wide Reading Programme of the school library and are encouraged to borrow books with themes related to mathematics from the school library. They also have to record their reading in the record book. They receive awards after the completion of the Programme and can earn an extra gift for every three mathematics related books read.

- Reading with buddies
  Students of P.5 and P.6 to be Mathematics Ambassadors are arranged to read books with P.2 students in the morning reading period once a week.

- Sharing of good books or passages
  Students share the content of the books or passages read during Mathematics lessons and elect the most popular book or passage after the sharing sessions in each term.

School B (Secondary)

Background: The school carried out a pilot reading scheme in S1 in the previous year to arouse students' interest in reading and to help students develop habits in reading mathematics books. Teachers participating in the pilot scheme share their students' achievement and the effectiveness of the pilot scheme with other teachers in the Mathematics Department. Built on their experience and good practices, they are keen to extend the reading scheme to all junior forms.

Implementation Plan for the Reading Scheme

Target: Junior secondary students

Objectives:

1. To enhance students' thinking skills, problem solving skills and creativity through reading texts with themes related to mathematics
2. To broaden students' exposure and their knowledge in mathematics
3. To nurture students' understanding of the cultural aspects of mathematics
4. To help students link the learning of Mathematics with other subjects, e.g. language subjects and History.

Activities and Strategies:

- Mathematics Reading Group

  In groups of four, students work on a theme set by the teachers. They are required to search and read relevant books or information from the Internet. The themes for S1, S2 and S3 are “Stories of mathematicians”, “Mathematics games and puzzles” and “Mathematics in daily life” respectively.

- Books Recommendation

  The teachers regularly share and recommend relevant books or reading materials from the Internet for students to read to help them accomplish their tasks.

- Reading Worksheet

  The teachers design different worksheets on each theme. For example, S1 students are required to select a mathematician, read books about his life and contributions and set a question on what they have found about the mathematician. For S2 students, they have to read books about mathematical games and puzzles, and then post challenging problems for their classmates.

- Sharing Sessions

  After the group work, students present and share their reading experiences and achievements in the class. They share the questions posted in the worksheet for class discussion.

- Publication of Students’ Work

  The teacher compile students’ work into a book to showcase students’ achievement in reading.

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Appendices

Appendices

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Appendix 1 : Seven Learning Goals of the Primary and Secondary Education

The two sets of learning goals for the primary and secondary education are updated in 2014 and 2017 respectively to respond to the latest changes from a natural continuum of progressive milestones that students are expected to achieve upon completion of the two levels of education based on the seven areas covering knowledge, skills, and values and attitudes for lifelong learning and whole-person development. The learning goals at the two levels are defined using slightly different diction and sequences due to consideration of the students' developmental and growth needs and learning experiences.

Seven Learning Goals of Primary Education

Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6) (2014)

To enable students to

- know how to distinguish right from wrong, fulfil their duties as members in the family, society and the nation, and show acceptance and tolerance towards pluralistic values
- understand their national identity and be concerned about society, the nation and the world, and to fulfil their role as a responsible citizen
- develop an interest in reading extensively and cultivate a habit of reading
- actively communicate with others in English and Chinese (including Putonghua)
- develop independent learning skills, especially self-management skills and collaboration skills
- master the basics of the eight Key Learning Areas to prepare for studying in secondary schools
- lead a healthy lifestyle and develop an interest in aesthetic and physical activities and an ability to appreciate these activities

Seven Learning Goals of Secondary Education

Secondary Education Curriculum Guide (2017)

To enable students to

- become an informed and responsible citizen with a sense of national and global identity, appreciation of positive values and attitudes as well as Chinese culture, and respect for pluralism in society
- acquire and construct a broad and solid knowledge base, and to understand contemporary issues that may impact on students' daily lives at personal, community, national and global levels
- become proficient in biliterate and trilingual communication for better study and life
- develop and apply generic skills in an integrative manner, and to become an independent and self-directed student for future study and work
- use information and information technology ethically, flexibly and effectively
- understand one's own interests, aptitudes and abilities, and to develop and reflect upon personal goals with aspirations for further studies and future career
- lead a healthy lifestyle with active participation in physical and aesthetic activities, and to appreciate sports and the arts

Appendix 2 : Development of Generic Skills in Mathematics Curriculum

Nine generic skills have been identified as essential for student learning for the 21st century in the school curriculum. They are grouped in three clusters of related skills, namely Basic Skills, Thinking Skills and Personal and Social Skills, for better understanding and application in a holistic manner:

Basic Skills	**Thinking Skills**	**Personal and Social Skills**
Communication Skills	Critical Thinking Skills	Self-management Skills
Mathematical Skills*	Creativity	Self-learning Skills*
Information Technology Skills	Problem Solving Skills	Collaboration Skills

Details of the 9 generic skills and how the Mathematics Education KLA contributes to the development of these skills are provided in this appendix.

* "Mathematical Skills" and "Self-learning Skills" have been referred to as "Numeracy Skills" and "Study Skills" respectively in earlier curriculum documents such as *Learning to Learn: Life-long Learning and Whole-person Development* (2001).

Collaboration Skills

Problem solving, planning and making decisions in a small group require collaboration skills, namely the skills of communication, appreciation, negotiation, making compromises and asserting leadership. Students with these skills will be able to effectively engage in and contribute to tasks involving teamwork.

The expected achievements of the students in this generic skill cannot be suitably classified according to Key Stages.

Expected Achievements	Examples of Implementation in Mathematics Education
1. Understanding the nature of group work Students will learn to • recognise the need for teamwork and that the team has a shared responsibility • recognise that individuals as well as the team have to take the consequences for their own actions	Students 1. participate in group work in mathematics like collecting data, measuring objects and presenting project results, and understand the benefits of working as a team 2. accept and follow the group decision on individual role in doing mathematical group work with the understanding that member's actions affect the product of group work (e.g. whether a correct solution could be found)
2. Desirable dispositions for group work Students will learn to • be open and responsive to others' ideas; appreciate, encourage and support the ideas and efforts of others • be active in discussing and posing questions to others, as well as in exchanging, asserting, defending and rethinking ideas • recognise and avoid stereotyping; withhold premature judgement until the facts are known • be willing to adjust their own behaviour to fit the dynamics of various groups and situations	Students 1. discuss and exchange ideas and findings openly with others in completing mathematical tasks and solving mathematical problems 2. exercise patience and listen to others in the discussion of mathematical problems (e.g. when investigating number patterns and formulating proofs of geometric problems) 3. value the contributions of others and put effort to achieve synergy in accomplishing mathematical tasks or solving mathematical problems together 4. appreciate different solutions to mathematical problems presented by others (e.g. using different approaches to prove mathematical theorems) 5. participate actively in discussions, pose questions in clarifying arguments in the solution of mathematical problems and offer improvements and corrections with justifications (e.g. discussing the strategies to be adopted in investigating statistical problems)

Expected Achievements	Examples of Implementation in Mathematics Education
3. Skills for group work Students will learn to <u>Goals setting</u> • select a strategy and plan cooperatively to complete a task in a team <u>Role taking</u> • understand the strengths and weaknesses of members and maximise the potential of the team • clarify and accept various roles and responsibilities of individual members in a team and be willing to follow team rules <u>Synergising</u> • liaise with members for views and resources • negotiate and compromise with others <u>Reflection</u> • reflect on and evaluate the strategy used by the group and make necessary adjustments	Students 1. share with groupmates their experience in solving mathematical problems and select a suitable strategy with agreed justifications 2. take up group tasks according to their strengths and weaknesses and are respectful to the roles assigned (e.g. members with good presentation skills, IT skills, modelling skills, etc. taking up related roles in accomplishing the mathematical tasks) 3. clarify arguments objectively and rationally in solving mathematical problems and look for a logical structure in addressing mathematical challenges (e.g. examining the appropriateness of a particular problem solving strategy) 4. liaise, negotiate and compromise with others in selecting suitable strategy for solving a mathematical problem (e.g. choosing between a deductive or an analytic approach to solving a geometrical problem, and negotiating with group members to set the theme and research method of a statistical study) 5. evaluate the strategy adopted and suggest improvements (e.g. evaluating the effectiveness of data collection by small teams, or presenting the results by a single member and suggesting a more effective method)

Communication Skills

Communication skills refer to the abilities to achieve the desired outcomes or goals in a process where two or more people interact (be it in a face-to-face or virtual context) through expressing or receiving messages using verbal and non-verbal means. To communicate effectively, students should learn to listen, speak, read and write competently. Not only should they express themselves in an accurate, organised and proper manner, but they should also understand and respect others' views and expectations, and use appropriate information and means to convey a message in accordance with the purpose, context and audience. They should also evaluate the effectiveness of their communication and identify areas for improvement to achieve the best results.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • comprehend and act appropriately on spoken instructions • comprehend the explicit messages conveyed in information from different media • use clear and appropriate means of communication, both verbal and non-verbal, to express meaning and feelings • work and discuss with others to accomplish simple tasks	Students 1. retrieve information from pictograms or block graphs according to the spoken instructions from teachers 2. interpret drawings, figures (e.g. rectangles and cubes) and symbols (e.g. +, –, ×, ÷) and describe them in plain language (e.g. 2 + 3 as 2 plus 3) and appropriate mathematical terms (e.g. a cube has six faces) 3. present findings with drawings and symbols 4. present data with tables and graphs (e.g. bar charts) 5. compare fractions and give verbal explanation to the result of comparison 6. express simple daily life problems in mathematical language (e.g. use symbols like $4×3) 7. work in small groups to discuss the method to measure the length and width of the classroom
Key Stage Two (Upper Primary) Students will learn to • comprehend and respond to different types of texts • comprehend and infer the messages conveyed in information from different media • use spoken, written, graphic and other	Students 1. interpret drawings, symbols (e.g. %), tables and graphs (e.g. read and discuss broken line graphs) 2. describe and explain findings/results of mathematical tasks in both oral and written forms (e.g. the average score of a student's performance in tests, the favorite fruit) 3. present results of tasks with appropriate drawings

Expected Achievements	Examples of Implementation in Mathematics Education
non-verbal means of expression to convey information and opinions, and to explain ideas • work and negotiate with others to develop ideas and accomplish tasks	and symbols and present data with tables, charts and graphs (e.g. bar charts, broken line graphs) 4. present solutions of problems logically (e.g. use of "=" properly) 5. express simple problems in mathematical language 6. discuss with others in accomplishing tasks such as projects on finding volumes of irregular objects
Key Stage Three (Junior Secondary) Students will learn to • understand, analyse, evaluate and respond to a range of different types of texts • synthesise the messages conveyed in information from different media • use appropriate language and/or other forms of communication to present information and different points of view, and to express feelings • work and negotiate with others to solve problems and accomplish tasks • reflect and improve on the effectiveness of their own communication	Students 1. interpret a daily life problem from the media and identify mathematical elements within (e.g. identify the spread rate of an epidemic) 2. describe findings or explain conjectures in both oral and written forms using mathematical language (e.g. the two triangles are congruent) 3. choose appropriate statistical charts to present data and use appropriate mathematical terminology or symbols in explaining ideas 4. formulate and write simple geometric proofs involving 2-D rectilinear figures with appropriate symbols, terminology and reasons 5. respond appropriately to others' mathematical arguments using precise mathematical vocabulary in both oral and written forms 6. recognise the differences between the language used in a mathematical context and that in daily life (e.g. the word "similar") and describe mathematical ideas and procedures in ordinary language 7. present solutions to a problem (e.g. explain calculation strategies) to others using mathematical language and ordinary language appropriately
Key Stage Four (Senior Secondary) Students will learn to • listen and read critically, evaluate the messages conveyed in information from different media and express ideas fluently in accordance with the audience and reader • use appropriate means of communication to inform, entertain, persuade and argue to achieve expected outcomes	Students 1. demonstrate a clear understanding of mathematical content or application of mathematics through logical presentation (e.g. write and present more complex geometric proofs with appropriate symbols and terminology) 2. intertwine ordinary language, mathematical language, diagrams and graphs to describe and explain mathematical concepts or solutions of real-life problems (e.g. to answer which group of athletes has a more stable performance by statistical

Expected Achievements	Examples of Implementation in Mathematics Education
• resolve conflicts and solve problems with others to accomplish tasks • evaluate the effectiveness of their communication with others from different perspectives for further improvement	measures) 3. explain how a mathematical concept is applied in an authentic situation in a way that a layman can understand (e.g. the Richter Scale) and further elaborate using appropriate graphs or charts after receiving feedback from others 4. produce concise and structured report on statistical survey for group discussion and evaluate the effectiveness of the graphs chosen for presenting the findings

Creativity

Creativity brings in changes or transformations and is manifested in new ideas, acts or products. It emerges spontaneously or through deliberate processes of divergent and convergent thinking. It involves the integration of general or domain-specific knowledge for a meaningful purpose.

Although the expected achievements of the students in this generic skill cannot be suitably classified according to Key Stages, development of creativity involves the following *abilities*, *dispositions* and *favourable factors for nurturing creativity*.

1. Abilities

Abilities	Descriptions
Sensitivity	To discern details from observation and quickly respond to stimulus
Fluency	To generate numerous ideas promptly
Flexibility	To adapt varied ideas and to initiate new thoughts for action
Originality	To produce unusual, novel and unique ideas
Elaboration	To expand, refine and embellish ideas

2. Dispositions

Dispositions	Description
Curiosity	To show interest and desire to find out more
Risk-taking	To show courage and determination to deal with uncertainties or ambiguities
Imagination	To enjoy fantasising and generating new ideas
Complexity	To be attracted to intricacies and novelty; to embrace challenges

3. Favourable Factors for Nurturing Creativity[13]

Factors	Corresponding actions
Place	• To foster a supportive environment (open, inviting and accepting atmosphere; resourceful, safe yet stimulating environment)
Person	• To recognise and accommodate the wide range of attributes and dispositions of students (strengths, weaknesses, learning styles, learning needs, motivation and readiness) • To identify and develop students' potential for creative acts

[13] Mooney, R. L. (1975). A conceptual model for integrating four approaches to the identification of creative talent. In C. W. Taylor & F. Barron (Eds.), *Scientific Creativity: Its Recognition and Development* (pp. 331-340). New York, NY: Robert E. Krieger.

Factors	Corresponding actions
Process	• To open up alternatives for students to explore personal interest • To provide interesting and stimulating themes conducive to arousing creative acts and satisfying a craving • To expose students to various stages of creating new ideas, acts or products (preparation, incubation, illumination and verification) • To value attempts to present new ideas and encourage further refinements
Product	• To encourage creative actions and output (ideas, plans, methods, solutions, products, theories) • To value the creative experience and celebrate students' creative output • To encourage students to persuade others (especially experts in the field) to accept the creative output

Examples of Implementation in Mathematics Education:

Key Stage 1	Students 1. propose applications of division in daily life 2. design a method to compare the capacities of two irregular containers 3. design methods to draw parallel lines and perpendicular lines using objects in simple 2-D or 3-D shapes
Key Stage 2	Students 1. create polygons with the same perimeter or area but in different shapes 2. design symmetrical figures or patterns 3. create different methods to find the perimeter of a circular object
Key Stage 3	Students 1. design patterns based on tessellations of a plane using triangles or quadrilaterals 2. design a card robot formed by different regular polyhedra and prisms 3. participate in group project to design a ramp for wheelchairs users for a building in their communities
Key Stage 4	Students 1. design encryption and decryption methods to transmit a message to their classmates 2. design a container for tennis balls that meets some preset requirements on shape, capacity and volume of material used 3. propose alternative solutions to problems

Critical Thinking Skills

Critical thinking is drawing out meaning from available data or statements, and examining and questioning their accuracy and credibility in order to establish one's views and evaluate the arguments put forward by oneself and others.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • extract, classify and organise information • identify and express main ideas, problems or core issues • understand straightforward cause-and-effect relationships • distinguish between obvious fact and opinion • notice obvious contradictions, seek clarifications and make simple predictions • draw simple but logical conclusions not contradictory to given data and evidence	Students 1. sort objects using various criteria such as shapes and sizes 2. choose the right tools to measure objects such as using measuring tapes to measure the circumference of a round table 3. reason inductively (e.g. when exploring the commutative property of addition) 4. choose relevant information in solving a problem (e.g. tell the number of toys from the information that there are 3 trains, 5 apples and 6 dolls in a box) 5. check the reasonableness of the answer to a problem (e.g. whether the amount of water drunk by a student per day is too large to be realistic)
Key Stage Two (Upper Primary) Students will learn to • make inductions/inferences from sources • cross-reference other sources to determine the reliability of a source • understand the concepts of relevance and irrelevance • distinguish between fact and opinion as well as source and evidence • recognise obvious inconsistencies, omissions, assumptions, stereotypes and biases • formulate appropriate questions, and make reasonable predictions and hypotheses • draw logical conclusions based on adequate data and evidence, and make predictions about consequences	Students 1. categorise information using various criteria such as considering properties of length and parallel sides when discussing properties of quadrilaterals 2. choose appropriate methods and units to measure objects, such as using the method of displacement to measure the volumes of irregular objects 3. reason inductively (e.g. when exploring the formula for the area of a rectangle) 4. check the reasonableness of the solution to a problem (e.g. the steps for solving a problem might be unreasonably complicated) 5. ask questions about mathematics during exploration (e.g. ask questions about the relation between circumferences and diameters of circles after related measurements of circular objects)

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Students will learn to • identify the issue at stake • clarify and define key words to guide thinking • compare information from different sources, note contrasts and similarities, and determine its reliability • differentiate between fact, opinion and reasoned judgement • recognise that information providers' value orientations and ideologies would affect the perspectives or judgement of sources • recognise and challenge stereotypes, emotional factors, propaganda and fallacies • draw and test conclusions as well as hypotheses, identify reasonable alternatives and predict probable consequences • admit their own limitations, shortcomings or errors of the thinking process	Students 1. use deductive reasoning to study the properties of geometric figures, such as "the sum of exterior angles of a convex polygon is four right angles" 2. generalise observations in symbolic forms from concrete experiences (e.g. when generalising the index laws from observing several examples) 3. judge whether the information given about a problem is relevant or not (e.g. identifying extraneous information given in a geometric problem) 4. examine the reasonableness of the solution to a problem and evaluate the strategy adopted (e.g. evaluate the effectiveness of using the graphical method to solve linear equations in one unknown) 5. compare different ways of approaching traditional mathematical problems (e.g. different proofs of Pythagoras' Theorem or different ways of tiling a floor using triangular tiles of a single size and shape)
Key Stage Four (Senior Secondary) Students will learn to • differentiate between real and stated issues, false and accurate representations, and relevant and irrelevant evidence • differentiate between sophisticated fact, opinion and reasoned judgement • recognise and challenge subtle or fundamental assumptions, permeating value orientations and ideologies • recognise that the selection and deployment of information/facts are affected by personal perspectives • draw warranted conclusions, predict and assess probable consequences and make reasoned judgement in reading, writing and speech • apply appropriate thinking skills to evaluate and reflect on their thinking process and suggest ways for improvement	Students 1. investigate and judge the validity of arguments derived from analysing data sets 2. assess statistical investigations presented in different sources such as news media and research reports, and be aware that different motives, perspectives and values may affect the conclusion (e.g. a company might abuse statistical measures to boost its sales) 3. compare and evaluate the effectiveness of graphical method and algebraic method in solving quadratic inequalities 4. construct, check and amend deductive proofs of geometric properties involving circles 5. reflect on problem solving method to see if there are any mistakes or imperfections (e.g. reflect on whether the value of common ratios is considered before applying the formulae for finding infinite sums of geometric sequences)

Information Technology Skills

Information technology skills are the ability to use IT critically to search, select, analyse, manage and share information. Mastery of IT skills facilitates collaborative learning, problem solving and self-directed learning.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • operate computers or mobile devices • input Chinese characters • use e-resources to support learning with the help of teachers • recognise some methods to locate and access information with given search criteria • generate, present, and safely share ideas with IT tools in learning activities	Students 1. use suitable application software to investigate number patterns and properties of numbers (e.g. odd, even, ascending, descending) 2. use suitable application software to create and explore geometric patterns (e.g. apps on tangram) 3. search from the Internet the lengths of the cross-harbour tunnels
Key Stage Two (Upper Primary) Students will learn to • use a variety of software packages for word-processing, calculation, image-processing and other learning activities • produce multimedia presentations with simple design • search, select and prudently share information via computer networks and other media • process information and produce user-generated content[14] using IT tools	Students 1. use suitable application software to investigate the properties of shapes, draw and create geometric patterns (e.g. draw squares, rectangles, triangles, circles and create geometric patterns with these figures) 2. use a spreadsheet to record data and create graphs for doing statistical projects (e.g. input data in a spreadsheet and present the data with line graphs or bar charts) 3. use the information obtained from the Internet for self-directed learning and when doing projects (e.g. symmetric objects in daily life) and share the results through school e-platforms

[14] User-generated content refers to content that is produced and shared by end-users of digital media.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Students will learn to • use appropriate IT tools to facilitate learning • use IT tools and strategies for processing and presenting information • produce multimedia presentations with appropriate design for different purposes • communicate and collaborate with others via computer networks and other media • verify and evaluate the accuracy and reliability of information	Students 1. use graphing software for various computational and exploratory activities (e.g. draw straight lines and explore their condition for perpendicularity) 2. use suitable application software to explore the relations of numbers (e.g. number patterns), formula (e.g. formulae of areas and volumes) and graphical representations (e.g. finding solutions of simultaneous equations by the graphical method) 3. use suitable application software to construct appropriate statistical diagrams (e.g. frequency polygon, histogram) to represent given data and to find simple statistical measures (e.g. mean, mode) 4. use dynamic geometry software to explore properties of 2-D rectilinear figures (e.g. the relationship among the angles or sides of a parallelogram) and to visualise geometric properties of 2-D and 3-D figures (e.g. uniform cross-section of prisms) 5. use the information obtained through the Internet, for doing projects (e.g. study the climate change in Hong Kong) and in self-directed learning, and discuss the reliability of information through school e-platforms
Key Stage Four (Senior Secondary) Students will learn to • strengthen capability in IT usage for lifelong learning • analyse and ethically use information from different sources for specific purposes • compare the effectiveness of various ways, including the use of IT tools, to solve a given problem • select and apply appropriate IT tools in different aspects of study, including processing information, generating and communicating original ideas artfully to audience with different backgrounds	Students 1. use dynamic geometry software to explore the locus of points satisfying given conditions and to visualise 3-D problems 2. use graphing software to sketch graphs of functions while tackling problems on transformations of graphs 3. integrate the use of a wide range of IT tools/ application software when doing statistical projects 4. select and obtain reliable data from the Internet (e.g. data from the Census and Statistics Department), choose appropriate graphs for presenting the data and discuss the results through school e-platforms 5. judge the appropriateness of using IT tools in solving mathematical problems (e.g. using dynamic geometry software to formulate a geometric proof)

Mathematical Skills[15]

Mathematical skills include the ability to perform computations and estimations of numbers in various forms, to describe spatial relationships between objects, to perform measurements, to manage data, to employ logical reasoning for drawing valid conclusions, and to apply mathematical concepts in different contexts.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • perform comparison and basic computations of whole numbers • describe shapes, sizes and positions • apply the knowledge of measurement and use appropriate units and tools for measurement • present data by means of and retrieve information from simple charts and graphs • perform simple deductions with the use of basic logical concepts, such as "and", "or", "all", "some", "because", "if … then" and "contradiction" • apply simple mathematical knowledge in daily life	Students 1. describe and compare the number of objects with natural numbers 2. perform properly basic computations involving whole numbers 3. sort and name objects (e.g. biscuits boxes, ice-cream cones) of different geometric shapes 4. recognise basic directions: east, south, west and north 5. measure the length and width of a classroom or a playground with suitable instruments (e.g. a metre ruler and a trundle wheel) to determine the amount of decoration materials 6. read simple statistical charts (e.g. simple bar charts) 7. form numbers that satisfy two conditions, e.g. use four numerals to form a 4-digit number which is odd and smaller than a fixed value
Key Stage Two (Upper Primary) Students will learn to • perform computations and simple estimations involving whole numbers, fractions, decimals and percentages, such as estimating expenses • use simple geometric properties, such as symmetry, parallel and perpendicular, to describe shapes, sizes and positions more accurately • apply strategies and formulae in measurement • collect and process data, present data	Students 1. choose the correct forms of numbers in presenting information (e.g. using percentages to describe the size of the portion of students with glasses in the class) 2. estimate the total expenditure of purchases made in a week 3. use the 8 compass points, e.g. north-east, south-west, for locating the relative positions of buildings 4. apply formulae to find the area of a desk top and the capacity of a locker in the classroom 5. collect classmates' preferences on food and drinks offered in the school Christmas party and present the data using suitable statistical graph for making

[15] In the context of generic skills, Mathematical Skills refer to the ability to apply mathematics in different key learning areas and subjects. The concepts and skills of the Mathematics subject to be applied are only those generally applicable to various disciplines.

Expected Achievements	Examples of Implementation in Mathematics Education
by means of suitable charts and graphs and retrieve information from charts and graphs • perform deductions, such as syllogism and provide counter examples • apply mathematical concepts in daily life	decisions 6. provide counter examples to disprove the statement "a 2-D figure with four equal sides must be a square" 7. use equations to solve simple problems
Key Stage Three (Junior Secondary) Students will learn to • handle very large or very small numbers and negative numbers with a sense of scale • perform numerical manipulations, such as percentage changes, and perform estimations with appropriate strategies • describe the rules of arrangement of objects or occurrence of events, such as the pattern formed by a set of shapes and the trend of population growth • describe spatial relationships between objects using distance, angle, scale, bearings and gradient • choose appropriate tools and strategies to find measurements according to the degree of accuracy required by the specific purpose • use different methods for handling (i.e. collecting, organising, analysing and presenting) quantitative information and make reasonable interpretation of the results • estimate risks and chances through the use of elementary probability • perform deductions and verifications, and check their validity • apply various mathematical concepts in authentic situations	Students 1. present the length of a plant cell and the speed of light using suitable unit and notation 2. calculate the compound interest of a saving scheme 3. estimate the amount of materials needed for decorating the classroom 4. describe the seat arrangement of a theatre 5. locate positions on a map by considering bearings of reference buildings 6. appreciate the proofs of the Pythagoras' Theorem and verify some Pythagorean triples 7. provide reference for the selection of school representatives for joint school athletic games by considering statistics of the performance of school team members

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage Four (Senior Secondary) Students will learn to • evaluate the appropriateness of tools and strategies for handling quantitative information • use quantitative information for making informed decisions in different contexts • evaluate processes of deductions to avoid committing logical fallacies • apply various mathematical concepts in different contexts with appropriate strategies and be aware of the need to make adaptations in new situations	Students 1. select and use the measures of central tendency and dispersion to compare students' performance in Mathematics assessments 2. carry out project learning by studying statistical information from the Hong Kong Yearbook or the website of the Hong Kong Observatory 3. check whether all conditions are fulfilled before applying a theorem 4. study on the wealth disparity between the more developed countries and the less developed ones (support the reasoning on social issues through organising and analysing numerical information in statisitcal charts)

Problem Solving Skills

Problem solving involves using various skills to resolve a difficulty. The process includes investigating the problem, synthesising information and generating ideas to determine the best course of action. Students need to adjust and evaluate strategies, as well as consolidate experience for knowledge construction.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • develop ideas about the problem and identify related sources of information • identify, under guidance, one or more ways of tackling the problem • choose and implement a solution plan, using support and advice given • follow the given step-by-step methods to check and describe the outcomes	Students 1. identify key information from word problems (e.g. operations required or quantities to be computed) 2. solve problems by simple computations (e.g. using addition to find the sum of money used in buying goods) 3. adopt various ways of solving problems (e.g. using drawing or manipulatives to do addition) 4. solve problems by choosing the correct data (e.g. identifying the maximum sale from a bar chart of daily sales)
Key Stage Two (Upper Primary) Students will learn to • identify the problem and describe its main features • propose alternative courses of action for solving it • plan and try out the selected option, obtain support and make changes when needed • develop an appropriate method to measure the effectiveness of the solution plan adopted • gain insights from the problem solving process	Students 1. use different approaches to solve problems (e.g. finding the area of a polygon by means of counting the number of squares, dissecting the polygon into parts or using formulae) 2. make use of various tools to solve problems (e.g. measuring tapes and metre rules) 3. choose the correct data and relevant information to solve problems (e.g. identify the correct height and base in finding the area of a triangle) 4. compare new problems with similar ones solved before and try to apply past experience to solve the problems (e.g. comparing the problem of finding the area of a trapezium to that of a triangle) 5. perform computations in different sequences and compare the computation processes

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Students will learn to • explore the problem and identify its main focus • suggest and compare the possible outcomes of each alternative course of action and justify the option selected • execute the planned strategy, monitor the progress and make adjustment when necessary • evaluate against established criteria the quality of outcomes, and review the effectiveness of the problem solving process • formulate personal views, and paraphrase or construct analogies to explain how the problem is solved	Students 1. understand the given conditions of a geometric problem, identify the proposition to be proved, and devise a plan for proving it using appropriate theorems 2. apply analytic and deductive approaches for solving geometric problems 3. judge and evaluate arguments of their own or others in presenting a solution to a mathematical problem (e.g. evaluating the reasons supporting that "mode" is the best measure of the most popular size of shoes in a market) 4. formulate a mathematical solution when tackling a problem in an authentic situation (e.g. finding the floor area of a flat from its layout plan)
Key Stage Four (Senior Secondary) Students will learn to • recognise the complexity of the problem and search for appropriate information required to solve it • formulate feasible strategies to achieve optimal results, considering both long and short term objectives • modify objectives or strategies and suggest remedial or enhancing measures to cope with circumstantial changes or difficulties • evaluate the overall strategy and outcomes, and anticipate future problems that may be incurred • consolidate experience on problem solving for knowledge construction	Students 1. search for appropriate information when carrying out a project on comparing mortgage plans of different banks 2. evaluate the anticipated amount of computations needed for solving a problem (e.g. solving simultaneous equations involving a quadratic one), and choose appropriate tools for computation or use another strategies to reduce computation time 3. review and modify the questionnaire design of a statistical survey on weekly time spent on mobile devices (e.g. number of options, ranges of time durations) before data collection 4. evaluate a sampling method and find out its limitations

Self-learning Skills

Self-learning skills refer to the ability to initiate, plan, carry out, evaluate and adjust learning activities autonomously. Students with advanced self-learning skills can select or design effective strategies for in-depth learning. These skills help students enhance their academic performance and self-efficacy. Self-learning skills form the core part of lifelong learning and help students acquire new knowledge to adapt to the fast changing world.

Expected Achievements	Examples of Implementation in Mathematics Education
Key Stage One (Lower Primary) Students will learn to • consciously listen and read to learn, and actively present their learning • concentrate and pay attention to instructions • identify and retain main ideas • collect information from given sources and organise it into pre-determined categories • try out different means to present ideas and demonstrate learning • develop simple learning plans to meet short term targets • show interest in enquiring further	Students 1. listen to instruction carefully, participate actively and show interest in learning activities in the class (e.g. exploring different types of triangles by using straws of equal or different lengths to form triangles) 2. understand concepts and do not learn only by rote memorisation (e.g. understand the concept of multiplication instead of just memorising the multiplication tables) 3. present simple problems in mathematical languages including symbols and graphs (e.g. pictograms)
Key Stage Two (Upper Primary) Students will learn to • take initiative in the enquiry learning area selected by themselves • actively locate required information from different media • take initiative in identifying and organising main points from different sources, e.g. note-taking, mind-mapping • decide on the most suitable means to present ideas and demonstrate learning • seek help appropriately when necessary • manage time to complete tasks according to a plan	Students 1. try to understand concepts and not to learn only by rote memorization (e.g. recognise the connection between the formulae of areas of different figures instead of memorise the formulae) 2. search for reference information from various sources including libraries, reference books, Internet, etc. 3. use different aids to build up mathematical concepts (e.g. use a table of integers from 1 to 100 to study the patterns of multiples) 4. use diagrams, pictures and charts to help understand mathematical concepts (e.g. mark the locations of moving objects at different time to understand the idea of speed) 5. learn from mistakes made in homework exercises, tests,

Expected Achievements	Examples of Implementation in Mathematics Education
• make use of feedback to reflect on the effectiveness of different learning tactics	etc. and learn from feedback given by teachers and classmates in solving mathematical problems
Key Stage Three (Junior Secondary) Students will learn to • initiate learning activities and apply relevant personal strengths to overcome challenges • set learning plans with stage-wise goals • identify lines of reasoning and possible hidden ideas in sources • function effectively in a group to achieve the learning goals • decide on the most suitable means to manage and present knowledge • adjust the learning strategies to improve learning effectiveness	Students 1. initiate and plan exploratory activities for knowledge construction (e.g. participate in a mathematics project competition) 2. learn from mistakes made in solving mathematical problems and identify their own incorrect concepts 3. identify key similarities and differences from working among various types of mathematical problems 4. use diagrams, pictures and charts to present and help understand mathematical concepts (e.g. use a diagram to present the classification of quadrilaterals) 5. contribute constructive ideas in group discussion for explaining mathematical results or suggesting strategies for solving mathematical problems
Key Stage Four (Senior Secondary) Students will learn to • initiate challenging learning activities and develop relevant personal strengths to overcome challenges • plan and set goals for self-initiated enquiries • autonomously select or design more effective learning strategies for • in-depth learning • evaluate key ideas, opinions and arguments identified from different sources independently, and synthesise them to construct and develop their own interpretation • evaluate and suggest ways to improve the effectiveness of learning strategies • learn beyond the prescribed curriculum and apply knowledge in a variety of contexts	Students 1. generalise observations to a higher level of abstraction for transferring strategies to a wider range of problems (e.g. recognise some typical counting problems for solving counting problems of similar structures) 2. initiate and plan exploratory activities for learning beyond the prescribed curriculum (e.g. project work on solving real-life problems, such as redesigning the layout of the school carpark) 3. select more effective learning strategies, such as exploration examples with increasing complexity for learning the properties of arithmetic sequences and geometric sequence instead of rote memorisation of related formulae 4. acquire further understanding of mathematical concepts by exchanging and discussing ideas with others, testing the hypothesis, searching for information from the Internet, etc., and evaluate effectiveness of different strategies 5. solve real-life and mathematical problems by studying the information for clues, presenting the problem in mathematical language, or integrating knowledge and skills of various strands of the curriculum

Self-management Skills

Self-management skills comprise essential life skills and desirable personal qualities such as maintaining emotional stability, making decisions and exercising self-discipline. Self-management skills enable students to embrace challenges encountered on a personal or team basis.

The expected achievements of the students in this generic skill are classified according to different levels of mastery.

Elements of Self-management Skills	**Beginning ——— Developing ——— Mastering** **Students will learn to**		
Self-worth	express positive statements about themselves	identify and apply personal skills, attitudes and values to overcome challenges	uphold, synthesise and renew their own beliefs and values
Goal setting and tracking	set goals to assist their learning and personal development	set and keep track of realistic goals	set, keep track of, and be reflective on and accountable for goals which work towards excellence in life
Decision making	make decisions in daily life situations with supporting reasons	list out and evaluate the pros and cons of a suggestion, and make prediction about the consequences of a decision	consider all factors, such as technical, ethical, resource and community considerations before making a decision
Confidence, resilience and adaptability	develop confidence and resilience in performing simple tasks and appreciate the progress made	demonstrate motivation, confidence, commitment and adaptability when faced with new or difficult situations, and derive satisfaction from accomplishments and efforts	demonstrate confidence and adaptability in adversities, tolerate ambiguities and appreciate lessons learnt from mistakes
Appropriate expression of emotions	understand, accept and appropriately express emotions	describe their feelings, such as joy and disappointment and identify factors contributing to these feelings	use appropriate means to contain or release their emotions
Managing resources	demonstrate care for personal properties and shared resources	treasure and make good use of time, money and other resources	suggest ways for effective, equitable and ethical use of resources

Elements of Self-management Skills	**Beginning** ——— **Developing** ——— **Mastering** **Students will learn to**		
Keeping promises to others	keep promises and fulfill obligations	assess feasibility before making promises	make determined efforts to keep promises; take responsibility and make up for broken promises obliged by circumstances
Self-discipline	exercise self-control against distractions, and focus on and complete given tasks at hand within a given time	extend self-control in scope and duration over personal impulses through developing positive thinking and self-affirmation	exercise self-control naturally as a habit of mind
Reflective Practice	review their learning readily to know more about themselves and how they work	form habits of reviewing their learning and identify factors that contribute to or hinder their learning effectiveness	sustain self-improvement by paying attention to and making judicious use of feedback

Elements of Self-management Skills	**Examples of Implementation in Mathematics Education**
Self-worth	Students: 1. appreciate their positive learning attitudes or effort put in learning mathematics 2. identify their strength among the strands of Number and Algebra, Measures, Shape and Space, and Data Handling 3. reflect on their learning attitude to different mathematics topics as well as their mathematics ability attained and plan for improvement
Goal setting and tracking	Students: 1. set learning goals in mathematics (e.g. understanding a difficult concept/theorem) 2. review their learning goals by considering their assessment results in Mathematics 3. consult teachers for advice on their learning goals in mathematics and monitor the progress for meeting the goals
Decision making	Students: 1. explain their solutions logically 2. choose appropriate data collection methods for statistical surveys 3. choose strategies for solving mathematical problems by considering various factors such as tools available, time constraints, presentation of solutions and whether a general solution is available

Elements of Self-management Skills	**Examples of Implementation in Mathematics Education**
Confidence, resilience and adaptability	Students: 1. verify their computation results to ensure their accuracy 2. apply mathematics to solve problems independently, being self-reliant and confident 3. examine and correct wrong calculations, algebraic manipulations or deductive proofs, discover incorrect concepts behind and correct their understanding on the related topics
Appropriate expression of emotions	Students: 1. communicate their own feeling to others when facing failure in solving problems 2. express the sense of accomplishment after solving a difficult mathematical question 3. release one's negative feeling of failure by taking a break when they get stuck by a difficult problem
Managing resources	Students: 1. bring along with set squares, rulers, calculators or other learning tools to the lessons when required 2. use calculators or application software, to facilitate their enquiry activities 3. apply suitable application software in project work, for sharing data with groupmates, conducting simulations, plotting graph, etc.
Keeping promises to others	Students: 1. solve difficult problems with persistence 2. discuss with groupmates of project work to arrange jobs and complete their own parts on schedule 3. inform groupmates of project work when faced with unexpected circumstances, such as break down of computer or loss of data
Self-discipline	Students: 1. work neatly and tidily in accomplishing tasks (e.g. statistical projects) and doing mathematical problems (e.g. drawing geometrical figures with rulers and pencils) 2. participate and focus on learning tasks even not being closely overseen by teachers 3. persist in following learning plans, overcome difficulties and distractions, enjoy the outcomes of studies and appreciate their own effort when goals are achieved

Elements of Self-management Skills	**Examples of Implementation in Mathematics Education**
Reflective practice	Students: 1. make use of assessment results to understand their learning progress 2. self-evaluate their own strengths and weaknesses in mathematics learning 3. evaluate and make use of others' opinions in accomplishing tasks (e.g. different ways of collecting data, different strategies for solving geometrical problems) in order to improve the problem solving methods applied

Appendix 3 : Integrative Application of Generic Skills

When completing a more complicated learning task, the generic skills are often applied in an integrative manner rather than in isolation. The descriptors of two such integrative applications of generic skills, namely "collaborative problem solving skills" and "holistic thinking skills", are presented below. Examples of implementation in the Mathematics Education KLA are also provided after each set of descriptors.

Collaborative Problem Solving Skills

Collaborative problem solving skills refer to students' ability to solve problems with synergised efforts through effective division of labour, as well as incorporation of information from multiple sources of knowledge, perspectives and experiences. Compared to individual problem solving, collaborative problem solving has distinct advantages because it enhances the creativity and quality of solutions through stimulation brought by the ideas of other group members[16]. In the 21st Century, it is particularly important for people with different perspectives and talents to solve problem as a team with the effective use of communication technology.

The expected achievements of the students in collaborative problem solving skills are classified according to different levels of mastery.

Students will learn to

Beginning	Developing	Mastering
Collaboration		
• be ready to act responsively and reach the goals with team members • follow the rules and instructions set for the team work • participate actively in the team and contribute to achievement of the team goals	• share other team members' perspectives on the problem and establish a common understanding • identify and capitalise on the talents and potential of members • be able to work with different people and accept the adjustments to plans or roles in changing situations	• treasure working as a team and take initiative to foster synergy for attaining the team goals • show mutual respect and support when dealing with difficult people and situations • take initiative to propose plans or make adjustments to the plans and roles in changing situations

[16] Adapted from OECD 2015 PISA Framework

Beginning	Developing	Mastering
Communication		
• comprehend messages with an open mind and ask questions to identify the problem and team goals • express oneself clearly to team members by verbal and/or non-verbal means • show courage in sharing new or unconventional ideas	• ask meaningful questions that clarify the vision, goals and viewpoints for better solutions • respond specifically to queries raised during the problem solving process • enhance mutual understanding through effective means and with a respectful attitude	• negotiate for consensus and foster a cooperative atmosphere to resolve conflicts • take the initiative in introducing new resources and exploring further ideas to facilitate the team to progress further
Problem solving		
• propose solutions or strategies to solve a problem • complete the task assigned to one's role in the team	• select a problem solving strategy and develop an action plan • execute actions that comply with the planned distribution of roles and make adjustments when necessary	• select a problem solving strategy and prepare alternative plans • monitor and evaluate individual and team effectiveness

Example of implementation in Mathematics

Key Stage: 1

Learning Unit: Fractions (I)

Description of Activity:

Students are required to form groups and design a vegetable garden in the backyard of the school. The piece of land of dimensions 6 m × 6 m has already been divided into 36 square pieces of dimensions 1 m × 1 m. Students need to decide which square pieces need to be planted with vegetables and the vegetables they would plant in based on these requirements: one-fourth of the pieces for tomatoes, at least 6 pieces for lettuce, double as much as lettuce for corn, the remaining for peppers, except for some square pieces

reserved as one or several paths to reach each kind of vegetable because the land is only accessible from one of the four sides.

The activity is designed to promote the integrative use of generic skills. The activity particularly facilitates the development of **collaborative problem solving skills**, which involve the application of collaboration, communication and problem solving skills. This learning activity emphasises synergy among students, team work and communication to enhance creativity and the quality of solutions.

Learning Activities	Generic Skills Involved
1. The groups are guided to calculate the number of small square pieces of land and the fractions of land for planting each kind of vegetable under the given requirements.	Problem solving
2. In each group, students ◆ share their garden design ideas and explain how the ideas meet the requirements; ◆ discuss and plan a vegetable garden for the group; ◆ discuss ways of enhancing their garden, such as the shape and position of each vegetable area;	Collaboration and communication
◆ evaluate the possible ways of enhancing their garden based on some criteria compromised by members; ◆ choose a plan, draw the garden for presentation and label each vegetable area.	Problem solving
3. The groups report on their design of the vegetable garden and explain the rationale behind their planning. They could use fractions to represent the quantities in their explanation.	Communication
4. Based on the ideas generated from the sharing of other groups, each group refines their design, such as adjusting the shape of the area for each kind of vegetable.	Collaboration and problem solving

Holistic Thinking Skills

Critical thinking skills, creativity and problem solving skills are conventionally categorised as higher order thinking skills. These three skills can be combined and employed integratively as holistic thinking skills to deal with complex issues. Holistic thinking skills enable students to deploy critical thinking skills to assess the validity of given information, creativity to explore other possibilities, and problem solving skills to examine the feasibility of each alternative.

The expected achievements of the students in holistic thinking skills are classified according to different levels of mastery.

Students will learn to

Beginning	Developing	Mastering
Critical Thinking: enquiring and assessing		
• ask questions to explore matters that attract interest • identify main ideas and clarify meaning in information	• pose questions to explore issues related to their immediate contexts • comprehend complementary and contradictory information	• pose questions that probe complex and abstract ideas about issues beyond local context and contemporary period • synthesise points from complementary and contradictory information
Creativity: generating		
• come up with new ideas by linking imagination and reality • create analogies by matching two ideas • brainstorm suggestions	• draw parallels between known and new scenarios and use ideas, patterns and trends to consider new possibilities • produce alternative or unconventional solutions • suspend judgement to consider alternative ideas and actions	• generate a large number of raw ideas • combine good ideas to make even better ideas • use existing knowledge in a novel way • temporarily suspend pragmatic and rational thinking to allow new possibilities to emerge
Critical Thinking and Problem Solving: analysing and comparing		
• realise real world constraints in drafting solutions	• estimate the cost and benefit of possible solutions from multiple perspectives	• compare the possible outcomes of each solution against both their own and prevailing values

Beginning	Developing	Mastering
• compare advantages and limitations of various solutions	• rate and select solutions according to criteria, such as feasibility, desirability and ethical considerations	• mediate opposing viewpoints and acknowledge the limitations of one's view • synthesise different considerations into a solution
Creativity and Problem Solving: predicting and fine-tuning		
• ask "what if" questions • consider ways of tackling possible consequences	• make adjustments to avoid possible pitfalls (e.g. ambiguity, stereotyping and misunderstandings) in planning and presentation of solutions • consider alternative courses of action in changing situations	• fine tune plans with reference to new developments • be sensitive to stakeholders' reactions • anticipate adverse impacts and suggest precautionary or compensatory measures accordingly
Problem Solving: executing and monitoring		
• choose a solution and devise an implementation plan, using support and advice given • turn the plan into workable parts with measures for implementation	• execute the plan, monitor progress and revise the strategies when necessary • realise the adverse effect of over-reacting and using emotional words	• monitor the progress with established check points or criteria • suggest ways to catch up with delays or optimise the results • manage over-reactions and strong emotions
Problem Solving and Critical Thinking: evaluating and reflecting		
• reflect on whether the task is accomplished • be open to comments and feedback	• evaluate the quality of outcomes and the solution process • invite and evaluate feedback	• evaluate the effectiveness of solutions with due regard for positive values • anticipate possible problems arising from the solution • make judicious use of comments and feedback

Example of implementation in Mathematics

Key Stage: 3

Learning Unit: 3-D Figure, Mensuration

Description of Activity:

Students are required to construct right circular cylinders using a piece of 20cm × 20cm paper. They need to formulate possible strategies for making right circular cylinders with the largest surface area from a piece of square-shaped paper.

The teacher can first familiarise students with the way to calculate the surface area of a right circular cylinder, and then guide students to compare the surface areas of the cylinders constructed. The activity facilitates students' development of **holistic thinking skills**, which involves the application of critical thinking skills to assess and synthesise given information, creativity to explore possibilities, and problem solving skills to examine the feasibility of each alternative of making the cylinder with the largest possible surface area.

Learning Activities	Generic Skills Involved
1. Each student brainstorms and explores as many as possible the net designs of right circular cylinders fitting the given pieces of square-shaped paper.	Creativity
2. Each student ◆ calculates and compares the surface areas of cylinders designed; ◆ shares with classmates the best design explored, discusses and compares the designs from multiple perspectives, e.g. whether a right circular cylinder can really be built from the net; and ◆ makes adjustments if needed to finalise the net designed.	Critical thinking and problem solving
3. The teacher chooses some students to present the nets designed. Students are required to compare their nets with those of other classmates. Students could also share	Problem solving

Learning Activities	Generic Skills Involved
with their classmates how they come up with the design, learn from other students and fine-tune their problem-solving strategies.	

Appendix 4 : Examples of Websites and Application Software (Apps) for Learning and Teaching of Mathematics

Some websites and application software (app) for supporting the learning and teaching of Mathematics are listed below for teachers' reference. Some of these websites and apps can be accessed by tablet computers. For a more comprehensive list of websites, please refer to the homepage of Mathematics Education Section (*http://www.edb.gov.hk/en/curriculum-development/kla/ma/link/index.html*).

	Examples of Website/Apps	Primary (P) Secondary (S)
(1)	**Apps of e-geoboard** **Geoboard, by The Math Learning Center** iOS *https://itunes.apple.com/hk/app/geoboard-by-math-learning/id519896952?mt=8* **Digital Geoboard** Android *https://play.google.com/store/apps/details?id=com.betodeoliveira.DigitalGeoboard*	P
(2)	**Apps for measuring angles** **Angle Meter FREE!** iOS *https://itunes.apple.com/hk/app/angle-meter-free/id422843391?mt=8* **Angle Meter PRO** Android *https://play.google.com/store/apps/details?id=iyok.com.anglemeterpro*	P, S
(3)	**GeoGebra** (multifunctional mathematics application software) Website: *https://www.geogebra.org* Apps: iOS *https://itunes.apple.com/us/app/geogebra/id687678494?mt=8* Android *https://play.google.com/store/apps/details?id=org.geogebra* Windows Mobile *https://www.microsoft.com/en-us/store/apps/geogebra/9wzdncrfj48n*	P, S

Examples of Website/Apps		Primary (P) Secondary (S)
(4)	**Mathway** (computer algebraic system with graphing tool) Website: *https://mathway.com/* Apps: iOS *https://itunes.apple.com/us/app/mathway/id467329677?ls=1&mt=8* Android *https://play.google.com/store/apps/details?id=com.bagatrix.mathway.android*	P, S
(5)	**Nrich enriching mathematics** (exploratory activities) Website: *http://nrich.maths.org/stemnrich*	P, S
(6)	**Thatquiz** (quiz on different topics) Website: *https://www.thatquiz.org/*	P, S
(7)	**Websites on STEM Education** **STEM Education** Website: *http://stem.edb.hkedcity.net/en/home/* **Intel Education** Website: *http://www.intel.com/content/www/us/en/homepage.html* (search for "Free teaching resources") **STEM Learning** Website: *https://www.stem.org.uk* (search for "Primary mathematics resource packages", access other resources by the link "RESOURCES")	P, S
(8)	**Center for Algebraic Thinking** (website on learning algebra) Website: *http://www.algebraicthinking.org/* Apps provided: *http://algebraicthinking.org/tech#Apps*	S
(9)	**Desmos** (graphing tool) Website: *https://www.desmos.com/* Apps: iOS *https://itunes.apple.com/us/app/desmos-graphing-calculator/id653517540*	S

Examples of Website/Apps	Primary (P) Secondary (S)
Android *https://play.google.com/store/apps/details?id=com.desmos.calculator*	
(10) **iCrosss Lite** (3-D figure tool) Apps: iOS *https://itunes.apple.com/us/app/icrosss-lite/id509927782?mt=8* Android *https://play.google.com/store/apps/details?id=com.olehyudin.icrosss.lite*	S
(11) **Minds of Modern Mathematics** (introduce the development of Mathematics) Apps: iOS *https://itunes.apple.com/hk/app/minds-of-modern-mathematics/id432359402?mt=8*	S
(12) **Sketchometry** (geometric construction tool) Website: *http://sketchometry.org/en/index.html* Apps: iOS *https://itunes.apple.com/us/app/sketchometry/id635195332?mt=8* Android *https://play.google.com/store/apps/details?id=com.sketchometry&hl=zh_TW*	S

Appendix 5 : Learning and Teaching Resources List for Mathematics

(A) Learning and teaching materials developed by the EDB

No.	Title / Description	CRC call no./Web link
Primary (P1 – P6)		
1	Addenda Series for Primary Mathematics (Volume 1) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities.	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/pri/addenda1.html*
2	Addenda Series for Primary Mathematics (Volume 2) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities.	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/pri/addenda2.html*
3	Addenda Series for Primary Mathematics (Volume 3) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities.	EDB 372.7044 X53 2004 *http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda3-index.html*
4	Addenda Series for Primary Mathematics (Volume 4) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities.	EDB 372.7044 X53 2005 *http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda4-index.html*
5	Addenda Series for Primary Mathematics (Volume 5) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities on fractions.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda5.html*
6	Addenda Series for Primary Mathematics (Volume 6) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities on numbers, addition and subtraction.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda6.html*

No.	Title / Description	CRC call no./Web link
7	Addenda Series for Primary Mathematics (Volume 7) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities on developing students' number sense.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda7.html*
8	Addenda Series for Primary Mathematics (Volume 8) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities on nurturing critical thinking skills and creativity.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda8.html*
9	Addenda Series for Primary Mathematics (Volume 9) (Chinese version only) This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of hands-on activities.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/addenda9.html*
10	Diversified Assessment Strategies in Primary Mathematics (Chinese version only) This booklet is published for sharing with teachers the ways to promote student learning through diversified modes of assessment.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/assessment.html*
11	Remedial Teaching in Primary Mathematics (Chinese version only) This booklet is published to assist schools to recognise students' learning difficulties and to provide follow-up actions.	*http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Primary Remedial/MPR index.htm*
12	Shape & Space (Chinese version only) This material aims at allowing students to learn the topics in the Sharp and Space Strand effectively through diversified activities and enjoy the learning process.	EDB 372.7044 K66 2000 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/S & Space/space index.htm*
13	The Mystery of Numbers (Chinese version only) This material is compiled for Key Stage 2, which covers the use of calculators for basic operations and conversions between fractions, decimals and percentages.	EDB 372.7044 Y86 1999 *http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/pri/mystery.html*

No.	Title / Description	CRC call no./Web link
14	Areas of Triangles (Chinese version only) This material aims at sharing with teachers some examples, exercises and resources on the learning and teaching of triangles.	EDB 372.7044 S26 2002 *http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/triangle_c/tri index.htm*
15	Plane Figure can be fun (Chinese version only) This material aims at sharing with teachers some examples, exercises and resources on the learning and teaching of quadrilaterals.	EDB 372.7044 P56 2002 *http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Plane/plane index.htm*
Junior Secondary (S1 – S3)		
16	Learning and Teaching Package on S1-5 Mathematics: Volume 1, Use of Information Technology This learning and teaching package contains brief notes on the use of IT in teaching Key Stage 3 Mathematics and the points of concern when using the examples in the teaching package.	EDB 510.712 Y86 2001 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/it_e/IT_e index.htm*
17	Learning and Teaching Package on S1-5 Mathematics: Volume 2, Catering for Learner Differences This learning and teaching package contains brief notes on the ways of catering learner diversity in teaching Key Stage 3 Mathematics and the points of concern when using the examples in the teaching package.	EDB 510.712 Z43 2001 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/ld_e/LD_e index.htm*
18	Learning and Teaching Package on S1-5 Mathematics: Volume 3, Fostering High Order Thinking Skills This learning and teaching package contains brief notes on the ways of fostering students' high order thinking skills in teaching Key Stage 3 Mathematics and the points of concern when using the examples in the teaching package.	EDB 510.712 P45 2001 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/hots_e/HOTS_e index.htm*
19	Learning and Teaching Package on S1-5 Mathematics: Volume 4, Number and Algebra Dimension This learning and teaching package contains learning targets, learning objectives, notes on teaching and examples for the learning and teaching of the Number and Algebra Strand in Key Stage 3.	EDB 510.712 S58 2002 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/NA_e/NA_e index.htm*

No.	Title / Description	CRC call no./Web link
20	Learning and Teaching Package on S1-5 Mathematics: Volume 5, Measures, Shape and Space Dimension This learning and teaching package contains learning targets, learning objectives, notes on teaching and examples for the learning and teaching of the Measures, Shape and Space Strand in Key Stage 3.	EDB 510.712 D85 2002 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/MSS_e/MSS_e index.htm*
21	Learning and Teaching Package on S1-5 Mathematics: Volume 6, Data Handling Dimension This learning and teaching package contains learning targets, learning objectives, notes on teaching and examples for the learning and teaching of the Data Handling Strand in Key Stage 3.	EDB 510.712 S58 2002 *http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/material/DH_e/DH_e index.htm*
22	Junior Secondary Mathematics This e-learning tool provides interactives activities of different scenarios, which facilitate students to learn the applications of percentages, coordinates, coordinate geometry of straight lines, approximation & errors and laws of indices.	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/treasure-trove.html*
23	Symmetry in 3-D Figures The package consists of learning and teaching aids to help students recognise the reflectional and rotational symmetries in cubes and regular tetrahedral.	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/js/symmetry.html*
Senior Secondary (S4 – S6)		
24 25 26	Explanatory Notes to Senior Secondary Mathematics Curriculum – Compulsory Part Explanatory Notes to Senior Secondary Mathematics Curriculum – Module 1 (Calculus and Statistics) Explanatory Notes to Senior Secondary Mathematics Curriculum – Module 2 (Algebra and Calculus) The explanatory notes aim at further explicating: ◆ the requirements of the Learning Objectives of the Compulsory Part, Module 1 and Module 2; ◆ the strategies suggested for the teaching of the Compulsory Part, Module 1 and Module 2; ◆ the connections and structures among different Learning Units of the Compulsory Part, Module 1 and Module 2; ◆ the context of development from different key stages, such as Key Stage 3, to the Compulsory Part; and	Compulsory Part: EDB 510.712 E97 2009 *http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/explancp_e.pdf* Module 1: EDB 510.712 E96 2010 *http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/m1_notes_e.pdf* Module 2: EDB 510.712 E96 2010 *http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/m2_eng.pdf*

No.	Title / Description	CRC call no./Web link
	◆ the curriculum articulation between the Compulsory Part and the Extended Part.	
Cross Levels		
27	Trigonometry in 3-Dimensional Space This e-learning tool aims to help teachers explain the concepts of angles in 3-dimensional figures with the use of animated examples.	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/treasure-trove.html*
28	Assessment for Learning (Secondary Mathematics) The Open-ended Questions The aim of this booklet is to introduce to teachers how open-ended questions can be used to assess learning.	EDB 510.712 A87 2003 *http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/js/teaching.html*
30	Remedial Teaching in Secondary Mathematics (Chinese version only) This booklet aims at assisting schools in promoting remedial teaching in secondary Mathematics by introducing the ways of implementation.	EDB 510.712 Z46 2001 *http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Secondary%20Remedial/SR%20index.htm*
31	Mathematics Cabinet Series 1 - 20 (Chinese version only) 1. NSSMC Compulsory Part 2. NSSMC Module 1 3. NSSMC Module 2 4. Mathematics Gems 5. Applications of Mathematics : Image Processing - Revolution of Matrix 6. Applications of Mathematics : Investment Portfolios and Market Efficiency 7. Applications of Mathematics : Analysis of Genes and Protein 8. Probability Kaleidoscope 9. Portrait of a Middle-aged Mathematics Teacher 10. Booklet of 2009 Statistics Creative-Writing Competition for Secondary School Students 11. Looking into the Conceptions of Mathematics and Mathematics Teaching from "Introduction of Calculus" 12. Booklet of 2010/11 Statistics Creative-Writing Competition for Secondary School Students 13. Booklet of 2011/12 Statistics Creative-Writing Competition for Secondary School Students 14. Mathematics teachers are no longer intimidated by students' questions - the mathematics knowledge required for primary and secondary teachers 15. Booklet of 2012/13 Statistics Creative-Writing Competition for Secondary School Students	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/treasure-trove.html*

No.	Title / Description	CRC call no./Web link
	16. Geometric Construction: Examples, Solutions and Proofs 17. Paper folding and Mathematics 18. Booklet of 2013/14 Statistics Creative-Writing Competition for Secondary School Students 19. Booklet of 2014/15 Statistics Creative-Writing Competition for Secondary School Students 20. Universal Scaling Laws 21. Three Crises in Mathematics 22. Booklet of 2015/16 Statistics Creative-Writing Competition for Secondary School Students The booklets of this series aim at providing reference materials on the learning and teaching of Mathematics to teachers.	
32	An English-Chinese Glossary of Terms Commonly Used in Mathematics This booklet aims at providing an English-Chinese glossary of commonly used mathematical terms in primary and secondary Mathematics for teachers' reference.	*http://www.edb.gov.hk/tc/curriculum-development/kla/ma/res/glossary-notes.html*

(B) Other useful resources / web links

No.	Title	Web link / Type
1	School Mathematics Newsletter	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/smn.html*
2	Inquiry in Mathematics Education	EDB 510.71 I57 2010 / Booklet
3	Hong Kong Mathematics Olympiad (HKMO) Past Papers	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/sa/hkmo-papers.html*
4	Examples on STEM Learning and Teaching Activities	*http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/STEMexamples.html*
5	TSA Question Papers and Marking Schemes (Primary)	*http://www.bca.hkeaa.edu.hk/web/TSA/en/PriPaperSchema.html*
6	TSA Question Papers and Marking Schemes (Secondary)	*http://www.bca.hkeaa.edu.hk/web/TSA/en/SecPaperSchema.html*
7	ETV	*http://resources.hkedcity.net/etv/*

(C) Community Resources

No.	Organisation	Web link
1	Census and Statistics Department	*http://www.info.gov.hk/censtatd*
2	Hong Kong Association for Mathematics Education	*http://www.hkame.org.hk*
3	Hong Kong Association for Science and Mathematics Education	*http://www.hkasme.org*
4	Hong Kong Mathematical Society	*http://www.hkms.org.hk*
5	Hong Kong Statistical Society	*http://www.hkss.org.hk*
6	International Mathematical Olympiad Hong Kong Committee	*http://www.imohkc.org.hk*

Appendix 6 : List of Collaborative Research and Development ("Seed") Projects for Mathematics

Year	"Seed" Project Title	Project Code
Projects for Primary Schools		
2001/2004	Developing Thinking Abilities through Primary Mathematics Curriculum	MA0101
2003/2004	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Primary Level	MA0103
2004/2005	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics	MA0104
2005/2006	Developing Students' Number Sense through Primary Mathematics Curriculum	MA0105
	Strengthening Teachers Mathematics Pedagogical Content Knowledge through Peer Lesson Observation	MA0205
	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Primary Level	MA0405
2006/2007	Developing Students' Number Sense through Primary Mathematics Curriculum	MA0106
	Assessment for Learning on Primary Mathematics	MA0206
2007/2008	Assessment for Learning on Primary Mathematics	MA0307
	Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space	MA0107
2008/2009	Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space	MA0108
	Alternative Assessment for Learning in Mathematics : Performance Task Assessment	MA0408
2009/2010	Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space	MA0109
	Alternative Assessment for Learning in Mathematics : Performance Task Assessment	MA0409
2010/2011	Fostering Students' Critical Thinking and Creativity through the Primary Mathematics Curriculum in the Dimensions of Measures, and Shape and Space	MA0110
2011/2012	Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape & Space	MA0111
2012/2013	Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape & Space	MA0112
2013/2014	Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape & Space	MA0113
2014/2015	Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Measures	MA0114
2015/2016	Exploration and Development of Learning and Teaching Strategies in the Dimension of Measures	MA0115
2016/2017	Exploration and Development of Learning and Teaching Strategies in the Dimension of Measures	MA0116
	Exploration and Development of Effective Strategies for Promoting and Implementing STEM Education in Primary Mathematics	MA0316

Year	"Seed" Project Title	Project Code
Projects for Secondary Schools		
2002/2003	Open-ended Assessment in the Learning and Teaching of Secondary Mathematics	MA0202
2003/2004	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Secondary Level	MA0203
	Learning and Teaching of Problematic Topics in the Revised Secondary Mathematics Curriculum (1999)	MA0303
2004/2005	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics	MA0104
	Exploration and Development of Effective Learning and Teaching Strategies in the Mathematics Curriculum at KS4	MA0204
2005/2006	Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Secondary Level	MA0505
	Exploration and Development of Effective Learning and Teaching Strategies in the Mathematics Curriculum at KS4	MA0305
	School-based Assessments of Secondary Mathematics	MA0605
2006/2007	Diversified Modes of Assessments in Mathematics	MA0306
2007/2008	Diversified Modes of Assessments in Mathematics	MA0407
	Incorporation of History of Mathematics into Learning and Teaching	MA0207
2008/2009	Investigation and Inquiry: Learning and Teaching of Mathematics through Modelling and Experiments	MA0208
	Interface on Learning and Teaching between Primary and Secondary Mathematics	MA0308
2009/2010	Interface on Learning and Teaching between Primary and Secondary Mathematics	MA0309
	Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3	MA0209
	Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices	MA0509
2010/2011	Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3	MA0210
	Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices	MA0310
2011/2012	Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3	MA0211
	Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices	MA0311
2012/2013	Applications of Mathematics in Everyday Life	MA0212
	Investigation and development of effective strategies on the learning and teaching of permutation, combination and probability in the Senior Secondary Mathematics Curriculum	MA0312
2013/2014	Applications of Mathematics in Everyday Life	MA0213
	Investigation and Development of Effective Strategies on the Learning and Teaching of Selected Topics of Senior Secondary Mathematics Curriculum	MA0313

Year	"Seed" Project Title	Project Code
2014/2015	Exploration and Development of Effective Self Directed Learning Strategies in the Dimension of Number and Algebra	MA0214
	Exploration and Development of Effective Strategies on the Learning and Teaching of Loci and Transformations of Functions	MA0314
2015/2016	Exploration and Development of Self-directed Learning Strategies in Mathematics	MA0215
	Exploration and Development of Strategies on the Learning and Teaching of Loci, Equations of Straight Lines and Transformations of Functions	MA0315
2016/2017	Exploration and Development of Effective Self-directed Learning Strategies in Junior Secondary Mathematics	MA0216
	Exploration and Development of Effective Strategies for Promoting and Implementing STEM Education in Secondary Mathematics	MA0416

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Bibliography

Bibliography

Ad hoc Committee on Holistic Review of the Mathematics Curriculum (2000). *Report on Holistic Review of the Mathematics Curriculum*. Hong Kong: Curriculum Development Council.

Atkinson, R. D. (2012). Why the current education reform strategy won't work. *Issues in Science and Technology*, Spring 2012: 29-36.

Bergmann, J. & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. Eugene, OR; Alexandria, VA: International Society for Technology in Education; Association for Supervision and Curriculum Development.

Brenda S. W. & Celestine H. P. (Ed.). (2014). *Models and approaches to STEM professional development*. Arlington, USA: NSTA press

Curriculum Development Council (1999). *Syllabuses for Secondary Schools – Mathematics (Secondary 1 - 5)*. Hong Kong: Author.

Curriculum Development Council (2000). *Mathematics Education Key Learning Area – Mathematics Curriculum Guide (P1 – P6).* Hong Kong: Author.

Curriculum Development Council (2001). *Learning to Learn –The Way Forward in Curriculum Development.* Hong Kong: Author.

Curriculum Development Council (2002). *Basic Education Curriculum Guide – Building on Strengths (Primary 1 – Secondary 3).* Hong Kong: Author.

Curriculum Development Council (2014). *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6).* Hong Kong: Author.

Curriculum Development Council (2017). *Secondary Education Curriculum Guide*. Hong Kong: Author.

Curriculum Development Council, Hong Kong Examinations and Assessment Authority (2007). *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)*. Hong Kong: Curriculum Development Council.

Clarke, S. (2001). *Recommendations for the Development of Formative Assessment in Hong Kong*. Hong Kong: Education Department.

Committee on Highly Successful Schools or Programs in K-12 STEM Education, Board on Science Education, Board on Testing and Assessment, Division of Behavioral and Social Sciences and Education, National Research Council. (2011). Successful K-12 STEM Education: Identifying Effective Approaches in Science, Technology, Engineering, and Mathematics. Retrieved 23 August 2016 from http://www.nap.edu/read/13158/chapter/1

Education Commission (2000). *Reform Proposals for the Education System in Hong Kong*. Hong Kong: The Printing Department.

Education Department (2001). *Information Guide to Support Services for Students with Special Educational Needs in Ordinary School.* Hong Kong.

Freeman, B., Marginson, S. & Tytler, R. (Ed.). (2015). *The age of STEM: educational policy and practice across the world in science, technology, engineering and mathematics*. New York, USA: Routledge.

Knowles, M. S. (1975). *Self-Directed Learning. A Guide for Learners and Teachers*. Englewood Cliffs: Prentice Hall/Cambridge.

Meghan M. M., Amanda E. G. & Terri G. W. (2014). What is STEM education? *Global Education Review*, 1(4), 1-6.

Mullis, I. V. S., Martin, M. O., Foy, P. & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College. Retrieved from http://timssandpirls.bc.edu/timss2011/reports/international-results-mathematics.html

OECD (2014). *PISA 2012 Results: What Students Know and Can Do (Volume I, Revised edition, February 2014): Student Performance in Mathematics, Reading and Science*. Paris: OECD Publishing. Retrieved from https://www.oecd.org/pisa/keyfindings/pisa-2012-results.htm

Wang, H., Moore, T. J., Roehrig, G. H. & Park, M. S. (2011). STEM integration: Teacher perceptions and practice. *Journal of Pre-College Engineering Education Research*. 1(2): 1-13.

Zimmerman, B. J. (1990). Self-regulated learning and academic achievement: An overview. *Educational Psychologist*, 25, 3-17.

Zimmerman, B. J. (2011). Motivational sources and outcomes of self-regulated learning and performance. In B. J. Zimmerman & D. H. Schunk (Eds.), *Handbook of self-regulation of learning and performance* (pp. 49–64). New York, NY: Routledge.

Zollman, A. (2012). Learning for STEM Literacy: STEM Literacy for Learning. *School Science and Mathematics*, 112(1), 12-19

余勝泉、胡翔（2015）。STEM 教育理念與跨學科整合模式。**開放教育研究**，21(4)，13-21。

教育統籌委員會（2000）。**全方位學習活動簡介**。香港：作者。

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Membership List

Membership of the Curriculum Development Council Committee on Mathematics Education (from September 2015 to August 2017)

Chairperson :	Mr LAM Ka-yiu NTHYK Yuen Long District Sec. School	(Member from 1.9.2015, Chairperson from 1.9.2016)
	Mr SUM Sing-wah Buddhist Kok Kwong Secondary School	(from 1.9.2015 to 31.8.2016)
Vice - Chairperson :	Mr WAI Kwok-keung Mathematics Education Section, Curriculum Development Institute, Education Bureau	(from 16.2.2016)
	Mr LEE Pak-leung Mathematics Education Section, Curriculum Development Institute, Education Bureau	(from 1.9.2015 to 15.2.2016)
Members from Tertiary Institutions :	Dr CHAN Wai-hong The Education University of Hong Kong	(from 1.9.2015)
	Prof CHING Wai-ki The University of Hong Kong	(from 1.9.2015)
	Dr LAW Huk-yuen The Chinese University of Hong Kong	(from 1.9.2015)
Members from Related Professional Bodies or Related Field :	Mr POON Wai-hoi, Bobby Hong Kong Association for Mathematics Education	(from 1.9.2015)
	Ms TSUI Kwan-yuk Hong Kong Association for Science and Mathematics Education	(from 1.9.2015)
Member from Committee on Special Educational Needs :	Ms WONG Tin-ling Ebenezer New Hope School	(from 6.10.2016)
	Ms CHEUNG Yuet-mei Ebenezer New Hope School	(from 1.9.2015 to 31.8.2016)
Member from Committee on Early Childhood Education :	Prof CHENG Zi-juan The Chinese University of Hong Kong	(from 1.9.2015)

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	Mr SHUM Yiu-kwong F.D.B.W.A. Chow Chin Yau School	(from 1.9.2015)
	Ms WONG Chui-han, Ellen Farm Road Government Primary School	(from 1.9.2015)
School Teachers :	Ms CHONG Hiu-li, Jackie HKBUAS Wong Kam Fai Secondary and Primary School	(from 1.9.2015)
	Mr LEE Wing-yeong STFA Cheng Yu Tung Secondary School	(from 1.9.2015)
	Mr SIU Kwok-leong Ho Ngai College (Sponsored by Sik Sik Yuen)	(from 1.9.2015)
	Mr TANG Hok-shu Po Leung Kuk Riverain Primary School	(from 1.9.2015)
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	Ms TSUI Fung-ming, Karin Hon Wah College	(from 1.9.2015)
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Member from Hong Kong Examinations and Assessment Authority:	Mr CHU Kan-kong Hong Kong Examinations and Assessment Authority	(from 1.9.2015)
Secretary:	Dr NG Yui-kin Mathematics Education Section, Curriculum Development Institute, Education Bureau	(from 1.9.2015)

