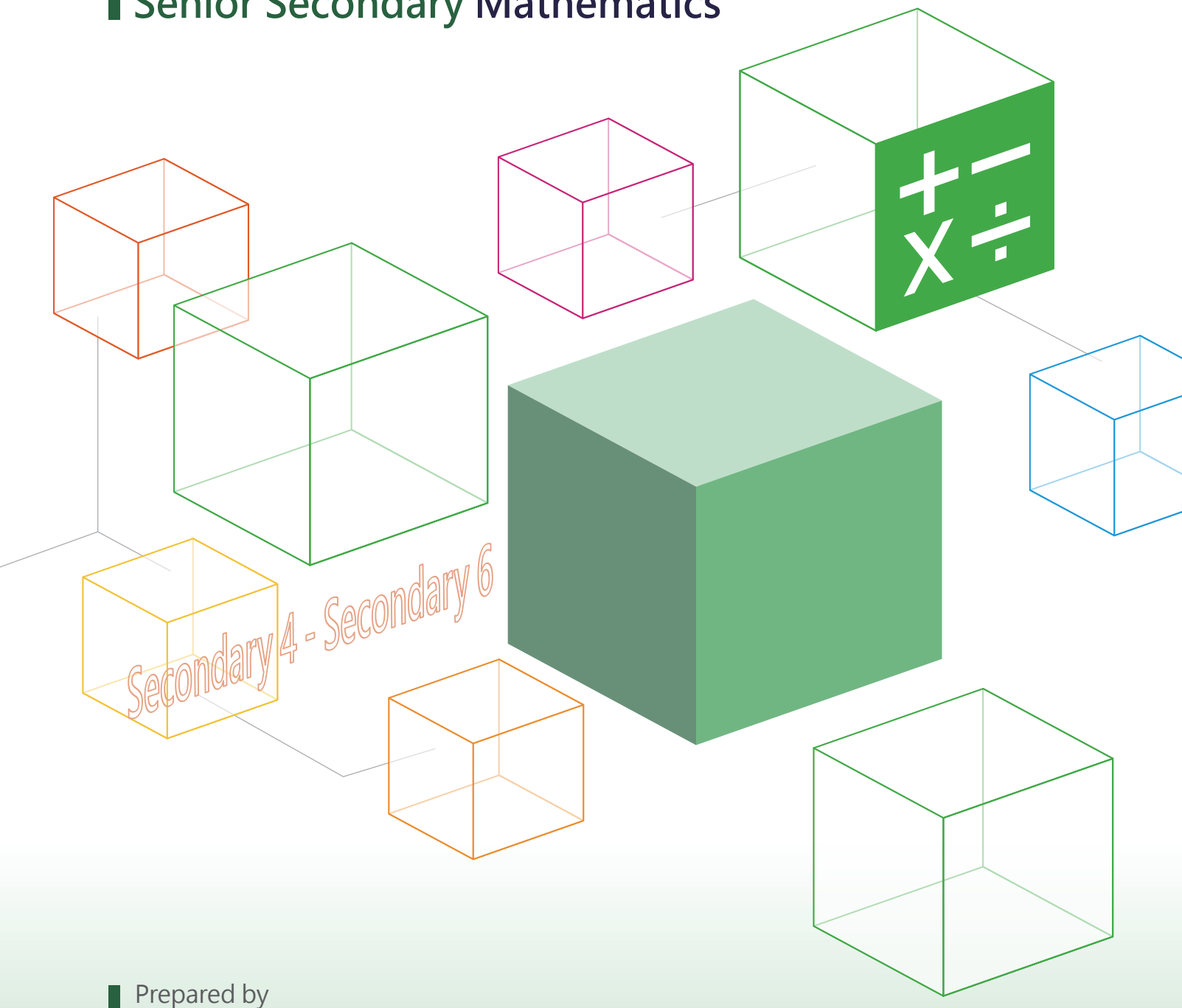
Supplement to Mathematics Education Key Learning Area Curriculum Guide

Learning Content of Senior Secondary Mathematics



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Contents

Preamble

In response to the need to keep abreast of the ongoing renewal of the school curriculum and the feedback collected from the New Academic Structure Medium-term Review and Beyond conducted from November 2014 to April 2015, and to strengthen vertical continuity and lateral coherence, the Curriculum Development Council Committee on Mathematics Education set up three Ad Hoc Committees in December 2015 to review and revise the Mathematics curriculum from Primary 1 to Secondary 6. The development of the revised Mathematics curriculum is based on the curriculum aims of Mathematics education, guiding principles of curriculum design, and assessment stipulated in *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6)* (2017).

This booklet is one of the series *Supplement to Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6)* (2017), aiming at providing a detailed account of:

1. the learning targets of the senior secondary Mathematics curriculum;
2. the learning content of the senior secondary Mathematics curriculum; and
3. the flow charts showing the progression pathways for the learning units of senior secondary Mathematics curriculum.

Comments and suggestions on this booklet are most welcomed. They may be sent to:

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Chapter 1 Learning targets

Learning Targets of the Compulsory Part of Senior Secondary Mathematics Curriculum		
Number and Algebra Strand	Measure, Shape and Space Strand	Data Handling Strand
Students are expected to:		
• extend the concepts of numbers to complex numbers; • further investigate and describe relationships between quantities using algebraic symbols; • generalise and describe patterns in sequences of numbers using algebraic symbols, and apply the results to solve problems; • interpret more complex algebraic relations from numerical, symbolic and graphical perspectives;	• use inductive and deductive approaches to study the properties of 2-dimensional figures; • perform geometric proofs involving 2-dimensional figures with appropriate symbols, terminology and reasons; • further inquire and describe geometric knowledge in 2-dimensional space using algebraic relations and apply the knowledge to solve problems; • inquire and describe geometric knowledge in 2-dimensional space and 3-dimensional space using trigonometric functions and apply the knowledge to solve problems; and	• understand the measures of dispersion; • select and use the measures of central tendency and dispersion to describe and compare data sets; • further investigate and judge the validity of arguments derived from data sets; • acquire basic techniques in counting; • formulate and solve more complex probability problems by applying simple laws; and • integrate the knowledge in statistics and probability to solve more complex real-life problems.

Learning Targets of the Compulsory Part of Senior Secondary Mathematics Curriculum		
Number and Algebra Strand	Measure, Shape and Space Strand	Data Handling Strand
Students are expected to:		
• manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve more complex real-life problems and justify the validity of the results obtained; and • apply the knowledge and skills in the Number and Algebra strand to generalise, describe and communicate mathematical ideas and further solve problems in other strands.	• apply the knowledge and skills in the Measures, Shape and Space strand to generalise, describe and communicate mathematical ideas and further solve problems in other strands.	

Learning Targets of Module 1 (Calculus and Statistics) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	Calculus	Statistics
Students are expected to:		
• apply binomial expansion for the study of probability and statistics; • model, graph and apply exponential functions and logarithmic functions to solve problems; and • understand the relationships between exponential and logarithmic functions and apply the two functions to solve real-life problems.	• recognise the concept of limits as the basis of differential and integral calculus; • understand the idea of differentiation and integration through consideration of concrete phenomena; • find the derivatives, indefinite integrals and definite integrals of simple functions; and • apply the knowledge of calculus to solve real-life problems.	• understand the concepts of probability, random variables, and discrete and continuous probability distributions; • understand the fundamental ideas of statistical reasoning based on the binomial, Poisson and normal distributions; • use statistical reasoning and thinking to know when and how to apply statistical methods to make inferences and justify conclusions; and • develop the ability to think mathematically about uncertainty and then apply such knowledge and skills to solve problems.

Learning Targets of Module 2 (Algebra and Calculus) of Senior Secondary Mathematics Curriculum		
Foundation Knowledge	Algebra	Calculus
Students are expected to:		
• recognise odd and even functions and their graphs; • understand the principle of mathematical induction; • expand binomials using the binomial theorem; • understand simple trigonometric functions, important trigonometric identities and formulae involving compound angles; and • recognise *e*.	• understand the concepts, operations and properties of matrices and the inverses of square matrices up to order 3; • solve systems of linear equations; • understand the concept, operations and properties of vectors; and • apply the knowledge of vectors to solve problems in 2-dimensional space and 3-dimensional space.	• understand the concept of limits as the basis of differential and integral calculus; • understand the concepts and properties of derivatives, indefinite integrals and definite integrals of functions; • find the derivatives, indefinite integrals and definite integrals of simple functions; • find the second derivatives of functions; • apply the knowledge of calculus to sketch curves; and • apply the knowledge of calculus to solve real-life problems.

Chapter 2 Learning content

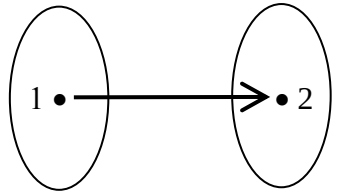
Learning Content of the Compulsory Part of Senior Secondary Mathematics Curriculum

Notes:

1. *Learning units are grouped under three strands ("Number and Algebra", "Measures, Shape and Space" and "Data Handling") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The learning objectives underlined are the Non-foundation Topics.*
4. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
5. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
6. *The total lesson time for the Compulsory Part of senior secondary Mathematics curriculum is 250 – 313 hours (i.e. 10% – 12.5% of the total lesson time available for the senior secondary curriculum). The total lesson time for the Compulsory Part and a module of the Extended Part of senior secondary Mathematics curriculum is 375 hours (i.e. 15% of the total lesson time available for the senior secondary curriculum).*

Learning Unit	Learning Objective	Time	Remarks
Number and Algebra Strand			
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method	19	
	1.2 form quadratic equations from given roots		The given roots are confined to real numbers.
	1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts		
	1.4 solve quadratic equations by the quadratic formula		The following are **not** required for students taking only the Foundation Topics: • expressing nonreal roots in the form $a \pm bi$ • simplifying expressions involving surds such as $2 \pm \sqrt{48}$
	1.5 understand the relations between the discriminant		When $\Delta < 0$, students have to point out that "the equation has **no real roots**" or "the

Learning Unit	Learning Objective	Time	Remarks
	of a quadratic equation and the nature of its roots		equation has **two nonreal roots**" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8.
	1.6 solve problems involving quadratic equations		Teachers should select the problems related to students' experiences. Problems involving complicated equations such as $\frac{6}{x}+\frac{6}{x-1}=5$ are required only in the Non-foundation Topics and dealt with in Learning Objective 5.4.
	1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations		The relations between the roots and coefficients include: • $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$, where α and β are the roots of the equation $ax^2+bx+c=0$ and $a\neq 0$.

Learning Unit	Learning Objective	Time	Remarks
	1.8 appreciate the development of the number systems including the system of complex numbers 1.9 perform addition, subtraction, multiplication and division of complex numbers		The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed. Complex numbers are confined to the form $a \pm bi$. Note: The coefficients of quadratic equations are confined to real numbers.
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables 2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions	10	Representations like  can also be accepted.

Learning Unit	Learning Objective	Time	Remarks
	2.3 understand the features of the graphs of quadratic functions		The features of the graphs of quadratic functions include: • the vertex • the axis of symmetry • the direction of opening • relations with the axes Students are required to find the maximum and minimum values of quadratic functions by the graphical method.
	2.4 find the maximum and minimum values of quadratic functions by the algebraic method		The method of completing the square is required. Students are required to solve problems related to maximum and minimum values of quadratic functions.

Learning Unit	Learning Objective	Time	Remarks
3. Exponential and logarithmic functions	3.1 understand the definitions of rational indices	16	The definitions include $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$.
	3.2 understand the laws of rational indices		The laws of rational indices include: • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $(a^p)^q = a^{pq}$ • $a^p b^p = (ab)^p$ • $\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$
	3.3 understand the definition and properties of logarithms (including the change of base)		The properties of logarithms include: • $\log_a 1 = 0$ • $\log_a a = 1$

Learning Unit	Learning Objective	Time	Remarks
	3.4 <u>understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs</u>		• $\log_a MN = \log_a M + \log_a N$ • $\log_a \frac{M}{N} = \log_a M - \log_a N$ • $\log_a M^k = k \log_a M$ • $\log_b N = \frac{\log_a N}{\log_a b}$ The properties and features include: • the domains of the functions • the function $f(x) = a^x$ and $f(x) = \log_a x$ increases (decreases) as x increases for $a > 1$ $(0 < a < 1)$ • $y = a^x$ is symmetric to $y = \log_a x$ about $y = x$ • the intercepts with the axes • the rate of increasing/the rate of decreasing of the functions (by direct inspection)

Learning Unit	Learning Objective	Time	Remarks
	3.5 solve exponential equations and logarithmic equations		Equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log(x-22) + \log(x+26) = 2$ are dealt with in Learning Objective 5.3.
	3.6 appreciate the applications of logarithms in real-life situations		The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed.
	3.7 appreciate the development of the concepts of logarithms		The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed.
4. More about polynomials	4.1 perform division of polynomials	14	Methods other than long division are also accepted.
	4.2 understand the remainder theorem		

Learning Unit	Learning Objective	Time	Remarks
	4.3 understand the factor theorem		Students are required to use factor theorem to factorise polynomials such as $x^3 \pm a^3$.
	4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials		The terms "H.C.F." , "gcd", etc. can be used.
	4.5 perform addition, subtraction, multiplication and division of rational functions		Computation of rational functions with more than two variables is **not** required. Rational functions refer to algebraic fractions at Key Stage 3.
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$	10	
	5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic		

Learning Unit	Learning Objective	Time	Remarks
	5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations		Solutions for trigonometric equations are confined to the interval from 0° to 360° .
	5.4 solve problems involving equations which can be transformed into quadratic equations		Teachers should select the problems related to students' experience.
6. Variations	6.1 understand direct variations and inverse variations, and their applications to solving real-life problems	7	
	6.2 understand the graphs of direct and inverse variations		
	6.3 understand joint and partial variations, and their applications to solving real-life problems		

Learning Unit	Learning Objective	Time	Remarks
7. Arithmetic and geometric sequences and their summations	7.1 understand the concept and the properties of arithmetic sequences 7.2 understand the general term of an arithmetic sequence 7.3 understand the concept and the properties of geometric sequences 7.4 understand the general term of a geometric sequence	17	The properties of arithmetic sequences include: • $T_n = \frac{1}{2}(T_{n-1} + T_{n+1})$ • if $T_1, T_2, T_3, \ldots$ is an arithmetic sequence, then $kT_1+a, kT_2+a, kT_3+a, \ldots$ is also an arithmetic sequence The properties of geometric sequences include: • $T_n^2 = T_{n-1} \times T_{n+1}$ • if $T_1, T_2, T_3, \ldots$ is a geometric sequence, then $kT_1, kT_2, kT_3, \ldots$ is also a geometric sequence

Learning Unit	Learning Objective	Time	Remarks
	7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems		Example: geometrical problems involving the sum of arithmetic or geometric sequences.
	7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems		Example: geometrical problems involving infinite sum of the geometric sequences.
	7.7 solve related real-life problems		Examples: problems about interest, growth or depreciation.
8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown	16	Compound inequalities involving logical connectives "and" or "or" are required. Solving the problems on triangle inequalities is required.
	8.2 solve quadratic inequalities in one unknown by the graphical method		

Learning Unit	Learning Objective	Time	Remarks
	8.3 solve quadratic inequalities in one unknown by the algebraic method 8.4 represent the graphs of linear inequalities in two unknowns in the rectangular coordinate plane 8.5 solve systems of linear inequalities in two unknowns 8.6 solve linear programming problems		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions 9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$ 9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$	11	Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity.

Learning Unit	Learning Objective	Time	Remarks
	9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives		
Measures, Shape and Space Strand			
10. Equations of straight lines	10.1 understand the equation of a straight line	7	Students are required to find the equation of a straight line from given conditions such as: • the coordinates of any two points on the straight line • the slope of the straight line and the coordinates of a point on it • the slope and the y-intercept of the straight line Students are required to describe the features of a straight line from its equation. The features include:

Learning Unit	Learning Objective	Time	Remarks
			• the slope • the intercepts with the axes • whether it passes through a given point The normal form is **not** required. Students are required to recognise the relation between slope and inclination.
	10.2 understand the possible intersection of two straight lines		Students are required to determine the number of intersection points of two straight lines from their equations. At Key Stage 3, students are required to solve simultaneous linear equations in two unknowns. Note: Teachers are suggested to arrange the teaching of this Learning Unit in the first term of S4.

Learning Unit	Learning Objective	Time	Remarks
11. Basic properties of circles	11.1 understand the properties of chords and arcs of a circle	23	The properties of chords and arcs of a circle include: • the chords of equal arcs are equal • equal chords cut off equal arcs • the perpendicular from the centre to a chord bisects the chord • the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord • the perpendicular bisector of a chord passes through the centre • equal chords are equidistant from the centre • chords equidistant from the centre are equal Students are required to understand that there is one and only one circle passing through given three non-collinear points.

Learning Unit	Learning Objective	Time	Remarks
	11.2 understand the angle properties of a circle		The property that the arcs are proportional to their corresponding angles at the centre should be discussed at Key Stage 3 when the formula for calculating arc lengths is being explicated. The angle properties of a circle include: • the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference • angles in the same segment are equal • the arcs are proportional to their corresponding angles at the circumference • the angle in a semi-circle is a right angle • if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter

Learning Unit	Learning Objective	Time	Remarks
	11.3 understand the properties of a cyclic quadrilateral		The properties of a cyclic quadrilateral include: • the opposite angles of a cyclic quadrilateral are supplementary • an exterior angle of a cyclic quadrilateral equals its interior opposite angle
	11.4 understand the tests for concyclic points and cyclic quadrilaterals		The tests for concyclic points and cyclic quadrilaterals include: • if A and D are two points on the same side of the line BC and $\angle BAC = \angle BDC$, then A, B, C and D are concyclic • if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic • if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic

Learning Unit	Learning Objective	Time	Remarks
	11.5 understand the properties of tangents to a circle and angles in the alternate segments		The properties include: • a tangent to a circle is perpendicular to the radius through the point of contact • the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle • the perpendicular to a tangent at its point of contact passes through the centre of the circle • if two tangents are drawn to a circle from an external point, then: - the distances from the external point to the points of contact are equal

Learning Unit	Learning Objective	Time	Remarks
			- the tangents subtend equal angles at the centre - the straight line joining the centre to the external point bisects the angle between the tangents • if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment • if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle
	11.6 use the basic properties of circles to perform simple geometric proofs		Knowledge on geometry learnt at Key Stage 3 can be involved in the geometric proofs.

Learning Unit	Learning Objective	Time	Remarks
12. Loci	12.1 understand the concept of loci	6	
	12.2 describe and sketch the locus of points satisfying given conditions		The conditions include: • maintaining a fixed distance from a fixed point • maintaining an equal distance from two given points • maintaining a fixed distance from a line • maintaining an equal distance from two parallel lines • maintaining an equal distance from two intersecting lines
	12.3 describe the locus of points with algebraic equations		Students are required to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $y = ax^2 + bx + c$).

Learning Unit	Learning Objective	Time	Remarks
13. Equations of circles	13.1 understand the equation of a circle	7	Students are required to find the equation of a circle from given conditions such as: • the coordinates of the centre and the radius of the circle • the coordinates of any three points on the circle Students are required to describe the features of a circle from its equation. The features include: • the centre • the radius • whether a given point lies inside, outside or on the circle
	13.2 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle		Students are required to find the equations of tangents to a circle.

Learning Unit	Learning Objective	Time	Remarks
14. More about trigonometry	14.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity 14.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from 0° to 360°) and other trigonometric equations (solutions in the interval from 0° to 360°) 14.3 understand the formula $\frac{1}{2} ab \sin C$ for areas of triangles 14.4 understand the sine and cosine formulae 14.5 understand Heron's formula 14.6 understand the concept of projection	25	Simplification of expressions involving sine, cosine and tangent of $-\theta$, $90° \pm \theta$, $180° \pm \theta$, … , etc. is required. Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and dealt with in Learning Objective 5.3.

Learning Unit	Learning Objective	Time	Remarks
	14.7 understand the angle between a line and a plane, and the angle between 2 planes 14.8 understand the theorem of three perpendiculars 14.9 solve related 2-dimensional and 3-dimensional problems		The concept of inclination is required. 3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between points, the distance between a point and a line, and the distance between a point and a plane.
Data Handling Strand			
15. Permutations and combinations	15.1 understand the addition rule and multiplication rule in the counting principle 15.2 understand the concept and notation of permutation	11	 Notations such as "P_r^n", "${}_nP_r$", "${}^nP_r$", etc. can be used.

Learning Unit	Learning Objective	Time	Remarks
	15.3 solve problems on the permutation of distinct objects without repetition		Problems such as "permutation of objects in which three particular objects are put next to each other" are required. Circular permutation is **not** required.
	15.4 understand the concept and notation of combination		Notations such as "C_r^n", "${}_nC_r$", "${}^nC_r$", "$\binom{n}{r}$", etc. can be used.
	15.5 solve problems on the combination of distinct objects without repetition		
16. More about probability	16.1 recognise the notation of set language including union, intersection and complement	10	The concept of Venn Diagram is required.
	16.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events		The addition law of probability refers to "$P(A \cup B) = P(A) + P(B) - P(A \cap B)$".

Learning Unit	Learning Objective	Time	Remarks
	16.3 <u>understand the multiplication law of probability and the concept of independent events</u> 16.4 <u>recognise the concept and notation of conditional probability</u> 16.5 <u>use permutation and combination to solve problems related to probability</u>		The multiplication law of probability refers to "$P(A \cap B) = P(A) \times P(B)$, where A and B are independent events". The rule "$P(A \cap B) = P(A) \times P(B \mid A)$" is required. Bayes' Theorem is **not** required.
17. Measures of dispersion	17.1 understand the concept of dispersion 17.2 understand the concepts of range and inter-quartile range 17.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data	13	A box-and-whisker diagram can also be called a "boxplot".

Learning Unit	Learning Objective	Time	Remarks
	17.4 understand the concept of standard deviation for both grouped and ungrouped data sets		Students are required to recognise the term "variance" and that variance equals to the square of standard deviation. Students are required to understand the following formula for standard deviation: $\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$.
	17.5 compare the dispersions of different sets of data using appropriate measures		
	17.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution		
	17.7 understand the effect of the following operations on the dispersion of the data: (i) adding a common constant to each item of the set of data		

Learning Unit	Learning Objective	Time	Remarks
	(ii) multiplying each item of the set of data by a common constant		
18. Uses and abuses of statistics	18.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 18.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations 18.3 assess statistical investigations presented in different sources such as news media, research reports, etc.	4	Students are required to recognise the concepts of "populations" and "samples". Students are required to recognise probability sampling and non-probability sampling. Students are required to recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability.

Learning Unit	Learning Objective	Time	Remarks
Further Learning Unit			
19. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learned in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	14	Examples: • solve simple financial problems in areas such as taxation and instalment payment • analyse and interpret data collected in surveys • explore and interpret graphs related to real-life situations • explore Ptolemy's Theorem and its applications • model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k\,a^x$ to linear relations

Learning Unit	Learning Objective	Time	Remarks
			• explore the relation between the Fibonacci sequence and the Golden Ratio • appreciate the applications of cryptography • explore the Ceva's Theorem and its applications • analyse mathematical games (e.g. explore the general solution of the water puzzle)
20. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.

Total lesson time: 250 hours

Learning Content of Module 1 (Calculus and Statistics) of Senior Secondary Mathematics Curriculum

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
5. *The total lesson time for the Compulsory Part and Module 1 of senior secondary Mathematics curriculum is 375 hours (i.e. 15% of the total lesson time available for the senior secondary curriculum).*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	Students are required to recognise the summation notation ($\sum$). The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of e and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$	8	
	2.2 understand exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are required to solve problems including those related to compound interest, population growth and radioactive decay.
	2.4 transform $y = ka^x$ and $y = k[f(x)]^n$ to linear relations, where a, n and k are real numbers, $a > 0$, $a \neq 1$, $f(x) > 0$ and $f(x) \not\equiv 1$		When experimental values of x and y are given, students are required to plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercepts.
	Subtotal in hours	11	

Learning Unit	Learning Objective	Time	Remarks
Calculus			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	5	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are **not** required).
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following algebraic functions are required: • polynomial functions • rational functions • power functions x^{α} • functions derived from the above ones through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$

Learning Unit	Learning Objective	Time	Remarks
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Students are required to recognise the notations: y', $f'(x)$ and $\frac{dy}{dx}$.
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Students are required to recognise the notations: $f'(x_0)$ and $\left.\frac{dy}{dx}\right\|_{x=x_0}$.

Learning Unit	Learning Objective	Time	Remarks
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	8	The rules include: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The formulae that students are required to use include: • $(C)'=0$ • $(x^n)'=nx^{n-1}$ • $(e^x)'=e^x$

Learning Unit	Learning Objective	Time	Remarks
			• $(\ln x)' = \frac{1}{x}$ • $(\log_a x)' = \frac{1}{x \ln a}$ • $(a^x)' = a^x \ln a$ Implicit differentiation and logarithmic differentiation are **not** required.
5. Second derivative	5.1 recognise the concept of the second derivative of a function	2	Students are required to recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Third and higher order derivatives are **not** required.
	5.2 find the second derivative of an explicit function		Students are required to recognise the second derivative test and concavity.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangent, rate of change, maximum and minimum	10	Local and global extrema are required.

Learning Unit	Learning Objective	Time	Remarks
7. Indefinite integration and its applications	7.1 recognise the concept of indefinite integration 7.2 understand the basic properties of indefinite integrals and basic integration formulae	10	Indefinite integration as the reverse process of differentiation should be introduced. Students are required to recognise the notation: $\int f(x)\,dx$. The properties include: • $\int kf(x)dx = k\int f(x)dx$ • $\int [f(x) \pm g(x)]dx$ $= \int f(x)dx \pm \int g(x)dx$ The formulae include: • $\int kdx = kx + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ • $\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ • $\int e^x dx = e^x + C$

Learning Unit	Learning Objective	Time	Remarks
			Students are required to understand the meaning of the constant of integration C.
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions		
	7.4 use integration by substitution to find indefinite integrals		Integration by parts is **not** required.
	7.5 use indefinite integration to solve problems		

Learning Unit	Learning Objective	Time	Remarks
8. Definite integration and its applications	8.1 recognise the concept of definite integration	12	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. Students are required to recognise the notation: $\int_a^b f(x)\,dx$. The concept of dummy variables is required, for example: $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. The Fundamental Theorem of Calculus that students are required to recognise is: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$.

Learning Unit	Learning Objective	Time	Remarks
	8.3 find the definite integrals of algebraic functions and exponential functions 8.4 use integration by substitution to find definite integrals		The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems		Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves.
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required. Students are required to determine whether an estimate is an over-estimate or under-estimate by using the second derivative and concavity.
	Subtotal in hours	51	
Statistics			
10. Conditional probability and Bayes' theorem	10.1 understand the concept of conditional probability 10.2 use Bayes' theorem to solve simple problems	6	

Learning Unit	Learning Objective	Time	Remarks
11. Discrete random variables	11.1 recognise the concept of discrete random variables	1	
12. Probability distribution, expectation and variance	12.1 recognise the concept of discrete probability distribution and represent the distribution in the form of tables, graphs and mathematical formulae 12.2 recognise the concepts of expectation $E[X]$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems	7	The formulae that students are required to use include: • $E[X]=\sum xP(X=x)$ • $\mathrm{Var}(X)=E\left[(X-\mu)^2\right]$ • $E[g(X)]=\sum g(x)P(X=x)$ • $E[aX+b]=aE[X]+b$ • $\mathrm{Var}(X)=E\left[X^2\right]-(E[X])^2$

Learning Unit	Learning Objective	Time	Remarks
			• $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$ Notation $E(X)$ can also be used.
13. The binomial distribution	13.1 recognise the concept and properties of the binomial distribution	5	The Bernoulli distribution should be introduced. The mean and variance of the binomial distribution are required (the proofs are **not** required).
	13.2 calculate probabilities involving the binomial distribution		Use of the binomial distribution table is **not** required.
14. The Poisson distribution	14.1 recognise the concept and properties of the Poisson distribution	5	The mean and variance of the Poisson distribution are required (the proofs are **not** required).
	14.2 calculate probabilities involving the Poisson distribution		Use of the Poisson distribution table is **not** required.

Learning Unit	Learning Objective	Time	Remarks
15. Applications of the binomial and the Poisson distributions	15.1 use the binomial and the Poisson distributions to solve problems	5	
16. Basic definition and properties of the normal distribution	16.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution 16.2 recognise the concept and properties of the normal distribution	3	Derivations of the mean and variance of the normal distribution are **not** required. Students are required to recognise that the formulae in Learning Objective 12.2 are also applicable to continuous random variables. The properties include: • the curve is bell-shaped and symmetrical about the mean • the mean, mode and median are all equal • the flatness can be determined by the value of σ • the area under the curve is 1

Learning Unit	Learning Objective	Time	Remarks
17. Standardisation of a normal variable and use of the standard normal table	17.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	
18. Applications of the normal distribution	18.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1 , x_2 , μ and σ , where $X \sim N(\mu, \sigma^2)$ 18.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$ 18.3 use the normal distribution to solve problems	7	

Learning Unit	Learning Objective	Time	Remarks
19. Sampling distribution and point estimates	19.1 recognise the concepts of sample statistics and population parameters	9	Students are required to recognise: If the population mean is μ and the population size is N, then the population variance is $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$.
	19.2 recognise the sampling distribution of the sample mean $\bar{X}$ from a random sample of size n		Students are required to recognise: • If the population mean is μ and the population variance is σ^2, then $E[\bar{X}] = \mu$ and $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$. • If $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ (the proof is **not** required).
	19.3 use the Central Limit Theorem to treat $\bar{X}$ as being normally distributed when the sample size n is sufficiently large		

Learning Unit	Learning Objective	Time	Remarks
	19.4 recognise the concept of point estimates including the sample mean and sample variance		Students are required to recognise: If the sample mean is $\bar{x}$ and the sample size is n, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Students are required to recognise the concept of unbiased estimator.
20. Confidence interval for a population mean	20.1 recognise the concept of confidence interval 20.2 find the confidence interval for a population mean	6	Students are required to recognise: • A $100(1-\alpha)\%$ confidence interval for the mean μ of a normal population with known variance σ^2, based on a random sample of size n, is given by $(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$. • When the sample size n is sufficiently

Learning Unit	Learning Objective	Time	Remarks
			large, a $100(1-\alpha)\%$ confidence interval for the mean μ of a population with unknown variance is given by $(\overline{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$, where s is the sample standard deviation.
	Subtotal in hours	56	
Further Learning Unit			
21. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Total lesson time: 125 hours

Learning Content of Module 2 (Algebra and Calculus) of Senior Secondary Mathematics Curriculum

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
5. *The total lesson time for the Compulsory Part and Module 2 of senior secondary Mathematics curriculum is 375 hours (i.e. 15% of the total lesson time available for the senior secondary curriculum).*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Odd and even functions	1.1 recognise odd and even functions and their graphs	2	Students are required to recognise that the absolute value function is an example of even functions.

Learning Unit	Learning Objective	Time	Remarks
2. Mathematical induction	2.1 understand the principle of mathematical induction	3	The First Principle of Mathematical Induction is required. Students are required to prove propositions related to the summation of a finite sequence. Proving propositions involving inequalities is **not** required.
3. The binomial theorem	3.1 expand binomials with positive integral indices using the binomial theorem	3	Proving the binomial theorem is required. Students are required to recognise the summation notation ($\sum$). The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
4. More about trigonometric functions	4.1 understand the concept of radian measure 4.2 understand the functions cosecant, secant and cotangent 4.3 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine	15	The formulae that students are required to use include: $1+\tan^2\theta=\sec^2\theta$ and $1+\cot^2\theta=\operatorname{cosec}^2\theta$ Simplifying trigonometric expressions by identities is required. The formulae include: • $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$ • $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$ • $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$ • $\sin 2A = 2\sin A\cos A$ • $\cos 2A = \cos^2 A-\sin^2 A$ $= 1-2\sin^2 A = 2\cos^2 A-1$

Learning Unit	Learning Objective	Time	Remarks
			• $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ • $\sin^2 A = \dfrac{1}{2}(1-\cos 2A)$ • $\cos^2 A = \dfrac{1}{2}(1+\cos 2A)$ • $2\sin A\cos B = \sin(A+B)+\sin(A-B)$ • $2\cos A\cos B = \cos(A+B)+\cos(A-B)$ • $2\sin A\sin B = \cos(A-B)-\cos(A+B)$ • $\sin A+\sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ • $\sin A-\sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ • $\cos A+\cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ • $\cos A-\cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ "Subsidiary angle form" is **not** required.

Learning Unit	Learning Objective	Time	Remarks
			$\sin^2 A = \frac{1}{2}(1-\cos 2A)$ and $\cos^2 A = \frac{1}{2}(1+\cos 2A)$ can be considered as formulae derived from the double angle formulae.
5. Introduction to *e*	5.1 recognise the definitions and notations of *e* and the natural logarithm	2	Two approaches for the introduction to *e* can be considered: • $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ These definitions may be introduced in Learning Objective 6.1.
	Subtotal in hours	25	

Learning Unit	Learning Objective	Time	Remarks
Calculus			
6. Limits	6.1 understand the intuitive concept of the limit of a function 6.2 find the limit of a function	3	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are **not** required). The formulae that students are required to use include: • $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.

Learning Unit	Learning Objective	Time	Remarks
7. Differentiation	7.1 understand the concept of the derivative of a function	13	Students are required to find the derivatives of elementary functions from first principles, for example: C, x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x and $\ln x$. Students are required to recognise the notations: y', $f'(x)$ and $\frac{dy}{dx}$. Testing differentiability of functions is **not** required.

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation		The rules include: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Learning Unit	Learning Objective	Time	Remarks
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		The formulae that students are required to use include: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ The following algebraic functions are required: • polynomial functions • rational functions • power functions x^α

Learning Unit	Learning Objective	Time	Remarks
			• functions formed from the above functions through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$
	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Students are required to recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Students are required to recognise the second derivative test and concavity. Third and higher order derivatives are **not** required.
8. Applications of differentiation	8.1 find the equations of tangents to a curve	14	
	8.2 find the maximum and minimum values of a function		Local and global extrema are required.

Learning Unit	Learning Objective	Time	Remarks
	8.3 sketch curves of polynomial functions and rational functions 8.4 solve the problems relating to rate of change, maximum and minimum		The following points should be considered in curve sketching: • symmetry of the curve • limitations on the values of x and y • intercepts with the axes • maximum and minimum points • points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students are required to deduce the equation of the oblique asymptote to the curve of a rational function by division.

Learning Unit	Learning Objective	Time	Remarks
9. Indefinite integration and its applications	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals	15	Indefinite integration as the reverse process of differentiation should be introduced. The formulae include: • $\int k\ dx = kx + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ • $\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ • $\int e^x dx = e^x + C$ • $\int \sin x\, dx = -\cos x + C$ • $\int \cos x\, dx = \sin x + C$ • $\int \sec^2 x\, dx = \tan x + C$

Learning Unit	Learning Objective	Time	Remarks
	9.3 understand the applications of indefinite integrals in mathematical contexts		Applications of indefinite integrals in some fields such as geometry is required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2 - x^2}$, $\frac{1}{\sqrt{a^2 - x^2}}$ or $\frac{1}{x^2 + a^2}$		Students are required to recognise the notations: $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$, and the concept of their related principal values.
	9.6 use integration by parts to find indefinite integrals		Teachers can use $\int \ln x \, dx$ as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time	Remarks
10. Definite integration	10.1 recognise the concept of definite integration	10	The definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The concept of dummy variables is required, for example, $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. Using definite integration to find the sum to infinity of a sequence is **not** required.
	10.2 understand the properties of definite integrals		The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
			• $\int_a^b [f(x) \pm g(x)]dx$ $= \int_a^b f(x)dx \pm \int_a^b g(x)dx$ • $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function • $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is an even function
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions		The Fundamental Theorem of Calculus that students are required to recognise is: $\int_a^b f(x)dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$.
	10.4 use integration by substitution to find definite integrals		

Learning Unit	Learning Objective	Time	Remarks
	10.5 use integration by parts to find definite integrals		The use of integration by parts is limited to at most two times in finding an integral.
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure 11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	4	"Disc method" is required.
	Subtotal in hours	59	
Algebra			
12. Determinants	12.1 recognise the concept of determinants of order 2 and order 3	2	Students are required to recognise the notations: $\|A\|$ and $\det A$.
13. Matrices	13.1 understand the concept, operations and properties of matrices	10	The addition, scalar multiplication and multiplication of matrices are required. The properties include:

Learning Unit	Learning Objective	Time	Remarks
			• $A+B=B+A$ • $A+(B+C)=(A+B)+C$ • $(\lambda+\mu)A=\lambda A+\mu A$ • $\lambda(A+B)=\lambda A+\lambda B$ • $A(BC)=(AB)C$ • $A(B+C)=AB+AC$ • $(A+B)C=AC+BC$ • $(\lambda A)(\mu B)=(\lambda\mu)AB$ • $\lvert AB\rvert=\lvert A\rvert\lvert B\rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The properties include: • the inverse of A is unique • $(A^{-1})^{-1}=A$ • $(\lambda A)^{-1}=\lambda^{-1}A^{-1}$ • $(A^n)^{-1}=(A^{-1})^n$ • $(A^T)^{-1}=(A^{-1})^T$

Learning Unit	Learning Objective	Time	Remarks
			• $\left\|A^{-1}\right\| = \left\|A\right\|^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.
14. Systems of linear equations	14.1 solve the systems of linear equations in two and three variables by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: A system of homogeneous linear equations has nontrivial solutions if and only if the coefficient matrix is singular.
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students are required to recognise some common notations of vectors in printed form (including **a** and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$) ; and some notations for magnitude (including $\left\|\mathbf{a}\right\|$ and $\left\|\vec{a}\right\|$).

Learning Unit	Learning Objective	Time	Remarks
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The properties include: • $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$ • $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$ • $\mathbf{a}+\mathbf{0}=\mathbf{a}$ • $0\mathbf{a}=\mathbf{0}$ • $\lambda(\mu\mathbf{a})=(\lambda\mu)\mathbf{a}$ • $(\lambda+\mu)\mathbf{a}=\lambda\mathbf{a}+\mu\mathbf{a}$ • $\lambda(\mathbf{a}+\mathbf{b})=\lambda\mathbf{a}+\lambda\mathbf{b}$ If $\alpha\mathbf{a}+\beta\mathbf{b}=\alpha_1\mathbf{a}+\beta_1\mathbf{b}$ ($\mathbf{a}$ and $\mathbf{b}$ are non-zero and are not parallel to each other), then $\alpha=\alpha_1$ and $\beta=\beta_1$

Learning Unit	Learning Objective	Time	Remarks
	15.3 understand the representation of a vector in the rectangular coordinate system		The formulae that students are required to use include: • $\left\|\overrightarrow{OP}\right\| = \sqrt{x^2 + y^2 + z^2}$ in $\mathbf{R}^3$ • $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$ and $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$ in $\mathbf{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not** required.

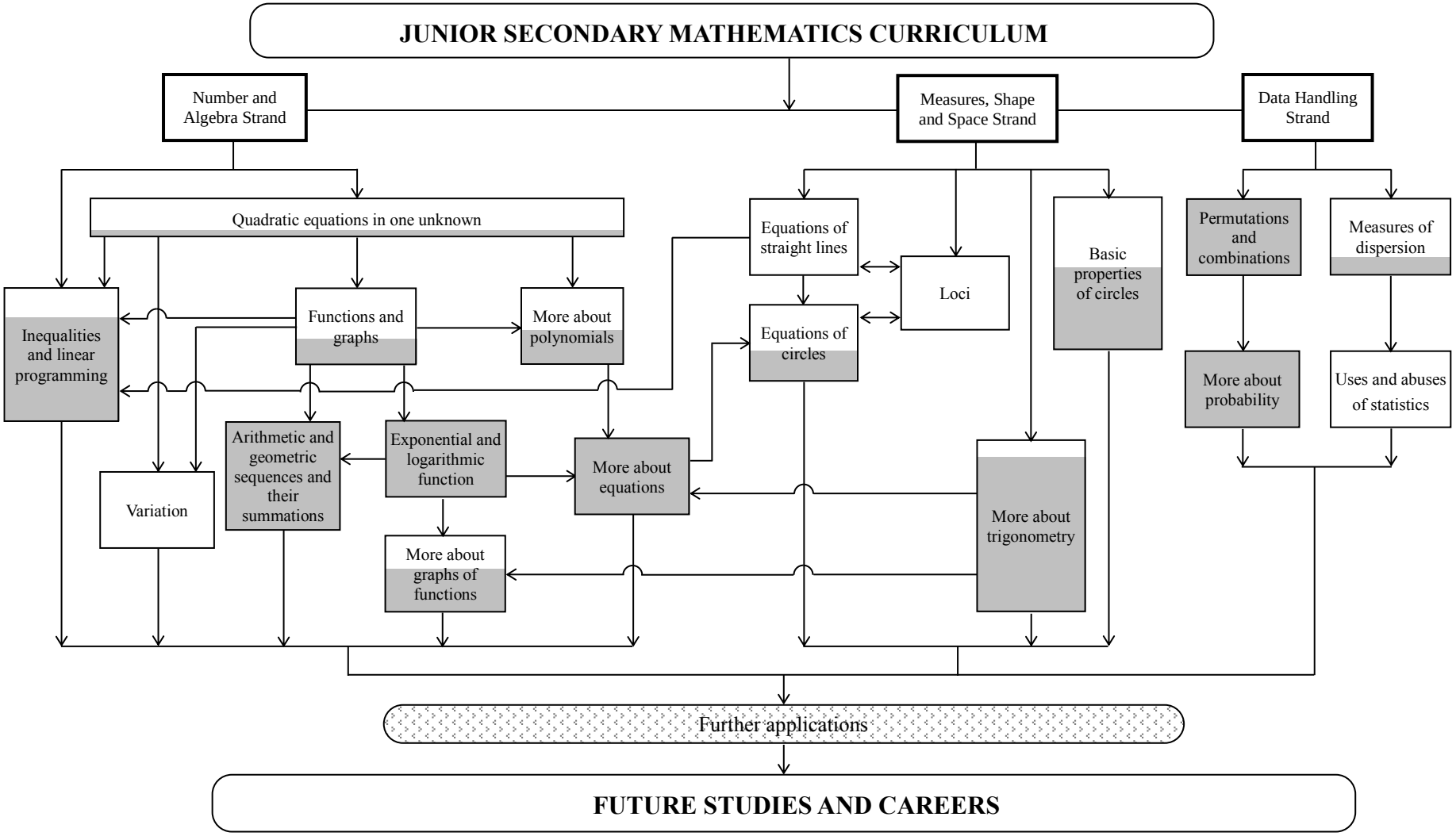
Learning Unit	Learning Objective	Time	Remarks
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors	5	The properties include: • $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$ • $\mathbf{a}\cdot(\lambda\mathbf{b})=\lambda(\mathbf{a}\cdot\mathbf{b})$ • $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ • $\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2\geq 0$ • $\mathbf{a}\cdot\mathbf{a}=0$ if and only if $\mathbf{a}=\mathbf{0}$ • $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\geq\lvert\mathbf{a}\cdot\mathbf{b}\rvert$ • $\lvert\mathbf{a}-\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-2(\mathbf{a}\cdot\mathbf{b})$
	16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$		The properties include: • $\mathbf{a}\times\mathbf{a}=\mathbf{0}$ • $\mathbf{b}\times\mathbf{a}=-(\mathbf{a}\times\mathbf{b})$ • $(\mathbf{a}+\mathbf{b})\times\mathbf{c}=\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$ • $\mathbf{a}\times(\mathbf{b}+\mathbf{c})=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ • $(\lambda\mathbf{a})\times\mathbf{b}=\mathbf{a}\times(\lambda\mathbf{b})=\lambda(\mathbf{a}\times\mathbf{b})$ • $\lvert\mathbf{a}\times\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2\lvert\mathbf{b}\rvert^2-(\mathbf{a}\cdot\mathbf{b})^2$

Learning Unit	Learning Objective	Time	Remarks
17. Applications of vectors	17.1 understand the applications of vectors	6	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector and the area of a triangle are required.
	Subtotal in hours	34	
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Total lesson time: 125 hours

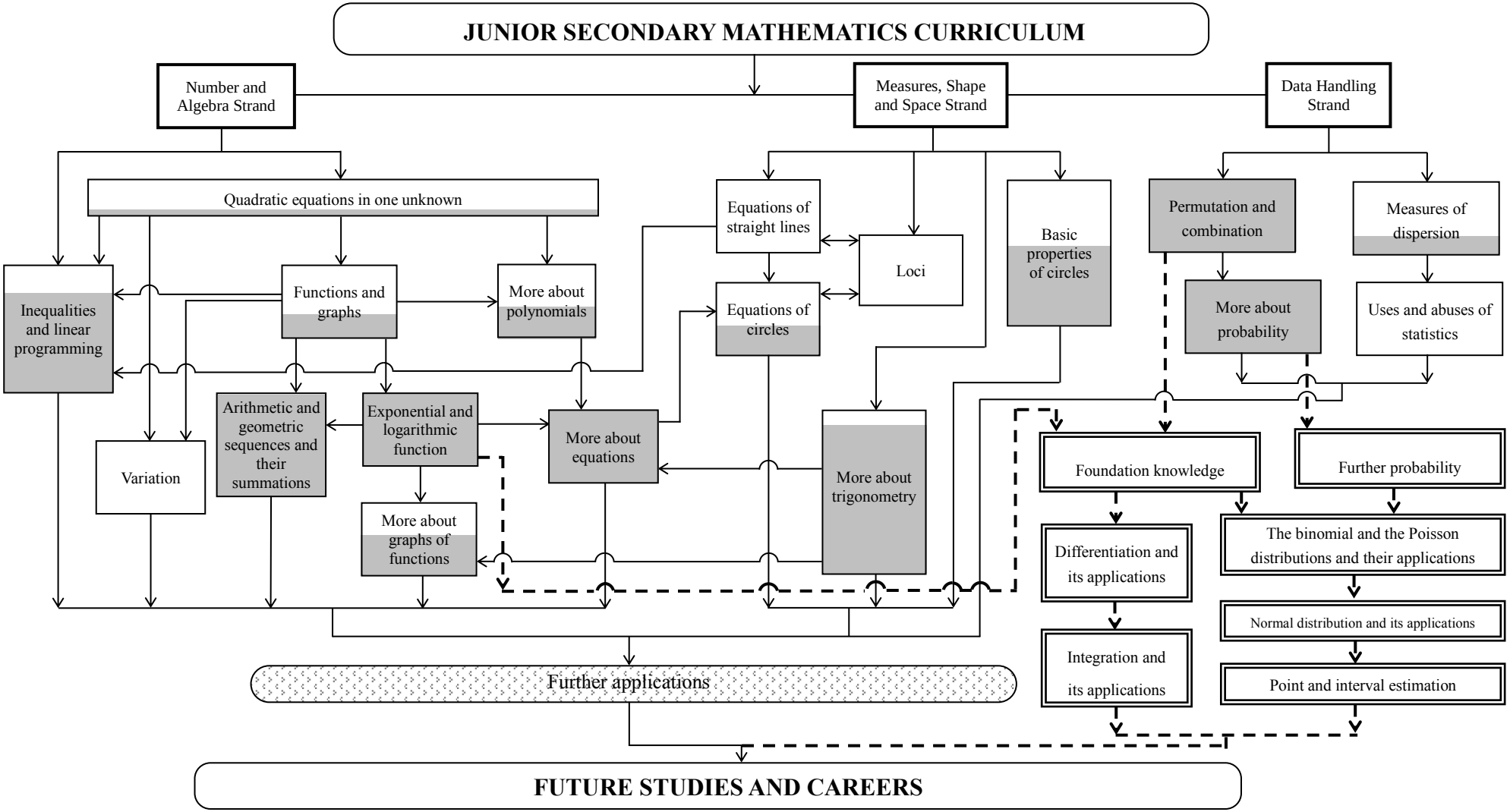
Chapter 3 Flow chart

Flow chart : Compulsory Part of Senior Secondary Mathematics Curriculum



Represents Non-foundation Topics.

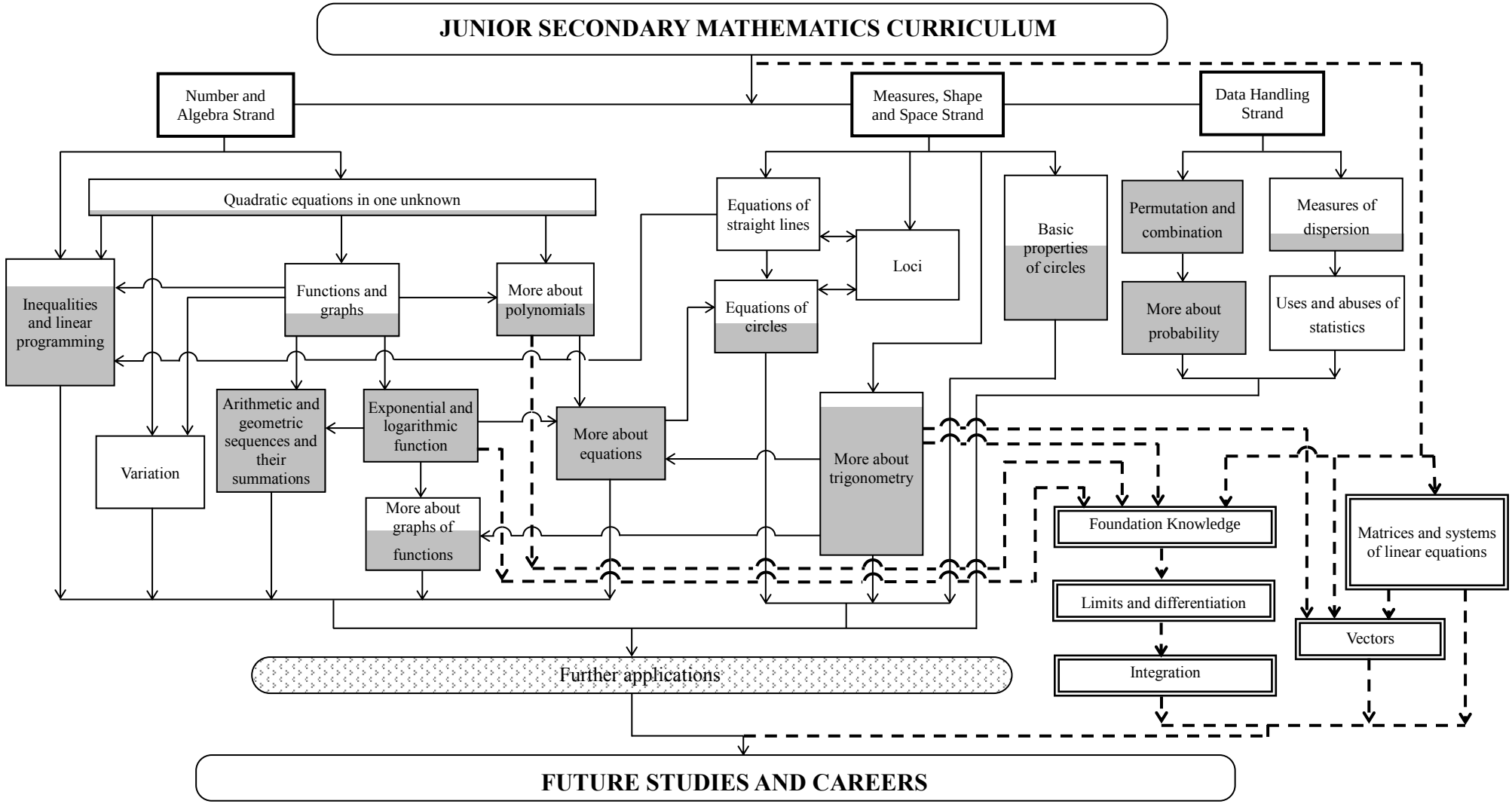
Flow chart : Compulsory Part with Module 1 (Calculus and Statistics) of Senior Secondary Mathematics Curriculum



represents Non-foundation Topics.

represents learning units in Module 1.

Flow chart : Compulsory Part with Module 2 (Algebra and Calculus) of Senior Secondary Mathematics Curriculum



represents Non-foundation Topics.

represents learning units in Module 2.

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(From September 2015 to August 2017)

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	Mr SHUM Yiu-kwong	
	Mr SIU Kwok-leong	
	Mr TANG Hok-shu	
	Mr TSANG Kin-fun	
	Ms TSUI Kwan-yuk	
	Ms TSUI Fung-ming, Karin	
	Ms WONG Chui-han, Ellen	
	Ms WONG Tin-ling	(from October 2016)
Secretary:	Dr NG Yui-kin (EDB)	

Membership of the CDC Committee on Mathematics Education

(From September 2017 to August 2019)

Chairperson:	Mr LAM Ka-yiu
Vice-chairperson:	Mr WAI Kwok-keung (EDB)
Members:	Mr CHAN Sai-hung
	Dr CHAN Wai-hong
	Ms CHAN Wai-yi
	Mr CHU Kan-kong (HKEAA)
	Mr CHU Lap-foo
	Ms CHUNG Po-loi
	Dr LEE Man-sang, Arthur
	Ms LEE Yuk-kit, Kitty
	Mr LEUNG Kwok-kei
	Dr LIU Kam-moon, Lester
	Mr MOK Sui-kei
	Mr NG Siu-kai (EDB)
	Mr PUN Chi-hang
	Ms WONG Chui-han, Ellen
	Mr YOUNG Chun-piu
	Dr YU Leung-ho Philip
Secretary:	Dr NG Yui-kin (EDB)

Membership of the CDC-HKEAA Committee on Mathematics Education

(From September 2015 to August 2017)

Chairperson:	Mr LAM Ka-yiu	
Vice-chairperson:	Mr WAI Kwok-keung (EDB)	(from March 2016)
	Mr LEE Pak-leung (EDB)	(until February 2016)
Members:	Mr CHEUNG Kam-tim, Thomas	
	Mr CHIU Kwok-sing	
	Mr CHIU Hong-ming	
	Mr CHU Kan-kong (HKEAA)	
	Mr LAU Chi-wah	
	Dr LEUNG Yuk-lun, Allen	
	Ms POON Suet-fan	
	Dr SHIU Wai-chee	
	Mr WONG Kwong-wing	
	Dr YU Leung-ho, Phillip	
Secretary:	Dr NG Yui-kin (EDB)	

Membership of the Ad Hoc Committee on Secondary Mathematics Curriculum (Junior Secondary and Compulsory Part of Senior Secondary)

(From December 2015)

Convenor	Mr LEE Kin-sum (EDB)	
Members:	Dr CHAN Yip-cheung	
	Mr CHIU Hong-ming	
	Mr CHIU Kwok-sing	
	Mr CHOW Kong-fai	
	Mr CHU Kan-kong (HKEAA)	
	Mr IP Che-ho	
	Mr LEE Wing-yeong	
	Mr LIU Hon-man	
	Dr LIU Kam-moon, Lester	
	Mr SIU Kwok-leong	
	Ms TSUI Fung-ming, Karin	
	Mr WONG Kwong-wing	
Secretary:	Mr LEE Chun-yue (EDB)	(from August 2017)
	Ms SIU Yuet-ming (EDB)	(until July 2017)

Membership of the Ad Hoc Committee on Secondary Mathematics Curriculum (Extended Part/Elective of Senior Secondary)

(From December 2015)

Convenor	Mr CHAN Sau-tang (EDB)
Members:	Dr CHAN Chi-kin
	Mr CHAN Sai-hung
	Mr CHEUNG Kam-tim, Thomas
	Mr CHU Kan-kong (HKEAA)
	Dr LAW Ka-ho
	Mr LEE Kwok-chu
	Mr POON Wai-hoi, Bobby
	Mr SUM Sing-wah
	Mr TANG Shu-yan
	Mr WONG Siu-hung
	Dr YU Leung-ho, Philip
Secretary:	Mr CHANG Kwok-kei (EDB)