Transforming a School Activity Room into a Café Corner

Key Stage: 3

Learning Unit: Inquiry and investigation
[This modelling activity mainly includes:
- Mensuration
- Rates, ratios and proportions]

Objectives:
(i) To enhance students' understanding of how to handle real-world problems through mathematical modelling
(ii) To enhance students' abilities in making assumptions and applying mathematical concepts in modelling
(iii) To enrich students' experience in applying mensuration and ratios in real-life scenarios

Prerequisite Knowledge:
(i) Solving problems involving surface areas
(ii) Solving problems involving ratios

Resources Required: Tape measures or digital measuring devices

Background Information:

Creating floor plans for spaces is an essential skill in interior design, architecture, and facilities management. It involves the application of mathematical knowledge and skills. Refining the floor plans based on real-world data aligns with the idea of mathematical modelling. Some scholars' proposed mathematical modelling tasks, such as the case described by Yeo (2015), propose progressively refining the floor plan design process based on the real-world scenario. This exemplar engages students in the imagination of transforming a school activity room into a comfortable café corner that promotes social interaction among students and teachers.

The main objective of this set of activities is to apply mathematical modelling approaches to address issues related to interior design. Specifically, students are tasked to think about how to use different factors to measure the comfort level of a space and design the floor plan for the café corner, which includes proposing the optimal arrangement of tables and chairs to better utilise the space and enhance the comfort level. Through group presentations and observing other classmates' designs, students

will further investigate how to determine the conditions for designing a comfortable café corner from a mathematical perspective.

Description of the Activities:

There are three main activities in this resource package:

- Activity 1: Guide students to think about how to describe mathematically the comfort level of a space and to create an initial floor plan for the café corner.
- Activity 2: Regarding the comfort level and budget of the furniture, collect real-world data related to café corners or restaurants (2A) and furniture (2B) and think about how to formulate conditions to determine the comfort level and furniture costs of the café corner.
- Activity 3: Using the outcomes from Activity 2 regarding the comfort level and furniture costs of the café corner, create a refined design for the café corner in the form of a scaled floor plan and establish a budget for the café corner's furnishings. Through group presentations, students observe different designs and think about how to optimise their own design.

To make this activity more challenging for more capable students, teachers can add constraints on the budget plan and incorporate the income of the café corner. This will require students to find a balance between ensuring comfort and accommodating a larger number of customers, thereby increasing the complexity of the context and creating more opportunities for mathematical modelling.

The following table summarises the modelling steps and elements that teachers can discuss with students in the corresponding activities.

Modelling Steps	Elements	WS1	WS2	WS3
Analysing the Real-world Problem	• Understand the real-world context • Clarify key factors of the problem • Identify related information/data and mathematical elements of the problem	1, 2a, 3a	1, 3ab	
Formulating a mathematical model	• Put forward assumptions to simplify the real-world problem • Convey the problem in a mathematical way • Determine relations between key quantities/variables	2bc, 3bc, 4	2, 4	1
Solving the Model for a Mathematical Solution	• Apply mathematical knowledge, skills and different tools to solve the model	4	2, 4	2
Interpreting the Mathematical Solution to Obtain a Real-world Solution	• Consider the meaning of the mathematical solution with respect to the real-world problem			2
Evaluating the Mathematical Model	• Validate the model against the real-world context • Reflect on the strengths and limitations of the model • Compare different models • Suggest refinements of the model	3d		3

Activity 1 (refer to Worksheet 1)

This activity begins by enhancing students' understanding of how to measure the comfort level of a space through different factors and then creating an initial design for the café corner in the form of a rough floor plan.

Pedagogical recommendations:

1. Teachers can introduce students to the context of transforming an activity room into a café corner. In Question 1, teachers can discuss with students how to describe mathematically the "comfort level" of the restaurant décor and identify the factors that are related to mathematics, which is the process of mathematising real-world problems in modelling. The following are some possible discussion outcomes.
    - Factors more directly related to mathematics:
        - The size of the café corner
        - The number of customers accommodated at one time
        - The numbers of tables and chairs
        - The distance between two sets of tables and chairs
    - Factors less directly related to mathematics:
        - Background music
        - Decoration
        - Colours

    Teachers can focus on two factors, including the passage width (Factor 1) and free-space ratio (Factor 2), that can help quantify the comfort level in a space.

2. Question 2 focuses on investigating the passage width (Factor 1). Teachers can facilitate students' discussion on how the passage width can affect the comfort level in a space in Question 2(a). The following are some possible discussion outcomes.
    - Generally speaking, the larger the passage width, meaning the more spacious the passage, and the higher the comfort level.
      Conversely, the smaller the passage width, meaning the more cramped the passage, and the lower the comfort level.

    In Question 2(b), students are tasked with measuring the passage width between two sets of tables and chairs in the activity room. They can use tape measures or digital measuring devices (e.g., Measurement Apps in tablets or smartphones) for the measurements. After that, teachers can encourage students to propose a

reasonable passage width, such as considering the passage width needed for a person holding a cup of coffee.

In Question 2(c), teachers can facilitate students' discussion on the difficulty and solution associated with quantifying the comfort level in a café corner or restaurant using the passage width during site visits. The following are some possible discussion outcomes.

- Difficulty: Typically, there are multiple passages with varying widths in a café corner or restaurant.
- Solution: We can formulate strategies to identify the most representative passage width, such as calculating the mean, modal class, or median of all passage widths in the space.

3. Question 3 focuses on investigating the free-space ratio (Factor 2). Teachers can facilitate students' discussion on the problems that arise when the free-space ratio is either close to 0 or close to 1 in Question 3(a). The following are some possible discussion outcomes.

   - If the free-space ratio is close to 0, the activity room becomes overcrowded, making customers' movement difficult.

     Conversely, if the free-space ratio is close to 1, it can result in a feeling of emptiness and underutilisation of the space.

   In Question 3(b), students are tasked with measuring the size of the activity room (the lengths of each wall segment) and the dimensions of the furniture inside, thereby calculating the free-space ratio of the current setting. After that, teachers can encourage students to refer to the current free-space ratio of the activity room and their perception of it to propose a reasonable free-space ratio for the café corner.

In Question 3(c), teachers can facilitate students' discussion on the difficulty and solution associated with quantifying the comfort level in a café corner or restaurant using the free-space ratio during site visits. The following are some possible discussion outcomes.

- Difficulty: Accurately measuring dimensions in an operating café corner or restaurant can be difficult due to the movement of customers and staff within the space.
- Solution: We can employ estimation techniques, such as measuring specific areas of the café corner or restaurant and then using these results to estimate the size of the entire space.

In Question 3(d), teachers can facilitate students' discussion on the limitations regarding the factor of the free-space ratio. The following are some possible discussion outcomes.

- The free-space ratio does not adequately reflect the effect of furniture shape and position or orientation on the comfort level. Besides, the free-space ratio is a 2D indicator and does not fully capture the extent of occupancy in a 3D space.

4. In Question 4, students use the dimension of the activity room as a blueprint and consider the existing equipment of the café corner to create an initial floor plan for the café corner. Also, teachers can guide students to discuss the comfort level of the café corner by considering Factors 1 and 2 or other relevant factors.

Activity 2A (refer to Worksheet 2)

This activity involves observing variables that affect the café corner design and customer experience, during which students are required to visit nearby café corners or restaurants for observation.

Pedagogical recommendations:

1. Students visit some nearby café corners or restaurants and take notes on
   (i) the number of customers accommodated at one time,
   (ii) the number of different types of tables available,
   (iii) the passage width and measurement strategy,
   (iv) the free-space ratio and measurement strategy, and
   (v) the perceived "comfort level" as a customer.

2. Based on their information collected during the site visits, students propose conditions that a comfortable café corner has to fulfil and decide whether to refine the values of Factor 1 (passage width) and Factor 2 (free-space ratio) in their design from Activity 1 (Question 4) or not. The following are some possible discussion outcomes.
   - A comfortable café corner has to fulfil the following conditions:
     - The number of customers ≤ 30
     - The number of tables ≤ 15
     - Factor 1: The passage width ≥ 1.5 m
     - Factor 2: The free-space ratio ≈ 0.5

Activity 2B (refer to Worksheet 2)

This activity involves collecting data about the furniture that best suit the new café corner. Students can visit a nearby furniture store or browse an online retailer to collect information.

Pedagogical recommendations:

3. Considering the comfort level and budget of the café corner, students are tasked to discuss the data/information of the furniture that they should collect. The following are some possible discussion outcomes.
    - Data/information related to the comfort level and budget:
        - Dimension
        - Function (e.g., the number of people per table)
        - Comfort level (e.g., the heights and materials of the tables and chairs)
        - Price

4. With the ideas in Question 3, students visit a nearby furniture store (or browse an online retailer) to collect information on several types of tables and chairs that could be considered for the café corner.

5. Based on the collected data, teachers can lead students to discuss the price ranges of different furniture and estimate the furniture costs based on the initial coffee corner design developed in Activity 1. This estimation forms one of the discussion criteria for Activity 3.

    Teachers and students can discuss the corresponding reasonable furniture costs based on the situation of the school activity room and the requirements of the café corner. The following are some possible discussion outcomes.
    - A typical small café corner that can provide 15 to 30 seats has a furniture cost of approximately $120,000 to $250,000. This includes a corresponding number of tables, chairs, and sofas, with the reference costs for each piece of furniture as follows:
        - Chairs/stools: $800 to $5,500 each
        - Tables (for 2 or 4 people): $3,000 to $12,000 each
        - Sofas/booths (for 2 to 4 people): $8,000 to $25,000 each
        - Bar counter (6 to 10 feet): $30,000 to $80,000 each

Activity 3 (refer to Worksheet 3)

In this activity, the problem focuses on floor plan design and the budget for the cost of the furniture using the data collected in Activities 1 and 2. Through group presentations, students observe different designs and think about how to optimise their own design.

Pedagogical recommendations:

1. Students are tasked to create a scaled floor plan of the café corner with reference to the following steps.
    i. Students use the measurements in Activity 1 to set the dimensions of the floor plan with a scale on a graph paper.
    ii. If necessary, students discuss whether to change the positions of the existing equipment in the café corner (see Question 4 in Activity 1).
    iii. Based on the observations from Activity 2A, students discuss how to arrange the furniture to enhance the comfort level of the café corner while also considering the operational needs of the café corner.
    iv. Based on the data in Activity 2B, students decide the types and quantities of tables and chairs that they are going to use.
    v. Students draw a scaled floor plan of the café corner that includes the layout of the entrance and furniture.

2. Based on the café corner's floor plan designed in Question 1, students are tasked to calculate or estimate (a) the maximum number of customers accommodated at one time, (b) the passage width of the café corner, (c) the free-space ratio of the café corner, and (d) the total cost for the furniture.

    After that, students are tasked to evaluate whether the design meets the conditions for a comfortable café corner as proposed in Activity 2. Teachers can invite students to present their outcomes.

3. Finally, Question 3 can be assigned as an after-class assignment. After observing classmates' café corner designs, teachers can recall those outstanding presentations, highlight their strengths and areas of further investigation, and then ask students to optimise their original conditions for a comfortable café corner and its design.

Reference:

Yeo, K. K. J. (2015). Learning through “Designing a Café”. In N. H. Lee, & K. E. D. Ng (Eds.), *Mathematical Modelling: From Theory to Practice* (pp. 163–173). Singapore: World Scientific.

Suggested lesson plans and teaching flow

Activity 1

- Teaching time: Approximately 40 minutes or a single lesson

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
5	• To arouse students' interest	1. Teachers arouse students' interest by discussing the real-world scenario. 2. Teachers discuss with students the factors that affect the "comfort level" of a café corner and their relevance to mathematics.	WS cover page WS1 Q1
20	• To explore measures to quantify the comfort level	1. Teachers introduce Factor 1: Passage Width and facilitates students' exploration of this factor. 2. Students measure the passage width between two sets of tables and chairs in the activity room and hence recommend an optimal passage width for the café corner. 3. Teachers discuss with students the difficulty and solution associated with quantifying the comfort level in a café corner or restaurant using the passage width during site visits.	WS1 Q2

Time (mins)	Objectives	Teaching activities and processes	Resources/ remarks
		4. Teachers introduce Factor 2: Free-space Ratio and facilitates students' exploration of this factor. 5. Students measure the size of the activity room and the dimensions of the furniture inside, thereby calculating its free-space ratio, and hence recommend an optimal free-space ratio for the café corner. 6. Teachers discuss with students the difficulty, solution, and limitations associated with quantifying the comfort level in a café corner or restaurant using the free-space ratio during site visits.	WS1 Q3
10	To create an initial design	1. Students create an initial floor plan for the café corner. 2. Teachers remind students to consider Factors 1 and 2, as well as other factors that are relevant.	WS1 Q4
5	• To introduce the subsequent activities	1. Teachers introduce the arrangement of Activities 2A and 2B. 2. Students form their groups.	

<u>Activity 2A</u>

- Activity 2A may be assigned as an outing activity. Students visit some nearby café corners or restaurants to carry out their investigation (WS2 Q1–2). The teacher may also ask students to collect relevant data through everyday observations.

Activity 2B

- Activity 2B may be assigned as an outing activity. Students visit a nearby furniture store to carry out their investigation (WS2 Q3–5). The teacher may also ask students to browse online retailers to collect information about tables, chairs, and other furniture.

Activity 3

- Teaching time: Approximately 80 minutes or a double lesson

Time (mins)	**Objectives**	**Teaching activities and processes**	**Resources/ remarks**
30	• To create a refined design	1. Students create a scaled floor plan of the café corner.	WS3 Q1
15	• To develop a budget for the café corner's furnishings	1. Students calculate or estimate (a) the maximum number of customers accommodated at one time, (b) the passage width of the café corner, (c) the free-space ratio of the café corner, and (d) the total cost for the furniture. 2. Students evaluate their design.	WS3 Q2
30	• To present outcomes	1. Students present their floor plan of the café corner and their budget report. 2. The teacher gives feedback on students' outcomes.	
5	• To conclude the activity	1. The teacher concludes the modelling activity. 2. The teacher recalls those outstanding presentations, highlighting their strengths and areas of further investigation 3. As an after-class assignment, the teacher asks students to optimise their original conditions for a comfortable café corner and its design.	WS3 Q3