

Predicting Trends in the Social Media Follower Counts of Singers

Key Stage: 3 or 4

Learning Unit: Inquiry and investigation

[This modelling activity mainly includes:

- Algebraic expressions
- Using percentages
- Arithmetic and geometric sequences and their summations (KS4)
- Exponential and logarithmic functions (KS4)]

Objectives:

- (i) To enhance students' understanding of how to handle real-world problems through mathematical modelling
- (ii) To enhance students' abilities in evaluating modelling approaches, making assumptions, and identifying limitations in modelling
- (iii) To enhance students' abilities in analysing data
- (iv) To enrich students' experience in applying the concepts of sequences of numbers, percentage changes, and exponential and logarithmic functions in real-life scenarios

Prerequisite Knowledge:

- (i) Recognising the concept of sequences of numbers
- (ii) Understanding the concept of percentage changes
- (iii) Understanding the concept of arithmetic and geometric sequences (KS4)
- (iv) Understanding the properties of exponential functions and recognising the features of their graphs (KS4)

Resources Required: Desktop or tablet computers with spreadsheet application and Internet connection

Background Information:

Investigating trends based on data is essential in daily life in areas such as finance and social media. Recognising these trends allows individuals and organisations to make informed decisions based on historical data and projected outcomes. The main goal of this exemplar is to enrich students' modelling experiences and to allow them to apply

mathematical concepts to solve real-life problems. Conducting this activity in KS3 will help students understand the practical application of the concepts related to algebraic expressions and percentage changes. If this activity is conducted in KS4, it will strengthen students' understanding of the practical application of the concepts related to arithmetic and geometric sequences and exponential functions.

This exemplar demonstrates how to use data as a foundation for mathematical modelling. Specifically, students are required to mathematically express the trends in social media follower counts based on the provided data (i.e., to formulate a model), thereby predicting the social media follower counts of different singers. In addition, by exploring different modelling approaches, students analyse the strengths and weaknesses of these approaches. Furthermore, students are prompted to consider the assumptions and limitations inherent in their formulated models, fostering a deeper understanding of the modelling process.

Description of the Activities:

There are four main activities in this resource package:

- Activity 1: To conduct preliminary analysis on the modelling context and make simple predictions.
- Activity 2: To explore modelling approaches applicable to linear trends.
- Activity 3: (Suitable for senior secondary students) To explore modelling approaches applicable to exponential trends.
- Activity 4: To further explore more complicated trends.

Remarks:

1. As the focus of this exemplar is to strengthen students' understanding of modelling approaches for linear trends and exponential trends, Activities 1 to 3 do not use real data. In Activity 4, teachers may consider replacing the provided data with real data according to students' abilities.
2. Teachers can use Activity 1 as a pre-class activity according to students' abilities.

The following table summarises the modelling steps and elements that teachers can discuss with students in the corresponding activities.

| Modelling Steps | Elements | WS1 | WS2 | WS3 | WS4 |
|--|--|-----|----------------------|----------------------|--------|
| Analysing the Real-world Problem | <ul style="list-style-type: none"> Understand the real-world context Clarify key factors of the problem Identify related information/data and mathematical elements of the problem | 1 | 1, 2, 3a, 4a, 5a, 8a | 1, 2, 3a, 4a, 5a, 6a | 1a, 2a |
| Formulating a mathematical model | <ul style="list-style-type: none"> Put forward assumptions to simplify the real-world problem Convey the problem in a mathematical way Determine relations between key quantities/variables | 2ab | | | |
| Solving the Model for a Mathematical Solution | <ul style="list-style-type: none"> Apply mathematical knowledge, skills and different tools to solve the model | 2b | 3bc, 4bc, 5bc, 8bc | 3bc, 4bc, 5bc, 6bc | 1b, 2b |
| Interpreting the Mathematical Solution to Obtain a Real-world Solution | <ul style="list-style-type: none"> Consider the meaning of the mathematical solution with respect to the real-world problem | | | | |
| Evaluating the Mathematical Model | <ul style="list-style-type: none"> Validate the model against the real-world context Reflect on the strengths and limitations of the model Compare different models Suggest refinements of the model | | 3d, 4d, 5d, 6, 7 | 3d, 4d, 5d, 7 | |

Activity 1 (refer to Worksheet 1)

This activity conducts preliminary analysis on the modelling context and then helps students make simple predications using the concept of sequences of numbers.

Pedagogical recommendations:

1. In Question 1, teachers can introduce students to the context of social media and facilitate students' discussion on the factors that influence the follower counts of singers. The following are some possible discussion outcomes.
 - Factors related to the singer themselves, such as the singer's age, personal charisma, and frequency of public appearances.
 - Factors related to social media, such as the quality of posts, posting frequency, interaction with fans on social media, and social media algorithms.
 - Other factors, such as the impact of the release of a movie starring the singer on the follower counts.

2. Question 2 shows the social media follower counts of Singer A and Signer B and asks students to predict the social media follower counts for both signers in the coming months based on the data. Students have to observed that the follower counts of Singer A follow an arithmetic sequence, whereas those of Singer B follow a geometric sequence. In Question 2(a), students discuss the assumptions involved in their predictions. In Question 2(b), students predict the follower counts for the next month based on the patterns observed in the follower counts of Singer A and Signer B. However, teachers should emphasise that in this series of activities, the primary governing factor focuses only on the previous numbers of followers. Meanwhile, it is not always possible to find consistent and obvious patterns in reality.

Suggested solution:

- (a) Assumptions:
 1. The primary governing factor is the previous numbers of followers.
 2. Consistent growth patterns.
 3. Linear growth for Singer A, while exponential growth for Singer B.

- (b) For Singer A, the follower count of each month is greater than that of its previous month by 25.

$$\begin{aligned}\text{The social media follower count of Singer A in June} &= 130 + 25 \\ &= 155\end{aligned}$$

For Singer B, the follower count of each month is double that of its previous month.

$$\begin{aligned}\text{The social media follower count of Singer B in June} &= 320 \times 2 \\ &= 640\end{aligned}$$

Activity 2 (refer to Worksheet 2)

This activity aims to establish students' modelling abilities in the context of linear trends. It focuses on exploring how to use the values of monthly changes as the key parameter of the model through different approaches, thereby formulating a mathematical model.

Pedagogical recommendations:

1. Question 1 shows the broken-line graph of the social media follower counts of Singer H. Teachers can facilitate students' discussion on the ways to predict the follower count in December 2023. The following are some possible discussion outcomes.
 1. Consider the change for the last two months; and then use this change to predict the follower count in December.
 2. Consider the changes from month to month and calculate an average; and then use this average to predict the follower count in December.
2. In Question 2, students are tasked with calculating the changes in the follower counts between months.

Suggested solution:

| Month | Change |
|--------------|---------------|
| Jan to Feb | <u>139</u> |
| Feb to Mar | <u>150</u> |
| Mar to Apr | <u>143</u> |
| Apr to May | <u>168</u> |
| May to Jun | <u>166</u> |

| Month | Change |
|--------------|---------------|
| Jun to Jul | <u>158</u> |
| Jul to Aug | <u>161</u> |
| Aug to Sep | <u>166</u> |
| Sep to Oct | <u>193</u> |
| Oct to Nov | <u>138</u> |

Teachers can introduce students to the IT skills involved in using spreadsheets to calculate these changes. Worksheet 2 uses Microsoft Excel as an example.

In Questions 3 to 5 and 8, teachers will guide students in exploring four possible approaches to predicting Singer H's follower counts in the future.

3. Modelling approach 1 only considers the change between the most recent two months for making predictions. In Question 3(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Change |
|------------|------------|
| Jan to Feb | <u>139</u> |
| Feb to Mar | <u>150</u> |
| Mar to Apr | <u>143</u> |
| Apr to May | <u>168</u> |
| May to Jun | <u>166</u> |

| Month | Change |
|------------|------------|
| Jun to Jul | <u>158</u> |
| Jul to Aug | <u>161</u> |
| Aug to Sep | <u>166</u> |
| Sep to Oct | <u>193</u> |
| Oct to Nov | <u>138</u> |

In Question 3(b), students are tasked with predicting Singer H's follower counts in December 2023 and January 2024.

Suggested solution:

Dec 2023: $2653 + 138 = 2791$

Jan 2024: $2653 + 138 \times 2 = 2929$

With the experience in answering Question 3(b), Question 3(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2653 + 138n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 3(d). The following are some possible discussion outcomes.

- Strengths:
 1. This model is easy to comprehend.
 2. This model can reflect the singer's current popularity.
- Weaknesses:
 1. This model overlooks the overall trend that includes earlier months.
 2. The change between the most recent two months can be unusually high or low due to specific events, which can result in overestimation or underestimation.

3. This model assumes that the follower counts change according to a single pattern.
4. Modelling approach 2 considers all of the change between months for making predictions. In Question 4(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Change |
|------------|------------|
| Jan to Feb | <u>139</u> |
| Feb to Mar | <u>150</u> |
| Mar to Apr | <u>143</u> |
| Apr to May | <u>168</u> |
| May to Jun | <u>166</u> |

| Month | Change |
|------------|------------|
| Jun to Jul | <u>158</u> |
| Jul to Aug | <u>161</u> |
| Aug to Sep | <u>166</u> |
| Sep to Oct | <u>193</u> |
| Oct to Nov | <u>138</u> |

In Question 4(b), students are tasked with predicting Singer H's follower counts in December 2023 and January 2024.

Suggested solution:

$$\text{Average change} = \frac{139 + 150 + \dots + 138}{10} = 158$$

$$\text{Dec 2023: } 2653 + 158 = 2811$$

$$\text{Jan 2024: } 2653 + 158 \times 2 = 2969$$

With the experience in answering Question 4(b), Question 4(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2653 + 158n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 4(d). The following are some possible discussion outcomes.

- Strengths:
 1. This model considers changes between months over a longer period, providing a more comprehensive analysis of the overall trend.
 2. Considering a longer time frame can mitigate the influence of short-term fluctuations.
- Weaknesses:
 1. Because this model relies on historical data over a longer period, it may

not fully reflect the signer's current popularity.

2. This model assumes that the follower counts change according to a single pattern.

5. Modelling approach 3 only considers the most recent three changes between months for making predictions. In Question 5(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Change |
|------------|------------|
| Jan to Feb | <u>139</u> |
| Feb to Mar | <u>150</u> |
| Mar to Apr | <u>143</u> |
| Apr to May | <u>168</u> |
| May to Jun | <u>166</u> |

| Month | Change |
|------------|------------|
| Jun to Jul | <u>158</u> |
| Jul to Aug | <u>161</u> |
| Aug to Sep | <u>166</u> |
| Sep to Oct | <u>193</u> |
| Oct to Nov | <u>138</u> |

In Question 5(b), students are tasked with predicting Singer H's follower counts in December 2023 and January 2024.

Suggested solution:

$$\text{Average change} = \frac{166 + 193 + 138}{3} = 166$$

$$\text{Dec 2023: } 2653 + 166 = 2819$$

$$\text{Jan 2024: } 2653 + 166 \times 2 = 2985$$

With the experience in answering Question 5(b), Question 5(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2653 + 166n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 5(d). The following are some possible discussion outcomes.

- Strengths:
 1. Considering the most recent changes can reflect the singer's current popularity.
 2. Considering the average change over a four-month time frame can mitigate the influence of short-term fluctuations.

- Weaknesses:
 1. The rationale for considering the three most recent changes is not clear.
 2. This model assumes that the follower counts change according to a single pattern.
6. After discussing the above three modelling approaches, teachers can facilitate the discussion on the common assumption and limitation of the formulated models. The following are some possible discussion outcomes.
- Assumption: These models assume that changes in follower counts are of a single pattern and follow a linear pattern.
 - Limitation: These models do not take into account important factors, such as the singer’s posting frequency and public appearances.

In Questions 7 and 8, teachers will guide students in incorporating the concept of “weight” into the modelling process.

7. Teachers can review Modelling approaches 1 and 2 from the perspective of “weight”.
- Model 1 only considers the change between the most recent two months (i.e., Oct and Nov 2023). Therefore, the weight assigned to this change is 1, while the weights for all other changes are set to 0.
 - Model 2 considers all of the changes between months (i.e., Jan to Feb, Feb to Mar, ..., and Oct to Nov). Therefore, the weights for all changes are set to 1.

Teachers can check students’ understanding of this approach by asking them to assign weights in the case of Modelling approach 3, where Model 3 only considers the most recent three changes between months (i.e., Aug to Sep, Sep to Oct, and Oct to Nov).

Suggested solution:

| Month | Change | Weight |
|------------|--------|----------|
| Jan to Feb | 139 | <u>0</u> |
| Feb to Mar | 150 | <u>0</u> |
| Mar to Apr | 143 | <u>0</u> |
| Apr to May | 168 | <u>0</u> |
| May to Jun | 166 | <u>0</u> |

| Month | Change | Weight |
|------------|--------|----------|
| Jun to Jul | 158 | <u>0</u> |
| Jul to Aug | 161 | <u>0</u> |
| Aug to Sep | 166 | <u>1</u> |
| Sep to Oct | 193 | <u>1</u> |
| Oct to Nov | 138 | <u>1</u> |

8. Modelling approach 4 considers the weights of changes between months. It calculates the weighted mean of these changes to obtain the weighted average change per month. In Question 8(a), teachers can facilitate students' discussion on the weights of these changes and their explanations. The following are some possible discussion outcomes.

| Month | Change | Weight |
|------------|--------|----------|
| Jan to Feb | 139 | <u>0</u> |
| Feb to Mar | 150 | <u>0</u> |
| Mar to Apr | 143 | <u>0</u> |
| Apr to May | 168 | <u>0</u> |
| May to Jun | 166 | <u>0</u> |

| Month | Change | Weight |
|------------|--------|----------|
| Jun to Jul | 158 | <u>0</u> |
| Jul to Aug | 161 | <u>0</u> |
| Aug to Sep | 166 | <u>1</u> |
| Sep to Oct | 193 | <u>2</u> |
| Oct to Nov | 138 | <u>3</u> |

Explanations:

- To reflect the extent of influence that more recent data has on the predictions, the most recent change is assigned the highest weight of 3.
- The second and third most recent changes are given weights of 2 and 1, respectively, indicating their lesser extent of influence.
- Changes other than these months are assigned a weight of 0, as they are considered no longer relevant.

In Question 8(b), students are tasked with predicting Singer H's follower counts in December 2023 and January 2024.

Suggested solution (based on the above outcomes):

$$\text{Average change} = \frac{166 \times 1 + 193 \times 2 + 138 \times 3}{1 + 2 + 3} = 161$$

$$\text{Dec 2023: } 2653 + 161 = 2814$$

$$\text{Jan 2024: } 2653 + 161 \times 2 = 2975$$

With the experience in answering Question 8(b), Question 8(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution (based on the above outcomes):

$$2653 + 161n$$

Activity 3 (refer to Worksheet 3)

This activity is similar to Activity 2, but it involves analysing an exponential trend. It focuses on exploring how to use the values of monthly percentage changes as the key parameter of the model through different approaches, thereby formulating a mathematical model.

Considering students' mathematical tools available, the arithmetic mean is used in the pedagogical recommendations. However, it is important to note that from a mathematical point of view, the geometric mean is preferred to the arithmetic mean though their difference is insignificant in this activity. Depended on the school context and students' ability, teachers can decide which mean to be used in modelling.

$$\text{Arithmetic mean} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\text{Weighted arithmetic mean} = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n}$$

$$\text{Geometric mean} = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n} \quad (\text{Note})$$

$$\text{Weighted geometric mean} = \sqrt{(w_1+w_2+\cdots+w_n)}{x_1^{w_1} \times x_2^{w_2} \times \cdots \times x_n^{w_n}} \quad (\text{Note})$$

(Note: Enrichment content, not included in the senior secondary mathematics curriculum)

Pedagogical recommendations:

1. Question 1 shows the broken-line graph of the social media follower counts of Singer K. Teachers can facilitate students' discussion on whether it is suitable to use the models formulated in Activity 2 for prediction. The following are some possible discussion outcomes.

- Unlike the data in Activity 2, the monthly changes in Singer K's follower counts are clearly inconsistent. Furthermore, the graph shows an accelerating growth in the follower counts.

Given this non-linear trend, Models 1 to 3 may not be suitable for prediction.

2. In Question 2, students are tasked with calculating the percentage changes in the follower counts between months.

Suggested solution:

| Month | Percentage change |
|------------|-------------------|
| Jan to Feb | <u>20.13%</u> |
| Feb to Mar | <u>21.20%</u> |
| Mar to Apr | <u>20.95%</u> |
| Apr to May | <u>21.43%</u> |
| May to Jun | <u>20.88%</u> |

| Month | Percentage change |
|------------|-------------------|
| Jun to Jul | <u>21.29%</u> |
| Jul to Aug | <u>21.66%</u> |
| Aug to Sep | <u>21.52%</u> |
| Sep to Oct | <u>21.57%</u> |
| Oct to Nov | <u>21.93%</u> |

Teachers can introduce students to the IT skills involved in using spreadsheets to calculate these changes. Worksheet 3 uses Microsoft Excel as an example.

In Questions 3 to 6, teachers will guide students in exploring four possible approaches to predicting Singer K's follower counts in the future.

3. Similar to Modelling approach 1, Modelling approach 5 only considers the percentage change between the most recent two months for making predictions. In Question 3(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Percentage change |
|------------|-------------------|
| Jan to Feb | <u>20.13%</u> |
| Feb to Mar | <u>21.20%</u> |
| Mar to Apr | <u>20.95%</u> |
| Apr to May | <u>21.43%</u> |
| May to Jun | <u>20.88%</u> |

| Month | Percentage change |
|------------|-------------------|
| Jun to Jul | <u>21.29%</u> |
| Jul to Aug | <u>21.66%</u> |
| Aug to Sep | <u>21.52%</u> |
| Sep to Oct | <u>21.57%</u> |
| Oct to Nov | <u>21.93%</u> |

In Question 3(b), students are tasked with predicting Singer K's follower counts in December 2023 and January 2024.

Suggested solution:

$$\text{Dec 2023: } 2185(1 + 21.93\%) = 2664$$

$$\text{Jan 2024: } 2185(1 + 21.93\%)^2 = 3248$$

With the experience in answering Question 3(b), Question 3(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2185(1 + 21.93\%)^n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 3(d). The following are some possible discussion outcomes.

- Strengths:
 1. This model is easy to comprehend.
 2. This model can reflect the singer's current popularity.
 - Weaknesses:
 1. This model overlooks the overall trend that includes earlier months.
 2. The percentage change between the most recent two months can be unusually high or low due to specific events, which can result in overestimation or underestimation.
 3. This model assumes that the follower counts change according to a single pattern.
4. Similar to Modelling approach 2, Modelling approach 6 considers all of the percentage changes between months for making predictions. In Question 4(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Percentage change |
|------------|-------------------|
| Jan to Feb | <u>20.13%</u> |
| Feb to Mar | <u>21.20%</u> |
| Mar to Apr | <u>20.95%</u> |
| Apr to May | <u>21.43%</u> |
| May to Jun | <u>20.88%</u> |

| Month | Percentage change |
|------------|-------------------|
| Jun to Jul | <u>21.29%</u> |
| Jul to Aug | <u>21.66%</u> |
| Aug to Sep | <u>21.52%</u> |
| Sep to Oct | <u>21.57%</u> |
| Oct to Nov | <u>21.93%</u> |

In Question 4(b), students are tasked with predicting Singer K's follower counts in December 2023 and January 2024.

Suggested solution:

$$\text{Average percentage change} = \frac{20.13\% + 21.20\% + \cdots + 21.93\%}{10} = 21.26\%$$

$$\text{Dec 2023: } 2185(1 + 21.26\%) = 2650$$

$$\text{Jan 2024: } 2185(1 + 21.26\%)^2 = 3213$$

With the experience in answering Question 4(b), Question 4(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2185(1 + 21.26\%)^n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 4(d). The following are some possible discussion outcomes.

- Strengths:
 1. This model considers percentage changes between months over a longer period, providing a more comprehensive analysis of the overall trend.
 2. Considering a longer time frame can mitigate the influence of short-term fluctuations .
- Weaknesses:
 1. Because this model relies on historical data over a longer period, it may not fully reflect the signer's current popularity.
 2. This model assumes that the follower counts change according to a single pattern.

5. Similar to Modelling approach 3, Modelling approach 7 only considers the most recent three percentage changes between months for making predictions. In Question 5(a), teachers can check students' understanding of this modelling approach by asking them to circle the data used for prediction.

Suggested solution:

| Month | Percentage change |
|------------|-------------------|
| Jan to Feb | <u>20.13%</u> |
| Feb to Mar | <u>21.20%</u> |
| Mar to Apr | <u>20.95%</u> |
| Apr to May | <u>21.43%</u> |
| May to Jun | <u>20.88%</u> |

| Month | Percentage change |
|------------|-------------------|
| Jun to Jul | <u>21.29%</u> |
| Jul to Aug | <u>21.66%</u> |
| Aug to Sep | <u>21.52%</u> |
| Sep to Oct | <u>21.57%</u> |
| Oct to Nov | <u>21.93%</u> |

In Question 5(b), students are tasked with predicting Singer K's follower counts in December 2023 and January 2024.

Suggested solution:

$$\text{Average percentage change} = \frac{21.52\% + 21.57\% + 21.93\%}{3} = 21.67\%$$

$$\text{Dec 2023: } 2185(1 + 21.67\%) = 2658$$

$$\text{Jan 2024: } 2185(1 + 21.67\%)^2 = 3235$$

With the experience in answering Question 5(b), Question 5(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution:

$$2185(1 + 21.67\%)^n$$

As a conclusion of this modelling approach, teachers can facilitate students' discussion on its strengths and weaknesses in Question 5(d). The following are some possible discussion outcomes.

- Strengths:
 1. Considering the most recent percentage changes can reflect the singer's current popularity.
 2. Considering the average percentage change over a four-month time frame can mitigate the influence of short-term fluctuations.
- Weaknesses:
 1. The rationale for considering the three most recent percentage changes is not clear.
 2. This model assumes that the follower counts change according to a single pattern.

6. Similar to Modelling approach 4, Modelling approach 8 considers the weights of percentage changes between months. It calculates the weighted mean of these percentage changes to obtain the weighted average percentage change per month. In Question 6(a), teachers can facilitate students' discussion on the weights of these percentage changes and their explanations. The following are some possible discussion outcomes.

| Month | % change | Weight |
|------------|----------|----------|
| Jan to Feb | 20.13% | <u>0</u> |
| Feb to Mar | 21.20% | <u>0</u> |
| Mar to Apr | 20.95% | <u>0</u> |
| Apr to May | 21.43% | <u>0</u> |
| May to Jun | 20.88% | <u>0</u> |

| Month | % change | Weight |
|------------|----------|----------|
| Jun to Jul | 21.29% | <u>0</u> |
| Jul to Aug | 21.66% | <u>0</u> |
| Aug to Sep | 21.52% | <u>1</u> |
| Sep to Oct | 21.57% | <u>2</u> |
| Oct to Nov | 21.93% | <u>3</u> |

Explanations:

- To reflect the extent of influence that more recent data has on the predictions, the most recent percentage change is assigned the highest weight of 3.
- The second and third most recent percentage changes are given weights of 2 and 1, respectively, indicating their lesser extent of influence.
- Percentage changes other than these months are assigned a weight of 0, as they are considered no longer relevant.

In Question 6(b), students are tasked with predicting Singer K's follower counts in December 2023 and January 2024.

Suggested solution (based on the above outcomes):

$$\text{Average percentage change} = \frac{21.52\% \times 1 + 21.57\% \times 2 + 21.93\% \times 3}{1 + 2 + 3} = 21.74\%$$

$$\text{Dec 2023: } 2185(1 + 21.74\%) = 2660$$

$$\text{Jan 2024: } 2185(1 + 21.74\%)^2 = 3238$$

With the experience in answering Question 6(b), Question 6(c) further requires students to formulate a mathematical model to mathematically express the predicted follower count at the n -th month after November 2023.

Suggested solution (based on the above outcomes):

$$2185(1 + 21.74\%)^n$$

7. Towards the end of Activity 3, teachers can facilitate the discussion on the common assumption and limitation of the formulated models. The following are some possible discussion outcomes.
- Assumption: These models assume that changes in follower counts are of a single pattern and follow an exponential trend.
 - Limitation: These models do not take into account important factors, such as the singer's posting frequency and public appearances.

Activity 4 (refer to Worksheet 4)

This activity involves analysing more complicated trends. It offers students opportunities to apply what they have learnt in the previous activities. Teachers should encourage students to use different approaches to modelling and predicting outcomes. Because there are no model answers to this kind of modelling and prediction problem, the focus of this activity should be on whether students can observe patterns, and then design corresponding assumptions and models, as well as to analyse whether the modelling process is reasonable or not.

Pedagogical recommendations:

1. Question 1 shows the broken-line graph of the social media follower counts of Singer G. In Question 1(a), teachers can encourage students to share their observations regarding the trend. Hence, teachers can facilitate students' discussion on making assumptions of modelling and predicting Singer G's follower counts in December 2023 and January 2024 in Question 1(b). **The following are some possible discussion outcomes.**
 - (a) Observations: Singer G's follower counts would experience a significant increase after every three months of relatively small growth.
 - (b) Assumptions: The trend follows patterns observed in the past, and more recent data has higher importance (weight).

Consider:

$$\text{Change from Mar to Apr} = 1469 - 933 = 536$$

$$\text{Change from Jun to Jul} = 2289 - 1521 = 768$$

$$\text{Change from Sep to Oct} = 3089 - 2449 = 640$$

$$\text{Change from Oct to Nov} = 3185 - 3089 = 96$$

$$\text{Dec 2023: } 3185 + 96 = 3281$$

$$\text{Jan 2024: } 3281 + \frac{536 \times 1 + 768 \times 2 + 640 \times 3}{1 + 2 + 3} = 3946$$

2. Similar to Question 1, Question 2 shows the broken-line graph of the social media follower counts of Singer Q. In Question 2(a), teachers can encourage students to share their observations regarding the trend. Hence, teachers can facilitate students' discussion on making assumptions of modelling and predicting Singer Q's follower counts in December 2023 and January 2024 in Question 2(b). **The following are some possible discussion outcomes.**

(a) Observations: From Jan to Aug, Singer Q's follower counts show a downward trend but do not exhibit a clear pattern. However, starting from Aug, the follower counts decline following an exponential trend.

(b) Assumptions: Due to the lack of a clear pattern in the changes from Jan to Aug, we exclude these data when formulating the model. Then, we assume that the follower counts will continue to decrease according to the pattern observed after Aug.

Consider:

$$\text{Percentage change from Aug to Sep} = \frac{1571-1785}{1785} \times 100\% = -12.0\%$$

$$\text{Percentage change from Sep to Oct} = \frac{1382-1571}{1571} \times 100\% = -12.0\%$$

$$\text{Percentage change from Oct to Nov} = \frac{1216-1382}{1382} \times 100\% = -12.0\%$$

$$\text{Dec 2023: } 1216(1 - 12.0\%) = 1070$$

$$\text{Jan 2024: } 1216(1 - 12.0\%)^2 = 942$$

Concluding remarks:

Overall, students engaged with various modelling approaches which can be used to predict future trends. To conclude this activity, teachers can:

- summarise and compare these modelling approaches;
- remind students that some assumptions and limitations may be involved in a mathematical model; and
- share that different models may have their own strengths and weaknesses. We have to continuously evaluate and refine them by comparing with more real-world data.

Suggested lesson plans and teaching flow

Activities 1 and 2

- Teaching time: Approximately 80 minutes or a double lesson

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|-------------|--|---|--------------------------------------|
| 15 | <ul style="list-style-type: none"> To arouse students' interest To establish the context of investigating trends | <ol style="list-style-type: none"> Teachers arouse students' interest by discussing the real-world scenario. Teachers use Q1 to introduce students to the context of investigating trends in the social media follower counts, and meanwhile, encourages students' discussion on the factors influencing the counts. Teachers use Q2 as a warm-up exercise to allow students to review the concept of sequences of numbers. Teachers emphasise that in this series of activities, the primary governing factor focuses only on the previous numbers of followers. | <p>WS cover page</p> <p>WS1 Q1–2</p> |
| 10 | <ul style="list-style-type: none"> To investigate a linear trend | <ol style="list-style-type: none"> Teachers use Q1 to show a linear trend. Students discuss in groups the ways of making predictions. Students are tasked to calculate the changes in the follower counts between months in Q2. Teachers may ask students to input data to spreadsheets and then introduce the use of spreadsheets to calculate these changes. | <p>WS2 Q1–2</p> |

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|--------------------|---|--|---------------------------------------|
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 1 and checks their understanding through Q3(a). Students are tasked to predict the counts in two subsequent months in Q3(b), which facilitates them to derive Model 1 in terms of n in Q3(c). Students discuss in groups the strengths and weaknesses of Model 1. | <p>WS2 Q3(a)–(c)</p> <p>WS2 Q3(d)</p> |
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 2 and checks their understanding through Q4(a). Students are tasked to predict the counts in two subsequent months in Q4(b), which facilitates them to derive Model 2 in terms of n in Q4(c). Students discuss in groups the strengths and weaknesses of Model 2. | <p>WS2 Q4(a)–(c)</p> <p>WS2 Q4(d)</p> |
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 3 and checks their understanding through Q5(a). Students are tasked to predict the counts in two subsequent months in Q5(b), which facilitates them to derive Model 3 in terms of n in Q5(c). Students discuss in groups the strengths and weaknesses of Model 3. | <p>WS2 Q5(a)–(c)</p> <p>WS2 Q5(d)</p> |

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|----------------|---|---|-----------------------|
| 5 | <ul style="list-style-type: none"> To enhance abilities in making assumptions and identifying limitations in modelling | <ol style="list-style-type: none"> Students discuss in groups the common assumptions and limitations of these models. Teachers may ask the following questions to stimulate thinking: Are the changes in the follower counts necessarily consistent? Are the factors considered by these models comprehensive? | WS2 Q6 |
| 15 | <ul style="list-style-type: none"> To formulate a mathematical model | <ol style="list-style-type: none"> Teachers review Modelling approaches 1 and 2 from the perspective of “weight”. Teachers check students’ understanding by asking them to assign weights in the case of Modelling approach 3. Teachers introduce students to Modelling approach 4. Students discuss in groups the weights involved and their explanations in Q8(a). Students are tasked to predict the counts in two subsequent months in Q8(b), which facilitates them to derive Model 4 in terms of n in Q8(c). | WS2 Q7–8 |
| 5 | <ul style="list-style-type: none"> To summarise Activities 1 and 2 | <ol style="list-style-type: none"> Teachers summarise the modelling skills involved in Activities 1 and 2. | |

Activities 3 and 4

- Teaching time: Approximately 80 minutes or a double lesson

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|--------------------|---|---|--------------------------------|
| 10 | <ul style="list-style-type: none"> To investigate an exponential trend | <ol style="list-style-type: none"> Teachers use Q1 to show an exponential trend. Students discuss in groups whether it is suitable to use Models 1 to 3 in prediction. Students are tasked to calculate the percentage changes in the follower counts between months in Q2. Teachers may ask students to input data to spreadsheets and then introduce the use of spreadsheets to calculate these percentage changes. | WS3 Q1–2 |
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 5 and checks their understanding through Q3(a). Students are tasked to predict the counts in two subsequent months in Q3(b), which facilitates them to derive Model 5 in terms of n in Q3(c). Students discuss in groups the strengths and weaknesses of Model 5. | WS3 Q3(a)–(c) WS3 Q3(d) |

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|------------------------|---|--|---------------------------------------|
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 6 and checks their understanding through Q4(a). Students are tasked to predict the counts in two subsequent months in Q4(b), which facilitates them to derive Model 6 in terms of n in Q4(c). Students discuss in groups the strengths and weaknesses of Model 6. | <p>WS3 Q4(a)–(c)</p> <p>WS3 Q4(d)</p> |
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model To evaluate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 7 and checks their understanding through Q5(a). Students are tasked to predict the counts in two subsequent months in Q5(b), which facilitates them to derive Model 7 in terms of n in Q5(c). Students discuss in groups the strengths and weaknesses of Model 7. | <p>WS3 Q5(a)–(c)</p> <p>WS3 Q5(d)</p> |
| 10 | <ul style="list-style-type: none"> To formulate a mathematical model | <ol style="list-style-type: none"> Teachers introduce students to Modelling approach 8. Students discuss in groups the weights involved and their explanations in Q6(a). Students are tasked to predict the counts in two subsequent months in Q6(b), which facilitates them to derive Model 8 in terms of n in Q6(c). | WS3 Q6 |

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|--------------------|---|---|---------------------------|
| 5 | <ul style="list-style-type: none"> To enhance abilities in making assumptions and identifying limitations in modelling | <ol style="list-style-type: none"> Students discuss in groups the common assumptions and limitations of these models. Teachers may ask the following questions to stimulate thinking: Are the percentage changes in the follower counts necessarily consistent? Are the factors considered by these models comprehensive? | WS3 Q7 |
| 20 | <ul style="list-style-type: none"> To explore more complicated trends | <ol style="list-style-type: none"> Teachers use Q1 to show a non-linear trend and encourages students to share their observations. Teachers facilitate students' discussion on making assumptions of modelling and predicting the follower counts. Similarly, teachers use Q2 to show a more complicated trend and encourages students to share their observations. Teachers facilitate students' discussion on making assumptions of modelling and predicting the follower counts. | WS4 Q1 WS4 Q2 |

| Time (mins) | Objectives | Teaching activities and processes | Resources/ remarks |
|------------------------|---|--|-------------------------------|
| 5 | <ul style="list-style-type: none"> • To conclude the entire activity | <ol style="list-style-type: none"> 1. Teachers recall the concept of a mathematical model which can help us make predictions. 2. Teachers summarise and compares the modelling approaches in this series of activities. 3. Teachers remind students that some assumptions and limitations may be involved in a mathematical model. 4. Teachers further share that different models may have their own strengths and weaknesses. We have to continuously evaluate and refine them using more real-world data. | |