# Trilateration and Global Positioning System

**Key Stage**: 4

**Strand**: Measures, Shape and Space

**Learning Units**: Equations of Straight Lines and Equations of Circles

**Objective**: To apply mathematical knowledge to solve real-life problems

**Pre–requisite Knowledge**:
(i) understand equations of circles
(ii) find the coordinates of the intersection points of two circles
(iii) understand the cosine formula

**Relationship with other KLA(s) in STEM Education:**
The Global Positioning System could be a topic for the Investigative Study in the curriculum of the elective subject Physics in senior secondary.

**Background information:**
Global Positioning System (GPS) is a satellite-based navigation system made up of more than 20 satellites. Each satellite transmits a unique signal and orbital parameters that allow GPS devices, such as mobile phones, watches and cars, to compute the precise locations of the devices. A GPS receiver uses this information and trilateration to calculate the distances between the receiver and several satellites, and determines a user's position.

**Description of the activity:**

## Activity 1: Trilateration

Positioning using GPS involves the application of a mathematical principle called trilateration. The teacher can introduce trilateration through the following activity.

1. Students are required to solve the following problem:
   There are three base stations that can send and receive signals from your mobile phone *P*. Suppose that, in a rectangular coordinate system, the locations of the three base stations *A*, *B* and *C* are (0,0), (36,0) and (16,32) respectively (1 unit represents 1 km). It is found that the distance between *P* and the three base stations *A*, *B* and *C* are

29 km, 25 km and 13 km respectively. Assume that *A*, *B*, *C* and *P* lie on the same horizontal plane. Find the coordinates of the point *P*.

2. In order to find the coordinates of *P*, the teacher may guide students to draw the points *A*, *B*, *C* and two circles with centres *A* and *B* respectively using GeoGebra or graph paper. The radii of circles centered at *A* and *B* are 29 units and 25 units respectively (Figure 1). As the two circles intersect at two points $P_1$ and $P_2$, the third given condition is necessary to determine the exact location of *P*.

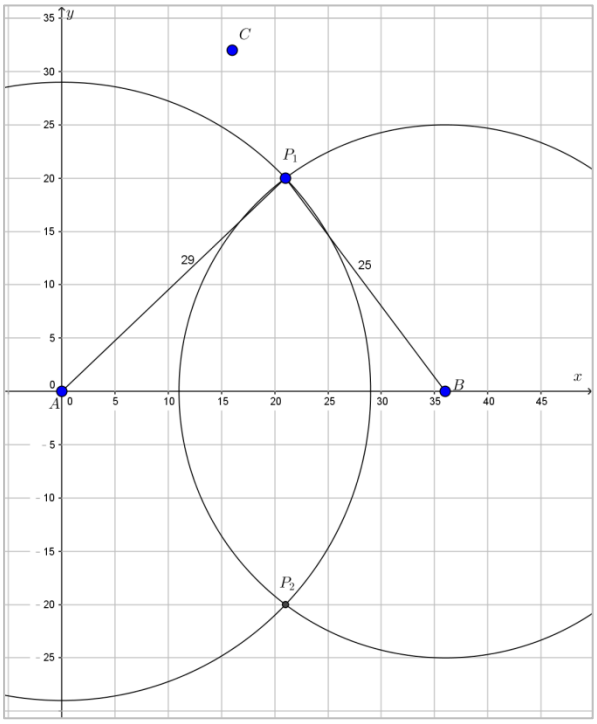


Figure 1

3. Once the third circle with centre *C* and radius 13 units is drawn, the location of *P* is determined, which is the intersection of three circles. Students can read from the graph that the approximate coordinates of *P* are (21,20) (See Figure 2).

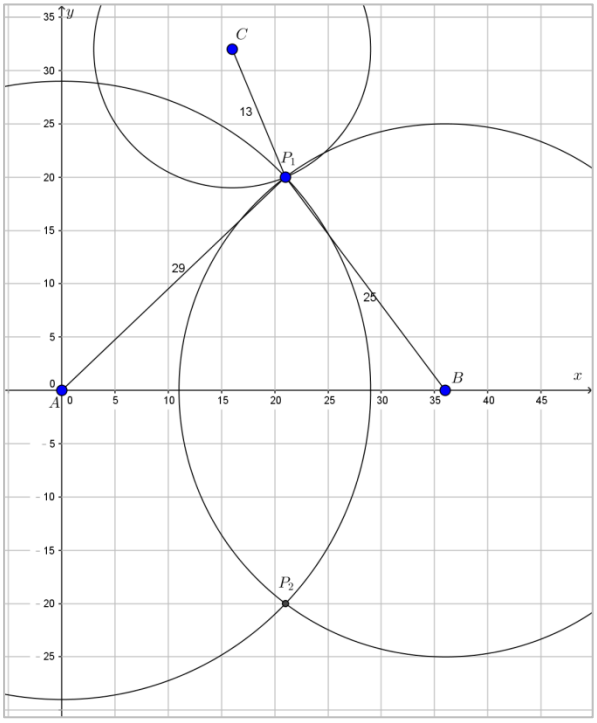


Figure 2

4. Apart from using graphical method, the teacher may guide students to use algebraic method to find the coordinates of *P*. Let the coordinates of *P* be (*x*,*y*). Three equations can be formed.

$$x^2 + y^2 = 29^2$$
$$(x - 36)^2 + y^2 = 25^2$$
$$(x - 16)^2 + (y - 32)^2 = 13^2$$

By solving the equations, $x = 21, y = 20$. The coordinates of *P* are (21,20).

**Notes for teachers:**

1. The teacher may explain to students that *A*, *B* and *C* in the problem represents GPS satellites in the real situation. The distance between a GPS satellite and a mobile phone is found by multiplying the signal transmission time and the speed of the signal, i.e. the speed of light.

2. The teacher should remind students that this is the simplified version of "Trilateration" used in GPS as we make use of circles to determine a location in a two-dimensional plane. In three-dimensional space, four spheres are needed to determine the location of a point in space. The teacher may refer to the following webpage for further explanations.
http://gisgeography.com/trilateration-triangulation-gps/

3. Students can watch the videos "How GPS Works 2003" and "How Do Global Positioning Systems Work? 2005" in the webpage: http://www.gps.gov/multimedia/videos/ to understand more about GPS and trilateration.

4. The teacher may discuss with students the widely use of GPS, including recording the running route, measuring the running speed, and finding the lost mobile phone, etc.

5. The teacher may use another example as below for illustration or as an activity:
If *A* = (0,0), *B* = (21,0), *C* = (9,15), *PA* = 13, *PB* = 20, *PC* = 5, the coordinates of *P*, after calculation, are (5,12). This example also shows that *P* may lie outside Δ*ABC*.

**Activity 2: Positioning by Trilateration**

1. This activity is carried out in the school basketball court and needs two volleyballs (or basketballs) and distance measuring tools.

2. The basketball court is labelled ABCD. The teacher asks students to measure the length and the width of the court. The teacher marks three points $A$, $P$ and $Q$ on the court. In order to simplify the situation, $A$ is chosen to be the origin whereas $P$ and $Q$ are the points on the sides opposite to A and near the mid-points of the sides of the court with coordinates $(p, r)$ and $(q, s)$ respectively. The values of $p$, $q$, $r$ and $s$ are found by measurements.

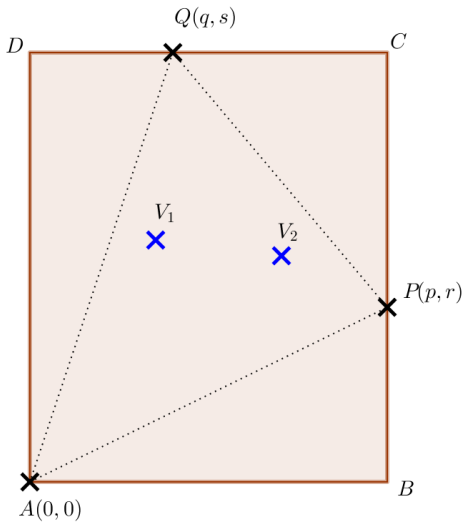


Figure 3

3. Two volleyballs $V_1$ and $V_2$ are placed inside the basketball court. (It would be better that the volleyballs are placed inside $\Delta APQ$.)

4. Students are required to find the distance between the two volleyballs without entering the basketball court. They can only measure the distance between $A$, $P$, $Q$ and the volleyballs.

**Notes for teachers:**

1. Students may use distance measure apps or laser measuring devices to measure the distance between two positions, for example $AB$, $AP$ and $AV_1$, etc.

2. In the real situation, there are errors in measurement, resulting in inconsistent simultaneous equations. To determine the coordinates of one volleyball, the teacher may guide students to solve the equations of circles centered at two of $A$, $P$ and $Q$ first. Afterwards, students could substitute the coordinates of the two interesting points calculated to the equation of the circle centered at the remaining point to determine which one is the correct position of the volleyball.

**Reference:**

1. http://electronics.howstuffworks.com/gadgets/travel/gps.htm
2. https://www.lelandwest.com/car-accessories-the-gps-device.cfm