Mathematical modelling on the accommodation demand of visitors to Hong Kong

Key stage: 3

Strand: Number and Algebra

Learning Unit: Using Percentages

Objective:
(i) To enrich students' experience in applying mathematics in handling daily life problems
(ii) To enhance students' abilities in applying the concepts of percentage in modelling real-life situations

Pre–requisite Knowledge:
(i) The concepts of percentage change and growth rate
(ii) Basic understanding in the collection and organisation of data

Relationship with other KLA(s):
The Learning Element "Business Environments, Operations & Organisations" under the Knowledge Context "Strategies & Management" at the Junior Secondary Level in the Technology Education (TE) KLA.

Resources Required:
(i) Internet resources on statistical data of local tourism industry [refer to Notes for teachers of the activities for suggested sources]
(ii) Tablet computers or smart phones with Internet connection

Scenario:
Tourism industry is a mainstay of Hong Kong's economy. In 2016, it accounted for 5% of Hong Kong's GDP and provided about 260 000 jobs, representing 7% of total employment in Hong Kong (HKSARG, 2018). The number of visitor arrivals rose consistently in recent years. In 2018, the total number of visitor arrivals reached a milestone of 65 million and over 29 million visitors stayed overnight in Hong Kong (Tourism Commission, Hong Kong, 2019). To meet the strong accommodation demands of tourists, reasonable estimation of the supply of hotel rooms and forecast of visitor arrivals in Hong Kong are very crucial for planning, quality assurance and resources allocation in tourism.

In this series of learning activities, mathematical models describing the number of visitor arrivals to Hong Kong and the hotel room supply under various factors and assumptions will be constructed. Students are expected to experience the process in constructing the mathematical models, such as (i) model formulation, (ii) model development, (iii) interpretation of solutions, (iv) model evaluation and (v) model refinements (Galbraith & Holton, 2018). It is a cyclical process in which real-life problems are mathematised, solved within a system of elements, operations and relationships, and then the model is checked back into the real-life situations (Verschaffel, Greer, & De Corte, 2002).

The following activities can be enriched by or connected with the content about business environments, decision making, planning, organisation, control and evaluation in business operations, which are included in the Knowledge Context "Strategies & Management" at the Junior Secondary Level in the TE KLA. Collaborations with TE teachers could be sought.

Description of the Activities:

Activity 1 (refer to Worksheet 1)

1. The teacher may arouse students' interest by asking them to estimate the number of visitor arrivals to Hong Kong in a whole year. Students should also be guided to think about the limitations and possible errors of their suggestions.

2. Teacher then gives a simplified scenario using annual percentage growth to familiarise students with the scenario and asks them to find the number of visitor arrivals next year under the given condition.

   The simplified scenario: Number of visitor arrivals this year = 3 000 000
   Annual growth rate = 2%

3. Mathematical modelling refers to using mathematical concepts and language to describe a real world situation, to test ideas and make estimations about the situation by mathematical computation and analysis. Teachers may explain to students that at a very basic and simplistic level, "mathematical model" has been used in the sense of a formula (Galbraith & Holton, 2018). To formulate a mathematical model to represent the annual number of visitor arrivals to Hong Kong, students are asked to use the data of 2012 and 2018 on the Internet to find the annual growth rate of the number of visitors.

4. By using the constructed mathematical model, students are requested to estimate the number of visitor arrivals in 2019 and 2020.

Questions for discussion:

1. What assumptions are made in your model?

2. Does your model fit the data in 2015, 2016 and 2017? If not, what are the reasons?

Notes for teachers:

1. Some suggestions on the ways for reasonable estimation of the number of visitor arrivals:
   (a) estimating the number of incoming flights or high-speed trains on a particular day, and multiplying it to the average capacity of the couriers for a rough estimate; or
   (b) search the number of visitors last year from the website of Immigration Department and take the same number for this year.

2. For the case of simplified scenario:
   Number of visitor arrivals this year = 3 000 000
   Annual growth rate = 2%
   Number of visitor arrivals next year = 3000000 (1 + 2%) = 3 060 000

3. For constructing the model, students may find the required data from the following website.
   Census and Statistics Department - Table E551: Visitor arrivals by country/region of residence
   https://www.censtatd.gov.hk/hkstat/sub/sp130.jsp?productCode=D5600551

   From the table in the above website, the number of visitor arrivals in 2012 and 2018 are 48 615 113 and 65 147 555 respectively.

   Let r% be the annual growth rate, we have
   $48615113\ (1 + r\%)^6 = 65147555$
   $r = 5.0$ (cor. to 1 d.p.)

   Formulate the mathematical model:
   V = the number of visitor arrivals at the 1st year

$r\%$ = the annual growth rate of visitors
n = the number of years after the 1st year
N_1 = the number of visitor arrivals at the nth year after the 1st year

$\therefore N_1 = V(1 + r\%)^n$
The model for Activity 1 is $N_1 = V(1 + 5\%)^n$.

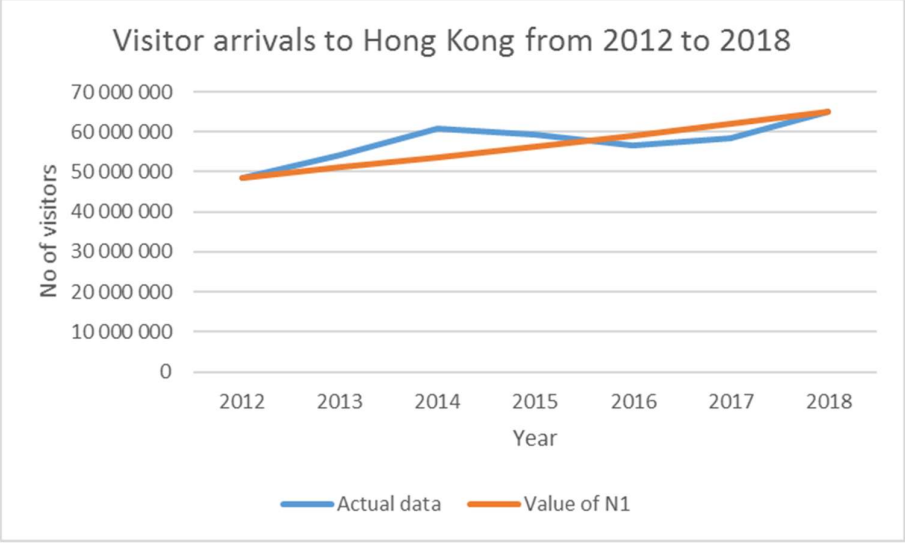
4. From the statistical information above, the total visitor arrivals in 2012 = 48615113

   The total visitor arrivals in 2019 = $48615113\ (1 + 5\%)^7$
   = 68406346 (cor. to the nearest integer)

   The total visitor arrivals in 2020 = $48615113\ (1 + 5\%)^8$
   = 71826663 (cor. to the nearest integer)

5. Assumptions:
   a) Visitor arrivals is growing at a compound annual growth rate.
   b) Annual growth rate between 2012 and 2018 and that beyond 2018 is a constant.

6. The model does not fit the data in 2015, 2016 and 2017. The number of visitors fluctuates from 2012 to 2018 and does not increase in compound growth rate in reality. In fact, making different assumptions and using different data set may devise different mathematical models.



7. Teacher may also discuss with students the pros and cons of using this simple model of constant annual growth rate. For example, the model is easy to construct without the needs of a large amount of different kinds of data. However, the model may have disadvantages of being oversimplified that it would not be able to reflect changing factors and fluctuations that affect the accuracy of short term forecast.

Activity 2 (refer to Worksheet 2)

1. The teacher askes students to formulate a mathematical model to represent the annual number of overnight visitor arrivals in Hong Kong by making use of the model in Activity 1.

2. The teacher may guide students to find the number of overnight visitor arrivals in 2018 and estimate the portion of them in the total number of visitor arrivals.

Question for discussion:

1. What assumptions are made in your model?

Notes for teachers:

1. Students may find the required data from the following website:
   Statistical Information of Visitor Arrivals to Hong Kong for 2017 and 2018
   https://partnernet.hktb.com/filemanager/intranet/pm/VisitorArrivalStatistics/ViS_Stat_E/VisE_2018/Tourism%20Statistics%2012%202018_R1.pdf

   From the above statistical data, the overnight visitor arrivals in 2018 = 29 262 701
   Portion of overnight visitor arrivals in 2018

   $= \frac{29262701}{65147555}$

   $\approx \frac{9}{20}$

   Formulate the mathematical model:
   V = the number of visitor arrivals at the 1st year
   $r\%$ = the annual growth rate of visitors
   N_1 = the number of visitor arrivals at the nth year after the 1st year
   N_2 = the number of overnight visitor arrivals at the nth year after the 1st year

From the above result, we have $N_2 = \frac{9}{20} N_1$

$\therefore \quad N_2 = \frac{9V(1+r\%)^n}{20}$

The model for Activity 2 is $N_2 = \frac{9V(1+5\%)^n}{20}$.

2. Assumptions:
   a) Visitor arrivals is growing at a compound growth rate.
   b) Annual growth rate between 2012 and 2018 and that beyond 2018 is a constant.
   c) The percentage of overnight visitor arrivals is a constant and the percentage in 2018 can be applied for every year.

Activity 3 (refer to Worksheet 3)

1. The teacher asks students to formulate a mathematical model to represent the annual number of hotel rooms provided in Hong Kong.

2. By using the constructed model, students are asked to estimate the number of hotel rooms in 2019 and 2020.

Questions for discussion:

1. What information is needed for constructing the model?

2. What assumptions are made in your model?

Notes for teachers:

1. Students are expected to be aware that the number of hotel rooms in Hong Kong in 2012 and 2018 are needed for constructing the model. Moreover, the rooms provided by guesthouses and other kinds of accommodation places are excluded in this model.

2. Students may find the required data from the following website:
   Census and Statistics Department : Hong Kong Monthly Digest of Statistics
   (February 2013, p. 240 and February 2019, Section 8-11, Table T10-14)
   https://www.censtatd.gov.hk/hkstat/sub/sp130_tc.jsp?productCode=B1010002

From the tables in the above website, we have
the estimated number of hotel rooms in 2012 = 67 394, and
the estimated number of hotel rooms in 2018 = 81 465

Let $R\%$ be the annual growth rate, we have
$67394\ (1 + R\%)^6 = 81465$
$R = 3.2$ (cor. to 1 d.p.)

3. Formulate the mathematical model:
   H = the number of hotel rooms at the 1st year
   $R\%$ = the annual growth rate of hotel rooms
   N_3 = the number of hotel rooms at the nth year after the 1st year

   Similar to the model in Activity 1,
   $N_3 = H(1 + R\%)^n$
   The model for Activity 3 is $N_3 = H(1 + 3.2\%)^n$.

4. Assumptions
   a) The number of hotel rooms is growing at a compound growth rate.
   b) Annual growth rate between 2012 and 2018 and that beyond 2018 is a constant.

5. The estimated number of hotel rooms in 2019 $= 67394\ (1 + 3.2\%)^7$
   $= 84019$ (cor. to the nearest integer)
   The estimated number of hotel rooms in 2020 $= 67394\ (1 + 3.2\%)^8$
   $= 86708$ (cor. to the nearest integer)

Activity 4 (refer to Worksheet 4)

1. The teacher may discuss with students what information is needed if we want to determine whether the total capacity of hotels and guesthouses is adequate for the accommodation demands of overnight visitors in 2022.

2. Making use of the models constructed in Activity 2 and 3 (i.e. N_2 and N_3), the teacher may guide students to construct two models to represent the average daily number of overnight visitor arrivals and the total capacity of hotels and guesthouses at the nth year after a particular year.

Questions for discussion:

1. What assumptions are made in your model?

2. When will the room supply be inadequate if all growth rates are unchanged?

3. How about the situation if each room can only accommodate 2 people per night on average? Is the room supply still adequate?

4. Which assumptions may be vulnerable to hold in real-life scenarios? Why? How can it be refined so as to improve the accuracy of the model?

Notes for teachers:

1. The data needed include (i) the number of rooms provided by hotels and guesthouses, (ii) the average number of people accommodated by each room per night, (iii) the number of overnight visitor arrivals and the average length of stay in Hong Kong, and (iv) the annual growth rates of visitors and rooms respectively. The places for accommodation other than hotels and guesthouses are excluded in this model.

2. Students may find the required data from the following websites:
   (i) Annual Reports of the Hong Kong Tourism Board
   http://www.discoverhongkong.com/tc/about-hktb/annual-report/index.jsp
   (ii) Tourism Commission
   https://www.tourism.gov.hk/tc_chi/statistics/statistics_perform.html
   (iii) Census and Statistics Department: Hong Kong Monthly Digest of Statistics (February 2019, Section 8-11, Table T10.14)
   https://www.censtatd.gov.hk/hkstat/sub/sp130_tc.jsp?productCode=B1010002
   From the above websites,
   a) the average length of stay among overnight visitor arrivals was around 3.2 nights
   b) the number of rooms provided by guesthouses in 2018 was 12526

3. Formulate the mathematical model:
   Variables for N_2
   V = the number of visitor arrivals at the 1st year
   r% = the annual growth rate of visitors
   N_2 = the number of overnight visitor arrivals at the nth year after the 1st year
   Variables for N_3* (with guesthouses)
   H* = the number of rooms provided at the 1st year (include hotels and guesthouses)
   R% = the annual growth rate of rooms

N_3* = the number of rooms provided by hotels and guesthouses at the nth year after the 1st year

New variables

A = the average number of people accommodated by each room per night

a = the average length of stay of overnight visitors (in number of nights)

N_4 = the average daily number of overnight visitors n years after the 1st year

C = the total capacity of hotels and guesthouses n years after the 1st year (in number of people)

We have $N_4 = \dfrac{a \cdot N_2}{365}$ and $C = AN_3$*

$$\therefore \quad N_4 = \frac{9aV\left(1+r\%\right)^n}{7300}$$

$$C = AH^*\left(1+R\%\right)^n$$

From Activity 2 and 3, we have r = 5.0 and R = 3.2 .

In Activity 4, we have a = 3.2 (see 2(a) above) and A = 3 (see 4(a) below). Therefore

$$N_4 = \frac{36V\left(1+5\%\right)^n}{9125}$$

$$C = 3H^*\left(1+3.2\%\right)^n$$

4. Assumptions:
   a) Each room can accommodate 3 people per night on average*.
   b) There are 365 days in one year.
   c) The average length of stay of overnight visitor arrivals are unchanged over the years.
   d) Annual growth rate for the rooms provided by guesthouses is also equal to 3.2%.
   e) There are roughly the same number of visitors visiting Hong Kong each day throughout the year.

[*Remark: The assumption in 4(a) is not directly come from the statistical data but based on the estimation from other sources of information. The following shows the hotel and guesthouse statistics of Hong Kong in 2018:

Room occupancy rate for hotels = 91%

Room occupancy rate for guesthouses = 84%

Number of overnight visitors = 29262701

Average length of stay of overnight visitors = 3.1 nights

Total number of rooms provided by hotels = 81465 rooms

Total number of rooms provided by guesthouses = 12526 rooms

Therefore, the estimated number of people accommodated by each room per night

$$= \frac{29262701 \times 3.1}{365\,(81465 \times 0.91 + 12526 \times 0.84)} \approx 3$$

We assume it is unchanged for 2018 onwards.]

5. From Activity 1, the number of visitors in 2018 is 65 147 555.

   Average daily number of overnight visitors in 2022

   $$= \frac{36 \times 65147555\ (1+5\%)^4}{9125}$$

   = 312 410 (cor. to the nearest integer)

   Total capacity of hotels and guesthouses at 2022

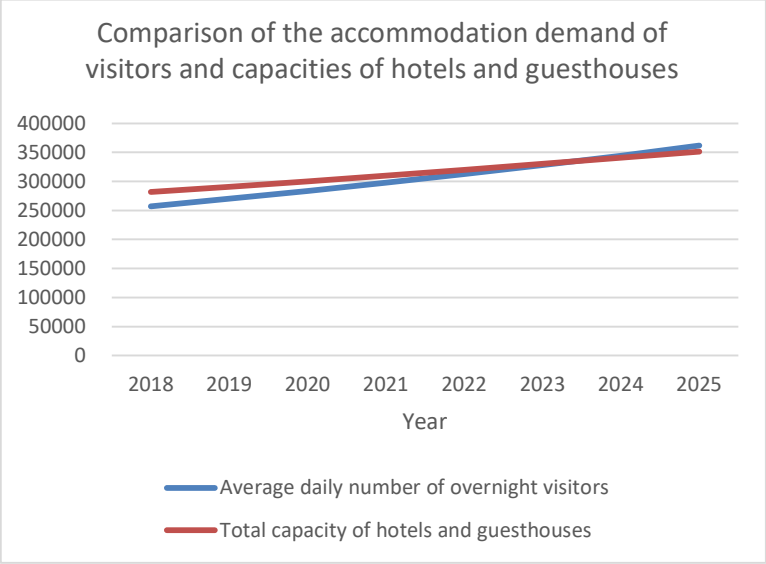
   = 3 (81465 + 12526)(1 + 3.2%)^4

   = 319 835 (cor. to the nearest integer)

   Therefore, the room supply is adequate for the accommodation demand of visitors in 2022.

6. If each room can only accommodate 2 visitors per night on average, the room supply is inadequate.

7. To determine when will the supply be inadequate if all growth rates are unchanged, students may use a spreadsheet such as the Excel to explore the number of years needed.



The following is for teachers' reference only.

$$\frac{36\times 65147555\ (1+5\%)^n}{9125} > 3\times 93991(1+3.2\%)^n$$

$$\left(\frac{1.05}{1.032}\right)^n > 1.097$$

$$n\log\left(\frac{1.05}{1.032}\right) > \log(1.097)$$

$$n > 5.358$$

Therefore, the supply is inadequate at 2024 if all growth rates are unchanged. Teacher may discuss with students how likely this situation will happen in reality. Model evaluation and refinement are encouraged by considering different assumptions and factors.

8. Teacher may discuss the limitations of assumptions made in constructing the model. For example, we use the average daily of overnight visitors to estimate the adequacy of room supply and we assume that there are roughly the same number of visitors visiting Hong Kong each day throughout the year. In reality, the tourism is characterised by seasonality which affects the occupancy rate of hotel rooms. The estimation of room supply is sufficient on average but it may be inadequate in peak reason. Moreover, the accommodation places other than hotels and guesthouses are also not considered in the model. For example, visitors may live in relatives' home.

Reference:

Galbraith, P., & Holton, D. (2018). *Mathematical Modelling: A Guidebook for Teachers and Teams*. Australian Council for Educational Research. Retrieved from https://www.immchallenge.org.au/files/IM2C-Teacher-and-student-guide-to-mathematical-modelling.pdf (accessed 2 September 2019).

The Government of the Hong Kong Special Administrative Region (HKSARG) (May, 2018). *Hong Kong: The Facts – Tourism*. Retrieved from https://www.gov.hk/en/about/abouthk/factsheets/docs/tourism.pdf (accessed 2 September 2019).

Tourism Commission, Hong Kong (August 2019). *Tourism Statistics*: *Tourism Performance in 2018*. Retrieved from https://www.tourism.gov.hk/english/statistics/statistics_perform.html (accessed 2 September 2019).

Verschaffel, L., Greer, B. & De Corte, E. (2002). *Everyday knowledge and mathematical modeling of school word problems*. In K. Gravemeijer, R., Lehrer, B., Oers, B., van and L. Verschaffel (Eds.), Symbolizing, modeling and tool use in mathematics education (pp. 257–276). Dordrecht, Netherlands: Springer.

Mathematical modelling for the accommodation of visitors in Hong Kong

Worksheet 1

Activity 1

Warm up activity

1. Please suggest some ways to estimate the number of visitor arrivals to Hong Kong in a whole year. Are there any limitations and possible errors of your suggestions?

_____

_____

2. You are given a simplified scenario below. Try to find the number of visitor arrivals next year.

   Number of visitor arrivals this year = 3 000 000

   Annual growth rate = 2%

_____

_____

To formulate a mathematical model to represent the number of visitor arrivals at the *n*th years after a particular year by assuming constant growth rate

- The following questions will guide you to build the mathematical model. Mathematics modelling refers to using mathematical language (e.g. a formula) to describe a real world situation for making estimation and prediction, solving problems, etc.
- For the above task, we may use the data from 2012 to 2018 for constructing the model. Try to find the related information on the Internet. The following is a useful website.

Census and Statistics Department - Table E551: Visitor arrivals by country/region of residence

https://www.censtatd.gov.hk/hkstat/sub/sp130.jsp?productCode=D5600551

1. a) What data are needed for constructing the model? (e.g. number of visitor arrivals in 2012)

_____

_____

1. b) Use letters to represent the variables that will be needed to model the number of visitor arrivals at the nth year after the first year (N_1), where $n = 0, 1, 2, \ldots$.

Let n be the number of years after the 1st year (i.e. $n = 0$ for the first year);
N_1 be the number of visitor arrivals at the nth year after the 1st year;
V be the number of visitor arrivals at the first year;
r% be the annual growth rate of visitor arrivals.

2. Use the number of visitor arrivals in 2012 and 2018 to estimate the annual growth rate.

_____

_____

3. Formulate a mathematical model to represent the number of visitor arrivals at the nth year after the first year (N_1), i.e. write a formula for N_1.

_____

_____

4. By taking 2012 as the first year, use your model to estimate the number of visitor arrivals in 2019 and 2020.

_____

_____

5. What assumptions are made in your model?

_____

_____

_____

6. Does your model fit the data in 2015, 2016 and 2017? If not, what might the reasons be?

_____

_____

_____

Mathematical modelling for the accommodation of visitors in Hong Kong

Worksheet 2

Activity 2

To formulate a mathematical model to represent the total number of overnight visitors at the *n*th year after a particular year by using the model in Activity 1.

We may use the data of 2018 for constructing the model. Try to find the related information on the Internet. You may find the required information from the following website:

Statistical Information of Visitor Arrivals to Hong Kong for 2017 and 2018

https://partnernet.hktb.com/filemanager/intranet/pm/VisitorArrivalStatistics/ViS_Stat_E/VisE_2018/Tourism%20Statistics%2012%202018_R1.pdf

1. What is the number of overnight visitor arrivals in 2018? Estimate the portion of them in the total number of visitor arrivals in 2018.

_____

_____

_____

2. Let N_2 be the total number of overnight visitors at the nth year after the 1st year. By using the model in Activity 1 and the result of question 1, write down the model for N_2.

_____

_____

_____

_____

3. What assumptions are made in your model?

_____

_____

_____

Mathematical modelling for the accommodation of visitors in Hong Kong

Worksheet 3

Activity 3

To formulate a mathematical model to represent the number of hotel rooms provided at the *n*th year after a particular year by considering growth rate.

1. We may use the data from 2012 to 2018 for constructing the model. What data are needed for constructing the model? Write down the corresponding variables for the data. The idea behind is similar to the model in Activity 1.

Let N_3 be the number of hotel rooms at the nth year after the 1st year;

_____

_____

You may find the required data from the following website:
Census and Statistics Department : Hong Kong Monthly Digest of Statistics (February 2013, p. 240 and February 2019, Section 8-11, Table T10-14)
https://www.censtatd.gov.hk/hkstat/sub/sp130_tc.jsp?productCode=B1010002

2. Formulate a mathematical model for N_3 .

_____

_____

_____

_____

_____

3. What assumptions are made in your model?

_____

_____

4. By taking 2012 as the first year, use your model to estimate the number of hotel rooms in 2019 and 2020.

_____

_____

Mathematical modelling for the accommodation of visitors in Hong Kong

Worksheet 4

Activity 4

Task: We want to determine whether the capacities of local hotels and guesthouses is adequate for the accommodation demands of overnight visitors in 2022. Thus we are going to compare (i) the capacity of hotels and guesthouses and (ii) the daily average number of visitors need to stay overnight in 2022 in this activity, based on the results in Activity 2 and 3.

To formulate mathematical models to represent the total capacity of hotels and guesthouses at the *n*th year after a particular year and the average daily number of overnight visitors at the *n*th year after a particular year.

1. What data are needed if we want to determine the two targeted numbers (e.g. the average number of nights of stay of overnight visitors in Hong Kong)? Write down the corresponding variables.

Let N_4 be the average daily number of overnight visitors at the nth year after the 1st year;

C be the total capacity (no. of people) of hotels and guesthouses at the nth year after the 1st year;

_____

_____

_____

You may find the required data from the following websites:

(i) *Annual Reports of the Hong Kong Tourism Board*
http://www.discoverhongkong.com/tc/about-hktb/annual-report/index.jsp

(ii) *Tourism Commission*
https://www.tourism.gov.hk/tc_chi/statistics/statistics_perform.html

(iii) *Census and Statistics Department : Hong Kong Monthly Digest of Statistics (February 2019, Section 8-11, Table T10-14)*
https://www.censtatd.gov.hk/hkstat/sub/sp130_tc.jsp?productCode=B1010002

2. From the above websites, find

(i) the average length of stay (no. of nights) among overnight visitor arrivals, and

(ii) the number of rooms provided by guesthouses in 2018.

3. Making use of the models constructed in Activity 2 and 3 (i.e. N_2 and N_3), formulate the models to represent N_4 and C.

4. What assumptions are made in your model?

5. By taking 2018 as the first year, use your model to estimate

(i) the average daily number of overnight visitors at 2022, and

(ii) the total capacity of hotels and guesthouses at 2022.

Is the room supply adequate for the accommodation demands of visitors?

6. When will the room supply is inadequate if all growth rates are unchanged? (You may use a spreadsheet such as the Excel to explore the no. of years needed)

_____

_____

_____

_____

7. Regarding Q5 above, what if each room can accommodate 2 people per night on average only?

_____

_____

_____

Evaluate the model

Which assumptions may be vulnerable to hold in real-life scenarios? Why? How can it be refined so as to improve the accuracy of the model?

_____

_____

_____

_____