Investigating Errors in Measurements of Distance Measuring Apps

Key Stage: 3

Mathematics Education
Strand: Measures, Shape and Space
(Learning Units: Errors in measurement and Trigonometry)

Technology Education
Knowledge contexts: Information and Communication Technology
(Module: Programming Concepts)

Objectives:
(i) To consolidate the concept of errors in measurement
(ii) To apply mathematical knowledge and computational thinking to design useful apps

Prerequisite Knowledge:
(i) Finding percentage errors of measurements
(ii) Using trigonometry to solve for unknown sides and angles in a right-angled triangle

Resources Required: Tablet computers or smart phones with distance measuring apps installed (The apps should make use of the principle described in this example, e.g. Smart Measure)

Description of the Activities:

Activity 1

1. The teacher guides students to investigate how the app calculates the distance between the user and an object.

2. Each student holds the tablet computer or smart phone at his/her eye level and aims the cross mark in the screen at the base of an object in the surroundings (e.g. the classroom door). The app will show the distance between the user and the object. (See Figure 1a and b)

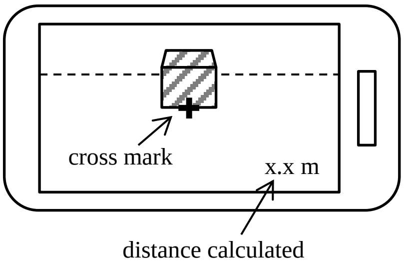

Figure 1a

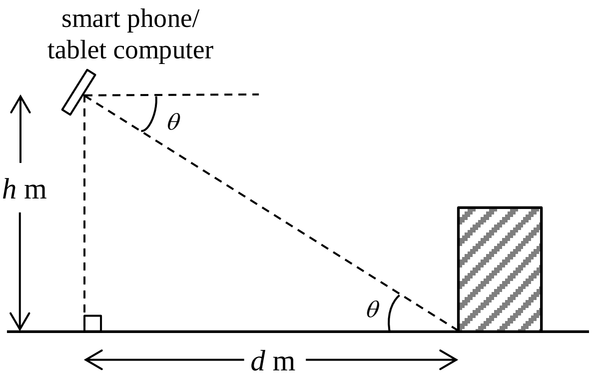

Figure 1b

In Figure 1b:

θ : the angle of depression of the base of the object from the smart phone/tablet computer

h : height of the device from the horizontal ground (usually assumed to be the eye level of the user)

d : distance of object from the device

3. Students are provided with the above figures and they are required to answer the following questions.

   a. By using trigonometry, write down the formula for the distance d in terms of h and θ.

   b. Should the object and the user of the app be situated at the same horizontal level?

   c. Suggest some sources of error in the measurement.

Notes for Teachers:

1. Suggested answers for the above questions.

   a. $d = \dfrac{h}{\tan\theta}$

   b. The object and the user of the app should be situated at the same horizontal level or otherwise h is not the vertical distance between the bottom of the object and the smart phone.

   c. Error in the distance calculated may due to the error in angle of depression resulted from inaccurate positioning of the cross mark in the screen (caused by hand-held shaking) or an inaccurate senor of the smart phone, and the error in eye level resulted from an incorrect input by the user.

2. The teacher should install the required distance measuring app in the tablet computers/smart phones before the lesson.

Activity 2

1. The teacher guides students to study how the error in the angle of depression measured by the digital device caused by hand-held shaking affects the distance calculated by the app. Assume that h is accurate and the error in θ caused by hand-held shaking is the same for all θ, we may study the error in distance by considering the total deviation in distance resulting from a +0.5° and –0.5° error in the angle of depression. Students are required to investigate this distance deviation, denoted by $L_{\pm 0.5°}$, for different angles of depression (see Figure 2).

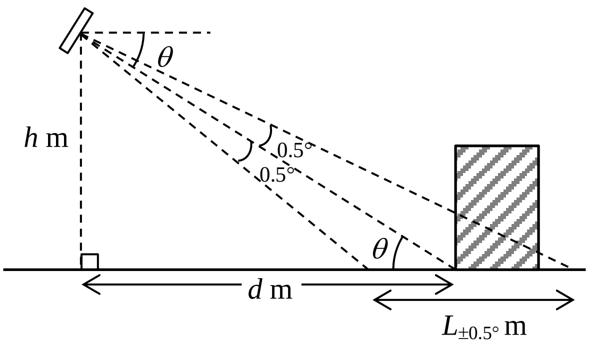

Figure 2

2. The teacher guides students to express $L_{\pm 0.5°}$ in terms of the angle of depression (θ) and the eye level (h).

3. The teacher can introduce $\dfrac{L_{\pm 0.5°}}{d}$ in percentage to students, which takes a concept similar to the relative error, to show how significant the deviation in distance is when compared with the distance calculated for different θ.

4. Students can use spreadsheet software to find $L_{\pm 0.5°}$ and the corresponding percentage of deviation for different values of θ to facilitate their investigation.

5. Students are required to set up a spreadsheet as below. Assume eye level of the user of the app to be 1.5 m, the students are required to set up formulae to calculate $L_{\pm 0.5°}$ and the percentage of deviation by the spreadsheet.

θ	*tan* θ	*tan* $(\theta + 0.5°)$	*tan* $(\theta - 0.5°)$	$L_{\pm 0.5°}$	Percentage of deviation
5					
10					
15					
⋮		⋮		⋮	
85					

6. Questions for discussion:

   a. Is the percentage of deviation the same for all values of θ? At which range of θ is the app less accurate?

   b. What is the relation between the percentage of deviation and h ?

Notes for teachers:

1. The value $L_{\pm 0.5°}$ is just for studying how errors in θ affect the calculated value of d and it is not a measure of the error in d .

2. $L_{\pm 0.5°} = \dfrac{h}{\tan(\theta - 0.5°)} - \dfrac{h}{\tan(\theta + 0.5°)}$ and

   Percentage of deviation $= \dfrac{L_{\pm 0.5°}}{\dfrac{h}{\tan\theta}} = \dfrac{\tan\theta}{\tan(\theta - 0.5°)} - \dfrac{\tan\theta}{\tan(\theta + 0.5°)}$.

3. The unit of angles is usually radian for spreadsheet software. As students have not learnt the conversion between degree and radian, the teacher should provide students with the formula for finding $\tan\theta$. Also, teacher may provide the following formulae for students who find it too difficult to set up the spreadsheet.

Values	Corresponding formulae	Remark
tan θ	=TAN(A2*PI()/180)	A2: θ
tan $(\theta + 0.5°)$	=TAN((A2+0.5)*PI()/180)	
tan $(\theta - 0.5°)$	=TAN((A2-0.5)*PI()/180)	
$L_{\pm 0.5°}$	=(G2/D2)-(G2/C2)	C2: tan $(\theta + 0.5°)$ D2: tan $(\theta - 0.5°)$ G2: h
Percentage of deviation	=E2*B2/G2	B2: $\tan\theta$ E2: $L_{\pm 0.5°}$ G2: h

4. For eye level of 1.5 m, the values of $L_{\pm 0.5^\circ}$ for different θ and the corresponding percentage of deviation are as follows:

	A	B	C	D	E	F	G
1	θ°	$\tan\,\theta$	$\tan\,(\theta + 0.5^\circ)$	$\tan\,(\theta - 0.5^\circ)$	$L_{\pm 0.5^\circ}$	percentage of deviation	eye level
2	5	0.087	0.096	0.079	3.481	20.3%	1.5
3	10	0.176	0.185	0.167	0.870	10.2%	1.5
4	15	0.268	0.277	0.259	0.391	7.0%	1.5
5	20	0.364	0.374	0.354	0.224	5.4%	1.5
6	25	0.466	0.477	0.456	0.147	4.6%	1.5
7	30	0.577	0.589	0.566	0.105	4.0%	1.5
8	35	0.700	0.713	0.687	0.080	3.7%	1.5
9	40	0.839	0.854	0.824	0.063	3.5%	1.5
10	45	1.000	1.018	0.983	0.052	3.5%	1.5
11	50	1.192	1.213	1.171	0.045	3.5%	1.5
12	55	1.428	1.455	1.402	0.039	3.7%	1.5
13	60	1.732	1.767	1.698	0.035	4.0%	1.5
14	65	2.145	2.194	2.097	0.032	4.6%	1.5
15	70	2.747	2.824	2.675	0.030	5.4%	1.5
16	75	3.732	3.867	3.606	0.028	7.0%	1.5
17	80	5.671	5.976	5.396	0.027	10.2%	1.5
18	85	11.430	12.706	10.385	0.026	20.1%	1.5

5. Suggested answers of the questions for discussion.

   a. The percentages of deviation for angles closer to 0° and 90° are greater. So the app is less accurate when measuring distance of objects that are away from or close to the user.

   b. The percentage of deviation is independent of *h*.

<u>Activity 3</u>

1. If students are interested, the teacher may ask students to investigate further:
   a. Students may explore how the error in eye-level (*h*) affects the distance calculated.

   b. Students may verify the conclusions about the relation between percentage of deviation and angle of depression drawn in Activity 2 by measuring objects placed at some preset distance, e.g. 0.3m, 1.5m and 5m.

2. (Challenge) Students may create a distance measuring app showing the percentage of deviation for users' reference.

Notes for Teachers:

1. The teacher may ask students to explore in groups.

2. The teacher should allow ample opportunities for students to discuss and draw a conclusion by themselves instead of giving them straightforward hints.

3. The teacher may explain more about the relation between error in d and error in h, i.e. assume that θ is accurate, the percentage error in d equals the percentage error in h, as for any particular measurement with angle of depression θ,

$$\frac{\Delta d}{d} = \frac{\frac{h+\Delta h}{\tan\theta} - \frac{h}{\tan\theta}}{\frac{h}{\tan\theta}} = \frac{\Delta h}{h}$$

, where Δd and Δh means errors in d and h respectively.

4. Measuring apps are widely used nowadays. Teachers could encourage students to find out and investigate the principles of other measuring apps from the Internet.

This example mainly involves the following generic skills:

1. Communication Skills
   - Understand, analyse and respond to teacher's spoken instructions and instructions on worksheets
   - Use appropriate language and mathematical expressions to present the methods and results of calculations

2. Critical Thinking Skills
   - Understand the limitations of distance measuring apps
   - Draw logical conclusions based on adequate data and evidence
   - By comparing the percentage of deviation for different angles of depression, comment on the performance of the distance measuring app for different angles of depression.

3. Information Technology Skills
   - Use distance measuring apps to carry out exploratory activities