

Linear Programming and Food Intake

- Key Stage:** 4
- Strand:** Number and Algebra
- Learning Units:** Linear Programming
- Objective:** To apply linear programming to solve real-life problems
- Pre-requisite Knowledge:** (i) Understanding linear inequality in two unknowns
(ii) Representing the graphs of linear inequalities in two unknowns on a plane
(iii) Solving systems of linear inequalities in two unknowns

Related Topics in other KLA(s) in STEM Education:

- (i) The topic “Nutrition in humans” under the learning content “Essential life processes in animals” in the Biology curriculum (Secondary 4 - 6)
- (ii) The topic “Nutrition and health” in Unit 12 “A healthy body” of Science at junior secondary

Description of the Activity:

The basic nutrients of food include carbohydrate, protein, fat, vitamins, etc. The body needs to obtain appropriate nutrients from food to maintain the normal functions of cells and organs, and to promote growth and development.

If the amounts of different nutrients of some food products are known, how should we select the food to meet the nutrient restrictions proposed by the government and experts and at the same time, minimise the expenditure in attaining this target?

Activity 1

1. The teacher distributes the information sheet to students. The information sheet contains the nutrition descriptions, weight and prices of 10 popular types of food, mostly obtained from their nutrition labels (營養標籤). Each nutrition label contains at least 6 common information as shown:

Energy (kcal)	Protein (g)	Sugar (g)
Carbohydrate (g)	Fat (g)	Sodium (mg)

The information sheet also contains the daily lower or upper limits of different nutrients suggested by the World Health Organization and the Department of Health of HKSARG.

2. The teacher then ask students to rank the 10 types of food from 1 – 10 (1 denotes the most favourite while 10 the least, etc.) and mark the numbers on the information sheet, above the names of the food products.
3. Referring to the given limits on nutrients as shown in the information sheet, students are required to find the amount of their two most favourite food (i.e. 1 and 2) to fulfill the following requirements:
 - a) If you are required to fulfill the **minimum energy intake** and **minimum protein intake**, find
 - i. the minimum cost incurred so as to meet this nutrition requirement,
 - ii. the corresponding quantity of each type of food consumed.
 - b) If you are required to fulfill the **minimum energy intake** and not to exceed the **maximum sodium intake**, find
 - i. the minimum cost incurred so as to meet this nutrition requirement,
 - ii. the corresponding quantity of each type of food consumed.
 - c) If you are required to fulfill the **minimum protein intake** and not to exceed the **maximum sodium intake**, find
 - i. the minimum cost incurred so as to meet this nutrition requirement,
 - ii. the corresponding quantity of each type of food consumed.

(Notes: Students are expected to tackle each of the above three problems by Linear Programming.)

Notes for Teachers:

1. Sample solution for 3(a):

Suppose the student selects Beef Hamburger (218 g, each piece \$ 12) and Malted Soya Drink (250 ml, each box \$ 5). Let x be the serving quantity of Beef Hamburger and y be the service quantity of Malted Soya Milk, while \$ C be the total amount of the food taken. As the amount taken in total needs to meet the minimum energy intake of 2,400 kcal and the minimum protein intake of 60 grams, we have

Objective Function: $C = 12x + 5y$ (corresponds to lines $y = -\frac{12}{5}x + \frac{C}{5}$)

Constraints: $512x + 128y \geq 2,400$ i.e. $16x + 4y \geq 75$ (Energy) ----- (1)

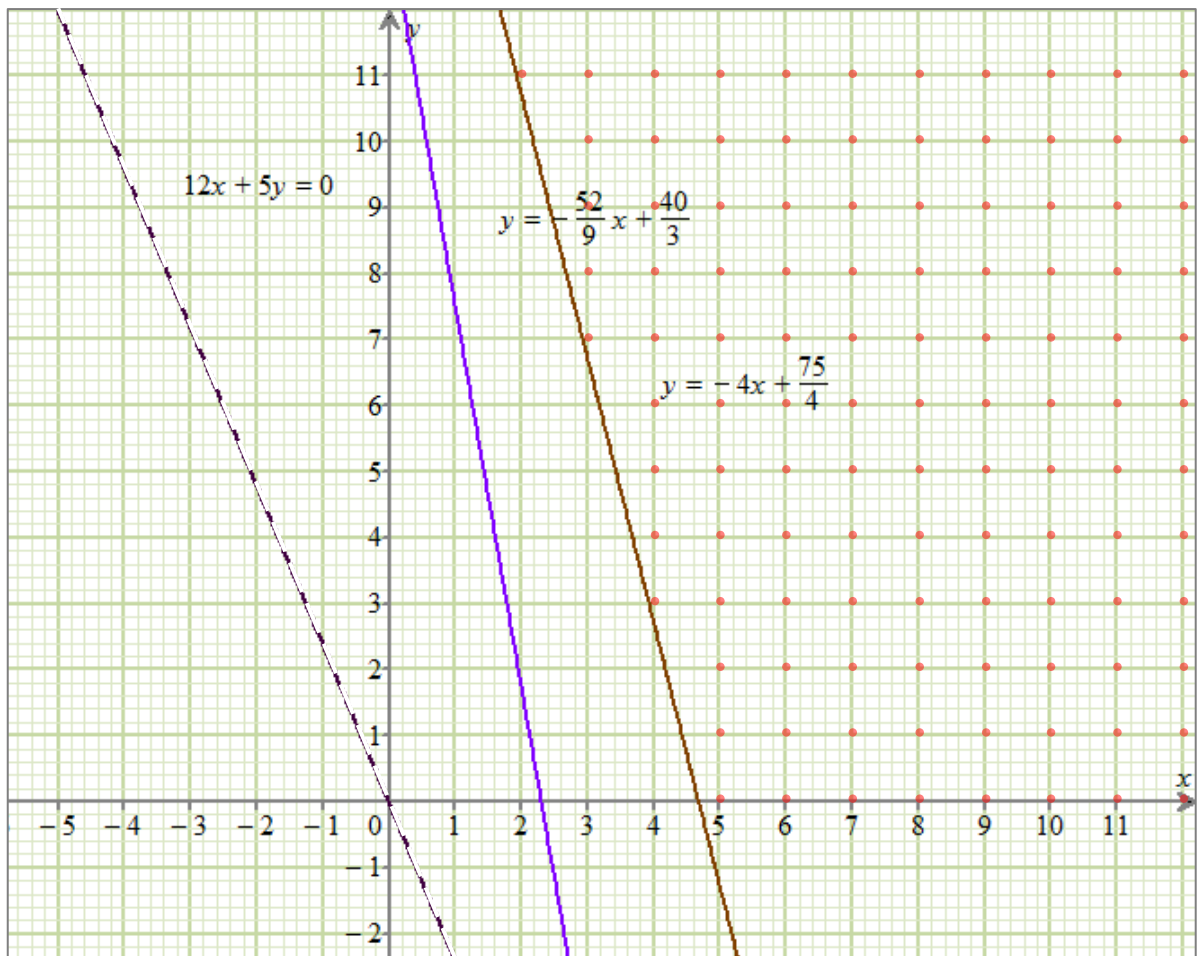
$26x + 4.5y \geq 60$ i.e. $52x + 9y \geq 120$ (Protein) ----- (2)

$x \geq 0, y \geq 0, x$ and y are non-negative integers. ----- (3)

Rewrite $16x + 4y \geq 75$ as $y \geq -4x + \frac{75}{4}$ and

$52x + 9y \geq 120$ as $y \geq -\frac{52}{9}x + \frac{40}{3}$.

The corresponding boundaries and line of objective function are plotted on the graph paper. The solution region is shown by the lattice points below.



Objective Function Protein Energy

By translating the line of objective function $12x + 5y = 0$ towards the right, the first lattice point interested is $(5, 0)$, which give rise to the minimum C among the lattice points. It means that **5 Beef Hamburgers and 0 boxes of Malted Soya Drink can meet the nutrition requirements in the minimum cost.**

The corresponding **Minimum Cost** = \$ [$12(5) + 5(0)$] = **\$ 60**

2. Instead of selecting food items among the given ten, students can freely choose other food products and collect their information from nutrient labels.
3. When more limits on nutrients are considered, there will be more constraints and the process of linear programming for finding the minimum cost and corresponding quantity of food will become more complicated.

Activity 2 (Extension)

1. The teacher may guide students to further investigate the problem if more than 2 types of food are chosen.
2. The teacher may explain that in real-life situation, we would not limit ourselves to consider two types of food only. But linear programming in Mathematics curriculum can manipulate two variables only.
3. The teacher can guide students to use an add-in for Microsoft Excel called Solver to tackle the optimisation problems with more than two variables. Procedures in using the Excel Solver are provided in the Annex.

Notes for teachers:

1. Teachers may assign students with questions similar to the following:

A student's daily intake is required to fulfill the following five nutrition constraints as stated in the information sheet:

Minimum Energy intake:	2,400 kcal
Minimum Protein intake:	60 grams
Minimum Carbohydrate intake:	350 grams
Maximum Fat intake:	65 grams
Maximum Sodium intake:	2,000 mg

Suppose you can consider taking any of the ten food products in the information sheet in a day, try to find out

- a) the minimum cost incurred so as to meet the above five nutrition requirements,
 - b) the corresponding quantity of each type of food that would be consumed.
2. Similar to Activity 1, students may design their own daily diet to meet all nutrition needs in a minimum cost using food products of their own choice.
 3. Students need to hand in screen captures of the solution provided by the Excel Solver and separate numerical calculations.

Reference:

1. Webpage on calories and different nutrients in foods in website of Department of Health, HKSARG
http://www.change4health.gov.hk/tc/healthy_diet/facts/calories_nutrients/index.html
2. Webpage on healthy diet in the website of the World Health Organization
<http://www.who.int> (Click “Media centre”, “All WHO fact sheets” and “Health and Diet”)

Information sheet

(i) 10 popular types of food with their nutrition facts are given as follows:

Food	Soft Drink: Cola (330 ml)	Malted Soya Drink (250 ml)	Calcium Milk (236 ml)	Instant Cereal Drink (30 g)	Sandwich Bread (100 g)
Energy (kcal)	139	128	94.4	135	248
Protein (g)	0	4.5	6.8	4	8.4
Sugar (g)	35	15	14.9	15	5.1
Carbohydrate (g)	35	19	14.9	23	44.6
Fat (g)	0	3.8	0	3	3.6
Sodium (mg)	13	188	118	150	422
Cost (\$)	6	5	7	4	4
Food	Biscuit / Cracker (100 g)	Bowl Noodle (100 g)	Instant Noodle (100 g)	Beef Hamburger (218 g)	Shrimp Shao Mai (150 g)
Energy (kcal)	275	301	463	512	279
Protein (g)	4.2	9	10.3	26	17
Sugar (g)	38	0.7	2.4	0	4.5
Carbohydrate (g)	64.8	35.2	58.9	40	20.5
Fat (g)	0.25	13.8	20.7	27	14.3
Sodium (mg)	188	1,140	1,936	824	855
Cost (\$)	5	7	3	12	8

(ii) According to the information from the World Health Organization and Department of Health of HKSARG, the recommended daily intake of different nutrients for an adult in a healthy and balanced diet are given as below:

Lower Limit (10 Types)		Upper Limit (5 Types)
Energy \geq 2,400 kcal	Carbohydrates \geq 350 g	Salt \leq 5 g
Protein \geq 60 g	Vegetable \geq 400 g	Sugar \leq 50 g
Dietary Fibre \geq 25 g	Calcium \geq 1,000 mg	Fat \leq 65 g
Iron \geq 20 mg	Vitamin A \geq 1,000 μ g	Sodium \leq 2,000 mg
Vitamin C \geq 75 mg	Vitamin E \geq 20 mg	Cholesterol \leq 200 mg / dL

Procedures in Using Excel Solver to solve Linear Programming problems

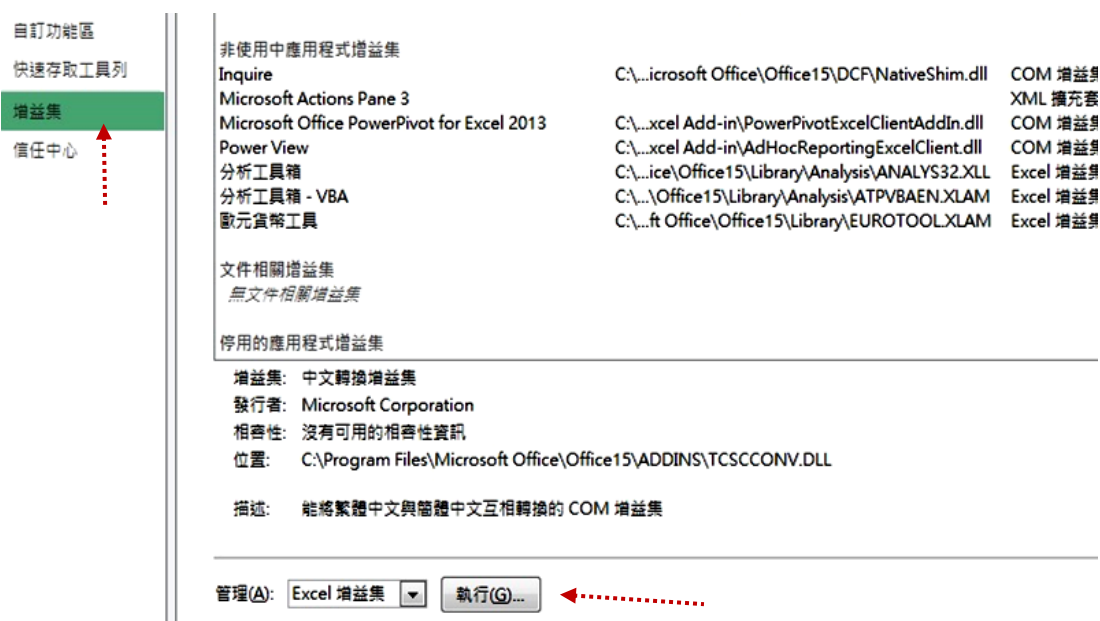
The following example considers the 10 types of food given in the information sheet for demonstration.

To install and use the Excel Add-in component Solver for solving linear programming related problems, please refer to the following steps:

Step 1: Open Microsoft Excel, select “File” and then “Option” (選項). [For Excel 2000 / 2003, click "Tools" (工具) at the top, then "Solver" (規劃求解).]

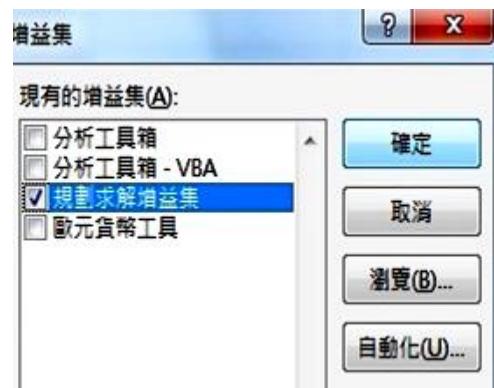


Step 2: Select “Add-Ins” (增益集) on the left and then click the button “Go” (執行) with the item “Excel Add-Ins” (Excel 增益集) shown next to it.



Step 3: Choose “Solver Add-Ins” (規劃求解增益集) as shown and press “OK”.

After having this add-in installed, an icon will be added to the Excel tool bar and you can click on it to solve advanced level linear programming problems even if the number of variables involved is more than 2.



Step 4: Input all relevant information as the raw data in the Excel spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2													
3		Soft Drink: Cola	Malted Soya Drink	Calcium Milk	Instant Cereal Drink	Sandwich Bread	Biscuit/ Cracker	Bowl Noodle	Instant Noodle	Beef Hamburger	Shrimp Shao Mai	Total	Min / Max Amount
4	Decision Variable	0	0	0	0	0	0	0	0	0	0		
5	Cost (\$)	6	5	7	4	4	5	7	3	12	8	0	
6													
7	Energy (kcal)	139	128	94.4	135	248	275	301	463	512	372	0	2400
8	Protein (g)	0	4.5	6.8	4	8.4	4.2	9	10.3	26	22.6	0	60
9	Carbohydrate (g)	35	19	14.9	23	44.6	64.8	35.2	58.9	40	20.5	0	350
10	Fat (g)	0	3.8	0	15	3.6	0.3	13.8	20.7	27	14.3	0	65
11	Sodium (mg)	13	188	118	150	422	188	1,140	1,936	824	855	0	2000

Step 5: Add the following 6 formulas into the cells L5 and L7 to L11, which calculate the total cost of food, total energy intake and the total intake of the four nutrients according to the quantities of food in row 4.

L5 total cost	=B5*\$B\$4+C5*\$C\$4+D5*\$D\$4+E5*\$E\$4+F5*\$F\$4+G5*\$G\$4+H5*\$H\$4+I5*\$I\$4+J5*\$J\$4+K5*\$K\$4
L7 total energy intake	=B7*\$B\$4+C7*\$C\$4+D7*\$D\$4+E7*\$E\$4+F7*\$F\$4+G7*\$G\$4+H7*\$H\$4+I7*\$I\$4+J7*\$J\$4+K7*\$K\$4
L8 total protein intake	=B8*\$B\$4+C8*\$C\$4+D8*\$D\$4+E8*\$E\$4+F8*\$F\$4+G8*\$G\$4+H8*\$H\$4+I8*\$I\$4+J8*\$J\$4+K8*\$K\$4
L9 total carbohydrate intake	=B9*\$B\$4+C9*\$C\$4+D9*\$D\$4+E9*\$E\$4+F9*\$F\$4+G9*\$G\$4+H9*\$H\$4+I9*\$I\$4+J9*\$J\$4+K9*\$K\$4
L10 total fat intake	=B10*\$B\$4+C10*\$C\$4+D10*\$D\$4+E10*\$E\$4+F10*\$F\$4+G10*\$G\$4+H10*\$H\$4+I10*\$I\$4+J10*\$J\$4+K10*\$K\$4
L11 total sodium intake	=B11*\$B\$4+C11*\$C\$4+D11*\$D\$4+E11*\$E\$4+F11*\$F\$4+G11*\$G\$4+H11*\$H\$4+I11*\$I\$4+J11*\$J\$4+K11*\$K\$4

Step 6: On the tab “Data” (資料) we can find the Add-Ins button “Solver” (規劃求解). Click on it and input all corresponding constraints and the objective cell (i.e. the total cost) as below.

	Soft Drink: Cola	Malted Soya Drink	Calcium Milk	Instant Cereal Drink	Sandwich Bread	Biscuit / Cracker	Bowl Noodle	Instant Noodle	Beef Hamburger	Shrimp Shao Mai	Total	Min / Max Amount
Decision Variable	0	0	4	1	0	7	0	0	0	0		
Cost (\$)	6	5	7	4	4	5	7	3	12	8	67	
Energy (kcal)	139										2437.6	2400
Protein (g)	0										60.6	60
Carbohydrate (g)	35										536.2	350
Fat (g)	0										17.1	65
Sodium (mg)	13										1938	2000

“Target Cell” (目標式) is set as \$L\$5 that is the total cost incurred, and choose “Minimum”. It means the final target is to evaluate the minimum total cost of the food at the cell L5.

Set “By Changing Cells” (藉由變更變數儲存格) as \$B\$4:\$K\$4 which covers all ten Decision Variables between the cells B4 and K4. (i.e. the final quantity for each type of food to be consumed that results in the minimum total cost).

“Subject to the Constraints” (設定限制式) are used to input all five constraints in the form of inequalities. Press “Add” and then input or choose “\$L\$7”, “>=”, “2400” to set the constraints that the total energy intake should be at least 2400 kcal. Constraints for cells L8 to L11 are inputted likewise. Be careful on the sign of inequality chosen to avoid mistakes.

As the quantity in cells B4 to K4 needs to be integers, a corresponding constraint needs to be inputted. Press “Add” and then input or choose “\$B\$4:\$K\$4”, “int”, “integer” to set the constraint.

For “Selecting a Solving Method” (選取求解方法), select “Simplex LP” (單純 LP).
 [For Excel 2000 / 2003 / 2007: Click "Options" (選項), then select "Assume Linear Model" and "Assume Non-Negative".]

Step 7: Press “Solve” (求解) and the solution is shown on the spreadsheet.

The final quantity of each type of food taken can be read from the Decision Variable row (i.e. B4 to K4) , and the total amount of energy and nutrient taken can be read from the 5 cells L7, L8, L9, L10 and L11 accordingly.

In optimisation, Energy and Protein intake are close to their respective limits, while Carbohydrate is taken more than the minimum requirement and Fats are taken much less than the maximum limit.

	Soft Drink: Cola	Malted Soya Drink	Calcium Milk	Instant Cereal Drink	Sandwich Bread	Biscuit / Cracker	Bowl Noodle	Instant Noodle	Beef Hamburger	Shrimp Shao Mai	Total	Min / Max Amount
Decision Variable	0	0	4	1	0	7	0	0	0	0		
Cost (\$)	6	5	7	4	4	5	7	3	12	8	67	
Energy (kcal)	139	128	94.4	135	248	275	301	463	512	372	2437.6	2400
Protein (g)	0	4.5	6.8	4	8.4	4.2	9	10.3	26	22.6	60.6	60
Carbohydrate (g)	35	19	14.9	23	44.6	64.8	35.2	58.9	40	20.5	536.2	350
Fat (g)	0	3.8	0	15	3.6	0.3	13.8	20.7	27	14.3	17.1	65
Sodium (mg)	13	188	118	150	422	188	1,140	1,936	824	855	1938	2000

In summary, we can read from Row 4, Decision Variable, of the spreadsheet that the optimum solution can be found by taking 4 boxes of Calcium Milk, 1 box of Instant Cereal Drink, 7 units of Biscuit / Cracker and 0 units of the remaining seven kinds of food.

The minimum total cost incurred = \$ 4(7) + \$ 1(4) + \$ 7(5) = \$ 67 .