Mathematical Modelling on Decision-making: Probabilistic Model

Key Stage: 4

Strand: Data Handling

Learning Unit: More about Probability

Objective: To allow students to understand the applications of probability in modelling real-life scenarios such as decision-making process to make reasonable forecast and nurture students' entrepreneurial spirit.

Pre–requisite Knowledge:
(i) Concepts of probability and relative frequency
(ii) Basic understanding in the collection and organisation of data

Relationship with other KLA(s) in STEM Education:
"Customer behavior" in Business Management Module of senior secondary Business, Accounting and Financial Studies (BAFS) in the Technology Education KLA.

Scenario:
Decision-making processes occur in all aspects of life and natural sciences. For example, the migration patterns of some species of animals could be analysed as the outcome of a series of decision-making process based on certain physical and physiological factors (Biebach et al, 1986). In human society, the process of decision-making is more obvious in many fields such as interpersonal communications, business activities and diplomatic negotiations.

One way to objectively forecast whether a decision would be made under various factors of consideration is to model the decision-making process mathematically and then quantitatively analyse the effects of the influencing factors towards the decision-making process. In this example, a probabilistic model is employed to model the decision-making process under the effect of brand loyalty. This customer behaviour that will affect the decision would be introduced in the subject BAFS of Technology Education KLA. Collaborations with teachers of BAFS could be sought.

The activities to be introduced are based on real-life scenario on the decision-making process of buying a new smartphone. Students are required to apply knowledge in probability to model the customer behaviour under the effect of brands. They would make use of the probabilistic model to be constructed to make forecasts and suggestions to increase the market share of a particular brand.

Description of the Activities:

Activity 1

1. The teacher may arouse students' interest with a real-life scenario on the decision-making process of buying the latest smartphone. In a simplified scenario that assumes only two brands (namely Brand I and Brand S) are available in the market, the teacher may ask students to discuss and comment whether the probability of following four types of decisions are roughly the same :

    1. A current Brand I smartphone user will buy a new Brand I smartphone,
    2. A current Brand I smartphone user will buy a new Brand S smartphone,
    3. A current Brand S smartphone user will buy a new Brand I smartphone, and
    4. A current Brand S smartphone user will buy a new Brand S smartphone.

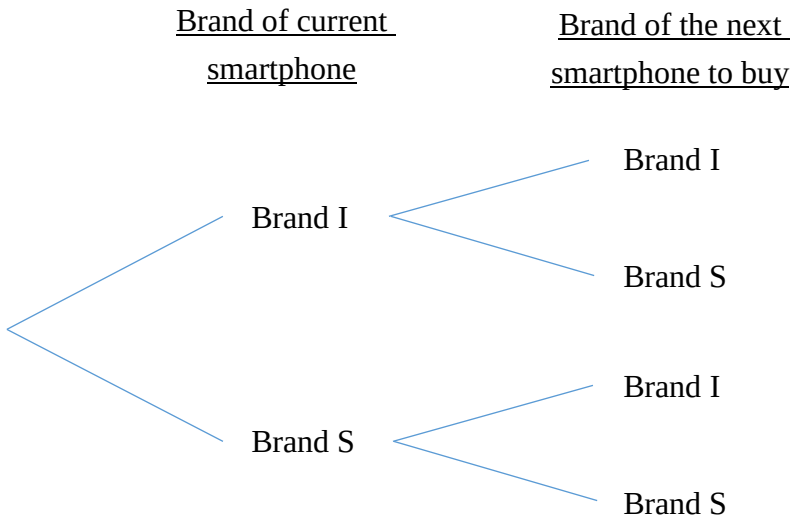
2. Then, the teacher may guide students recognise that the different preferences of the consumers could be represented by conditional probability, and ask students to represent the scenario with a tree diagram with suitably defined events.

Questions for discussion:

1. How is the real-life scenario of buying the latest smartphone simplified in this activity? How would the simplification(s) affect the forecast of a person's decision using the above model?

Notes for Teachers:

1. Students should construct a tree diagram similar to the one below:



2. In this model, the scenario is simplified to two brands only, and assumes all smartphone users will replace their smartphones. This model cannot forecast whether a person would decide not to replace his/her smartphone at the moment of discussion.

Activity 2

1. After the tree diagram in Activity 1 is constructed, the teacher may discuss with students the limitations and assumptions made so that the model may stand. For example, students are expected to figure out that there are more than two brands in the market, and that not all smartphone users would consider buying the latest smartphone at the present stage of the discussion.

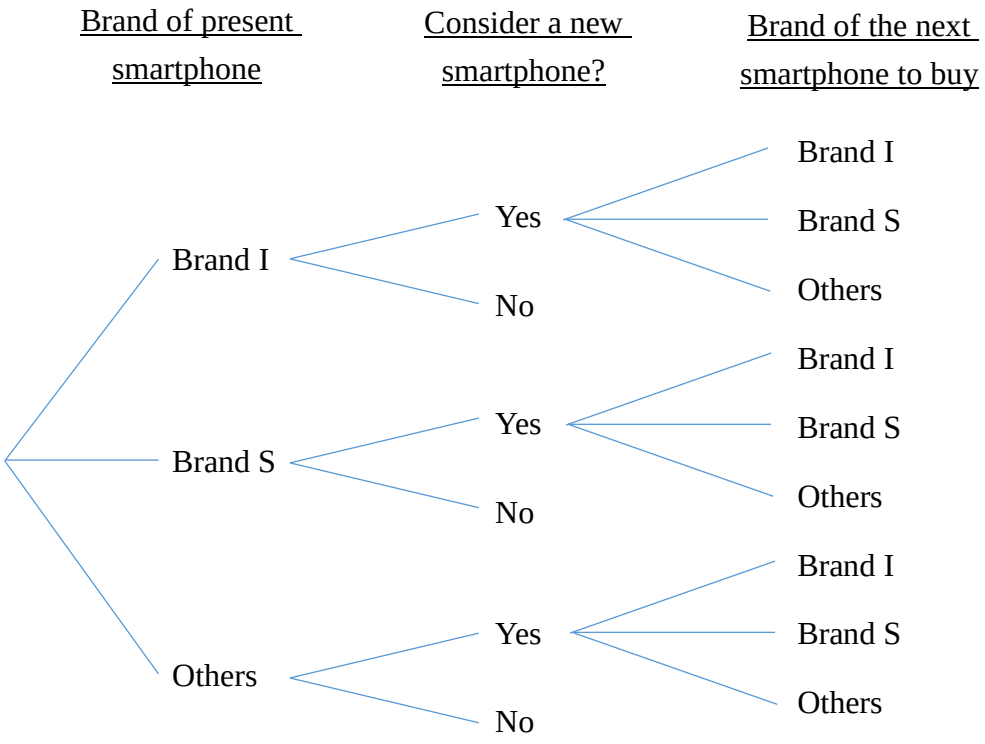
2. The teacher then guides students to discuss how to refine the model so that some limitations and assumptions of the model in Activity 1 may be relieved, and present the refined model with an appropriate tree diagram.

Questions for discussion:

1. Are there still any limitations, assumptions and constraints in the refined mathematical model? What are the pros and cons to construct a more complicated tree diagram to relieve some of the limitations, assumptions and constraints?

Notes for Teachers:

1. Students are expected to draw tree diagrams similar to the following:



2. In the refined model, assumptions such as the new smartphones of both Brand I, Brand S and other brands are available at roughly the same time have been made. The refined model also cannot take care of the fact that behavior of different users of a certain brand may be very diverse in buying new smartphone. For example, the users of Brand I smartphone using earlier generations may have different considerations from those using the latest available generation. It may not be reasonable to put these two groups of Brand I smartphone users into one category.

3. Mathematical modelling is always a simplification of the real-life scenario so that the general pictures can be discussed mathematically. An oversimplified model may depend on unrealistic assumptions and there would be more limitations and constraints to use. In other words, it has less practical use in forecasting the real-life scenario. On the other hand, a complicated model may rely on more information and data to be collected in the real-life scenario, and some data may even be not available. Hence, there may be cases that even one can model the real-life scenario very accurately, the model may not be solvable mathematically.

Activity 3

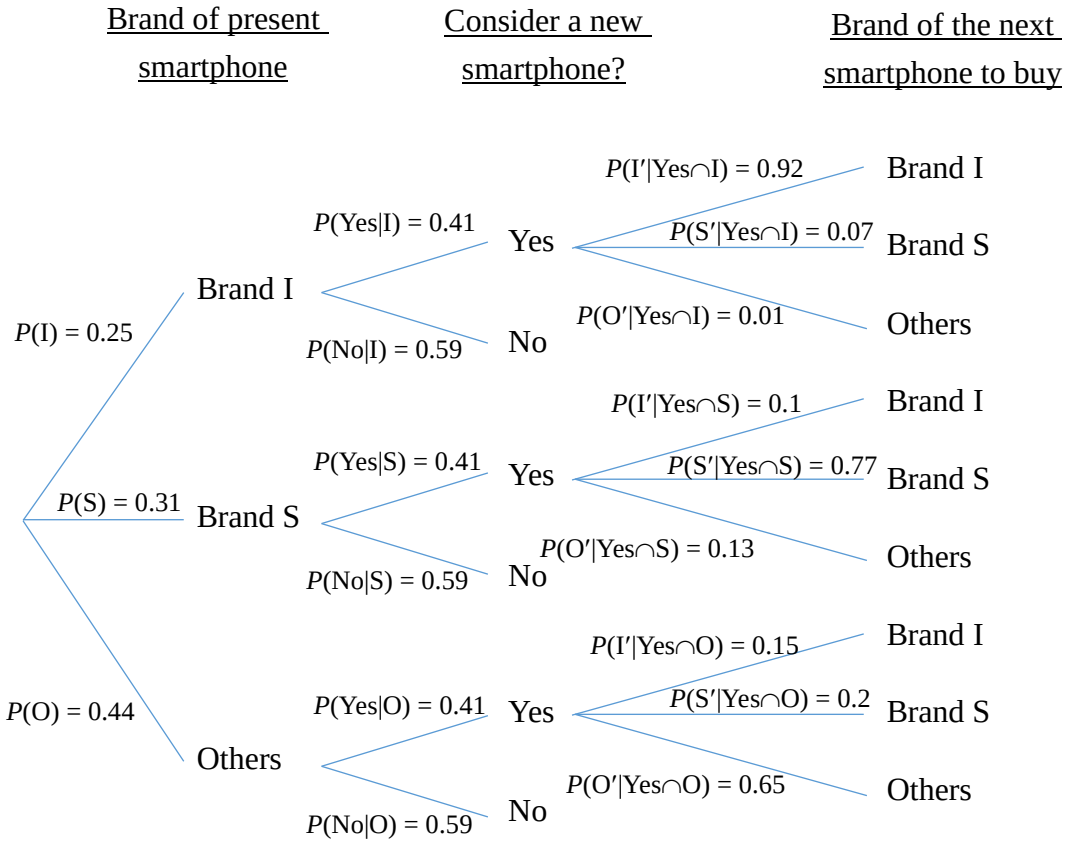
1. After students have refined their tree diagrams, the teacher may guide them to look for the probabilities of each branch in the tree diagram to forecast quantitatively the probability of a particular decision would be made under different conditions.

2. The teacher may first ask students to suggest a sensible way to enumerate the probabilities of different events and hence complete the mathematical model. Normally statistical methods (such as conducting statistical survey) would be proposed. In order to save time for more practices on the manipulations of conditional probability, the teacher may ask students to search for relevant statistical reports on customer behaviour in smartphone brand, or provide for students the information sheet in the Annex with data drawn from various marketing researches accessible in the Internet.

3. The teacher then ask students to use the data in the information sheet to complete the tree diagram, and by using the tree diagram, answer the following questions:
   (a) Which group of smartphone users has greater brand loyalty, users of Brand I or Brand S?
   (b) Predict the future market share of Brand I and Brand S after their release of new models.

Notes for Teachers:

1. Using the given data, the complete tree diagram in Activity 2 becomes:



where I, S and O in the probability expressions represent the events of currently

using smartphones of Brand I, Brand S and others respectively, whereas I′, S′ and O′ represent the events of choosing Brand I, Brand S and others respectively when buying new smartphones.

2. (a) As $P(\text{I}' \mid \text{Yes} \cap \text{I}) = 0.92 > 0.77 = P(\text{S}' \mid \text{Yes} \cap \text{S})$, users of Brand I has greater brand loyalty.

   (b) After the release of new models, the market share of Brand I and Brand S is expected to be 0.28 and 0.32 respectively. Calculation for the new market share of Brand I is provided below as an illustration:

$$\begin{aligned} P(\text{I}') = \; & P(\text{I}) \times P(\text{Yes} \mid \text{I}) \times P(\text{I}' \mid \text{Yes} \cap \text{I}) + P(\text{I}) \times P(\text{No} \mid \text{I}) \\ & + P(\text{S}) \times P(\text{Yes} \mid \text{S}) \times P(\text{I}' \mid \text{Yes} \cap \text{S}) \\ & + P(\text{O}) \times P(\text{Yes} \mid \text{O}) \times P(\text{I}' \mid \text{Yes} \cap \text{O}) \end{aligned}$$

$$\begin{aligned} P(\text{I}') &= 0.25 \times 0.41 \times 0.92 + 0.25 \times 0.59 + 0.31 \times 0.41 \times 0.1 + 0.44 \times 0.41 \times 0.15 \\ &= 0.28 \end{aligned}$$

3. Using the inquiry and investigation approach, teacher may discuss with students the linkage between statistics, probability and making sensible forecast on decision-making.

4. Teacher may collaborate with teachers of BAFS and discuss with students if they were the marketers of Brand I, what they could do to increase the market share according to the model.

This exemplar mainly involves the following generic skills:

1. Mathematical skills
   - Apply mathematical concepts in probability and statistics to translate a real-life scenario into mathematical representation

2. Critical thinking
   - Observe critically the assumptions, constraints and limitations of the mathematical models so as to refine the models to better fit the real-life scenarios

3. Problem solving skills
   - Use mathematical tools such as tables and diagrams to represent probability, and statistical methods to define and illustrate real-life problems
   - Study and propose suitable and solvable mathematical models as possible methods in solving a real-life problem

Reference:

Biebach H, Friedrich B, Heine G (1986). Interaction of body mass, fat, foraging and stopover period in trans-Sahara migrating passerine birds. Oecologia (Berl) 69:370–379

Annex: Information sheet on some marketing researches on Smartphone

1. **A marketing research on global mobile market by an international games, esports & mobile market intelligence in April 2017:**
   - The global market share of Brand I and Brand S are roughly 25% and 31% respectively

2. **A marketing tracker on smartphone market by an international service provider of market intelligence, advisory services in information technology, telecommunications and consumer technology markets in December 2017:**
   - Roughly 41% of smartphone users expect to replace their smartphones in the future 12 months respectively

3. **A survey on brand loyalty of smartphones by an international investment bank in May 2017:**
   - 92% and 7% of Brand I smartphone users who expect to replacement their smartphones in the future 12 months will respectively go for a new Brand I and Brand S smartphone
   - 10% and 77% of Brand I smartphone users who expect to replacement their smartphones in the future 12 months will respectively go for a new Brand I and Brand S smartphone
   - Roughly 15% and 20% of other customers who expect to replacement their smartphones in the future 12 months will respectively go for a new Brand I and Brand S smartphone

Remark:

The information in this Annex is retrieved and modified from authentic marketing research reports on the smartphone market.