<u>Investigation on the relation between the maximum walking velocity and the length of legs</u>

Key Stage: 4

Strand: Number and Algebra

Learning Units: Functions and graphs
Exponential functions and logarithmic functions
Equations of straight lines

Objective: To explain some phenomena in real-life situations through mathematical modelling

Pre-requisite Knowledge:
(i) recognise the concept of functions
(ii) understand exponential and logarithmic functions
(iii) understand the equations of straight lines

Relationship with other KLA(s) in STEM Education:
Uniform circular motion in the topic "Force and Motion" of the Compulsory Part of Physics at senior secondary.

Background information:
When you are hurry to somewhere, you may walk very fast. Do you observe that no matter how you walk, you would have limitation on your walking velocity? It means that everyone has his/her own maximum walking velocity. You may agree that if your legs are longer, your maximum walking velocity is greater. Is there a linear relation between the maximum walking velocity and the length of legs? In this investigation, students are going to explore a suitable simple model to express the relation between the maximum walking velocity and the length of legs.

Description of the Activities:

<u>Activity 1:</u>

1. The class is divided into several groups (it would be preferable to having at most 4 members in a group).
2. In each group, every student walks as fast as possible along a straight path 10 m long for 3 times and the walking time for each student is recorded.

3. In each group, the length of the legs of each group member is calculated through the measurements of the lengths of left and right legs and finding the average of these two measured values. The length of each leg of a group member is measured from the hip joint to the sole of the leg of the group member vertically when he or she stands straight.
4. The data obtained in 2 and 3 above are recorded on a spreadsheet which is shown as follows:

Group ______________					
Member's Name	Length of the legs of the member L m	Distance D m for walking	Time t s for walking 10 m	Approximate maximum walking velocity $\frac{D}{t}$ m/s	Average velocity V m/s (= $\frac{(\text{i})+(\text{ii})+(\text{iii})}{3}$)
		10		(i)	
		10		(ii)	
		10		(iii)	
		10		(i)	
		10		(ii)	
		10		(iii)	
		10		(i)	
		10		(ii)	
		10		(iii)	
		10		(i)	
		10		(ii)	
		10		(iii)	
		10		(i)	
		10		(ii)	
		10		(iii)	

Notes for Teachers:

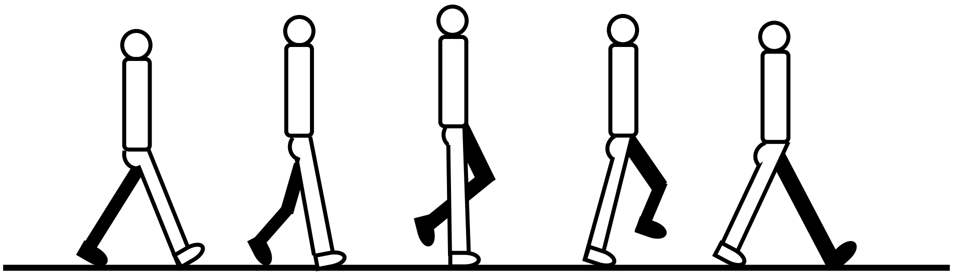
1. To be more accurate, the length of the legs L m of a group member is defined as the average of the lengths of left and right legs of the group member as the lengths of left and right legs of some people may not be equal.

2. The measurements of the lengths of legs (in metres) should be correct to 3 significant figures and the measurements of time (in seconds) should be correct to

the nearest integer.

3. For Activity 1, there are some requirements for walking:
   - When each group member walks along a straight path 10 m long, he or she cannot run and even do the hips swing to the left or right.
   - The walking style should be in the following way:
     - One foot is on the ground as the other is lifted.
     - While the foot is on the ground, the leg is straight and the body rises and falls in a shape of an arc of a circle with centre at the foot.

   The related diagram is as follows:



4. The teacher may create an online Google form and students in each group can use tablets to input the measured lengths of the legs L m and the time used t s into the Google form.

<u>Activity 2:</u>

1. The teacher merges all the spreadsheets collected from groups through Activity 1 into a new spreadsheet.
2. The teacher guides students to extract the data of the lengths of legs L m and the corresponding average velocities V m/s from the new spreadsheet and plot the graph of V against L with the aid of dynamic mathematics software.
3. Through the dynamic software, students in each group examine whether the linear model of function, i.e. $V = mL + c$ where m and c are constants, is suitable to describe the relation between V and L.
4. If the linear model is not suitable, students have to examine other models of functions to find one whose graph best fits all the points (L, V). According to students' mathematical knowledge involving functions and indices, students may try the following functions for explorations:
   - $V = aL^2 + bL + c$ where a, b and c are constants;
   - $V = ab^L$ where a and b are constants;

- $V = kL^n$ where k and n are constants;
- $V = a\log_b L$ where a and b are constants.

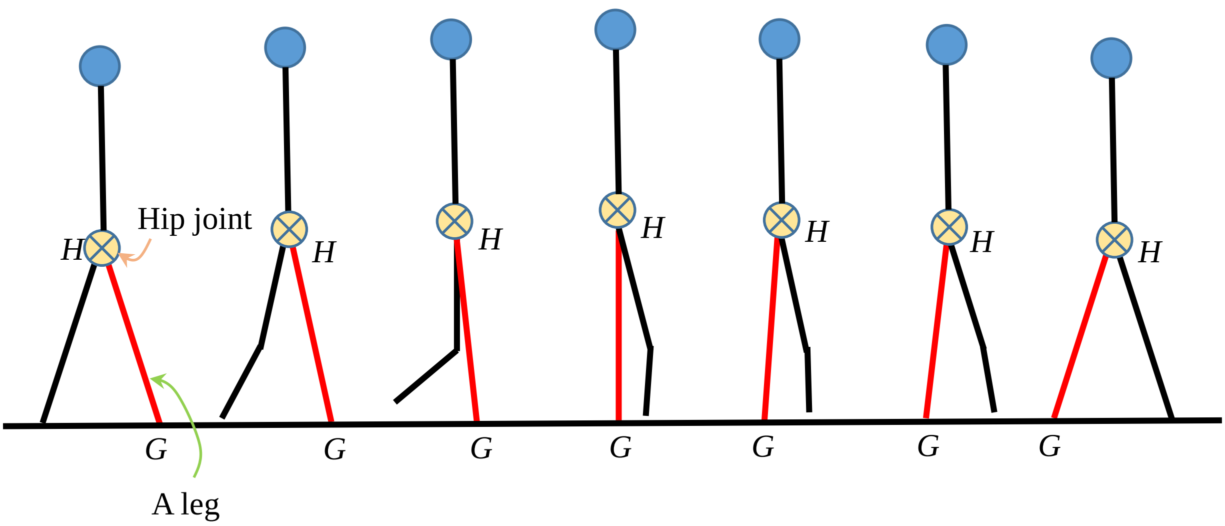
5. After the explorations, each group should present a suitable model of function to illustrate the relation between the maximum walking velocity and the length of legs.

Notes for Teachers:

For Activity 2, teachers have to merge all data from groups and guide students to extract the data of the lengths of legs L m and the corresponding average velocities V m/s to plot the graph of V against L, with the aid of dynamic mathematics software, such as GeoGebra, Desmos, etc. In addition, students are guided to use moving sliders provided by the dynamic mathematics software to find a suitable function whose graph best fits all the points (L, V).

<u>Activity 3 (Enrichment: For students who study Physics):</u>

According to the walking style provided in Notes for Teachers in Activity 1, human walking is modelled as the movement of the hip joint H in an arc of a circle with a leg HG as radius and G as the centre of the circle, as in the following diagram:



Assume that

(i) H is the hip joint and the centre of mass of the body is at H.

(ii) H is moving around G in uniform circular motion with centre G, radius HG.

Let ℓ m, v m/s and a m/s^2 be the length of HG, the velocity of H and the acceleration of H towards G respectively.

(a) Express v in terms of a and ℓ .

(b) Assume that the downward acceleration is not greater than the gravitational acceleration g at H when H is vertical above G, show that $v \le g^{\frac{1}{2}}\ell^{\frac{1}{2}}$.

(c) From (b), suggest a suitable function to express the relation between L and V? In addition, compare this result with the function obtained in Activity 2 to see if it fits the data collected better.

Notes for Teachers:

1. For Activity 3,

(a) By using the formula of centripetal acceleration in uniform circular motion,

$$a = \frac{v^2}{\ell}$$

$$v = \sqrt{a\ell}$$

$$v = a^{\frac{1}{2}}\ell^{\frac{1}{2}}$$

(b) Since the downward acceleration is not greater than the gravitational acceleration g at H when H is vertical above G, by the formula of centripetal acceleration in uniform circular motion, we have:

$$\frac{v^2}{\ell} \le g$$

$$v \le \sqrt{g\ell}$$

$$v \le g^{\frac{1}{2}}\ell^{\frac{1}{2}}$$

(c) By assumption, H is moving around G in uniform circular motion with velocity v m/s. The walking velocity should be less than or equal to v m/s as the direction of the walking velocity is horizontal. By (b), $V = kg^{\frac{1}{2}}L^{\frac{1}{2}}$ may be a suitable function to describe the relation between L and V, where k is a constant.

2. The result from Activity 2 may be different from the above function as:
   (i) there are measurement errors;
   (ii) students' walking styles are different from the suggested model, such as both legs not being stretched straight or wiggling of hips happened when walking;
   (iii) some students are running but not walking.

3. It should be noted that the style of walking required in Activity 1 was to facilitate the comparison with the result in Activity 3. The teacher may take other walking

styles that students could try to examine other models on the relation between the maximum walking velocity and the length of legs.

Reference:

1. Harrison, A. J., Molloy, P. G., & Furlong, L-A. M. (2016). Does the McNeill Alexander model accurately predict maximum walking speed in novice and experienced race walkers? *Journal of Sport and Health Science, 7*, 372-377

2. McNeill, A. R. (1996). Walking and running. *The Mathematical Gazette, 80*, 262 – 266

3. McNeill, A. R. (2001). *Modelling, Step By Step*. Retrieved 21 December 2018 from Plus Magazine Web site: https://plus.maths.org/content/os/issue13/features/walking/index