Supplementary Notes
to Senior Secondary Mathematics Curriculum

for the 2013/14 Secondary 4 cohort

2013

(Blank Page)

Contents

(Blank Page)

1. Introduction

1.1 Background

After the first cycle of the implementation of the New Senior Secondary Curriculum, Education Bureau (EDB), Curriculum Development Council (CDC) and Hong Kong Examinations and Assessment Authority (HKEAA) have joined hands to review the curricula and assessments of all Key Learning Areas. Based on the feedback collected, some recommendations on addressing the concerns and enhancing the implementation of the curriculum and assessment are made.

1.2 Issues and Considerations

The following feedback and concerns with respect to the implementation of the New Senior Secondary Mathematics Curriculum (NSSMC) and assessment are noted:

- further clarification regarding the breadth and depth of certain topics in the curriculum;
- inadequate allocation of lesson time; and
- views on the implementation of School-based Assessment (SBA).

In making recommendations to address the concerns above, our major considerations are:

- Introducing major changes in the short term would require further intensive preparation again when teachers have just familiarised themselves with the subject. More data collection and deliberations are required to explore suggestions that may have major impact on the curriculum and assessment design.
- The assessment objectives of SBA could be met by conducting diversified modes of assessment in schools.

1.3 Summary of Short-term Recommendations

The short-term recommendations to be implemented at S4 in the 2013/14 school year leading to 2016 HKDSE Examination are:

- To address the issues of the breadth/depth of certain topics in and lesson time of the curriculum, the learning contents of Compulsory Part, Module 1 and Module 2 are trimmed down;
- No changes to 2016 HKDSE Mathematics Assessment Framework are made; and
- SBA will not be implemented in 2016 HKDSE Mathematics Examination and thereafter.

1.4 Outline of Assessment Design

Compulsory Part

Component		Weighting	Duration
Public Examination	Paper 1 Conventional questions	65%	2¼ hours
	Paper 2 Multiple-choice questions	35%	1¼ hours

Module 1 (Calculus and Statistics)

Component		Weighting	Duration
Public Examination	Conventional questions	100%	2½ hours

Module 2 (Algebra and Calculus)

Component		Weighting	Duration
Public Examination	Conventional questions	100%	2½ hours

2. Revised Learning Contents of Senior Secondary Mathematics Curriculum

The learning contents of Compulsory Part, Module 1 and Module 2 listed in *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)* (2007) are revised. Contents deleted are enclosed in boxes like [ ]. Newly added remarks are enclosed in boxes like [ ]. Revised lesson time is enclosed by [ ].

Remark:

As a core subject, Senior Secondary Mathematics Curriculum accounts for up to 15% (approximately 375 hours) of the total lesson time available in the senior secondary curriculum. The suggested time allocations for Compulsory Part and Extended Part are as follows:

	Lesson time (Approximate number of hours)
Compulsory Part	10% – 12.5% (250 hours – 313 hours)
Compulsory Part with a module	15% (375 hours)

(Blank Page)

2.1

Revised Learning Contents of Compulsory Part

The Learning Contents of Compulsory Part

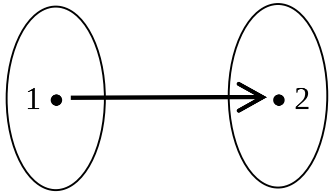
Notes:

1. *Learning units are grouped under three strands (“Number and Algebra”, “Measures, Shape and Space” and “Data Handling”) and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The learning objectives underlined are the Non-foundation Topics.*
4. *The notes in the “Remarks” column of the table may be considered as supplementary information about the learning objectives.*
5. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*
6. *Schools may allocate up to 313 hours (i.e. 12.5% of the total lesson time) to those students who need more time for learning.*

Learning Unit	Learning Objective	Time	Remarks
Number and Algebra Strand			
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method	19	
	1.2 form quadratic equations from given roots		The given roots are confined to real numbers.
	1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts		

Learning Unit	Learning Objective	Time	Remarks
	1.4 solve quadratic equations by the quadratic formula		The following are **not** required for students taking only the Foundation Topics: • expressing nonreal roots in the form $a \pm bi$ • simplifying expressions involving surds such as $2 \pm \sqrt{48}$
	1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots		When $\Delta < 0$, students have to point out that "the equation has **no real roots**" or "the equation has **two nonreal roots**" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8.

Learning Unit	Learning Objective	Time	Remarks
	1.6 solve problems involving quadratic equations		Teachers should select the problems related to students' experiences. Problems involving complicated equations such as $\frac{6}{x}+\frac{6}{x-1}=5$ are required only in the Non-foundation Topics and tackled in Learning Objective 5.4.
	1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations		The relations between the roots and coefficients include: • $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$, where α and β are the roots of the equation $ax^2+bx+c=0$ and $a \neq 0$.
	1.8 appreciate the development of the number systems including the system of complex numbers		The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed.

Learning Unit	Learning Objective	Time	Remarks
	1.9 <u>perform addition, subtraction, multiplication and division of complex numbers</u>		Complex numbers are confined to the form $a \pm bi$. Note: The coefficients of quadratic equations are confined to real numbers.
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables 2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions 2.3 understand the features of the graphs of quadratic functions	10	Finding the domain of a function is required but need **not** be stressed. Representations like  are also accepted. The features of the graphs of quadratic functions include: • the vertex • the axis of symmetry • the direction of opening • relations with the axes Students are expected to find the maximum

Learning Unit	Learning Objective	Time	Remarks
			and minimum values of quadratic functions by the graphical method.
	2.4 <u>find the maximum and minimum values of quadratic functions by the algebraic method</u>		Students are expected to solve problems relating to maximum and minimum values of quadratic functions.
3. Exponential and logarithmic functions	3.1 <u>understand the definitions of rational indices</u>	16	The definitions include $\sqrt[n]{a}$, $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$. Students are also expected to evaluate expressions such as $\sqrt[3]{-8}$.
	3.2 <u>understand the laws of rational indices</u>		The laws of rational indices include: • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $(a^p)^q = a^{pq}$ • $a^p b^p = (ab)^p$ • $\frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$

Learning Unit	Learning Objective	Time	Remarks
	3.3 understand the definition and properties of logarithms (including the change of base)		The properties of logarithms include: • $\log_a 1 = 0$ • $\log_a a = 1$ • $\log_a MN = \log_a M + \log_a N$ • $\log_a \frac{M}{N} = \log_a M - \log_a N$ • $\log_a M^k = k \log_a M$ • $\log_b N = \frac{\log_a N}{\log_a b}$
	3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs		The following properties and features are included: • the domains of the functions • the function $f(x) = a^x$ increases (decreases) as x increases for $a > 1$ $(0 < a < 1)$ • $y = a^x$ is symmetric to $y = \log_a x$ about $y = x$ • the intercepts with the axes • the rate of increasing/the rate of decreasing (by direct inspection)

Learning Unit	Learning Objective	Time	Remarks
	3.5 solve exponential equations and logarithmic equations		Equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log(x - 22) + \log(x + 26) = 2$ are tackled in Learning Objective 5.3.
	3.6 appreciate the applications of logarithms in real-life situations		The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed.
	3.7 appreciate the development of the concepts of logarithms		The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed.
4. More about polynomials	4.1 perform division of polynomials	14	Methods other than long division are also accepted.
	4.2 understand the remainder theorem		
	4.3 understand the factor theorem		

Learning Unit	Learning Objective	Time	Remarks
	4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials		The terms "H.C.F." , "gcd", etc. can be used.
	4.5 perform addition, subtraction, multiplication and division of rational functions		Computation of rational functions with more than two variables is **not** required.
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$	10	
	5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic		
	5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations		Solutions for trigonometric equations are confined to the interval from 0° to 360° .
	5.4 solve problems involving equations which can be transformed into quadratic equations		Teachers should select the problems related to students' experience.
6. Variations	6.1 understand direct variations (direct proportions) and inverse variations (inverse proportions), and their applications to solving real-life problems	9	

Learning Unit	Learning Objective	Time	Remarks
	6.2 understand the graphs of direct and inverse variations 6.3 understand joint and partial variations, and their applications to solving real-life problems		
7. Arithmetic and geometric sequences and their summations	7.1 understand the concept and the properties of arithmetic sequences 7.2 understand the general term of an arithmetic sequence 7.3 understand the concept and the properties of geometric sequences 7.4 understand the general term of a geometric sequence	17	The properties of arithmetic sequences include: • $T_n = \frac{1}{2}\ (T_{n-1} + T_{n+1})$ • if $T_1, T_2, T_3, \ldots$ is an arithmetic sequence, then $k\,T_1 + a$, $k\,T_2 + a$, $k\,T_3 + a, \ldots$ is also an arithmetic sequence The properties of geometric sequences include: • $T_n^2 = T_{n-1} \times T_{n+1}$ • if $T_1, T_2, T_3, \ldots$ is a geometric sequence, then $k\,T_1$, $k\,T_2$, $k\,T_3, \ldots$ is also a geometric sequence

Learning Unit	Learning Objective	Time	Remarks
	7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems 7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems 7.7 solve related real-life problems		Example: geometrical problems relating to the sum of arithmetic or geometric sequences. Example: geometrical problems relating to infinite sum of the geometric sequences. Examples: problems about interest, growth or depreciation.
8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown 8.2 solve quadratic inequalities in one unknown by the graphical method 8.3 solve quadratic inequalities in one unknown by the algebraic method 8.4 represent the graphs of linear inequalities in two unknowns on a plane	16	Compound inequalities involving logical connectives “and” or “or” are required.

Learning Unit	Learning Objective	Time	Remarks
	8.5 solve systems of linear inequalities in two unknowns 8.6 solve linear programming problems		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions 9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$ 9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$ 9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives	11	Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity.
Measures, Shape and Space Strand			
10. Basic properties of circles	10.1 understand the properties of chords and arcs of a circle	23	The properties of chords and arcs of a circle include: • the chords of equal arcs are equal • equal chords cut off equal arcs

Learning Unit	Learning Objective	Time	Remarks
			• the perpendicular from the centre to a chord bisects the chord • the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord • the perpendicular bisector of a chord passes through the centre • equal chords are equidistant from the centre • chords equidistant from the centre are equal Students are expected to understand why there is one and only one circle passing through given three non-collinear points. Note: the property that the arcs are proportional to their corresponding angles at the centre should be discussed at Key Stage 3 when the formula for calculating arc lengths is being explicated.

Learning Unit	Learning Objective	Time	Remarks
	10.2 understand the angle properties of a circle		Angle properties of a circle include: • the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference • angles in the same segment are equal • the arcs are proportional to their corresponding angles at the circumference • the angle in a semi-circle is a right angle • if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter
	10.3 understand the properties of a cyclic quadrilateral		The properties of a cyclic quadrilateral include: • the opposite angles of a cyclic quadrilateral are supplementary • an exterior angle of a cyclic quadrilateral equals its interior opposite angle

Learning Unit	Learning Objective	Time	Remarks
	10.4 understand the tests for concyclic points and cyclic quadrilaterals		The tests for concyclic points and cyclic quadrilaterals include: • if A and D are two points on the same side of the line BC and $\angle BAC = \angle BDC$, then A, B, C and D are concyclic • if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic • if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic

Learning Unit	Learning Objective	Time	Remarks
	10.5 understand the properties of tangents to a circle and angles in the alternate segments		The properties include: • a tangent to a circle is perpendicular to the radius through the point of contact • the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle • the perpendicular to a tangent at its point of contact passes through the centre of the circle • if two tangents are drawn to a circle from an external point, then: - the distances from the external point to the points of contact are equal - the tangents subtend equal angles at the centre - the straight line joining the centre to the external point bisects the angle between the tangents

Learning Unit	Learning Objective	Time	Remarks
	10.6 use the basic properties of circles to perform simple geometric proofs		• if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment • if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle

Learning Unit	Learning Objective	Time	Remarks
11. Locus	11.1 understand the concept of loci	7	
	11.2 describe and sketch the locus of points satisfying given conditions		The conditions include: • maintaining a fixed distance from a fixed point • maintaining an equal distance from two given points • maintaining a fixed distance from a line • maintaining a fixed distance from a line segment • maintaining an equal distance from two parallel lines • maintaining an equal distance from two intersecting lines
	11.3 describe the locus of points with algebraic equations		Students are expected to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $y = ax^2 + bx + c$).

Learning Unit	Learning Objective	Time	Remarks
12. Equations of straight lines and circles	12.1 understand the equation of a straight line	14	Students are expected to find the equation of a straight line from given conditions such as: • the coordinates of any two points on the straight line • the slope of the straight line and the coordinates of a point on it • the slope and the *y*-intercept of the straight line Students are expected to describe the features of a straight line from its equation. The features include: • the slope • the intercepts with the axes • whether it passes through a given point The normal form is **not** required.

Learning Unit	Learning Objective	Time	Remarks
	12.2 understand the possible intersection of two straight lines		Students are expected to determine the number of intersection points of two straight lines from their equations. Note: Solving simultaneous linear equations in two unknowns is a learning objective at Key Stage 3.
	12.3 understand the equation of a circle		Students are expected to find the equation of a circle from given conditions such as: • the coordinates of the centre and the radius of the circle • the coordinates of any three points on the circle Students are expected to describe the features of a circle from its equation. The features include: • the centre • the radius • whether a given point lies inside, outside or on the circle

Learning Unit	Learning Objective	Time	Remarks
	12.4 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle		Finding the equations of tangents to a circle is required.
13. More about trigonometry	13.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity 13.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from $0°$ to $360°$) and other trigonometric equations (solutions in the interval from $0°$ to $360°$) 13.3 understand the formula $½ ab \sin C$ for areas of triangles 13.4 understand the sine and cosine formulae 13.5 understand Heron's formula	21	Simplification of expressions involving sine, cosine and tangent of $-\theta$, $90° \pm \theta$, $180° \pm \theta$, … , etc. is required. Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and tackled in Learning Objective 5.3.

Learning Unit	Learning Objective	Time	Remarks
	13.6 use the above formulae to solve 2-dimensional and 3-dimensional problems		The “above formulae” refer to those mentioned in Learning Objectives 13.3 – 13.5. 3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between a point and a line, and the distance between a point and a plane. Note: Exploring the properties of simple 3-D figures is a learning objective at Key Stage 3.
Data Handling Strand			
14. Permutation and combination	14.1 understand the addition rule and multiplication rule in the counting principle	11	
	14.2 understand the concept and notation of permutation		Notations such as “ P_r^n ” , “${}_nP_r$” , “ ${}^nP_r$” , etc. can be used.

Learning Unit	Learning Objective	Time	Remarks
	14.3 solve problems on the permutation of distinct objects without repetition		Problems such as "permutation of objects in which three particular objects are put next to each other" are required. Circular permutation is **not** required.
	14.4 understand the concept and notation of combination		Notations such as " C_r^n " , " ${}_nC_r$ " , " ${}^nC_r$ " , " $\binom{n}{r}$ " , etc. can be used.
	14.5 solve problems on the combination of distinct objects without repetition		
15. More about probability	15.1 recognise the notation of set language including union, intersection and complement	10	The concept of Venn Diagram is required.
	15.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events		The addition law of probability refers to " $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ".
	15.3 understand the multiplication law of probability and the concept of independent events		The multiplication law of probability refers to " $P(A \cap B) = P(A) \times P(B)$, where A and B are independent events".

Learning Unit	Learning Objective	Time	Remarks
	15.4 recognise the concept and notation of conditional probability 15.5 use permutation and combination to solve problems relating to probability		The rule "$P(A \cap B) = P(A) \times P(B \mid A)$" is required. Bayes' Theorem is **not** required.
16. Measures of dispersion	16.1 understand the concept of dispersion 16.2 understand the concepts of range and inter-quartile range 16.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data 16.4 understand the concept of standard deviation for both grouped and ungrouped data sets 16.5 compare the dispersions of different sets of data using appropriate measures	14	A box-and-whisker diagram can also be called a "boxplot". The term "variance" should be introduced. Students are required to understand the following formula for standard deviation: $\sigma = \sqrt{\dfrac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$.

Learning Unit	Learning Objective	Time	Remarks
	16.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution 16.7 explore the effect of the following operations on the dispersion of the data: (i) adding an item to the set of data (ii) removing an item from the set of data (iii) adding a common constant to each item of the set of data (iv) multiplying each item of the set of data by a common constant		
17. Uses and abuses of statistics	17.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 17.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations	4	The concepts of “populations” and “samples” should be introduced. Probability sampling and non-probability sampling should be introduced. Students should recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability.

Learning Unit	Learning Objective	Time	Remarks
	17.3 assess statistical investigations presented in different sources such as news media, research reports, etc.		
Further Learning Unit			
18. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learnt in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	14	Examples: • solve simple financial problems in areas such as taxation and instalment payment • analyse and interpret data collected in surveys • explore and interpret graphs relating to real-life situations • explore Ptolemy's Theorem and its applications • model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k a^x$ to linear relations • explore the relation between the Fibonacci sequence and the Golden

Learning Unit	Learning Objective	Time	Remarks
			Ratio • appreciate the applications of cryptography • explore the Ceva's Theorem and its applications • investigate the causes and effects of the three crises in mathematics • analyse mathematical games (e.g. explore the general solution of the water puzzle)
19. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.

Grand total: 250 hours

(Blank Page)

2.2

Revised Learning Contents of Module 1

Learning Contents of Module 1 (Calculus and Statistics)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	The use of the summation notation ($\sum$) should be introduced. The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of the number *e* and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$	7	
	2.2 recognise exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are expected to know how to solve problems including those related to compound interest, population growth and radioactive decay.

Learning Unit	Learning Objective	Time	Remarks
	2.4 transform $y = kx^n$ and $y = ka^x$ to linear relations, where *a*, *n* and *k* are real numbers, $a > 0$ and $a \neq 1$		When experimental values of *x* and *y* are given, students can plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercept.
	Subtotal in hours	10	
Calculus Area			
Differentiation and Its Applications			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	5	Students should be able to distinguish "continuous functions" and "discontinuous functions" from their graphs. The concepts of continuous function and discontinuous function are **not** required. Theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions should be stated without proof.

Learning Unit	Learning Objective	Time	Remarks
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following types of algebraic functions are required: ● polynomial functions ● rational functions ● power functions x^{α} ● functions derived from the above ones through addition, subtraction, multiplication, division and composition, for example, $\sqrt{x^2+1}$
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced.
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Notations including $f'(x_0)$ and $\left.\frac{dy}{dx}\right\|_{x=x_0}$ should be introduced.

Learning Unit	Learning Objective	Time	Remarks
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	7	The following rules are required: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ • $\frac{dx}{dy}=\frac{1}{\frac{dy}{dx}}$

Learning Unit	Learning Objective	Time	Remarks
	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The following formulae are required: • $(C)'=0$ • $(x^n)'=nx^{n-1}$ • $(e^x)'=e^x$ • $(\ln x)'=\frac{1}{x}$ • $(\log_a x)'=\frac{1}{x\ln a}$ • $(a^x)'=a^x\ln a$ Implicit differentiation is **not** required. ~~Logarithmic differentiation is required.~~ Logarithmic differentiation is **not** required.

Learning Unit	Learning Objective	Time	Remarks
5. Second derivative	5.1 recognise the concept of the second derivative of a function 5.2 find the second derivative of an explicit function	2	Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangents, rates of change, maxima and minima	9	Local and global extrema are required.
	Subtotal in hours	23	
Integration and Its Applications			
7. Indefinite integrals and their applications	7.1 recognise the concept of indefinite integration	10	Indefinite integration as the reverse process of differentiation should be introduced.

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the basic properties of indefinite integrals and basic integration formulae		The notation $\int f(x)\,dx$ should be introduced. The following properties are required: • $\int k\,f(x)dx = k\int f(x)\,dx$ • $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$ The following formulae are required and the meaning of the constant of integration C should be explained: • $\int k\,dx = kx + C$ • $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\,dx = \ln\lvert x \rvert + C$ • $\int e^x\,dx = e^x + C$

Learning Unit	Learning Objective	Time	Remarks
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions 7.4 use integration by substitution to find indefinite integrals 7.5 use indefinite integration to solve problems		 Integration by parts is **not** required.
8. Definite integrals and their applications	8.1 recognise the concept of definite integration	12	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. The notation $\int_a^b f(x)\,dx$ should be introduced. The knowledge of dummy variables, i.e. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ is required.

Learning Unit	Learning Objective	Time	Remarks
	8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals		The Fundamental Theorem of Calculus refers to $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$. The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	8.3 find the definite integrals of algebraic functions and exponential functions		
	8.4 use integration by substitution to find definite integrals		
	8.5 use definite integration to find the areas of plane figures		Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves.
	8.6 use definite integration to solve problems		
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required.
	Subtotal in hours	26	

Learning Unit	Learning Objective	Time	Remarks
Statistics Area			
Further Probability			
10. Conditional probability and independence	10.1 understand the concepts of conditional probability and independent events 10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events C and D to solve problems	3	
11. Bayes' theorem	11.1 use Bayes' theorem to solve simple problems	4	
	Subtotal in hours	7	
Binomial, Geometric and Poisson Distributions and Their Applications			
12. Discrete random variables	12.1 recognise the concept of a discrete random variable	1	
13. Probability distribution, expectation and variance	13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae 13.2 recognise the concepts of expectation $E(X)$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems	5	

Learning Unit	Learning Objective	Time	Remarks
	13.3 use the formulae $E(aX+b)=aE(X)+b$ and $\text{Var}(aX+b)=a^2\,\text{Var}(X)$ to solve simple problems		
14. Binomial distribution	14.1 recognise the concept and properties of the binomial distribution	5	Bernoulli distribution should be introduced. The mean and variance of the binomial distribution should be introduced (proofs are **not** required).
	14.2 calculate probabilities involving the binomial distribution		Use of the binomial distribution table is **not** required.
15. Geometric distribution	15.1 recognise the concept and properties of the geometric distribution 15.2 calculate probabilities involving the geometric distribution	4	The mean and variance of geometric distribution should be introduced (proofs are **not** required).
16. Poisson distribution	16.1 recognise the concept and properties of the Poisson distribution	4	The mean and variance of Poisson distribution should be introduced (proofs are **not** required).

Learning Unit	Learning Objective	Time	Remarks
	16.2 calculate probabilities involving the Poisson distribution		Use of the Poisson distribution table is **not** required.
17. Applications of binomial, geometric and Poisson distributions	17.1 use binomial, geometric and Poisson distributions to solve problems	5	
	Subtotal in hours	24	
Normal Distribution and Its Applications			
18. Basic definition and properties	18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution	3	Derivations of the mean and variance of the normal distribution are **not** required. The formulae written in Learning Objective 13.3 are also applicable to continuous random variables.

Learning Unit	Learning Objective	Time	Remarks
	18.2 recognise the concept and properties of the normal distribution		Properties of the normal distribution include: ● the curve is bell-shaped and symmetrical about the mean ● the mean, mode and median are equal ● the dispersion can be determined by the value of σ ● the area under the curve is 1
19. Standardisation of a normal variable and use of the standard normal table	19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	

Learning Unit	Learning Objective	Time	Remarks
20. Applications of the normal distribution	20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu,\ \sigma^2)$ 20.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu,\ \sigma^2)$ 20.3 use the normal distribution to solve problems	7	
	Subtotal in hours	12	
Point and Interval Estimation			
21. Sampling distribution and point estimates	21.1 recognise the concepts of sample statistics and population parameters	7	
	21.2 recognise the sampling distribution of the sample mean from a random sample of size n		If the population mean is μ and population variance is σ^2, then the mean of the sample mean is μ and the variance of the sample mean is $\frac{\sigma^2}{n}$.

Learning Unit	Learning Objective	Time	Remarks
	21.3 recognise the concept of point estimates including the sample mean, sample variance and sample proportion 21.4 recognise Central Limit Theorem		The concept of “estimator” should be introduced. If the population mean is μ and the population size is N, then the population variance is $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$. If the sample mean is $\bar{x}$ and the sample size is n, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Recognising the concept of “unbiased estimator” is required.
22. Confidence interval for a population mean	22.1 recognise the concept of confidence interval 22.2 find the confidence interval for a population mean	6	• a $100(1 - \alpha)\%$ confidence interval for the mean μ of a normal population with known variance σ^2 is given by

Learning Unit	Learning Objective	Time	Remarks
			$(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$ ● when the sample size n is sufficiently large, a 100(1 − α)% confidence interval for the mean μ of a population with unknown variance is given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$, where s is the sample standard deviation
23. Confidence interval for a population proportion	23.1 find an approximate confidence interval for a population proportion	3	For a random sample of size n, where n is sufficiently large, drawn from a Bernoulli distribution, a 100(1 − α)% confidence interval for the population proportion p is given by $(\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$, where $\hat{p}$ is an unbiased estimator of the population proportion.
	Subtotal in hours	16	

Learning Unit	Learning Objective	Time	Remarks
Further Learning Unit			
24. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours

2.3
Revised Learning Contents of Module 2

Learning Contents of Module 2 (Algebra and Calculus)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1 Surds	1.1 rationalise the denominators of expressions of the form $\dfrac{k}{\sqrt{a} \pm \sqrt{b}}$	1.5	This topic can be introduced when teaching limits and differentiation.

Learning Unit	Learning Objective	Time	Remarks
2. Mathematical induction	2.1 understand the principle of mathematical induction	3	Only the First Principle of Mathematical Induction is required. Applications to proving propositions related to the summation of a finite sequence ~~and divisibility~~ are included. Proving propositions involving inequalities is **not** required.
3. Binomial Theorem	3.1 expand binomials with positive integral indices using the Binomial Theorem	3	Proving the Binomial Theorem is required. The use of the summation notation ($\sum$) should be introduced. The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
4. More about trigonometric functions	4.1 understand the concept of radian measure	11	
	4.2 find arc lengths and areas of sectors through radian measure		
	4.3 understand the functions cosecant, secant and cotangent and their graphs		
	4.4 understand the identities $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$		Simplifying trigonometric expressions by identities is required.
	4.5 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine		The following formulae are required: • $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ • $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ • $\sin 2A = 2 \sin A \cos A$ • $\cos 2A = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A = 2\cos^2 A - 1$ • $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ • $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

Learning Unit	Learning Objective	Time	Remarks
			• $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ • $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ • $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$ • $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ • $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ • $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ "Subsidiary angle form" is **not** required. $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ and $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ can be considered as formulae derived from the double angle formulae.

Learning Unit	Learning Objective	Time	Remarks
5. Introduction to the number *e*	5.1 recognise the definitions and notations of the number *e* and the natural logarithm	1.5	Two approaches for the introduction to *e* can be considered: • $e = \lim_{n \to \infty}(1 + \frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ This section can be introduced when teaching Learning Objective 6.1.
	Subtotal in hours	20	

Learning Unit	Learning Objective	Time	Remarks
Calculus Area			
Limits and Differentiation			
6. Limits	6.1 understand the intuitive concept of the limit of a function	3	Students should be able to distinguish "continuous functions" and "discontinuous functions" from their graphs. The absolute value function $\lvert x \rvert$, signum function $\operatorname{sgn}(x)$, ceiling function $\lceil x \rceil$ and floor function $\lfloor x \rfloor$ are examples of continuous functions and discontinuous functions. Students are **not** required to distinguish "continuous functions" and "discontinuous functions" from their graphs. The theorem on the limits of sum, difference, product, quotient, scalar multiple and composite functions should be introduced but the proofs are **not** required.

Learning Unit	Learning Objective	Time	Remarks
	6.2 find the limit of a function		The following formulae are required: • $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.
7. Differentiation	7.1 understand the concept of the derivative of a function	14	Students should be able to find the derivatives of elementary functions, including C, x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x, $\ln x$ from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced. Testing differentiability of functions is **not** required.

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation		The following rules are required: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		The following formulae are required: • $(C)' = 0$ • $(x^n)' = n\,x^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(\cot x)' = -\operatorname{cosec}^2 x$ • $(\sec x)' = \sec x \tan x$ • $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$

Learning Unit	Learning Objective	Time	Remarks
			• $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ The following types of algebraic functions are required: • polynomial functions • rational functions • power functions x^{α} • functions formed from the above functions through addition, subtraction, multiplication, division and composition, for example $\sqrt{x^2+1}$
	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.

Learning Unit	Learning Objective	Time	Remarks
8. Applications of differentiation	8.1 find the equations of tangents and normals to a curve	14	
	8.2 find maxima and minima		Local and global extrema are required.
	8.3 sketch curves of polynomial functions and rational functions		The following points are noteworthy in curve sketching: • symmetry of the curve • limitations on the values of x and y • intercepts with the axes • maximum and minimum points • points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students may deduce the equation of the oblique asymptote to the curve of a rational function by division.
	8.4 solve the problems relating to rate of change, maximum and minimum		
	Subtotal in hours	31	

Learning Unit	Learning Objective	Time	Remarks
Integration			
9. Indefinite integration	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals	16	Indefinite integration as the reverse process of differentiation should be introduced. The following formulae are required: • $\int k\,dx = kx + C$ • $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\,dx = \ln\lvert x \rvert + C$ • $\int e^x\,dx = e^x + C$ • $\int \sin x\,dx = -\cos x + C$ • $\int \cos x\,dx = \sin x + C$ • $\int \sec^2 x\,dx = \tan x + C$ • $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$ • $\int \sec x \tan x\,dx = \sec x + C$ • $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$

Learning Unit	Learning Objective	Time	Remarks
			For more complicated calculations, see Learning Objectives 9.4 to 9.6.
	9.3 understand the applications of indefinite integrals in real-life or mathematical contexts		Applications of indefinite integrals in some fields such as geometry and physics are required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$ or $\sqrt{a^2+x^2}$		Notations including $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and their related principal values should be introduced.
	9.6 use integration by parts to find indefinite integrals		$\int \ln x \, dx$ can be used as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time	Remarks
10. Definite integration	10.1 recognise the concept of definite integration	11	The definition of the definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The use of dummy variables, including $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$, is required. Using definite integration to find the sum to infinity of a sequence is **not** required.

Learning Unit	Learning Objective	Time	Remarks
	10.2 understand the properties of definite integrals		The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [\,f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions 10.4 use integration by substitution to find definite integrals		Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$, should be introduced.

Learning Unit	Learning Objective	Time	Remarks
	10.5 use integration by parts to find definite integrals 10.6 understand the properties of the definite integrals of even, odd and periodic functions		The use of integration by parts is limited to at most two times in finding an integral. The following properties are required: • $\int_{-a}^{a} f(x)\,dx = 0$ if f is odd • $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ if f is even • $\int_{0}^{nT} f(x)\,dx = n\int_{0}^{T} f(x)\,dx$ if $f(x+T) = f(x)$, i.e. f is periodic
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure 11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	4	~~Both~~ "disc method" ~~and "shell method"~~ is required. Finding the volume of a hollow solid is required.
	Subtotal in hours	31	

Learning Unit	Learning Objective	Time	Remarks
Algebra Area			
Matrices and Systems of Linear Equations			
12. Determinants	12.1 recognise the concept and properties of determinants of order 2 and order 3	3	The following properties are required: • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$ • $\begin{vmatrix} a_1 & kb_1 & c_1 \\ a_2 & kb_2 & c_2 \\ a_3 & kb_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & kb_1 \\ a_2 & b_2 & kb_2 \\ a_3 & b_3 & kb_3 \end{vmatrix} = 0$ • $\begin{vmatrix} a_1 + a_1' & b_1 & c_1 \\ a_2 + a_2' & b_2 & c_2 \\ a_3 + a_3' & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1' & b_1 & c_1 \\ a_2' & b_2 & c_2 \\ a_3' & b_3 & c_3 \end{vmatrix}$

Learning Unit	Learning Objective	Time	Remarks
			• $\begin{vmatrix} a_1+kb_1 & b_1 & c_1 \\ a_2+kb_2 & b_2 & c_2 \\ a_3+kb_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ Notations including $\lvert A \rvert$ and $\det(A)$ should be introduced.

Learning Unit	Learning Objective	Time	Remarks
13. Matrices	13.1 understand the concept, operations and properties of matrices	9	The addition, scalar multiplication and multiplication of matrices are required. The following properties are required: • $A + B = B + A$ • $A + (B + C) = (A + B) + C$ • $(\lambda + \mu)A = \lambda A + \mu A$ • $\lambda(A + B) = \lambda A + \lambda B$ • $A(BC) = (AB)C$ • $A(B + C) = AB + AC$ • $(A + B)C = AC + BC$ • $(\lambda A)(\mu B) = (\lambda\mu)AB$ • $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The following properties are required: • the inverse of A is unique • $(A^{-1})^{-1} = A$ • $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$ • $(A^n)^{-1} = (A^{-1})^n$ • $(A^t)^{-1} = (A^{-1})^t$

Learning Unit	Learning Objective	Time	Remarks
			• $\lvert A^{-1} \rvert = \lvert A \rvert^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.
14. Systems of linear equations	14.1 solve the systems of linear equations of order 2 and order 3 by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: • A system of homogeneous linear equations in three unknowns has nontrivial solutions if and only if the coefficient matrix is singular The wording "necessary and sufficient conditions" could be introduced to students.
	Subtotal in hours	18	
Vectors			
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students should recognise some common notations of vectors in printed

Learning Unit	Learning Objective	Time	Remarks
			form (including **a** and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$); and some notations for magnitude (including $\lvert \mathbf{a} \rvert$ and $\lvert \vec{a} \rvert$).
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The following properties are required: • $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ • $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ • $\mathbf{a} + \mathbf{0} = \mathbf{a}$ • $0\,\mathbf{a} = \mathbf{0}$ • $\lambda(\mu\mathbf{a}) = (\lambda\mu)\mathbf{a}$ • $(\lambda + \mu)\mathbf{a} = \lambda\mathbf{a} + \mu\mathbf{a}$ • $\lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b}$ • If $\alpha\mathbf{a} + \beta\mathbf{b} = \alpha_1\mathbf{a} + \beta_1\mathbf{b}$ (**a** and **b** are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$

Learning Unit	Learning Objective	Time	Remarks
	15.3 understand the representation of a vector in the rectangular coordinate system		The following formulae are required: • $\lvert\overrightarrow{OP}\rvert = \sqrt{x^2 + y^2 + z^2}$ in $\mathbf{R}^3$ • $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$ and $\cos\theta = \dfrac{x}{\sqrt{x^2 + y^2}}$ in $\mathbf{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not** required.

Learning Unit	Learning Objective	Time	Remarks
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors	5	The following properties are required: • $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$ • $\mathbf{a} \cdot (\lambda\mathbf{b}) = \lambda(\mathbf{a} \cdot \mathbf{b})$ • $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ • $\mathbf{a} \cdot \mathbf{a} = \lvert \mathbf{a} \rvert^2 \geq 0$ • $\mathbf{a} \cdot \mathbf{a} = 0$ if and only if $\mathbf{a} = \mathbf{0}$ • $\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \geq \lvert \mathbf{a} \cdot \mathbf{b} \rvert$ • $\lvert \mathbf{a} - \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 + \lvert \mathbf{b} \rvert^2 - 2(\mathbf{a} \cdot \mathbf{b})$
	16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$		The following properties are required: • $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ • $\mathbf{b} \times \mathbf{a} = -(\mathbf{a} \times \mathbf{b})$ • $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$ • $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ • $(\lambda\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\lambda\mathbf{b}) = \lambda(\mathbf{a} \times \mathbf{b})$ • $\lvert \mathbf{a} \times \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 \lvert \mathbf{b} \rvert^2 - (\mathbf{a} \cdot \mathbf{b})^2$ The following properties of scalar triple products should be introduced: • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$

Learning Unit	Learning Objective	Time	Remarks
17. Applications of vectors	17.1 understand the applications of vectors	8	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector, the volume of a parallelepiped and the area of a triangle are required.
	Subtotal in hours	18	
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours

(Blank Page)

3. Summary of Changes to Senior Secondary Mathematics Curriculum

3.1 Changes to Compulsory Part

Learning Unit	Learning Objective	Original Time	Amended Time
Data Handling Strand			
17. Uses and abuses of statistics	17.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 17.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations 17.3 assess statistical investigations presented in different sources such as news media, research reports, etc.	8	4
Further Learning Unit			
18. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learnt in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	20	14
19. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	20	10

3.2 Changes to Module 1

Learning Unit	Learning Objective	Original Time	Amended Time
Calculus Area			
Differentiation and Its Applications			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	6	5
Remark: The concept of continuous function and discontinuous function is not required.			
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation 4.2 find the derivatives of algebraic functions, exponential functions and logarithmic function	10	7
Remark: The following rule is not required: $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ Logarithmic differentiation is not required.			
Integration and Its Applications			
8. Definite integrals and their applications	8.5 use definite integration to find the areas of plane figures	15	12
Remark: Students are not required to use definite integration to find the area between a curve and the y-axis and the area between two curves.			
Further Learning Unit			
24. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	7

3.3 Changes to Module 2

Learning Unit	Learning Objective	Original Time	Amended Time
Foundation Knowledge Area			
2. Mathematical induction	2.1 understand the principle of mathematical induction	5	3
Remark: The application to proving divisibility is not required.			
Calculus Area			
Limits and Differentiation			
6. Limits	6.1 understand the intuitive concept of the limit of a function	5	3
Remark: Students are not required to distinguish "continuous functions" and "discontinuous functions" from their graphs.			
Integration			
11. Applications of definite integration	11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis	7	4
Remark: The "shell method" is not required.			
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	7