SUN 日	MON 一	TUE 二	WED 三	THU 四	FRI 五	SAT 六

AUGUST
八月 2009

Mastermind

Mastermind is a simple two-player code-breaking board game invented in 1970 by Mordecai Meirowitz. The generalized mastermind problem can be formulated as a search for a hidden code which uses hints provided by a black box. With 4 pegs and 6 colours, there are 6 x 6 x 6 x 6 = 1296 different patterns. Koyama and Lai (1993), the mathematician described a strategy that minimizes the average number of guesses, requiring on average 4.340 guesses, although may require up to six in the worst case. A slight modification also described by Koyama and Lai (1993) increases the average to 4.341, but reduces the maximum number of guesses required to five.

Find the least value of $\frac{5}{5-2\sin\theta}$ for $0° \le \theta \le 90°$.

1 十一

By definition, which of the following is true?
(1) $z^{x^x} = (z^x)^x$, or
(2) $z^{x^x} = z^{(x^x)}$.

2 十二

For any $n > 1$,
$\sqrt{2\sqrt{3\sqrt{4...\sqrt{(n-1)\sqrt{n}}}}} < x$.
What is the minimum integral value of x?

3 十三

《九章算術》勾股：勾股容圓
八尺為股六尺勾，內容圓徑怎生求？
有人算得如斯妙，算學方為第一籌。

4 十四

If ${}_5C_0 + {}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5 = 2^n$, then n = ?

5 十五

Given $m^n = n^m$, for $0 < m$, $n < 10$ and $m \ne n$, find $m + n$.

6 十六

The letters represent different single-digit numbers.
If $\overline{ABCDEF} \times F = \overline{GGGGGG}$, find F.

7 十七

$X + \overline{XXX} + \overline{XXXXX} + \overline{XXXXXXX} + \ldots + \overline{XXXXXXXXXXXXXXXXX} = 89786756453423112$.
Find X.

8 十八

Find the minimum value of $2x^2 - 20x + 59$.

9 十九

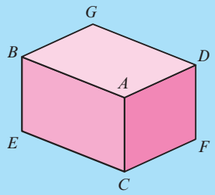
If $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right) = \frac{5}{x}$, find x.

10 二十

If $\frac{x+2y}{3x-4y} = 5$, $x : y = k : 7$, then k = ?

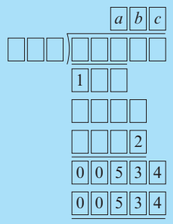
11 廿一

The areas of the faces $ABEC$, $ACFD$ and $ADGB$ are 216, 96 and 144 respectively. Find the length of AC.



12 廿二

Find $a + b + c$.

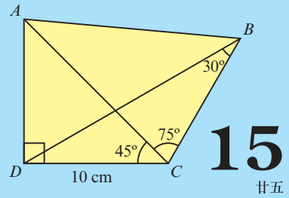


13 廿三

If $\sum_{n=1}^{b^2} n = \overline{bbb}$, where b is a digit and $\overline{bbb}$ is a 3-digit integer, find $\frac{7b}{3}$.

14 廿四

Refer to the figure, find the length of AB, correct to the nearest integer.

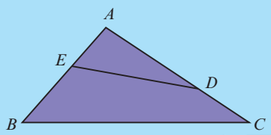


15 廿五

Find the coefficient of x in the expansion of $(1 + 2x)^2 (1 + 3^x)^4$.

16 廿六

In $\triangle ABC$, $AE = 3$, $AD = 6$ and $DC = 4$. If $\angle AED = \angle ACB$, then EB = ?



17 廿七

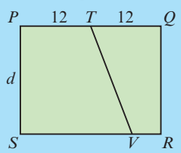
Given $\log_{2009}(x + 9) - \log_{2009} x = \log_{2009} 3 - \log_{2009} 2$, find x.

18 廿八

If the sum of 6 consecutive odd numbers = 144, find the smallest number.

19 廿九

In the figure, $PQRS$ is a rectangle with $PQ = 24$ and $PS = d$. T is the mid-point of PQ. V is a point on SR and $\frac{\text{area of } PTVS}{\text{area of } TQRV} = 2$. SV = ?

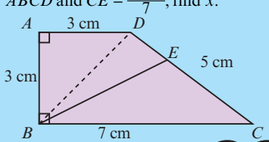


20 七月

The radius of a circle is 8 cm. A new circle is formed by increasing the radius by 10%, find the percentage increase in the area of the circle.

21 初二

If BE is the bisector of quadrilateral $ABCD$ and $CE = \frac{x+3}{7}$, find x.



22 初三

23 處暑

Given a sequence –111, –106, –101, –96, …, find the number of terms that are negative in this sequence.

Given $2\sin^2\theta + 5\sin\theta - 3 = 0$, $0° \le \theta \le 90°$, $\theta = ?°$.

30 十一

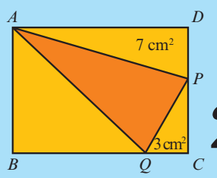
24 初五

To complete a job, A needs 10 days, B needs 15 days and C needs 24 days. It takes 14 days if A, B and C works, in turn, x, y and z days respectively. Find $6z$.

The ratio of the number of \$5 coins to that of \$2 coins is 11 : 20. The total amount is \$95, what is the total number of coins?

31 十二

If $DP : PC = 2 : 3$, find twice the area of $\triangle APQ$.



25 初六

a, b and c are 3 consecutive even numbers and $(c^b)^a = 1\,679\,616$. What is $a + b \times c$?

26 初七

The difference of L.C.M. and H.C.F. of x and y is 45, where $x + y$ is also equal to 45. If $x > y$, find x.

27 初八

Given a sequence 1, 5, 6, 11, 17, x, find x.

28 初九

If the average of 5 distinct numbers is 33, and the averages of the largest four and the smallest four are 34 and 31 respectively, find the smallest number.

29 初十

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