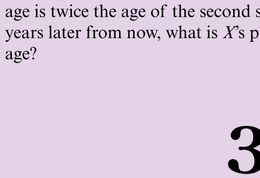
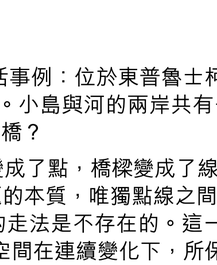
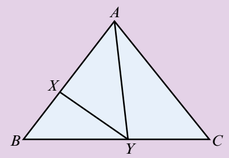
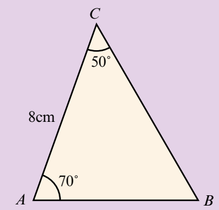
JUNE
六月 2009

SUN 日	MON 一	TUE 二	WED 三	THU 四	FRI 五	SAT 六
	In a right-angled triangle, where $C = 90°$, $2\sin A\cos B\sin C - (\sin 2A - \sin 2B + \cos 2B) = ?$ **1** 初九	Find the maximum value of $y = \dfrac{2}{3\sin x + 4}$. **2** 初十	Given $4^{x^2+2} = 8^{2x}$, find the sum of the roots. **3** 十一	Find k for $\log_9 81^k = 8$. **4** 十二	Find the last digit of $1^{2009} + 2^{2009} + 3^{2009} + 4^{2009} + 5^{2009}$. **5** 芒種	If the median of the 5 different integers 2, 7, 10, x, $2x - 3$ is 7, then $x = ?$ **6** 十四
The graph of $y = 2x^2 - 9x + 4$ cuts the x-axis at A and B, and the y-axis at C. Find the area of triangle ABC. **7** 十五	Solve for a: ${}_{24}C_0 + {}_{24}C_1 + {}_{24}C_2 + \ldots + {}_{24}C_{24} = a^a$. **8** 十六	If area of triangle CDE : area of triangle $BCE = 1 : 2$, area of triangle CDE : area of trapezium $ABCD$ is $1 : p$, $p = ?$  **9** 十七	Two fair dice are thrown, the probability that only one '6' on top is $\dfrac{k}{36}$, $k = ?$ **10** 十八	 Find $\dfrac{x}{5}$. **11** 十九	輯錄自絹布問題《古算題》 今有絹布三十疋，共賣價錢五百七， 四疋絹價九十貫，三疋布價該五十， 欲問絹幾何？價鈔各該分端的， 若人算無差錯，堪把芳名題郡邑。 **12** 二十	Given $\log_{(2x-5)}(3x^2 - 9x + 51) = 2$, find x for $x > 0$. **13** 廿一
Given $BE = EF = FC$. The area of $\triangle AHG$ is $\dfrac{x}{5}$. Find x. **14** 廿二	Given $1^3 + 2^3 + \ldots + n^3 = 120^2$, find n. **15** 廿三	Solve for n: ${}_{17}C_0 + {}_{17}C_1 + {}_{17}C_2 + \ldots + {}_{17}C_8 = 2^n$. **16** 廿四	Given $10 < R < 20$, find R for $R^5 = 1419857$ without using a calculator. **17** 廿五	$D(x, y)$ is the intersection point of the angle bisectors of $\triangle ABC$ with vertices $A(25, 7)$, $B(18, a)$ and $C(11, 7)$. Find x. **18** 廿六	There are 4 positive integers with one of them being even. The sums of any 2 of them are 54, 63, 75, 86, 98 and 107. If x is the sum of the 3 odd numbers among those 4 positive integers, find the value of $x - 100$. **19** 廿七	Given $\dfrac{9}{10} = \dfrac{1}{A} + \dfrac{1}{B} + \dfrac{1}{C}$ where A, B, C are all distinct integers, find $A + B + C$. **20** 廿八
A straight line $y = p$ $(p > 0)$ intersects the curve $y = x^2 - 4x + (21 + p)$ at $A(\alpha, p)$ and $B(\beta, p)$, find the value of $\alpha\beta$. **21** 夏至	If $\tan A = \dfrac{-5}{4}$, then $\dfrac{2\sin A - 3\cos A}{-3\sin A + 2\cos A} = \dfrac{k}{23}$. Find k. **22** 三十	Solve for n: $1 + 3 + 5 + \ldots + 45 = n^2$. **23** 閏五月	In the figure, X and Y are points on AB and BC respectively such that $AX : XB = 3 : 2$ and $BY : YC = 4 : 3$. If the area of $\triangle ABC = 70$, find the area of $\triangle AXY$.  **24** 初二	The length of a rectangle is decreased by 20%. Assume the area of the rectangle remains unchanged, find the percentage increase of its width. **25** 初三	《九章算術》勾股： 今有圓材，埋在壁中，不知大小。以鐻鐻之，深一寸，鐻道長一尺。問徑幾何？ **26** 初四	Find the area of the triangle ABC, correct to the nearest integer.  **27** 初五
If $28^a = 6$, $28^b = 18$, find the value of $3^{\frac{1}{b-a}}$. **28** 初六	Among 10 consecutive positive integers, the sum of multiples of 3 is 99 and the sum of multiples of 2 is 170. Find the smallest integer. **29** 初七	X's age is 5 times the sum of his two sons' ages now and it reduces to twice of their sum 6 years later. If the elder son's age is twice the age of the second son 3 years later from now, what is X's present age? **30** 初八				

拓樸學（Topology）

拓樸學（Topology）始於一個現實生活事例：位於東普魯士柯尼斯堡（今日俄羅斯加里寧格勒）有一條河，河中心有兩個小島。小島與河的兩岸共有七條橋連接。怎樣才能在不重覆路線的情況下，一次逛完這七座橋？

歐拉（Leonhard Paul Euler）把陸地變成了點，橋樑變成了線。在這個問題中，點的具體座標，線的長短曲直等都不改變問題的本質，唯獨點線之間的相關位置，或相互連結的情況不能變，歐拉證明七橋問題中的走法是不存在的。這一類問題後來被命名為「拓樸學」。粗略地來說，拓樸學研究的是空間在連續變化下，所保持不變的那些性質。

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