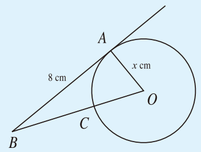
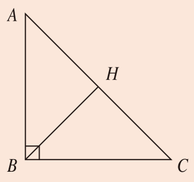
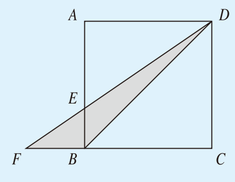
JULY
七月 2010

SUN 日	MON	TUE 二	WED 三	THU 四	FRI 五	SAT 六
	拋物線與建築 有很多熟悉的建築物都以數學形式表達出來，如正方形、矩形、錐形，而位於三藩市的聖瑪利亞大教堂就以雙曲拋物面設計，以一個雙曲拋物面與一條空間中的雙曲線的結合。在聖瑪利亞大教堂的剪綵儀式上，有賓客問建築師，如果著名建築師米高柯哲奴還在世，你認為他對這教堂的設計有什麼看法，建築師便答道：「他是不可能想到這個設計的，因這幾何理論在他的時代是尚未證明。」 			Birthday of **Gottfried Wilhelm von Leibniz**. Major philosopher and a polymath, Leibniz invented differential calculus independently of Newton. He introduced a consistent notation for integrals and infinitesimals (1675). Unlike d'Alembert, Leibniz never thought of the derivative as a limit. **1** 香港特別行政區成立紀念日	Given that $x + y = 1$ and $x^2 + y^2 = 2$. If $x^3 + y^3 = \frac{5}{m}$, find m. **2** 廿一	Find $3\left(1n\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)\right)$. **3** 廿二
Two positive numbers are such that their difference is equal to 1 and their product is also equal to 1. Find the difference of the cubes of these two numbers. **4** 廿三	Find the last digit of $3^2 + 6^2 + 9^2 + ... 2010^2$. **5** 廿四	Let the curve $y = x^2 + 7x + 12$ intersect the x-axis at points A and B, and intersect the y-axis at point C. Find the area of ΔABC. **6** 廿五	In the figure, AB is a tangent to the circle, with centre O. Given that AB = 8 cm, BC = 5 cm and $OA = x$ cm, find the area bounded by AB, BC and the arc AC (Correct to the nearest integer).  **7** 廿六	There are 4 balls with labels A, B, C, D and 4 pockets also with labels A, B, C, D. A ball is put into each pocket. Find the number of ways in which exactly 1 ball has label matching the label on the pocket. **8** 廿七	The sequence $\{a_n\}$ is defined as: $a_1 = c$, $a_{n+1} = a_n + 2n (n \geq 1)$. If $a_{100} = c + \overline{yy00}$, find the digit y. **9** 廿八	Suppose the straight line $y + 3x - 4 = 0$ intersects the parabola $y = x^2$ at points A and B. If O is the origin, find the area of ΔOAB. **10** 廿九
If $R^{2000} < 5^{3000}$, where R is a positive integer, find the largest value of R. **11** 三十	$\lim_{x\to 0}\frac{\cos 5x - \cos x}{x^2} = -b$. Find b. **12** 六月	If $13^a = 6$ and $13^b = 18$, find the value of $3^{\frac{1}{b-a}}$. **13** 初二	A person borrows $100 at nominal interest rate of 13% per year. How much more interest in % has he paid in that year if the interest rate is continuous compounding? (Round up to the nearest integer). **14** 初三	Given that $\left(5^{\log 30}\right)\left(\frac{1}{3}\right)^{\log 0.5} = a$, find a. **15** 初四	Find p if $\log_4(\log_2 p) = \log_2(\log_4 p)$. **16** 初五	If $A = \begin{pmatrix} 3 & -5 \\ 1 & 4 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} \frac{4}{n} & \frac{5}{n} \\ \frac{-1}{n} & \frac{3}{n} \end{pmatrix}$, find n. **17** 初六
Given the area of a square $ABCD$ is d. If E, F, G, H are the mid-points of AB, BC, CD and DA respectively, and $EF = 3$, find d. **18** 初七	Given that $Q = 19 \times (\frac{1}{100} + \frac{99}{100^2} + \frac{99^2}{100^3} + \frac{99^3}{100^4} + ...)$, find Q. **19** 初八	If a regular 8-sided polygon has p diagonals, find p. **20** 初九	How many right angles are here?  **21** 初十	Starting at 4:00pm, how many minutes will the minute-hand take to overlap the hour-hand (Correct the answer to the nearest integer)? **22** 十一	If the 50th power of a 2-digit number P is a 69-digit number, find P. **23** 大暑	Find the total number of arrangements if 5 students sit in a round table. **24** 十三
In the figure, ΔABC is a right-angled triangle and $BH \perp AC$. If $AB = 15$, $HC = 16$ and area of ΔABH : area of ΔABC is 9 : m, find m.  **25** 十四	Let a be any real number such that $0 \leq a \leq 26$. Find the minimum value of $f(x) = \lvert x - a \rvert + \lvert x - 26 \rvert + \lvert x - a - 26 \rvert$. **26** 十五	If $[x]$ represents the integral part of x, find $\sum_{k=0}^{\infty}\left[\frac{27 + 2^k}{2^{k+1}}\right]$. **27** 十六	A domino is a thin rectangular piece of wood whose top face is divided into two squares. Each of the squares is either blank or contains between one to six dots. How many different domino faces are possible? **28** 十七	Given that n and k are natural numbers and $1 < k < n$. If $\frac{(1+2+3+...+n)-k}{n-1} = 10$, find $n + k$. **29** 十八	In the figure, $ABCD$ is a square of length 10 cm. AEB, FED and FBC are straight lines. The area ΔAED is larger than that of ΔFEB by 20 cm^2. Find the area of ΔDFB.  **30** 十九	Find n such that $\frac{1}{2} - \frac{1}{6} - \frac{1}{12} - \frac{1}{20} - ... - \frac{1}{930} = \frac{1}{n}$. **31** 二十

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