

**OCT**  
十月 2010

**音樂中的週期函數**

週期函數揉合現代樂器設計和電腦音響設計的精髓。許多樂器的製造都是先把週期函數所產生聲音的圖像，與這些樂器所發出的最佳聲音圖像作出比較後，再加以改進而成。電子音樂的原音再生也是跟週期圖像有著緊密聯繫。

Agent A is deciding whether to become an undercover by flipping a coin three times. He will be an undercover if the number of head obtained is greater. How many coins arrangement are there for A to become an undercover?

**3**  
廿六

**4**  
廿七

It is given that  $4 \cos^4 \theta + 5 \sin^2 \theta - 4 = 0$ , where  $0 < \theta < 2\pi$ . If the maximum value of  $\theta$  is  $\frac{m\pi}{3}$ , find  $m$ .

**5**  
廿八

A bag contains  $d$  balls of which  $x$  are black,  $x + 1$  are red and  $x + 2$  are white. If the probability of drawing a black ball randomly from the bag is not more than  $\frac{1}{6}$ , find the value of  $d$ .

**6**  
廿九

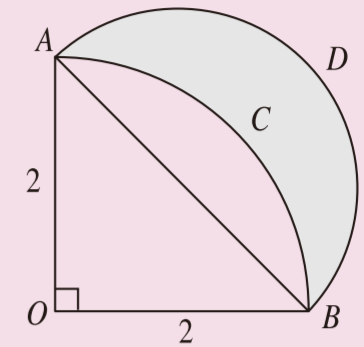
Given that  $(x - 1)^2 + y^2 = 4$ , where  $x$  and  $y$  are real numbers. Find the maximum value of  $2x + y^2$ .

**7**  
三十

Let  $x \oplus y = x + y - xy$ , where  $x, y$  are real numbers. Find the value of the expression  $1 \oplus (0 \oplus 1)$ .

**1**  
國慶日

If  $ABD$  is a semi-circle and  $OACB$  is a sector, find the area of shaded region  $ACBD$ .



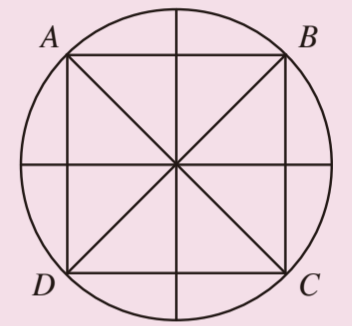
**2**  
廿五

Given that  $m, c$  are positive integers less than 10. If  $m = 2c$  and  $\overline{0.mcmc\dots} = \frac{c+4}{c+8}$ , find  $c$ .

There are two sequences: Sequence A: 1, 2, 3, 4, ... and Sequence B: 1, 2, 4, 8, ... Find the smallest integer  $n$  such that  $\sum_{k=1}^n b_k > 5 \sum_{k=1}^n a_k$ .

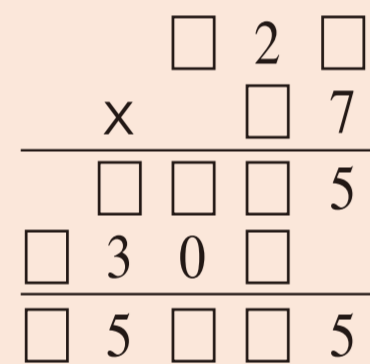
**8**  
寒露

Find the area of the square  $ABCD$ , if radius of the circle is  $\frac{3}{\sqrt{2}}$ .



**9**  
初二

找出算式中最后一行方格中数字的和。



**10**  
初三

Given that the difference between two 3-digit natural numbers  $\overline{xyz}$  and  $\overline{zyx}$  is a positive integer greater than 600. Find the greatest value of  $x + z$ .

**11**  
初四

Let  $A = -1^2 + 2^2 - 3^2 + 4^2 - \dots - 2003^2 + 2004^2$ . Find the sum of all the digits of  $A$ .

**12**  
初五

1 到  $x$  的整數全部相乘，其積用 12 來除。得到的商若是可以被 12 整除，則再用 12 除。連續用 12 除，直到商不被 12 整除。最後的商為 25025。求  $x$ 。

**13**  
初六

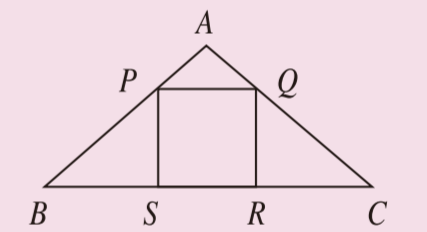
Mr. Chan has 8 sons and  $k$  daughters. Each of his sons has 8 sons and  $k$  daughters. Each of his daughters has  $k$  sons and 8 daughters. It is known that the number of his grandsons is one more than the number of his granddaughters and  $k$  is a prime number. Find  $2k$ .

**14**  
初七

一個圓的周長是 5.4 米，兩隻螞蟻從一條直徑的兩端同時出發沿圓周相向爬行，這兩隻螞蟻每分鐘分別爬行 5.5 厘米和 3.5 厘米。它們每次爬行 1 秒、3 秒、5 秒...就掉頭爬行。那麼兩隻螞蟻第一次相遇的爬行時間在開始後多少分鐘？(答案準確至整數)

**15**  
初八

Square  $PQRS$  is inscribed in  $\Delta ABC$ . The areas of  $\Delta APQ$ ,  $\Delta PBS$  and  $\Delta QRC$  are 4, 4 and 12 respectively. Find the area of the square  $PQRS$ .



**16**  
重陽節

Find the first positive integer that can be written as the sum of a positive cube and a perfect square in two different ways; that is, the smallest  $n$  such that  $x^3 + y^2 = n$  has two different solutions of positive integers  $x$  and  $y$ . Note that the next such number is 65.

**17**  
初十

Find the smallest integer  $p$  such that  $\frac{p^2}{12} - 3p + 24$  is a negative integer.

**18**  
十一

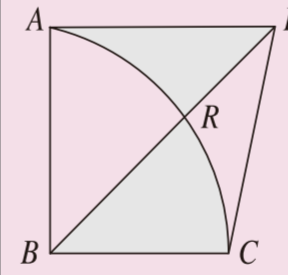
If  $\alpha$  and  $\beta$  are roots of quadratic equation  $4x^2 - 10x + 3 = 0$ , find  $4(\alpha^2 + \beta^2)$ .

**19**  
十二

Given that  $a, b, c$  are positive integers and  $a < b < c = 11$ . Find the number of triangles formed with sides equal  $a$  cm,  $b$  cm and  $c$  cm.

**20**  
十三

The sector  $ABC$  is one quarter of a circle with radius 4 cm and  $AD$  is a tangent to sector  $ABC$  at point  $A$ . Suppose the area of two shaded parts are equal, find the area of trapezium  $ABCD$  (Correct to the nearest integer).

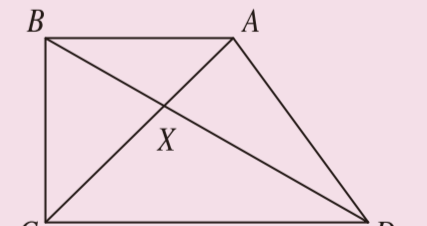


**21**  
十四

Given  $C_3^n = 70n$ . Find  $n$ .

**22**  
十五

$ABCD$  is a trapezium, the segments  $AB$  and  $CD$  are both perpendicular to  $BC$  and the diagonals  $AC$  and  $BD$  intersect at  $X$ . If  $AB = 5$  cm,  $BC = 12$  cm,  $CD = 16$  cm and the area of  $\Delta BXC$  is  $w$  cm<sup>2</sup>, find  $w$  (Correct to the nearest integer).



**23**  
霜降

**24** 一排 6 張椅子上坐 3 個人，每 2 人之間至少有一張空椅子，問共有多少種不同的坐法？

**31**  
廿四

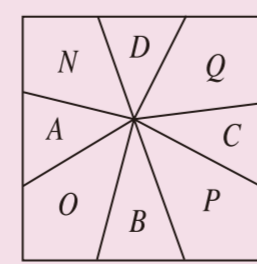
Birthday of **Évariste Galois**. While still in his teens, he was able to determine a necessary and sufficient condition for a polynomial to be solvable by radicals, thereby solving a long-standing problem. He died from wounds suffered in a duel under shadowy circumstances at the age of twenty.

**25**  
十八

Given that the letters in the following long division problem represent digits from 0 to 9. Find the quotient  $MA$ .

**26**  
十九

如圖所示，將正方形的每條邊都各自三等分，其邊上的點與正方形內部的一點相連結，形成 4 個四邊形和 4 個三角形。若正方形的邊長為 12 cm，四邊形  $N, O, P$  的面積之和為 69 cm<sup>2</sup>，求四邊形  $Q$  的面積是多少？



**27**  
二十

Let  $[x]$  be the integral part of  $x$ .

$$\text{Find } M = \frac{1}{293} \sum_{k=1}^{1024} [\log_2 k].$$

**28**  
廿一

有 10 張寫著連續整數的卡片。從中挑出 3 之倍數的卡片，相加起來是 99。此外，將所有 2 之倍數相加，和則為 170。求所有卡片中最小的數。

**29**  
廿二

Suppose  $\{X_i, i = 1, 2, \dots, n\}$  is a random sample of size  $n$  from  $f(x) = 2x; 0 < x < 1$  and zero otherwise. Find  $n$  if

$$E(Y_n = \max\{X_1, X_2, \dots, X_n\}) = \frac{60}{61}.$$

**30**  
廿三