Reference Materials for Secondary School Mathematics

Assessment for Learning (Secondary Mathematics)

The Open-ended Questions

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Introduction

Teachers are currently in the midst of the curriculum reform. They face several issues that require particular attention. One of these is the change in the concept of assessment: from "assessment of learning" to "assessment for learning" (which is recommended and emphasized in the CDC Report "Learning to Learn – The Way Forward in Curriculum Development" (CDC, 2001)). To realize this assessment concept, diversified assessment tools and strategies are encouraged to assess students' performance on different aspects of their learning.

Purpose of assessment

In the past, assessment was used as a measure of success and failure by simply giving a grade to students and was regarded as a terminal activity. However, assessment has nowadays taken on a broader meaning than what we usually perceive. For example, Rowntree (1977) considered assessment as a way of getting to know students and the quality of their learning and Ramsden (1992) described it as a way of teaching more effectively through understanding what students know and do not know while Wiggins (1998, p.7) held that the primary aim of assessment is to "*educate and improve* student performance, not merely to *audit* it".

Assessment is not the product but a means to an educational purpose. Teachers should look at the **process** of how to come up with the **product** and try to strike a balance between these two. Assessment is the process of collecting evidence of students' learning. Its focus is to provide information to show where and why students do not learn well and help them improve (assessment for learning) rather than just find out what mathematical knowledge students have learnt (assessment of learning).

To embody the spirit of assessment for learning in classroom, formative assessment, which is designed to measure what students know and are learning as they go along, has become more and more essential, as the information gathered can be immediately fed back into the learning/teaching cycle to help students learn better. Well-organized formative assessment not only helps students understand their strengths and weaknesses, but also provides teachers with information on the efficiency of their teaching.

Assessment in mathematics

Mathematics is not simply a collection of isolated pieces of facts, formulas and information, but a field of inquiry that builds on a network of interrelated ideas (Baroody & Coslick, 1998). In the "Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 3)", it is considered as "an intellectual endeavour and a mode of thinking" (CDC, 2002, p.2). Therefore, mathematics assessment should describe students' abilities in acquiring a variety of mathematical concepts, in carrying out a variety of mathematical procedures and applying them to solve problems in both familiar and unfamiliar situations. The revised secondary mathematics curriculum, implemented in all secondary schools of Hong Kong (starting at Secondary One in 2001 and progressing to other levels year by year), emphasizes, apart from the subject content, on fostering students' high order thinking skills and developing students' positive attitudes towards mathematics (CDC, 1999). The emphasis is fully reflected in the curriculum document (CDC, 1999). Words such as "explore", "discuss", "investigate", "compare", construct" etc., are used in the curriculum to outline the learning objectives, which intend to highlight the importance of students' skills on critical thinking, problem-solving, creativity and communication.

Traditional assessment items in mathematics such as multiple-choice items, fill-in-blank items and close-ended items are popularly used in schools of Hong Kong. However, complex thinking and learning involve processes that cannot be reduced to a routine and knowledge is a complex network of information and abilities rather than a series of isolated facts and skills (Gibbons, 1992). There are aspects like the 3Cs (i.e. creativity, critical thinking skill and communication skill) which could not be appropriately assessed by these items. Moreover, the report on the Third International Mathematics and Science Study (TIMSS) 1999 (Mullis, et.al., 2000) pointed out that Hong Kong students performed well in solving routine and computational questions, but they did not perform equally well on non-routine questions which require them to think and communicate their thoughts and ideas. The report "Secondary Analysis of the TIMSS-R Data for Hong Kong" released in April 2002 also pointed out that performance-based assessment[1] enhanced students' achievement if they were frequently used. Therefore, there is an increasing need to employ alternative item types to fill in the gaps.

Open-ended question is one form of performance-based assessments and is particularly useful to assess students' thinking abilities including communicating and reasoning skills in mathematics. This booklet aims at providing basic information on setting, administering

[1] A performance-based assessment refers to an assessment that "requires a student to create an answer or a product that demonstrates his/her knowledge/skills (cited in Gibbons, 1992).

and marking open-ended questions for teachers. Sample activities, assessments and exemplars of different levels of work are displayed to teachers who are going to be involved in using open-ended questions in class. Analysis on students' work to provide feedback to students and teachers is also included. The details are necessary as they provide teachers with relevant information (such as difficulties encountered) on employing open-ended questions in student assessment.

Open-ended questions

It is generally agreed that conceptual understanding is not just an accumulation of knowledge but the restructuring of old ideas to accommodate new experiences. Moreover, the continued expansion of knowledge makes people aware of the increasing need to manage information, see patterns, identify needs and solve problems. As a result of this need, there is a shift in focus from learning as content knowledge to learning as the ability to use and interpret knowledge critically and thoughtfully.

If people accept or are convinced that the goal of education has changed, there should be some implication on evaluation and assessment. If subject knowledge in itself is not a sufficient criterion for achievement, judgments of correct and incorrect responses to simple test of skills and knowledge are not enough either. To measure how well a student performs, teachers should examine the process rather than just the final product. Furthermore, people try to make sense of their perceptions and experiences, and the associations that students make are personal and may differ considerably from the one we intended. This creates the demand for a more open-ended form of testing along with a more complex scheme of evaluation.

Open-ended questions focus on students' understanding and their ability to reason and apply knowledge in less routine contexts. Such questions can inform more clearly the levels of student achievement. California State Department of Education opined that open-ended questions describe "tasks for which many answers are appropriate and imply that higher order processes are necessary to respond" (cited in Kulm, 1990, p.8) while 劉學質 has also the following view on open-ended questions: "Open-ended questions have many answers. This type of questions provides students with chances to solve problems in their own ways. During the process of problem solving, students integrate in different ways their knowledge and skills to discover new ways of thinking) (cited in 戴再平, 1999, p.28).

In general, open-ended questions require complex thinking and may yield multiple solutions. They require teachers to interpret and use multiple criteria in evaluating students' responses.

Instead of simple memorization, they require students to construct their own responses (e.g. drawing a rectangle with perimeter 16 cm) and hence open a window to students' thinking and understanding. Such tasks become vehicles for communicating actual achievement to parents, teachers and the students themselves. Teachers at the same time have to prepare to change their modes of assessing students. They have to accept any sensible solutions which are different from the model answers and should be ready to give useful feedback to students.

Open-ended questions can also cater for different abilities of students. Able students enjoy working on open-ended questions as they can tackle a problem with different approaches. For students who are relatively weak in mathematics, appropriately designed open-ended questions could arouse their curiosity and hence stimulate them to learn better. However, it should be pointed out that open-ended questions could be more time-consuming in terms of the lengths of time in arriving answers and scoring. Consequently, adopting only open-ended questions in tests/examinations might not cover all the topics to be assessed and hence affect the validity of the tests/examinations.

Guidelines on using open-ended questions to assess students' performance

The following points should be paid attention to when using open-ended questions to assess students' performance.

- Stress communication – Teachers should keep on asking students to explain and expand on the ideas (orally or verbally).
- Apply skills in practical context – Teachers should set problems in the context of current affairs or daily-life situations that require decision-making. That will motivate students, help them realize the relevancy of the school learning and encourage them to begin transferring that knowledge to different contexts.
- Evaluate at suitable times – The information helps to focus on instruction and provides evidence of students' understanding. To make valid and reliable judgment about student attainment, evidence in a range of contexts should be used.

Teachers should note that as open-ended questions are non-routine, students are not familiar with their format and requirement. Therefore, teachers need to explain to students at suitable times the related issues of open-ended questions such as their requirements.

Creating open-ended questions

Some simple methods applied in creating open-ended questions are:

- Adapting a question from tests or homework

- Identify the content
- Create a context
- Require students to produce a realistic product related to the problem situation (a model, a map, a chart etc.)

- Generalizing a problem
- Making the context, style and the result open.

To make questions open-ended, the following statements/questions could be used:

- Describe or explain how you find your answers.
- Write a story or a description on the information given in a graph.
- Do you agree with the solution? Explain why you think this is correct.
- Use the information to show how you obtain your answer.
- Explain your answer and give examples.

Examples on developing open-ended questions

Traditional close-ended questions could sometimes be changed to open-ended questions by rephrasing the questions or merely replacing numbers by letters. The following examples show how it is done.

Example 1

Solve the equation $2x + 3 = 5$.

It could be changed to:

Find all possible integral values of x and y satisfying the equation $2x + 3y = 50$.

Example 2

Factorize the following expressions:

(a) $x^2 + 3x - 18$;

(b) $x^2 + 7x + 12$.

It could be changed to:

Find the possible integral values of a and b in the following expressions so that they can be factorized.

(a) $x^2 + ax - 18$;

(b) $x^2 + 7x + b$.

Example 3

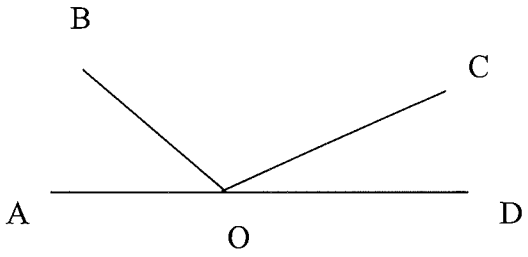
Find the number of obtuse angle(s) in the given figure.

(a) 1

(b) 2

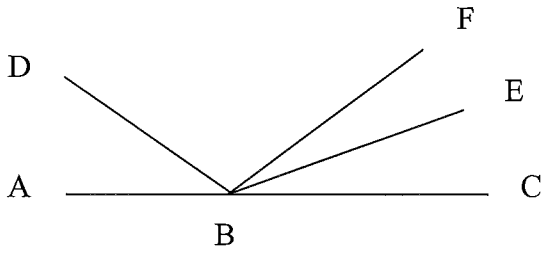
(c) 3

(d) 4



It could be changed to:

Name any two possible obtuse angles in the given figure.



Example 4

Five girls and three boys received 18, 20, 19, 21, 22, 23, 24 and 29 Christmas cards respectively from their friends. Find the mean number of cards received by the eight children.

It could be changed to:

Five girls and three boys received Christmas cards from their friends. The mean number of cards received by the five girls was 20 while the mean number of cards received by the three boys was 28. Is "The person who has received the largest number of cards must be a boy" a true statement? Explain why.

Example 5

My mother went to a shop which was on sale yesterday. Everything in the shop had been reduced by 20%. How much did she pay for a set of tea cups which marked $70?

This is a typical question on discount. Some students will simply use the rule. It is rather difficult for teachers to check students' understanding on applying percentages. This question could be changed slightly so that students require a little bit of thought before getting the answer.

It could be changed to:

My mother went to a shop which was on sale yesterday. Everything in the shop had been

reduced by 20%. After spending sometime in the shop, she purchased a set of tea cups. Which of following is the price most likely paid by her? Give reasons for your choice.

(a) $50

(b) $54

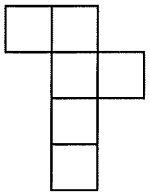
(c) $56

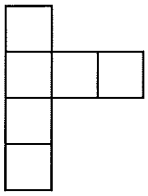
(d) $58

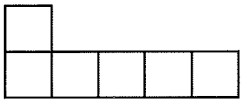
(e) $60

Example 6

Which of the following diagrams represents the net of a cube?

(a) 

(b) 

(c) 

It is hard to tell whether students really understand the meaning of a net for a cube or just pick the choice at random. The question could be modified by asking students to draw the net of a cube. If they do not know what a net means, they cannot draw it.

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It could be changed to:

Draw a possible net for a cube.

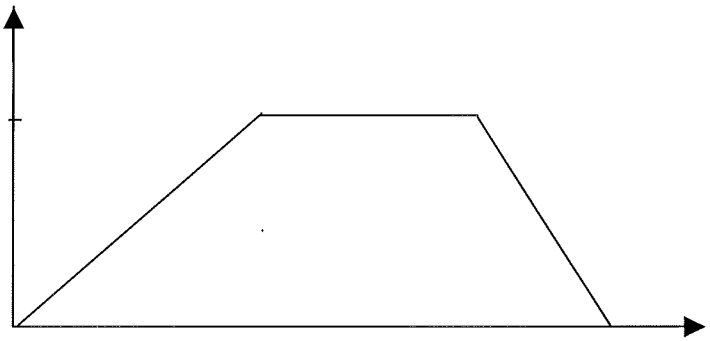
or

Draw three possible nets for a cube.

These two questions eliminate the element of guessing. If a student can produce three correct nets, he/she should understand how to make a cube by using six identical squares.

Example 7

Referring to the diagram below, what is the highest water level in the tank?



It could be changed to:

Label the axes by names and mark them with scales. Write a story to describe the graph.

The followings are two possible responses from students:

(a) A car increases its speed from rest to 60 km/h in the first 10 minutes. It then travels at a constant speed for another 10 minutes. It starts to reduce its speed and stops in 5 minutes.

(b) An empty cistern (a water-tank in the toilet) fills water at a constant rate until it is full in two minutes. Water in the tank remains unchanged for another two minutes. Water is discharged at a constant rate until it is empty again in 1 minute. The rate of discharge is faster than the rate of filling.

As reflected from the answers, especially the second one, the student tries to use a daily example to fit into the situation. It is original and creative. Open-ended questions like this provide students with the opportunity to think creatively as long as it describes what the diagram tries to convey.

Example 8

Solve the equation $4x - 7 = 9$

There are a number of ways to modify this type of question to make it more appealing to students and get them involved.

It could be changed to:

Use the information below to construct a linear equation in one unknown in which the solution is a positive integer.

+4	-7	+9	x	=

or

Use the information below to construct a linear equation in one unknown in which the solution is maximum.

+4	-7	+9	x	=

Further examples of open-ended questions

Example 9 (Statistics and the Truth)

There were 100 complaints by consumers about the overcharging on commission by real estate agents in Hong Kong in 2000 and the Consumer Council prosecuted 2 cases. There were 300 such cases in 2001 and they resulted in 3 prosecutions. The Chairman of the Consumer Council decided not to prosecute the other cases.

Mr T.M. Chan was the Chairman in 2000 and 2001. He is running for re-election. The challenger is Ms Carrie Pun.

Ms Pun made this statement in her latest campaign advertisement, "The current Chairman of the Consumer Council has a terrible record in fighting against unfair treatment by real estate agents to their customers. He only prosecuted 2 % of those reported in 2000, and only 1 % of those reported in 2001. Vote for me so that I can make the change. I will be serious about enforcing the law!"

This was Mr Chan's response, "I have worked hard to protect the right of our customers against unfair treatment. In spite of inadequate budget, we have been improving our record of prosecutions. In 2001, we prosecuted 50 % more on unfair treatment than in the previous year. Vote for me so that I can continue to improve law enforcement in Hong Kong!"

Ignoring the opinions, which candidate's statement is true in the sense of being mathematically correct? Explain.

Example 10

The Hong Kong Youth's Club holds weekly meetings. What is the greatest number of meetings that the club could have in March? What is the least number?

Example 11

Amy ate an average of two hamburgers a day in January. What was the possible number of hamburgers that she might have eaten on January 15?

Example 12

Karl baked an apple pie. He claimed that he had eaten $\frac{1}{2}$, Mary $\frac{2}{8}$ and John $\frac{25}{100}$ of the pie. Is that possible? Explain your answer.

Example 13

10 people went to a restaurant last night. One member got a VIP card which entitled a 20% discount on the bill. The restaurant also added 10% service charges.

(a) Did it affect the amount on the bill if a 20% discount is followed by a 10% surcharge or the other way round? Explain your answer.

(b) They ordered 10 dishes each valued $100. That means the total value on food was $1000. The bill came up to be $900. Can you comment on this amount?

Example 14

A rectangle 12 cm by 5 cm is cut once by a straight line to form two pieces. If the two pieces are rearranged to form a parallelogram, how can this be done?

Example 15

Is the statement "The sum of any two angles of a triangle is greater than or equal to the third angle" always true? Explain your answer.

Example 16

Is "The square root of any number less than the number itself" a true statement? Explain your answer.

Some considerations in setting open-ended questions

The following questions should be addressed when setting open-ended questions:

1. Are the questions set too open in nature?
   If the questions are too open, it will be difficult to match with the intended learning objectives and you will obtain results which you do not want. For example, the question "If the answer is 11, what could the question be?" is very open. Teachers will certainly receive quite a number of answers such as $4+7=11$ or $13-2=11$ which may not be the answers they expect. To avoid this to happen, teachers should let students know what you expect from them. For example, if you want your students to make use of indices or surds, you have to mention them in your question.

2. Will all students have fair and equal access to information used in the question?

3. Do the wording and instructions in the question make it clear to students to allow them to use different strategies?

4. Do you present a situation that is suitable to the abilities of your students?

Scoring open-ended questions

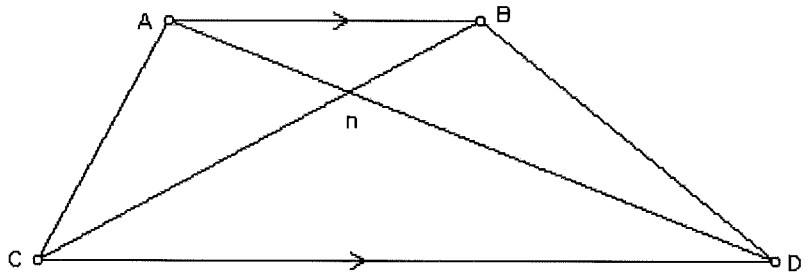
One essential element in scoring open-ended questions is the preparation of a scoring rubric. A rubric, in brief, is a set of scoring guidelines for evaluating students' work (Wiggins, 1998, p.154). It indicates the criteria on which students' performance will be judged. The following criteria could be considered in preparing a scoring rubric for open-ended questions:

- Conceptual understanding – Showing an understanding of what is asking in the task
- Processes and Strategies – Choosing strategies that can work, and carrying out the strategies chosen
- Communication – Explaining "why" at each step (using pictures, symbols and/or vocabulary)
- Interpreting Reasonableness – After completing the task, reviewing the work and showing why the solution is reasonable in relation to the task.

The following is an example on marking open-ended questions.

Example 17

Write down as many equalities (such as lengths, angles, areas etc.) as possible from the diagram below. Justify your answers.



The scoring rubrics of this problem could be as follows:

Description	Points
List ALL pairs of equal angles and triangles equal in area with appropriate reasons	5
List ALL pairs of equal angles and more than one pair of triangles equal in area with appropriate reasons	4
List ALL pairs of equal angles with reasons	3

List ALL pairs of equal angles without reasons	2
List ANY geometric equality	1
No idea	0

Example 18

Given that $x : y = 4 : 9$

(a) Write down three pairs of x and y.

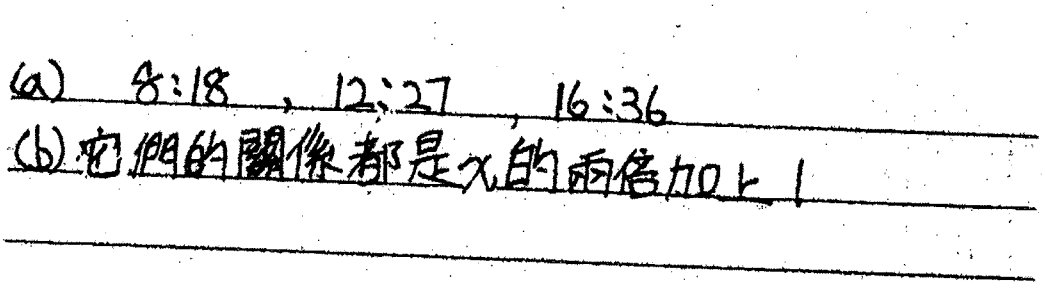
(b) What is the relationship between the numbers? Explain briefly.

The following is a suggested assessment rubrics:

Description	Points
List THREE pairs of x and y and give a relationship between x and y with valid reasons	3
List THREE pairs of x and y and give a relationship between x and y but without justifications	2
List THREE pairs of x and y only	1
No idea	0

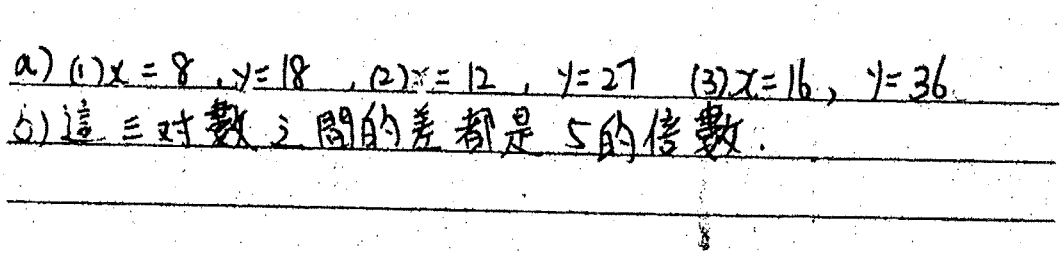
The followings are three samples of students' work on this question

Sample 1



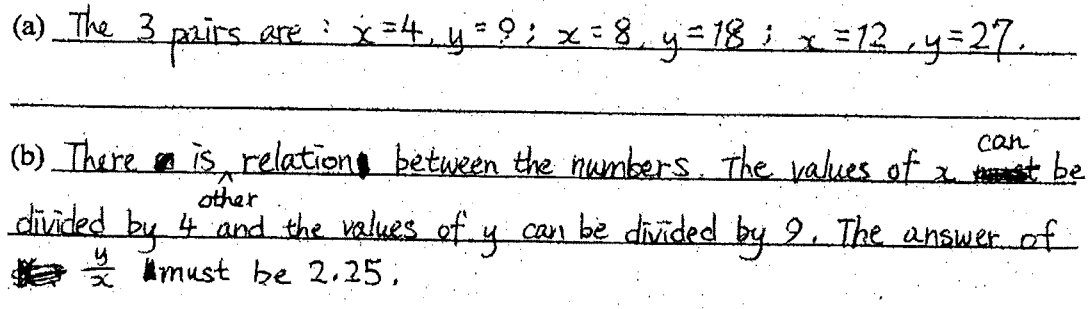
This student gets one mark only for listing three pairs of values correctly. The relationship suggested here is not correct.

Sample 2



This student gets two marks for naming three pairs of values of x and y correctly as well as suggesting a reasonable relationship between x and y. However, he/she fails to give any support on how to come up with this result.

Sample 3



This student gets three marks for naming three pairs of values of x and y correctly, giving a reasonable relationship between x and y and a simple proof on how this result is derived

Annotated student work samples

Some teachers resist the use of open-ended questions because scoring is relatively more subjective when compared with the close-ended ones. However, the quality of mathematics assessment should not rely on its accessibility of objective scoring (de Lange, 1995). It is particularly true in schools' internal assessments which are of low-stake. Nevertheless, annotated student work could be made use of to help teachers (of the same level, say) develop common understanding on learning objectives and make reliable judgments on students' achievement against the learning objectives.

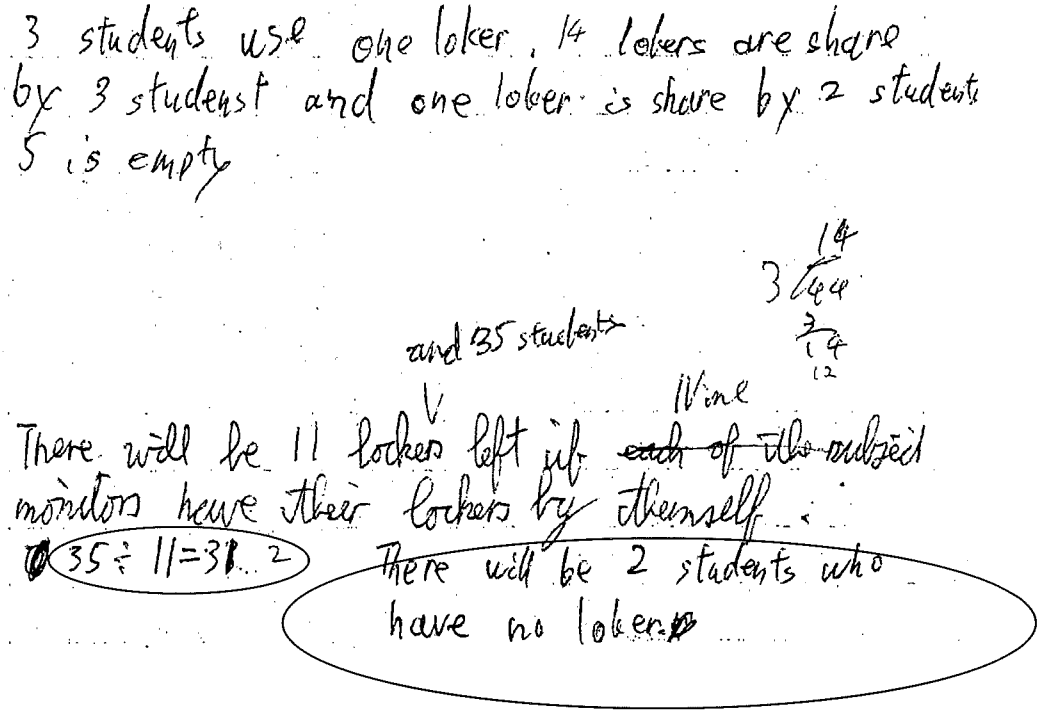
Example 19

There are 20 lockers for 44 students. Each locker could be shared among at most 3 students.

(a) Suggest a rule to allocate the lockers among the 44 students.

(b) Monitor says, "9 subject monitors should have lockers by themselves." Can this statement be done? Justify your answer.

Student work



Commentary

In this example, the student has correctly:

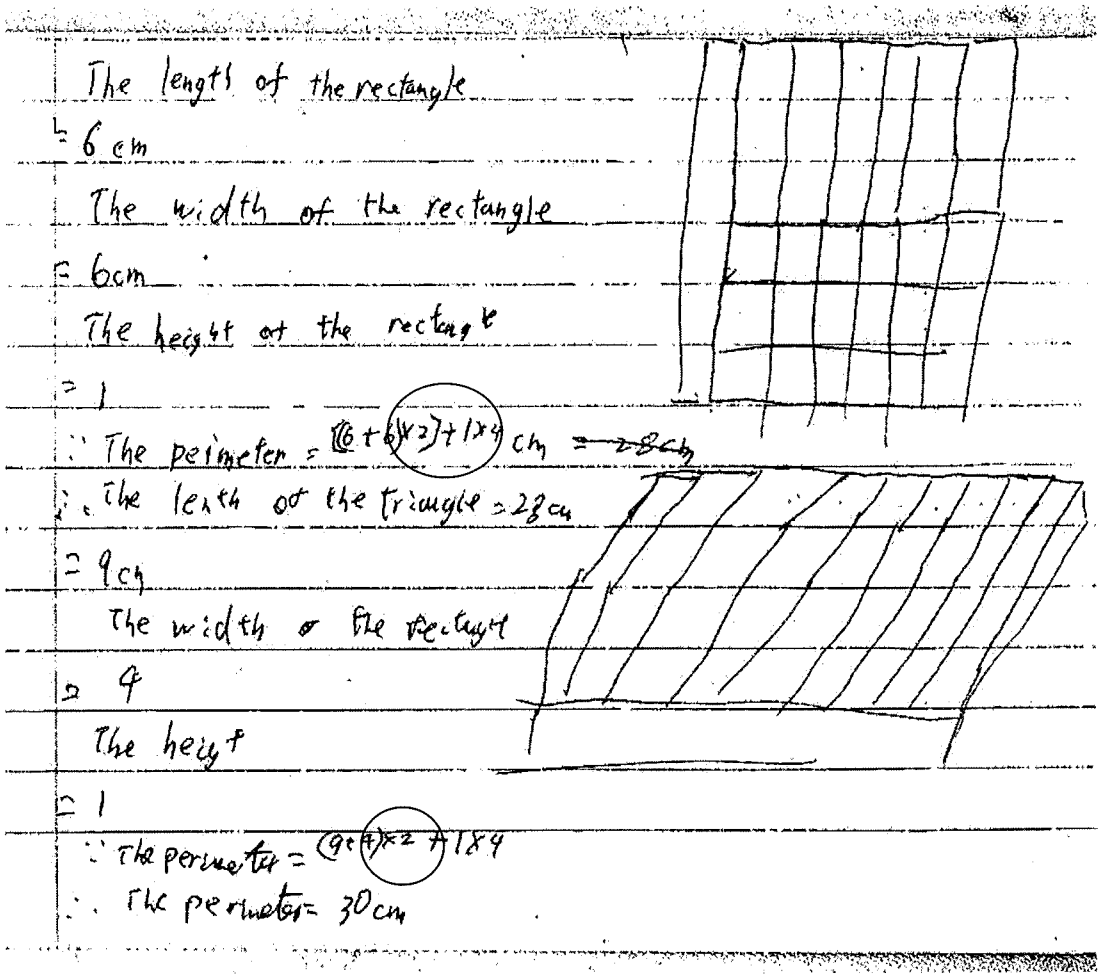
- described a way of assigning lockers to the class which satisfies the condition stated in the question;
- recognized that in part (b), the suggestion does not work by pointing out that some students will not have their lockers under such an arrangement;
- worked out that at least two students in class will not have lockers assigned to them.

By looking at the student's response, teachers can easily understand that this student understood the question well and managed to take into consideration the constraint and information. This student also attempted to use mathematical evidence to justify his/her answer.

Example 20

There are 36 squares. Each square is of length 1 cm. All the 36 squares are put together to form a rectangle. Find the perimeters of any two rectangles formed.

Student's work



Commentary

In this example the student has correctly:

- worked out two different rectangles using all 36 squares;
- identified the length and width for each rectangle.

By looking at the solution, it is quite sure that this student knew how to form two rectangles using all the 36 squares. From the working, teachers also knew that this student had got problems in finding correctly the perimeter concerned.

Example 21

If ABC is an isosceles triangle with AB = AC and N is the mid-point of BC, write down two properties relating to the segment AN.

Students' work

1.

AN divide △ABC into two equally.
BN = CN

2.

AN divides ~~∠~~ ∠A equally.
If AN is longer, it will ~~divide~~ be the mid-point of BC

3.

Its is the reflectional symmetry of the figure.
It divides the figure into two right-angled triangles.

4.

(1) ∠ANB = ∠ANC = 90° because BC ⊥ AN.
(2) △ABN ≅ △ACN (1) AB = AC (given) (2) ∠ANB = ∠ANC (proved above)
(3) AN = AN (common side) ∴ △ABN ≅ △ACN (SAS)

Commentary

In the above samples, students failed to provide two properties relating to the line segment AN. By looking at the responses, teachers should be aware of students' limited knowledge in describing geometrical properties. Students should be helped to expand their vocabularies and terminologies such as perpendicular bisector, angle bisector, median, altitude, reflectional symmetry, rotational symmetry etc. in order to better equip them with the necessary tools when they are required to describe geometrical properties in future.

Analysis on students' work is important in scoring open-ended questions. It helps teachers understand students' strengths and weaknesses and their difficulties of employing open-ended questions in their assessment.

Providing feedback to students

If assessment is to obtain information on how well a student learns and to improve a student's performance, it is not enough to just give student a grade. It is important to give quality and constructive feedback to students, which help them recognize their strengths and weaknesses and hence know how to make improvement. It should be noted that feedback is not

guidance, praise or blame. It is neither a judgment. It is a visible fact that tells a student what resulted from his/her action and shows the student a standard.

The following two examples are to illustrate how feedback could be provided to students based on their performance:

1. Referring to Example 19, the teacher needs to confirm that the student has done the right thing. The calculation given in his/her work is appropriate and reveals that under the said condition, there will be insufficient lockers for the whole class.

2. Referring to Example 20, it is obvious that the student mixes up some formulas in finding the perimeter of a rectangle. To help this student, the teacher may direct the student's attention to his/her experience in finding perimeters of simple two-dimensional figures. For example, the teacher may ask the student to add up the lengths of all the sides of a rectangle. Once the student can recall his/her experience on this topic, he/she should have no problem to get the answer.

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More examples of open-ended questions in the 3 learning dimensions, namely "Number and Algebra", "Measures, Shape & Space" and "Data Handling" can be found in Appendix 1. Two sample test papers including both multiple-choice items and open-ended questions are also included in Appendix 2. These papers are suitable to students of S2 or S3 levels. Teachers are free to try these two papers out in their schools to see if there are any significant differences in students' performance in multiple-choice items and open-ended questions.

Exchange of ideas and sharing of experience are welcome. Please direct all enquiries to the Mathematics Education Section, Education and Manpower Bureau, 4/F Kowloon Government Offices, 405 Nathan Road, Kowloon.

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Appendix 1

More Examples on Open-ended Questions

Number and Algebra Dimension

1. Write down any two numbers in two decimal places such that the value of each number is 7.2 when rounded to 1 decimal place.

2. Find three different prime numbers such that their sum is 41.

3. Find two numbers such that their quotient and difference are the same.

4. The cost on fuel is \$24/m^3 and that on electricity is \$0.25/unit. If the monthly expenditure of a family on fuel and electricity is \$250. What are the respective possible expenditures of the family on fuel and electricity?

5. Write down two inequalities which have the same solution to $\frac{x+1}{7} > 3$.

6. There are 40 students in a class. They are asked to sit in x rows and y columns. If the difference between x and y is greater than 1, find the possible values of x and y.

7. Write two sets of Pythagorean numbers (i.e. numbers a, b and c such that $a^2 + b^2 = c^2$) with numbers greater than 5.

8. If $2^a + b = 20$, find the possible values of a and b.

9. Use the information below to construct a linear equation in one unknown with a negative solution.

+3 、 –7 、 +9 、 x 、 =

10. Find the possible integral values of a and b in the following expressions so that they can be factorized.
    (a) $x^2 + ax - 32$
    (b) $x^2 + 11x + b$

11. At most 50 apples were put into a box. If the apples are divided evenly among 7 children, there are 3 left. What are the possible numbers of apples in the box?

12. There are some \$1, \$2, \$5 and \$10 coins. List out 5 possible combinations to get a total of \$50.

13. In a quiz contest, 20 marks will be awarded for each correct answer, 10 marks and 5 marks will be deducted for each wrong answer and for not giving any response respectively. If there are 10 questions in the contest, a team gets 50 marks. What are the possible numbers of correct answer, wrong answer and no response of the team?

14. 27 children stand in a line. There are 8 children in between George and Peter. Give three possible positions for George and Peter? (Write down their positions from the left hand side to the right hand side.)

15. John will go to a bank once every 30 days. Andy will go to the same bank once every 45 days. If John and Andy both went to the bank on 1 January 2003, write down three dates on which they will visit the bank in the same day.

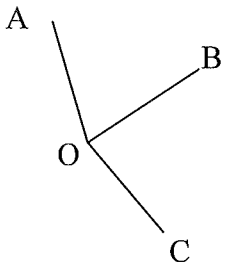
16. 50 students visit the Hong Kong Annual Book Fair. The fares of tram, bus and mini-bus per passenger are \$2, \$4 and \$6 respectively. The total fares of 50 students are \$150. It is known that most students take tram and the next is bus. Find the respective number of students taking tram, bus and mini-bus.

17. –4, 2 and 8 are three terms (not necessarily consecutive but in that order) in a sequence. Suggest two ways to continue the sequence.

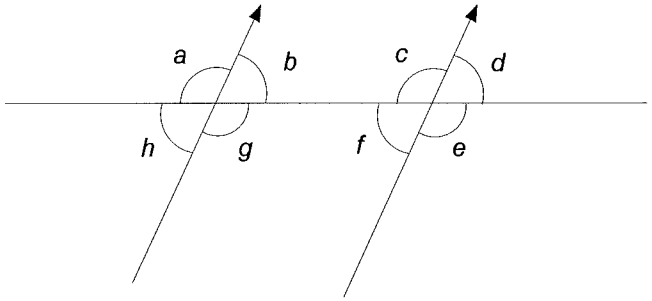
18. By inserting $+$, $-$, $\times$, $\div$ or ( ) between the numbers 2, 4, 5 and 6, and by using the given numbers once, write an expression with 24 as its resulting value.

Measures, Shape and Space Dimension

19. Name any two reflex angles in the figure below.



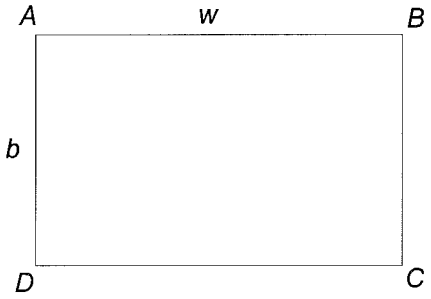
20. The figure shows a line which cuts two parallel lines. Point out 5 pairs of angles such that the sum of each pair is 180° .



21. ΔABC is an obtuse-angled triangle. If $\angle A$ is an obtuse angle, point out two relations connecting $\angle A$, $\angle B$ and $\angle C$.

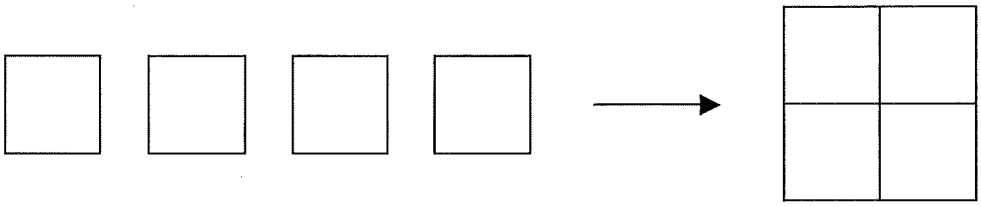
22. Write down two conditions such that a quadrilateral is a parallelogram.

23. Given a rectangle with dimensions $w \times b$ where $w > b$. By drawing line segments on the given figure, suggest two ways to divide the rectangle into four identical parts.



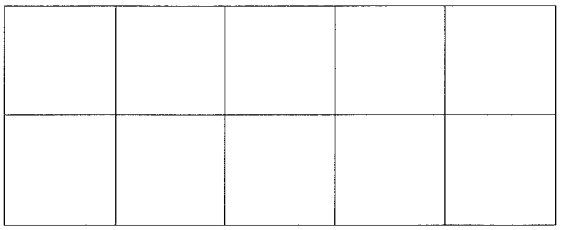
24. The sides of ΔABC are of lengths 2 cm, 3 cm and 4 cm. The sides of ΔPQR are of lengths 8cm, x cm and y cm. If the two triangles are similar, find two sets of possible values of x and y.

25.



Four identical squares can be combined to form a figure as shown above. Draw three other possible figures formed by using the four identical squares (with at least one side or one point in common between any two squares).

26.



Shade three appropriate cells in the figure above so that it has

(a) one axis of symmetry;

(b) no axis of symmetry.

Give two possible answers to both (a) and (b).

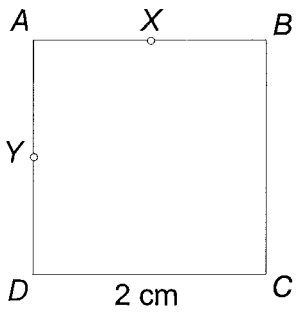
27. Name three points on a line which is parallel to the x-axis.

28. Name two points which lie on the line $y = 3x + 2$.

29. Describe the transformations on point A (–4, 4) which produce the image A' (–6, –3).

30. Write down the coordinates of two distinct points on the line which passes through (2, 3) and (–4, 1).

31. ABCD is a square of side 2 cm. X and Y are the midpoints of AB and AD respectively as shown in the figure below. Draw 5 triangles (with either X or Y as one of the vertices) whose area is 1 cm^2 in the square ABCD.



32. In the rectangular coordinate plane, draw six points which are 10 units from the point A (0, 4).

33. P (5, 2), Q (7, 2) and R lie on the rectangular coordinate plane. If the area of ΔPQR is 12 square units, find two possible coordinates of R.

34. The coordinates of the position of John are P(3, 4). He wants to go back to O (0, 0). If he could only walk horizontally or vertically, write down three possible routes.

35. Draw three figures which possess the property of reflectional symmetry.

36. Draw three figures which has 2-fold rotational symmetry (i.e. identical images appear 2 times as the figure is rotated one complete turn about its center of rotation).

Data Handling Dimension

37. The following table shows the number of family members of a group of Form One students.

Number of family members	2	3	4	5	6
Frequency	5	14	10	x	4

Give three possible values of x

(a) if the mode is 3;

(b) if the median is 3.

38. The mean age of the people attending the HK Educational Expo was 18. Does this mean that most of the people were teenagers? Give reasons to support your answer.

39. Jack took six tests. The full mark of each test was 100. His average score was found to be 80. What were his possible scores in each test?

40. John, Peter and Mary have created a new game. Each player tosses two coins. John makes a move if he gets 2 heads. Peter makes a move if he gets 2 tails. Mary makes a move if she gets 1 head and 1 tail. One who moves farthest will win the game. After five rounds, who will win the game? Explain.

Mathematics Attainment Test (Sample 1)
Time Allowed : 40 min

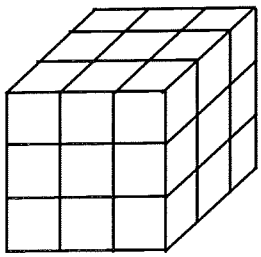
Section A Multiple Choice Questions (10 questions)
Answer ALL questions.
Each question carries 2 marks.

1. Arrange the terms $3x$, $-2x^2$, x^5 and 7 in ascending order of x.
   A. $7+3x-2x^2+x^5$
   B. $7+3x+2x^2+x^5$
   C. x^5-2x^2+3x+7
   D. x^5+2x^2+3x+7

2. If $a^8 \div a^5 = a^n$, find the value of n.
   A. 13
   B. 40
   C. 3
   D. 1.6

3. Given a sequence with terms 1, 2, 4, 8, $\cdots$, write down the 5^{th} and the 6^{th} terms of the sequence.
   A. 10, 12
   B. 10, 16
   C. 12, 16
   D. 16, 32

4. The large cube in the figure below is composed of 27 small cubes. All surfaces of this large cube are painted red. Find the ratio of the number of small cubes having one red surface to that having two red surfaces to that having three red surfaces.
   A. 3 : 6 : 4
   B. 7 : 12 : 8
   C. 10 : 9 : 7
   D. None of the above



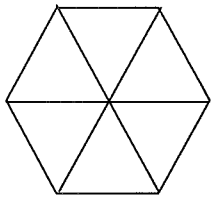
5. The figures shows a regular hexagon. How many trapeziums are there in the hexagon?

A. 6

B. 8

C. 10

D. 12



6. ABCDE and ABPQR are two regular pentagons with AB as their common side, find $\angle PBC$.

A. 144°

B. 86°

C. 44°

D. 186°

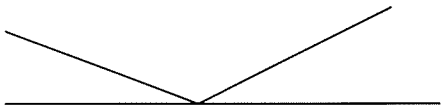
7. Find the number of obtuse angle(s) in the given figure.

A. 1

B. 2

C. 3

D. 4



8. The height of a triangular prism is 12 cm and the lengths of the sides of its base are 6 cm, 8 cm and 10 cm, find the volume of the prism.

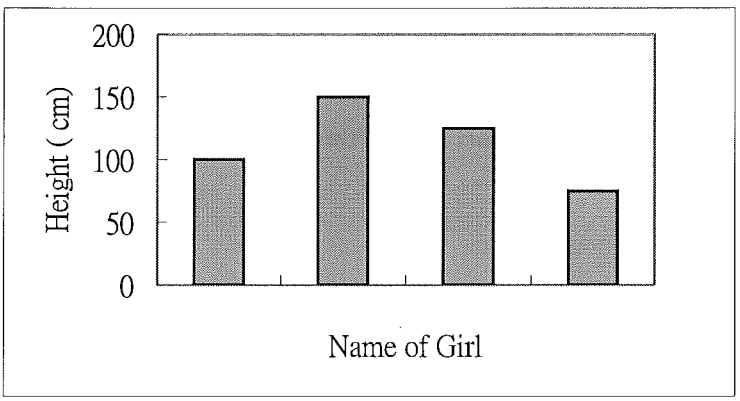
A. 480 cm^3

B. 360 cm^3

C. 144 cm^3

D. 288 cm^3

9. The graph shows the heights of four girls Amy, Debbie, Dawn and Sarah.



The names are missing from the graph. Amy is the tallest. Debbie is the shortest. Dawn is shorter than Sarah. How tall is Sarah?

A. 57 cm

B. 100 cm

C. 125 cm

D. 150 cm

10. The result of 40 students in a test is given in the following table.

Grades	A	B	C	D	E	F
No. of students	4	5	10	11	4	6

If a pie chart is drawn to show the above result, what is the central angle of the sector showing the students attaining grade 'B'?

A. 40°

B. 45°

C. 60°

D. 50°

Section B Short Questions (10 questions)

Answer ALL questions

Each question carries 5 marks.

11. Write down two binomial expressions in one unknown.

12. If x and y are positive integers, solve the equation $2x + 5y = 40$.

13. ΔABC is a right-angled triangle. It is known that A = (2, –1) and B = (–1, 2).

(a) Find three possible coordinates of C.

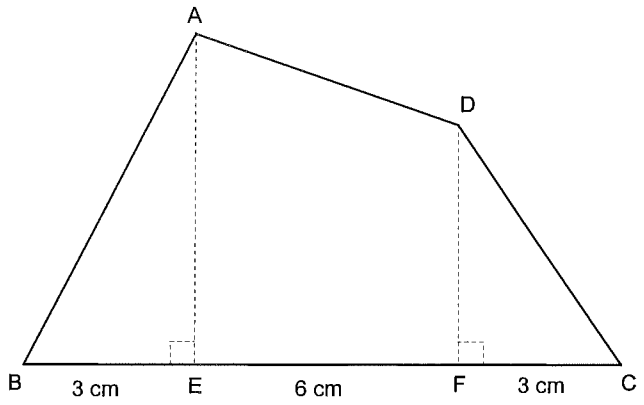
(b) Are there any more answers? Explain.

14. Find three sets of integral values of x and y satisfying $-8 < x < y < xy < 8$.

15. If ABC is an isosceles triangle with AB = AC and N is the mid-point of BC, write down two properties of the line segment AN.

16. In Δ ABC, AB = 17, BC = 15 and AC = 8. State two properties of Δ ABC.

17.



The area of the figure is 54 cm^2. How long are AE and DF?

18. Joe believes that the product of any two positive numbers (not necessarily integral) is always greater than either one of the two numbers. Write a note to Joe explaining to him when his belief is true and when it is false by giving:
    (a) an example of a product of two positive numbers in which the product is greater than either one of the two numbers;
    (b) an example of a product of two positive numbers in which the product is greater than one of the numbers and less than the other;
    (c) an example of a product of two positive numbers in which the product is greater than two of the numbers.

19. A Fibonacci sequence starts with two numbers and the subsequent numbers in the sequence is the sum of the two previous numbers. For example, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... is a Fibonacci sequence. Can you give two starting numbers that would give 100 as the sixth term of the sequence?

20. There were 50 apples. John ate some of them. The rest are then evenly put into seven boxes. What are the possible numbers of apples eaten by John?

END

Mathematics Attainment Test (Sample 2)
Time Allowed : 40 min

Section A Multiple Choice Questions (10 questions)

Answer ALL questions.

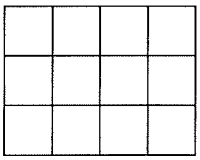
Each question carries 2 marks.

1. The constant term of the polynomial $3x - 2x^2 + x^4 - 4x$ is
   A. 0
   B. –4
   C. –2
   D. 3

2. $(3a^2b)^3 =$
   A. $9a^2b$
   B. $27a^2b$
   C. $9a^6b^3$
   D. $27a^6b^3$

3. Given that $1^3 = 1$, $1^3 + 2^3 = 9$, $1^3 + 2^3 + 3^3 = 36$ and $1^3 + 2^3 + 3^3 + 4^3 = 100$, deduce the value of $1^3 + 2^3 + 3^3 + 4^3 + \ldots + 10^3$.
   A. 1000
   B. 2500
   C. 3000
   D. 3025

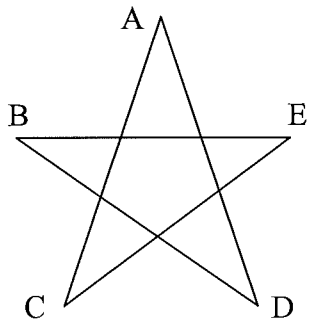
4. In the figure, there are three rows of squares. The total number of squares so formed are:



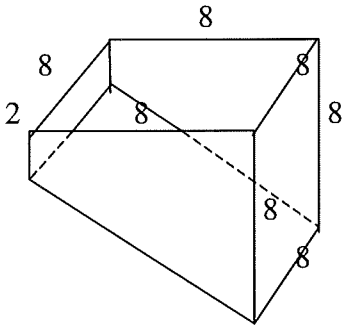
   A. 18
   B. 19
   C. 20
   D. 21

5. The number of axes of symmetry of a circle is
   A. 1
   B. 2
   C. 4
   D. None of the above

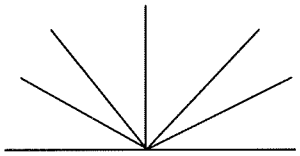
6. A star is drawn as shown.
   $\angle A+\angle B+\angle C+\angle D+\angle E=$
   A. 180°
   B. 270°
   C. 360°
   D. 540°



7. The cross-section of a solid prism is a trapezium. The dimensions, in metres, of the prism are shown in the diagram. The total surface area of the prism is
   A. 264 m^2
   B. 240 m^2
   C. 304 m^2
   D. 280 m^2



8. Find the number of acute angles in the given figure.
   A. 5
   B. 6
   C. 10
   D. 11



9. The table shows the heights of 50 students.

Heights x (m)	No. of students
$1.45 \le x < 1.55$	8
$1.55 \le x < 1.65$	12
$1.65 \le x < 1.75$	15
$1.75 \le x < 1.85$	8
$1.85 \le x < 1.95$	5
$1.95 \le x < 2.05$	2

Find the average height of the students.

A. 1.852 m

B. 1.692 m

C. 1.732 m

D. 1.752 m

10. Which of the following gives the best estimate of 184×57?

A. 200×57

B. 190×60

C. 180×60

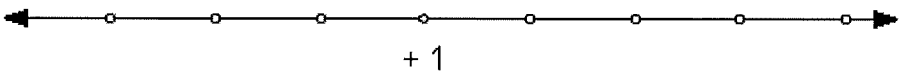
D. 180×50

Section B Short Questions (10 questions.)

Answer ALL questions.

Each question carries 5 marks.

11.

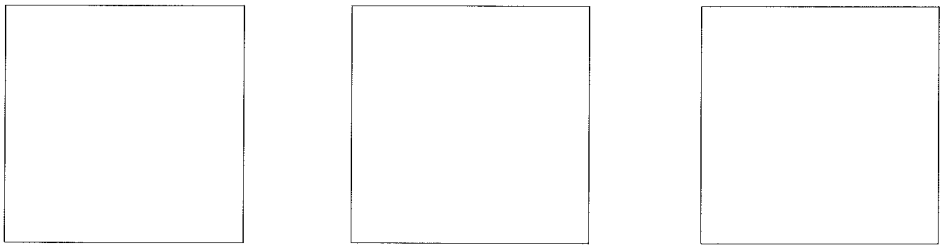


(a) Complete the scale of number line.

(b) Find the difference between the largest number and the smallest number.

12. A 12 cm by 12 cm square is divided into 12 small and congruent rectangles. Find the possible value(s) of the perimeter(s) of one of the small rectangles?

13. Suggest 3 different ways to cut a square into two congruent pieces.



14. A watch marked \$480 was on sale. The shopkeeper gave John a discount. John paid \$400 and got back the change. What was the possible discount?

15. Two consecutive terms of a sequence is 2 and 4. Give the next two possible terms.

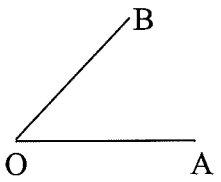
16. Given $\frac{1}{3} < \frac{p}{q} < \frac{1}{2}$ where p and q are positive integers, find two sets of possible values of p and q.

17. The table shows the distribution of wages of workers in a factory.

Wages ($)	No. of workers
2001-2200	10
2201-2400	2x
2401-2600	3y
2601-2800	4
2801-3000	8
Total	35

Find a set of possible values of x and y.

18. In the figure, the arm OA can be turned in clockwise or anticlockwise direction to OB, find three possible angles of revolution of the motion.



19. If m is not divisible by n and $m \times n = 5544$ where m and n are positive integers, find 3 possible sets of values of m and n.

20. There are 10 balls labeled 1 to 10 in the bag.
    (a) Two balls are drawn at random, one after the other, with replacement. Give three possible outcomes to get a sum of 8.
    (b) Three balls are drawn at random, one after the other, with replacement. Give three possible outcomes to get a sum of 8.

END

Appendix 3

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