Example 1: Statistics and life insurance

Key Stage : 3

Strand: Number and Algebra, Data Handling

Learning Unit: Inquiry and Investigation

Objectives: Learn how statistics are related to our daily lives and how they can be applied in real life situation

Pre-requisite Knowledge:

(1) Recognise the concept of weighted mean
(2) Solve application problems about weighted mean
(3) Understand the concepts of rate, ratio and proportion

Activity Content:

(i) Teachers present to students a feature article on "The Mortality Trend in Hong Kong, 1986 to 2020" compiled by the Census and Statistics Department (C&SD), which can be downloaded from the web-site: https://www.censtatd.gov.hk/en/EIndexbySubject.html?scode=160&pcode=FA100094

(i) Based on the data from Table 1 (Number of deaths, crude death rate and standardised death rate, 1986 to 2020) on page 5 and Table 2 (Age-sex specific mortality rates in selected years) on page 6, teachers can first discuss the difference between **crude death rate** and **standardised death rate** with students.

a) **Crude death rate:** Crude death rate is the ratio of deaths per 1,000 population in a given year to the total number of population in that year.

Referring to the 2020 data in Table 1, students can try how to find out the population of Hong Kong in 2020:

Deaths = 50,666
Crude death rate = 6.8
Let the population of Hong Kong be n in 2020, $\frac{50666}{n} \times 1000 = 6.8$ n = 7450882

By comparing the population of Hong Kong in 2020 = 7,484,800,

(Reference: https://www.info.gov.hk/gia/general/202108/12/P2021081200387.htm?fontSize=1)

$$\text{Crude death rate} = \frac{50666}{7487800} \times 1000 = 6.77$$

Teachers may discuss with students why the results are different, guiding them to think about the meaning of significant figures when taking approximate values.

b) **Standardised death rate:** The standardised death rate is calculated based on the age and sex distribution of the population at a given time. Here we take the 2016 Population By-census to obtain the population distribution by age and sex as a reference.

$$\text{Standardised death rate (\%)} = \frac{\sum(\text{death rate in each age group} \times \text{number of population in each age group in 2016})}{\text{total population in 2016}} \times 1000$$

Teachers can guide students to calculate the standardised death rate for men in 2020 and enhanced their understanding of the standardised death rate. The standardised death rate can be compared with the death rate measured in any year with the death rate in the reference year. In a reference year, the crude death rate is equivalent to the standardised death rate. If the standardised death rate in a given year is greater than the death rate in the reference year, it means that the number of deaths in that year is higher than expected; and conversely, lower than expected.

From the 2016 Population By-census Main Results (https://www.bycensus2016.gov.hk/data/16bc-main-results.pdf), we refer to Table 3.2 on page 61, taking male population in 2016 as the reference population, and then apply the 2020 male death rates of each age group to the reference population structure to derive the expected death number for male in each age group. The standardised death rate for male in 2020 is obtained by dividing the expected total number of deaths by the total number of population in the reference population.

The process in calculating the standardised death rate for male in 2020 is as follows:

Age group	Death by age group (per 1000 males) in 2020	Male Population (Reference Population) 2016	Expected number of deaths
0	2.1	27691	$27691 \times \frac{2.1}{1000} = 58$
1–4	0.1	117132	12
5–9	0.1	151175	15
10–14	0.1	132291	13
15–19	0.2	176507	35
20–24	0.3	220122	66
25–29	0.4	226992	91
30–34	0.5	232664	116
35–39	0.8	227333	182
40–44	1.2	233942	281
45–49	2.0	240072	480
50–54	3.3	291891	963
55–59	5.1	306795	1565
60–64	7.5	243951	1830
65–69	12.2	196395	2396
70–74	18.8	112184	2109
75–79	34.2	101223	3462
80–84	52.5	74103	3890
85+	111.6	59013	6586
total		3371476	24150

Total expected number of deaths in male = 24150

Standardised death rate for male in 2020 (per 1000 males) = $\frac{24150}{3371476} \times 1000 = 7.16$

(ii) Teachers may introduce the concept of life insurance to students. The insurance company collects a certain premium from the policyholder. When the policyholder dies, the insurance company pays a sum of insurance to the beneficiary designated by the policyholder. Such contract comprises a life insurance plan. Teachers can estimate the total amount of insurance premiums by age group from the death rates in Table 2, so that students can estimate the minimum premium that insurance companies should charge.

Calculate insurance premium:

(iii) Matching the data in Table 2, we choose the data from the 2016 Population By-census for reference. Students are required to download Table E2021A : 2021 Population Census - Main Tables (Demographic) from the website: https://www.censtatd.gov.hk/en/EIndexbySubject.html?scode=600&pcode=D5212101, selected Table A103ae: Population and Population (excluding foreign domestic helpers) by Sex, Age and Year, and process the feature article "The Mortality Trend in Hong Kong, 1986 to 2020" and the 2016 Population By-census data comprehensively, and create a spreadsheet to integrate the population by sex and age group, standardised death rate and death population in 2016.

Suggested Solution: See Age population by group and death rate_2016.xls

(iv) If an insurance company wants to market a one-year life insurance policy with an insured amount of $1,000,000 to all males in the age group aged 40-44, assuming that a male in this age group who purchases the life insurance will be compensated $1,000,000 if he dies within one year, otherwise the insurance company will acquire the whole payment. How should insurance companies calculate the premiums to be paid by policyholders?

Taking the data of male from an Excel file, we get:

Male		
Age group	**Standardised death rate**	**Population (excluding foreign domestic helpers)**
0	2.1	27691
1 – 4	0.2	117132
5 – 9	0.1	151175
10 – 14	0.1	132291
15 – 19	0.2	176507
20 – 24	0.3	220122
25 – 29	0.5	226992
30 – 34	0.6	232664
35 – 39	0.8	227333
40 – 44	1.3	233942
45 – 49	2.0	240072
50 – 54	3.1	291891
55 – 59	5.3	306795
60 – 64	8.4	243951
65 – 69	12.7	196395
70 – 74	21.6	112184
75 – 79	34.6	101223
80 – 84	60.3	74103
85+	123.0	59013
All ages	7.7	3371476

According to the data, the number of deaths = $\frac{1.3}{1000} \times 233942 \approx 304$

It is assumed that all males in this age group purchase this type of life insurance.

The amount of compensation from the insurance company = 304×1000000
$= \$304000000$

There are 233942 males in the 40-44 age group, and if the compensation is shared by them, they will pay the minimum insurance contribution for that year
$= \frac{304000000}{233942} \approx 1299.47$

$\approx \$1300$

(v) Using a similar approach, if an insurance company turns to all females in the 40-44 age group to promote the same life insurance, how much will each person pay?

Female		
Age group	**Standardised death rate**	**Population (excluding foreign domestic helpers)**
0	1.9	25644
1 – 4	0.2	109003
5 – 9	0.1	140592
10 – 14	0.1	126927
15 – 19	0.2	164246
20 – 24	0.2	215834
25 – 29	0.2	235048
30 – 34	0.3	264012
35 – 39	0.4	262495
40 – 44	0.7	287350
45 – 49	1.0	298831
50 – 54	1.7	333714
55 – 59	2.6	310673
60 – 64	4.0	249396
65 – 69	5.9	198687
70 – 74	9.6	108578
75 – 79	17.1	105151
80 – 84	33.4	92865
85+	89.3	114 268
All ages	5.2	3643314

Number of deaths in this group = $\frac{0.7}{1000} \times 287350 \approx 201$

The amount of compensation from the insurance company = 201×1000000

$= 201000000$

There are 287,350 females in the 40-44 age group, so they need to pay insurance premiums $=\frac{201000000}{287350} \approx 699.50$

≈ 700

The amount of insurance that male have to pay is almost double that of female.

Extended Question:

If the insurance company changes to a 10-year term life insurance policy with $1,000,000 sum insured for 50,000 males aged 43, what additional data are needed to determine the annual premium payable by each customer?

Suggested solution:

✧ If you can find the standardised death rate for each age between the ages of 43 and 53 in details, the insurance amount calculated will be more accurate.

✧ We build an Excel file and record the number of deaths each year, and then calculate the total insurance premium payable by the insurance company over a 10-year period, and then calculate the equivalent premium of HK $*x* for each survivor. The formula is as follows:

The total premium paid by the insurance company over a period of 10 years
= 1,000,000 × deaths of customers who purchased insurance over the 10-year period
= 1000000 × 1084
= 10840000000

The standardised death rate for the males of age group 40-44 is 1.3, for that of age group 45-49 is 2.0, and for that of age group 50-54 is 3.1. Based on these data, we can calculate the number of deaths per year, please refer to the insurance payment 10 years.xls for detailed figures, some of which are excerpted below:

Period	Age	Death number	Number of survivors
0	43	0	50000
1	44	$50000 \times \frac{1.3}{1000} = 65$	49935
2	45	$49935 \times \frac{1.3}{1000} = 65$	49870
3	46	100	49770
4	47	100	49671
5	48	99	49571
6	49	99	49472
7	50	99	49373
8	51	153	49220
9	52	153	49068
10	53	152	48916

$$1084000000 = 50000x + 49935x + 49870x + 49770x + 49671x + 49571x + 49472x + 49373x + 49220x + 49068x$$

$$1084000000 = (50000 + 49935 + 49870 + 49770 + 49671 + 49571 + 49472 + 49373 + 49220 + 49068)x$$

$$1084000000 = 495950x$$

$$x = 2185.70$$

The annual premium paid by the insured ≈ HK$ 2186

Note:

1. Insurance companies market a life insurance policy of $1,000,000 for a period of one year and $1,000,000 to 50,000 males at the age of 43. The sum insured is to be paid annually as follows:

$$= \frac{\frac{50000 \times 1.3}{1000} \times 1000000}{49935} \approx 1301.7$$

2. Why is the premium of 10-year term life insurance higher than that of 1-year term life insurance?

   It is due to death rate increases with age. The number of people who survive to buy insurance is decreasing, so everyone's premiums are getting higher and higher. Life insurance that protects 10 years should include future premiums, and compared with life insurance that protects l year, the insurance premium of the former is higher for products with the same insurance amount of 1000000.

3. The premium given in the example is called "pure premium", and the insurance premium of the actual insurance product sold by the insurance company is also added to the operating expenses of the insurance company, and the "additional premium" added to the reserve fund (to cope with the increase in the amount of insurance insured when the death rate is higher than predicted).

4. When calculating the actual premium, the insurance company may also take into account the investment income on the insurance premium that the insurance company will receive, so that it can charge a cheaper insurance premium.

Reference:

(i) Feature article “The Mortality Trend in Hong Kong, 1986 to 2020” in Hong Kong Monthly Digest of Statistics November 2021

(ii) Table 3.2 of the main results of the 2016 Population By-census

(iii) “Statistics: Mathematical Tools for Decision Making” in the 3rd issue of 2014, published on March 3 in the Newton Journal.