## The Thirty-third Hong Kong Mathematics Olympiad (HKMO) (2015/16)

#### Aims / Objectives

This is to invite secondary schools to participate in the captioned competition.

#### Details

2. The Thirty-third Hong Kong Mathematics Olympiad (33<sup>rd</sup> HKMO) is jointly organised by the Mathematics Education Section, Education Bureau and the Department of Mathematics and Information Technology of The Hong Kong Institute of Education. The aim of the competition is to develop students' mathematical abilities and foster their interest in mathematics.

3. The Heat Event will be held on 27 February 2016 (Saturday). The <u>50</u> teams with the highest scores (sum of the scores in the <u>Individual Event, Group Event</u> and <u>Geometric</u> <u>Construction Event</u>) in the Heat Event will enter into the Final Event which is scheduled for April 2016.

4. To enhance the competition, the Organising Committee of the 33<sup>rd</sup> HKMO would like to invite teachers interested in the competition to propose question(s) for the Heat or the Final Event of HKMO. Teachers interested in doing so may write down both the question (one to two questions, in English, Chinese or bilingual) and the full solution on Appendix 5d, and **FAX** it to the Mathematics Education Section, Education Bureau <u>on or before 11 December 2015 (Friday)</u>. The problem selection working group will select suitable questions for the competition. All unselected questions will be sealed and kept for one year, which will then be destroyed afterwards.

5. With commencement from the 33<sup>rd</sup> HKMO, there will be a slight change in the mode of the Heat Event. The Individual Event will include 2 parts, part A consists of 10 questions with 1 mark each and part B consists of 5 questions with 2 marks each. The total score for the Individual Event will be 20 marks. A sample paper is attached (Appendix 5c) for teacher's reference. The modes of Group Event and Geometric Construction Event will remain unchanged.

6. Schools interested in sending a team to take part in the  $33^{rd}$  HKMO are requested to complete the attached Application Form (Appendix 5a) or the e-form which is available from the website (<u>http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/sa/hkmo-33-hkmo.html</u>) and return it via <u>Fax</u> to the Mathematics Education Section, Education Bureau <u>on or before 11</u> <u>December 2015 (Friday)</u>.

7. Regulations for the captioned competition are attached (Appendix 5b).

### **Contact Person**

8. For enquiries, please contact Mr CHENG Sze-man of the Mathematics Education Section, Curriculum Development Institute, Education Bureau on 2153 7436.

### **Application Form**

Mathematics Education Section Education Bureau 4/F, Kowloon Government Offices 405 Nathan Road Kowloon Fax No.: 3426 9265 (Attn: Mr CHENG Sze-man)

#### The Thirty-third Hong Kong Mathematics Olympiad (2015/16)

My school would like to nominate the following students to take part in the captioned competition:

No.	Name of Student	Name of Student		
	Name in English	Name in	(M/F)	Class
	(BLOCK LETTERS)	Chinese		
1				
2				
3				
4				
5				
6				

Dr / Mr /Ms /Miss\* \_\_\_\_\_\_ will be in charge of the school team. \* *Delete whichever inapplicable*.

	Signature of	Principal:		
	Name of Prin	ncipal:		
	School:			
	Address:			
	Tel. No.:		Fax No.:	
School Chop	Date:			

#### <u>The Thirty-third Hong Kong Mathematics Olympiad (2015/16)</u> <u>Regulations (Heat Event)</u>

- 1. The Heat Event consists of three parts: 60 minutes for the individual event, 20 minutes for the group event and 20 minutes for the geometric construction event.
- Each team should consist of 4 to 6 members who are students of <u>Secondary 5</u> level or below. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event and the geometric construction event. Teams of less than 4 members will be disqualified.
- Members of each team, <u>accompanied by the teacher-in-charge, should wear proper school</u> <u>uniform</u> and present <u>ID Card or student identification document</u> when registering at the venue reception not later than 9:00 a.m. Failing to do so, the team <u>will be disqualified</u>.
- 4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.
- 5. Each member of a team has to solve 15 questions in the individual event (*10 questions in Part A* and *5 questions in Part B*) and each team has to solve 10 questions in the group event and 3 questions in the geometric construction event.
- 6. In the group event and geometric construction event, discussions among participating team members are allowed provided that the voice level is kept to a minimum.
- 7. Please note that
  - (a) for the individual and group events, devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will not be allowed to be used; and
  - (b) or the geometric construction event, only writing instruments (e.g. pens, pencils, etc), straightedge provided and compasses will be allowed to be used;

otherwise the team will be disqualified or risk deduction of marks.

# 8. <u>All answers in the individual event and the group event should be numerical and reduced</u> to the simplest form unless stated otherwise. No proof or demonstration of work is required.

- 9. Participants having electronic communication devices should have them turned off (including the alarm function) and put them inside their bags or under their chairs.
- 10. For the individual event, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The maximum score for a team should be 80.

- 11. For the group event, 2 marks will be given to each correct answer. The maximum score for a team should be 20.
- 12. For the geometric construction event, the maximum score for a team should be 20 (all working, including construction work, must be clearly shown).
- 13. No mark for speed will be awarded in the Heat Event.
- 14. Participants should bring along their own instruments, e.g. ball pens, pencils and compasses.
- 15. The 50 teams with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) will be qualified for the Final Event.
- 16. Awards of the Heat Event:
  - (a) For the individual event,
    - (i) candidates obtaining full score will be awarded Best Performance and Score certificates;
    - (ii) apart from the best performer(s) in (i),
      - (1) the first 2% of top scoring candidates will be awarded First-class honour certificates;
      - (2) the next 5% of top scoring candidates will be awarded Second-class honour certificates ; and
      - (3) the next 10% of top scoring candidates will be awarded Third-class honour certificates ;
  - (b) For the group event, teams obtaining full marks will be awarded Best Performance and Score certificates.
  - (c) For the geometric construction event, teams having outstanding performance will be awarded certificates of merit.
  - (d) About 10% of participating schools with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) in each region will be awarded certificates of merit.
- 17. Should there be any queries, participants should reach the Centre Supervisor immediately after the competition. The decision of the Organising Committee on the queries is final.

#### Appendix 5c

# Hong Kong Mathematics Olympiad (2015 / 2016) Heat Event (Individual) Sample Paper 香港數學競賽 (2015 / 2016) 初賽項目(個人) 模擬試卷

#### Part A 甲部

- 除非特別聲明,答案須用數字表達,並化至最簡。
   Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
- 本部各題佔分相等,每題1分。
   All questions in this section are of equal marks, each question carries 1 mark.
- 整數 x 減去 12 後是一個整數的平方。將 x 加上 19 後則是另一個整數的平方。 求 x 的值。
   An integer x minus 12 is the square of an integer. x plus 19 is the square of another integer. Find the value of x. (2010/11 Heat Event (Individual) Qu. 5)
- 2. 已知  $(10^{2015})^{-10^2} = 0.000\cdots01$ ,求 *n* 的值。 Given that  $(10^{2015})^{-10^2} = 0.000\cdots01$ . Find the value of *n*.

(2014/15 Heat Event (Individual) Qu. 2)

3. 如圖一所示, *ABCD* 為圓內接四邊形, 其中 *AD* = 5、*DC* = 14、 *BC* = 10 及 *AB* = 11。求 四邊形 *ABCD* 的面積。

As shown in Figure 1, *ABCD* is a cyclic quadrilateral, where AD = 5, DC = 14, *BC* = 10 and *AB* = 11. Find the area of quadrilateral *ABCD*.

(Modified from 2013/14 Heat Event (Individual) Qu. 5)



4. 圖二所示為一直角三角形 ACD,其中 B 是 AC 上的點且 BC = 2AB。已知 AB = a 及  $\angle ACD = 30^{\circ}$ ,求  $\theta$  的值。 Figure 2 shows a right-angled triangle ACD where B is a point on AC and BC = 2AB. Given that

AB = a and  $\angle ACD = 30^\circ$ , find the value of  $\theta$ .

(Modified from 2012/13 Heat Event (Individual) Qu. 3)



5. 學校推出每張面值為 \$10、\$15、\$25 及 \$40 的四種賣物券。甲班用若干張 \$100 紙幣買了 30 張賣物券,包括其中兩種賣物券各 5 張及另外兩種賣物券各 10 張。問甲班共用了多少張 \$100 紙幣購買賣物券?
A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40. Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types. How many one-hundred dollar notes does Class A use to buy the raffle tickets?(2010/11 Heat Event (Individual) Qu. 8)

6. 求 2<sup>2011</sup> 除以 13 的餘數。

Find the remainder when  $2^{2011}$  is divided by 13. (2010/11 Heat Event (Individual) Qu. 1)

- 7. 2<sup>20</sup>×25<sup>12</sup> 是一個多少個位的數?
   Find the number of places of the number 2<sup>20</sup>×25<sup>12</sup>.
   (2011/12 Heat Event (Individual) Qu. 4)
- 8. 甲、乙及丙三人互相傳球。甲首先將球傳出。有多少不同方案使得經過 5 次傳球後, 球會回傳給甲?

A, B and C pass a ball among themselves. A is the first one to pass the ball to the other one. In how many ways will the ball be passed back to A after 5 passes? (2010/11 Heat Event (Individual) Qu. 6)

9. 已知  $a \not \ge b$  為不相同質數,且 $a^2 - 19a + m = 0 \not \ge b^2 - 19b + m = 0$ ,求 $\frac{a}{b} + \frac{b}{a}$ 的值。 Given that a and b are distinct prime numbers,  $a^2 - 19a + m = 0$  and  $b^2 - 19b + m = 0$ . Find the value of  $\frac{a}{b} + \frac{b}{a}$ . (2011/12 Heat Event (Individual) Qu. 6)

10. 已知 
$$a_1, a_2, ..., a_n, ...$$
為一正實數序列,其中  $a_1 = 1$  及  $a_{n+1} = a_n + \sqrt{a_n} \frac{1}{4} \circ \bar{x}$   
 $a_{2015}$  的值。  
It is given that  $a_1, a_2, ..., a_n, ...$  is a sequence of positive real numbers such that  $a_1 = 1$   
and  $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$ . Find the value of  $a_{2015}$ .

(2014/15 Heat Event (Individual) Qu. 5)

### Part B 乙部

- 除非特別聲明,答案須用數字表達,並化至最簡。
   Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
- 本部各題佔分相等,每題2分。
   All questions in this section are of equal marks, each question carries 2 marks.
- 11. 若方程  $(k^2-4)x^2-(14k+4)x+48=0$  有兩個相異的正整數根,求 *k* 的值。 If the quadratic equation  $(k^2-4)x^2-(14k+4)x+48=0$  has two distinct positive integral roots, find the value(s) of *k*. (2011/12 Heat Event (Individual) Qu. 8)
- 12. 已知 y = (x+1)(x+2)(x+3)(x+4) + 20, 求 y 的最小值。 Given that y = (x+1)(x+2)(x+3)(x+4) + 2013, find the minimum value of y. (2012/13 Heat Event (Individual) Qu. 5)
- 13. 在 1 至 2015 之間(包括1及2015在內)有多少對相異整數的積是 5 的倍數?
   How many pairs of distinct integers between 1 and 2015 inclusively have their products as multiples of 5? (2014/15 Heat Event (Individual) Qu. 1)
- 14. 設 x 為實數。求  $\sqrt{x^2 4x + 13} + \sqrt{x^2 14x + 130}$  的最小值。

Let x be a real number. Find the minimum value of  $\sqrt{x^2 - 4x + 13} + \sqrt{x^2 - 14x + 130}$ . (2014/15 Heat Event (Individual) Qu. 9) 15. 如圖三, AE = 14, EB = 7, AC = 29 and  $BD = DC = 10 \circ 求$   $BF^2 \circ$ In Figure 3, AE = 14, EB = 7, AC = 29 and BD = DC = 10. Find  $BF^2$ . (Modified from 2011/12 Heat Event (Individual) Qu. 10)



完 END

# **Questions Proposal Form**

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Fax No.: 3426 9265
(Attn: Mr CHENG Sze-man)

School: \_\_\_\_\_

# The Thirty-third Hong Kong Mathematics Olympiad (2015/16)

Question:

#### Solution:

(Please use additional sheets of paper if necessary)