

Hong Kong Mathematics Olympiad (2021/22)

Individual Paper 2

香港数学竞赛 (2021/22)

个人项目卷二

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

The diagrams are not necessarily drawn to scale.

除特别指明外，所有答案须以数字的真确值表达，并化至最简。

不接受近似值。

所有附图不一定依比例绘成。

Part A

甲部

1. Let $\frac{A}{2022} = \frac{1}{1+1 \times 2 \times 3 \times \cdots \times 2022} + \frac{1}{1 + \frac{1}{1 \times 2 \times 3 \times \cdots \times 2022}}$. Find the value of A .

设 $\frac{A}{2022} = \frac{1}{1+1 \times 2 \times 3 \times \cdots \times 2022} + \frac{1}{1 + \frac{1}{1 \times 2 \times 3 \times \cdots \times 2022}}$ 。求 A 的值。

2. Both \overline{AB} and \overline{CB} are two-digit positive integers, where A , B and C are different digits. Let $d = \overline{AB} + \overline{CB}$. If $\overline{AB} \times \overline{CB} = \overline{BCBB}$ is a four-digit number, find the value of d .

\overline{AB} 和 \overline{CB} 均为两位正整数，其中 A , B 和 C 是不同的数字。设 $d = \overline{AB} + \overline{CB}$ 。若 $\overline{AB} \times \overline{CB} = \overline{BCBB}$ 是四位数，求 d 的值。

3. Suppose the equation $x^2y - 2x^2 - 3y - 13 = 0$ has only one pair of positive integer solution (x_0, y_0) . If

$a = y_0 - x_0$, find the value of a .

假设方程 $x^2y - 2x^2 - 3y - 13 = 0$ 只有一对正整数解 (x_0, y_0) 。若 $a = y_0 - x_0$ ，求 a 的值。

4. Figure 1 shows a square. The midpoint of each side is joined to the two end points of the opposite side and a four-pointed star is thus formed (the shaded part). Find the value of $\frac{\text{Area of the four-point star}}{\text{Area of the square}}$.

图一所示为一正方形。每一条边的中点都连接对边的两 endpoints，由此形成一个四角星（着色部分）。求 $\frac{\text{四角星的面積}}{\text{正方形的面積}}$ 的值。

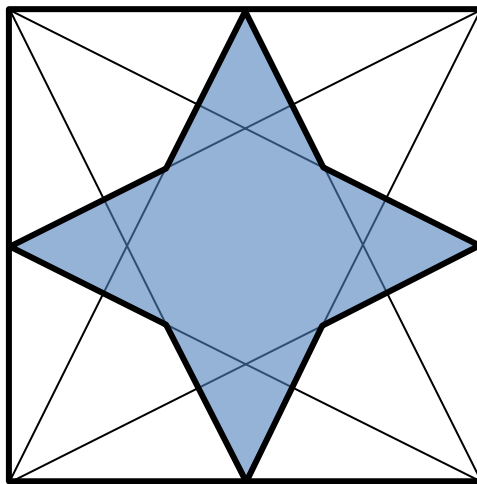


Figure 1

图一

Part B

乙部

5. $VABC$ is a right pyramid with $VA = VB = VC$ and $AB = BC = CA = a$ m. Let its height be h m and its total surface area and volume are the same. If a and h are both positive integers, find the sum of all possible values of h .

$VABC$ 为一个锥体，其中 $VA = VB = VC$ 及 $AB = BC = CA = a$ m。设它的高为 h m 及它的总表面积及体积相等。若 a 和 h 均为正整数，求 h 的可能值之和。

6. In Figure 2, $ABCD$ is a parallelogram. E is the midpoint of BC , AE and BD intersect at H , AC and DE intersect at F , AC and BD intersect at G . If the area of the quadrilateral $EFGH$ and $ABCD$ are 10 cm^2 and $k \text{ cm}^2$ respectively, find the value of k .

图二中, $ABCD$ 是平行四边形。 E 为 BC 的中点, AE 和 BD 相交于 H , AC 和 DE 相交于 F , AC 和 BD 相交于 G 。若四边形 $EFGH$ 的面积及 $ABCD$ 的面积分别为 10 cm^2 及 $k \text{ cm}^2$, 求 k 的值。

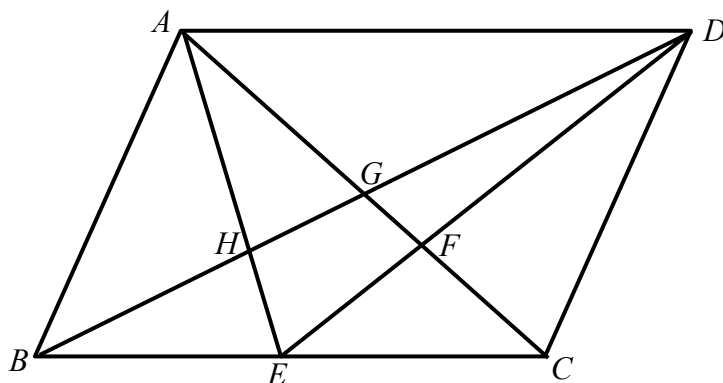


Figure 2

图二

7. Given that $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$ and $x^3 + y^3 + z^3 = 3$. Find the value of $x^4 + y^4 + z^4$.

已知 $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$ 及 $x^3 + y^3 + z^3 = 3$ 。求 $x^4 + y^4 + z^4$ 的值。

8. For all positive integers $n > 1$, a function f is defined as

$$f(1) = 2021 \text{ and } f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n).$$

Find the value of $f(2021)$.

对所有正整数 $n > 1$, 函数 f 定义如下:

$$f(1) = 2021 \text{ 及 } f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n)。$$

求 $f(2021)$ 的值。

完
END