

Hong Kong Mathematics Olympiad (2021/22)

Individual Paper 2

香港數學競賽 (2021/22)

個人項目卷二

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

The diagrams are not necessarily drawn to scale.

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。

不接受近似值。

所有附圖不一定依比例繪成。

Part A

甲部

1. Let $\frac{A}{2022} = \frac{1}{1+1 \times 2 \times 3 \times \cdots \times 2022} + \frac{1}{1 + \frac{1}{1 \times 2 \times 3 \times \cdots \times 2022}}$. Find the value of A .

設 $\frac{A}{2022} = \frac{1}{1+1 \times 2 \times 3 \times \cdots \times 2022} + \frac{1}{1 + \frac{1}{1 \times 2 \times 3 \times \cdots \times 2022}}$ 。求 A 的值。

2. Both \overline{AB} and \overline{CB} are two-digit positive integers, where A , B and C are different digits. Let $d = \overline{AB} + \overline{CB}$. If $\overline{AB} \times \overline{CB} = \overline{BCBB}$ is a four-digit number, find the value of d .

\overline{AB} 和 \overline{CB} 均為兩位正整數，其中 A , B 和 C 是不同的數字。設 $d = \overline{AB} + \overline{CB}$ 。若 $\overline{AB} \times \overline{CB} = \overline{BCBB}$ 是四位數，求 d 的值。

3. Suppose the equation $x^2y - 2x^2 - 3y - 13 = 0$ has only one pair of positive integer solution (x_0, y_0) . If

$a = y_0 - x_0$, find the value of a .

假設方程 $x^2y - 2x^2 - 3y - 13 = 0$ 只有一對正整數解 (x_0, y_0) 。若 $a = y_0 - x_0$ ，求 a 的值。

4. Figure 1 shows a square. The midpoint of each side is joined to the two end points of the opposite side and a four-pointed star is thus formed (the shaded part). Find the value of $\frac{\text{Area of the four-point star}}{\text{Area of the square}}$.

圖一所示為一正方形。每一條邊的中點都連接對邊的兩端點，由此形成一個四角星(著色部分)。

求 $\frac{\text{四角星的面積}}{\text{正方形的面積}}$ 的值。

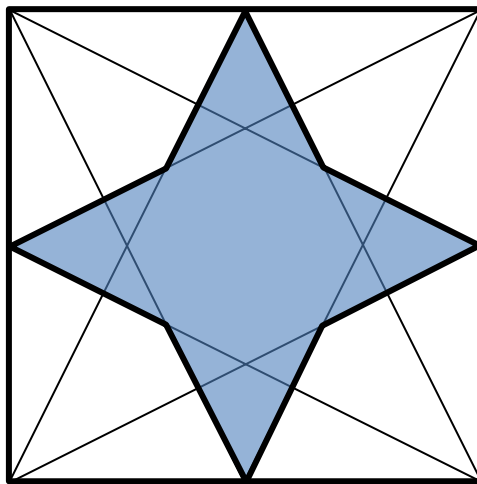


Figure 1

圖一

Part B

乙部

5. $VABC$ is a right pyramid with $VA = VB = VC$ and $AB = BC = CA = a$ m. Let its height be h m and its total surface area and volume are the same. If a and h are both positive integers, find the sum of all possible values of h .

$VABC$ 為一個錐體，其中 $VA = VB = VC$ 及 $AB = BC = CA = a$ m。設它的高為 h m 及它的總表面積及體積相等。若 a 和 h 均為正整數，求 h 的可能值之和。

6. In Figure 2, $ABCD$ is a parallelogram. E is the midpoint of BC , AE and BD intersect at H , AC and DE intersect at F , AC and BD intersect at G . If the area of the quadrilateral $EFGH$ and $ABCD$ are 10 cm^2 and $k \text{ cm}^2$ respectively, find the value of k .

圖二中， $ABCD$ 是平行四邊形。 E 為 BC 的中點， AE 和 BD 相交於 H ， AC 和 DE 相交於 F ， AC 和 BD 相交於 G 。若四邊形 $EFGH$ 的面積及 $ABCD$ 的面積分別為 10 cm^2 及 $k \text{ cm}^2$ ，求 k 的值。

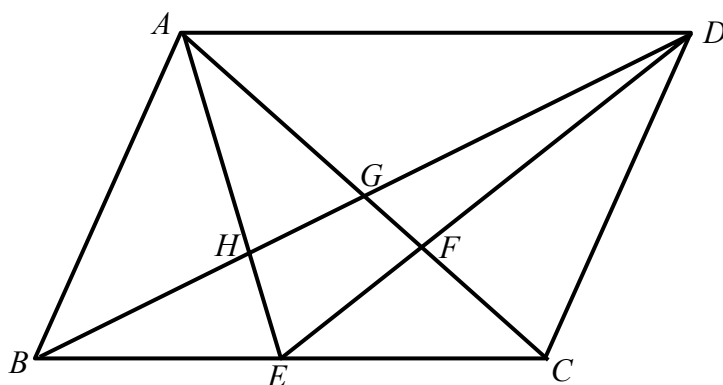


Figure 2

圖二

7. Given that $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$ and $x^3 + y^3 + z^3 = 3$. Find the value of $x^4 + y^4 + z^4$.

已知 $x + y + z = 1$ ， $x^2 + y^2 + z^2 = 2$ 及 $x^3 + y^3 + z^3 = 3$ 。求 $x^4 + y^4 + z^4$ 的值。

8. For all positive integers $n > 1$, a function f is defined as

$$f(1) = 2021 \text{ and } f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n).$$

Find the value of $f(2021)$.

對所有正整數 $n > 1$ ，函數 f 定義如下：

$$f(1) = 2021 \text{ 及 } f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n)。$$

求 $f(2021)$ 的值。

完
END