# 學校數學通訊

# School Mathematics Newsletter

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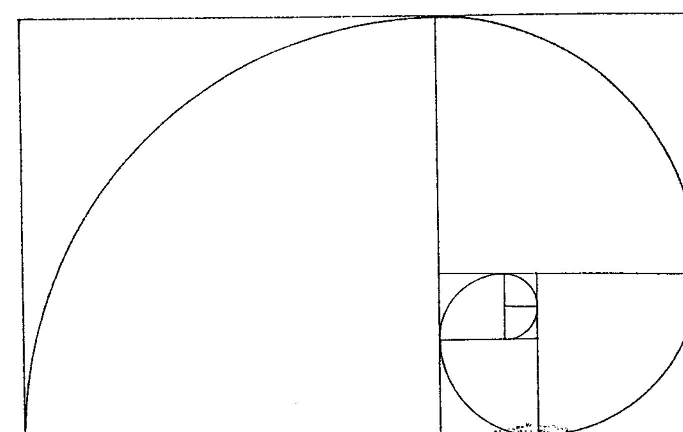

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The School Mathematics Newsletter aims at serving as a channel of communication in the mathematics education of Hong Kong. School principals are therefore kindly requested to ensure that every member of their mathematics staff has an opportunity to read this Newsletter.

We welcome contributions in the form of articles on all aspects of mathematics education as the SMN is meant for an open forum for teachers of mathematics, however, the views expressed in the articles in the SMN are not necessarily those of the Education Department, Hong Kong.

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《學校數學通訊》旨在爲香港數學教育界提供一個溝通渠道，故此懇請各校長將本通訊交給貴校所有數學科教師傳閱。

爲使本通訊能成爲教師的投稿公開園地，歡迎讀者提供任何與數學教育有關的文章。唯本通訊內所發表的意見，並不代表教育署的觀點。

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# FOREWORD

Welcome to the thirteenth issue of the School Mathematics Newsletter (SMN).

As in the past issues of the SMN, the articles in the present issue are contributed by professionals interested in mathematics education, many of whom have rendered uninterrupted support to the publication of the SMN in the past years. The Mathematics Section of the Advisory Inspectorate Division wishes to thank them sincerely for their contributions, without which the publication of this issue of the SMN will not be possible.

Mathematics teachers in Hong Kong most likely have to face unprecedented changes/challenges in their classroom teaching in the near future. The Target Oriented Curriculum (TOC) Initiative will start to be implemented in primary schools from September, 1995 onwards. The development of the TOC Programme of Study for secondary schools will soon begin. Ways of assisting students to develop higher order process skills in mathematics learning, e.g. inquiry, reasoning, problem solving, etc., through various content areas and assessing these skills formatively and summatively will be of key concern to mathematics teachers in the years to come. There is a consensus that a core curriculum, which constitutes 60 - 70% of the main stream curriculum, is needed for the 'average' students. However, how to make the implementation of the core curriculum compatible with the TOC Initiative of setting common targets of learning and assessment for all students at different stages of schooling needs thorough consideration. A small group of enthusiastic mathematics teachers and educators are now proposing to restructure the existing primary, secondary and sixth form mathematics curricula to make them more coherent and up-to-date to meet the needs of students and the modern society.

All the above issues are important ones that will shape our future mathematics curricula in primary and secondary schools. Tackling these issues demands the collecting of ideas and suggestions from different sectors involved in the mathematics education of our youngsters and seriously discussing these ideas/suggestions. A lot of experience sharing and debating are needed before concerted efforts from all parties concerned help to develop new curricula which front line mathematics teachers are happy to implement in classrooms.

The SMN has a wide circulation as it is distributed to all primary and secondary schools, and also to professionals interested in mathematics education. Hence the SMN is best served as an open forum for expressing ideas/suggestions, sharing experiences and holding constructive debates on issues mentioned above. The Editorial Board of the SMN sincerely invites readers to make use of the publication to voice their opinions and contribute to the formulation of the forthcoming new mathematics curricular initiatives.

Mathematics Section
Advisory Inspectorate Division



# 把數學變成有趣的一科：從香港大學「數學漫話」公開講座談起

香港中文大學課程與數學學系
黃毅英

香港大學數學系於三月十二及廿六日舉辦兩次的「數學漫話」公開講座，反應空前熱烈，可謂一時無兩。

該次講座原定於香港大學梁球琚樓舉行，可是消息公布後不久，查詢電話已不絕於耳。有見及此，有關方面決定將兩次講座分別移師至較大的圖書館新翼及新落成的第四期大樓，並再關毗鄰的班房作即場電視播放。事實證明轉移陣地確有必要，兩日來兩個房間近六百座位不只瞬即擠滿，不少後來者更蓆地而坐或站在墻邊……。參與此盛會的包括教師、中學生、一般聽眾及其他教育界人士，場面之鼎盛可謂近年難得一見。

## 費馬最後定理

第一講乃由數學系梁鑑添博士漫話三百五十年的數學懸案：費馬最後定理。

費馬本來爲法國一名律師，但酷愛數學。他在書中扉白寫滿了筆記，其中亦有論及 $x^n+y^n=z^n$ 的整數解。

眾所周知，勾股定理說明，若 x, y, z 爲直角三角形的邊長,其中z爲斜邊的長度，則 $x^2+y^2=z^2$ 。且我們可找出不少邊長爲整數的直角三角形。如《周髀算經》所云：「勾三、股四，弦五」，即有 $3^2+4^2=5^2$。然費馬在扉白中卻指出，若 $n\geq3$,則 $x^n+y^n=z^n$ 不可能有整數解。他把此事寫出後，並謂：「因爲空位不足，故無法將證明寫下。」由於費馬的其他猜想都被證明了，唯獨缺此，故名爲「費馬最後定理」。三百五十年來，無數數學工作者一直不能證明費馬最後定理是對還是錯。

這數學史上的最大懸案，直至去年六月，普林斯頓大學的懷爾教授發表了一道定理之證明，才得到突破。因爲大家知道，該定理證畢亦即費馬定理也得到證實了。然而至今各地數學家仍在細心核實中。在三個半世紀以來，此定理之推敲引動了無數有力數學工具的發展，並開拓了不少的數學分支。

梁博士更借用了王國維《人間詞話》中所言三個階段來描述：首先，數學家致力開首的幾個 n，可謂「昨夜西風凋碧樹，欲上高樓，望盡天涯路」，不知何時才把所有 n 都證畢；其後數學工作者用不同的方法，處理一組組的 n，可謂「衣帶漸寬終不悔，爲伊消得人憔悴」而突飛猛進。最後則致力打破最後的難關，也可說是到了「眾裏尋他千百度，驀然回首，那人卻在燈火闌珊處」的地步。



杵臼關節、阿基米德、多面體

數學就是這樣，有其引人入勝之處，而且又常常在我們身邊。正如蕭文強教授在第二講「杵臼關節、阿基米德、多面體」一開始便引用中國數學家華羅庚所說：「宇宙之大、粒子之微、火箭之速、化工之巧、地球之變、生物之謎、日用之繁，無處不用數學」。

數年前，蕭教授收到醫學院某同事的一個電話，詢及一個問題。原來人體中杵臼關節磨損程度和兩者相接的面積有關。這變成了從平面照片投射反過來計算球體上面積的問題。一時間蕭教授還需點思索才能解答，當夜回家與其兒子玩耍，剛好有一件球狀之玩具，由此實物之研究，他便得到了答案。

然而蕭教授卻不因此滿足。他從文獻中發現古希臘阿基米德已獲至如此美妙的結果，但對多面體的一些性質卻要待二千年後方見眉目。追本溯源，這些性質原來要跳出傳統度量幾何才能眞相大白，此亦尋出幾何發展的一條脈絡。

從數學發展的歷程回顧，整個線路即清淅可見。從歐拉給歌德巴赫的信中可以看到多面體角和原來與一道與角度及長度無關的性質等價。此爲頂點、棱與面的關係。而個中卻須跳出傳統度量幾何、規限長度（距離）不變的局限才能見其眞貌，由距離不變進而到其他不變量的「幾何」體系,亦是近世拓樸學萌芽的其中一個催化劑。

## 把數學變得有趣點

自從普及教育實施之後，學校教學頗有惶惶不可終日之感。學生成績低落、學習動機薄弱、如是衍生種種課堂紀律問題、學生情緒問題和學生行爲問題，其中又伴隨著家庭問題等等。

無數的教育研究及教育政策的因應推行，新的教學法、編班制度、加強輔導人員與駐校社工，以至甚麼通達學習、愉快學習、輔導教學、按能力分班、校本課程、目標爲本課程、核心課程……如此種種，都是希望解決普及教育中的不同學習問題。

然而其中有多少措施能直接攻入問題的核心，多少則只是圍著邊皮、期待著主力軍的出現呢？

學習就是那麼奇妙的一件事：在校園裏多種些花、在運動場上多加數張椅，其功效有時不容低估。種多些花又有什麼教育效果呢？說不定，種多些花可能增加了學生的歸屬感、陶冶了他們的性情……教育本來就是一件潛移默化的事。

學習目標的細心釐定、課程的設計、課本與教具的運用和教師的敷陳固甚重要，但學生能生活在一有助學習的環境是必需的。

只要能引發學生的學習興趣與動機，他們就如迷上了歌星一樣，會自動尋找多些資料，從而知多點、識多點……。其他甚麼讀書方法與材料等都顯得次要，一切就是這般的毋須加工便奇妙地自然啓動。



引發興趣的問題，作爲必修科的數學尤爲首要。筆者近數年因工作關係每年都收到近百份數學教師精心設計的教學計劃，每份都著意教學過程的編排，如何把課題清晰地闡述給學生。然而筆者以爲，如何把數學變成一個學生樂於學習、甚至扣人心絃的學科是同樣重要的。數學向來給人高不可攀的印象以致製造了不少憎惡數學者。如何撇開生硬的式子與怕人的繪圖，將數學平實的，以普及漫話的方式告訴學生早已成爲數學教學者的一大課題。

從兩次「數學漫話」公開講座的反應可以清楚看到，數學講座等活動實在大受同學的歡迎。不少同學是十分希望能知多點課本以外的數學知識，除了公開講座之外，普及課外讀物亦是提高學習數學興趣的一個方法。此次講者之一的蕭教授所著的《一、二、三……以外》普及讀物便是爲人所津津樂道者。此外，只要大家能從「學會了些甚麼」此類有形的成效釋放出來，現時校內數學學會亦大有進一步發展的潛質。

有些人誤以爲數學所重視的只是一大堆的公式，這些公式的眞確性和利用這些公式得到準確的答案。從費馬定理的探求正正顯示在這些之外還有更廣闊的天地。

費馬最後定理提出以來，至今雖然仍未證實得到完全的解決，然而，在這數百年的求證過程中，數學家成功地找出了一些新的方法和開拓了一些新的數學領域。正如梁博士在講座中說，數學的探究是累積性的，而梁博士當天正是從歷史的發展縷述箇中的來龍去脈。

不單如此，梁博士更細意闡述數題的思路，讓聽眾所聽到的不光是一堆數學事實，而能掌握其中的原委。其中涉及的一些結果更非長時間細心閱讀無法明瞭，梁博士卻細心分析和鋪陳其中證明的策略，使聽眾亦可在很短時間內深入問題的關鍵與核心。

在上述種種之中，聽眾更被梁博士之笑容與風度所感染，大家不其然浸淫在數學探究的氣氛裏。

從蕭教授的講座中亦可感受到數學在枯槺的操練以外，亦有有趣的天地。

他不只將歷史步伐清楚勾畫，更使聽眾體驗到數學在繁複運算之外，還有類比、歸納、找尋佐證等探索與思路。蕭教授更特別指出不少人忽略學習數學中親自動手研究實物與實例，處理個別情況等活動的重要性。從歸納眾多的例子，實有尋出通則的可能。

更可一提的是蕭教授的講述與講義的一大特色是在其內容裏埋下不少伏線，給有興趣者甚多繼續探求的空間。「舉一」希望能達至「反三」不正是教育的一個重大原則嗎？



# 120°三元數組

水戈木

【前言：畢氏三元數組】

眾所周知，在三角形ＡＢＣ中，如果∠Ｃ爲直角，則三角形的三條邊長有以下的關係：

$$a^2+b^2=c^2 \ldots\ldots [1]$$

這就是著名的「畢達哥拉斯定理」,或簡稱爲「畢氏定理」(國內則稱之爲「勾股定理」。）。相傳在西方，這定理最先是由畢達哥拉斯證明的，因此這定理就以他的姓氏來命名。畢氏的另一貢獻，就是指出有不少三元整數組合能夠滿足〔1〕式,如(3,4,5)、(5,12,13)等等。他並且提出了一個有效的方法來推算某些三元數組（詳情請閱參考書目（一））。因此，人們亦稱那些數組爲「畢氏三元數組」。

事實上，到了今天，已經有人研究出求「畢氏三元

數組」的具體方法。

首先，由三角函數的關係式中，我們知道

$\cos^2\theta+\sin^2\theta=1$

假如令 $t=\tan\frac{1}{2}\theta$，則

$$\left(\frac{1-t^2}{1+t^2}\right)^2+\left(\frac{2t}{1+t^2}\right)^2=1$$

或 $(1-t^2)^2+(2t)^2=(1+t^2)^2$

如果 t 爲一有理數，t 便可改寫爲 $\frac{v}{u}$，而 u、v 則爲整數。將 $t=\frac{v}{u}$ 代入上式，並將全式倍大 $u^4$ 倍，便可得到

$$(u^2-v^2)^2+(2uv)^2=(u^2+v^2)^2$$

比較[1]式，並不妨設 $u>v>0$，便得

$a=(u^2-v^2),\quad b=2uv,\quad c=u^2+v^2$......〔2〕

因爲 u、v 爲整數的關係,〔2〕式無疑就是製造「畢氏三元數組」的工具。藉此,我們可以將「畢氏三元數組」列出。(見《表一》)。

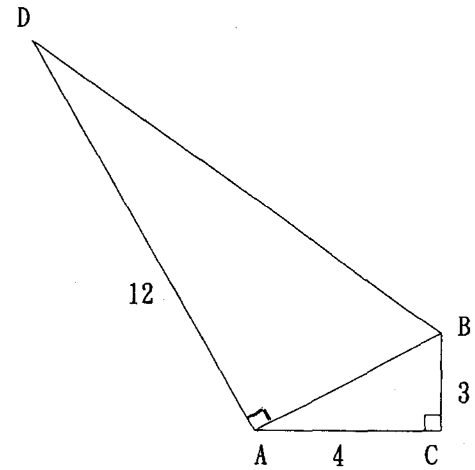


| v \ u | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | (3,4,5) | (8, 6,10) | (15, 8,17) | (24,10,26) | (35,12,37) |
| 2 | --- | (5,12,13) | (12,16,20) | (21,20,28) | (32,24,40) |
| 3 | --- | --- | ( 7,24,25) | (16,30,34) | (27,36,45) |
| 4 | --- | --- | --- | ( 9,40,41) | (20,48,56) |
| 5 | --- | --- | --- | --- | (11,60,61) |

表一：畢氏三元數組

對我來說，《表一》是非常有價值的。因爲基於一些固執的想法，我總希望由我所擬定的測驗或考試題目，每當應用完「畢氏定理」和開方後，最後的結果依然是一個整數。而《表一》就能夠提供非常有用的數據，讓我去設計這類試題。下題就是其中之一個例子：

在圖二中，求ＢＤ的長度。



圖二

【問題提出：　120°三元數組】

在一般的三角形ＡＢＣ中，如果要表達它的三條邊長的關係，就需要借助「餘弦定律」

$$c^2 = a^2 + b^2 - 2ab\cos\angle C \ldots\ldots.[3]$$

明顯地，假如∠Ｃ爲直角，則[3]式就等同於[1]式；換句話說，「餘弦定律」就是「畢氏定理」的一個推廣。

幾年前，在一次偶然的情況下，意外地發現了一個結果：假若∠Ｃ爲 120°，則(3,5,7) 是滿足[3]式的三元整數組合。這個發現，刺激起我對尋找那些當∠Ｃ等於120°，又能夠滿足[3]式的三元整數組合的興趣；在此統稱它們爲「120°



三元數組」。

首先，將∠Ｃ＝120°代入[3]式，得

$$c^2 = a^2 + b^2 + ab \ldots\ldots.(4)$$

下一步，當然就是想辦法構作一套類似[2]式的工具，令我們可將那些三元數組逐一列出。很可惜，（可能因爲我天資不夠的緣故！）經過多次嘗試和研究，也無法得到任何成果。繼而唯有退而求其次，嘗試用電腦去進行搜尋。以下就是一個我用來搜尋「120°三元數組」的GW-BASIC程式：

```
100 REM SEARCH FOR 120-TRIPLETS
110 C = 1
120 AGAIN = 1
130 WHILE AGAIN = 1
140  B = 1
150 WHILE B < C AND AGAIN = 1
160    A = 1
170    WHILE A < B AND AGAIN = 1
180      T = A*A + B*B - C*C + A*B
190      IF T = 0 THEN GOSUB 280
200      A = A + 1
210      IF INKEY$ = " " THEN AGAIN = 0
220     WEND
230     B = B + 1
240     WEND
250     C = C + 1
260 WEND
270 END
280 PRINT "A =";A;", B = ";B;", C = ";C
```

290 RETURN
從上述程式所得的結果，我們不難找出下面習題的答案。

> 兩艘船A和B正向著一港口C航行。由船A測得C的方位角爲070°而與C的距離爲24km。又由船B測得C的方位角爲310°而與C的距離爲21km。求兩艘船之間的距離。

利用電腦去計算〔4〕式的整數解，始終不夠「數學化」。到底有沒有類似〔2〕式的方法去計算「120°三元數組」呢？現在我們知道∠C＝120°時的結果，那麼，還有沒有其他的∠C，有如此美妙的結果呢？又或說得準確些，有沒有其他的整數角C，同樣可以令〔3〕式有整數解呢？以上兩個問題，都是有待解答的。

【初步解答： 60°三元數組】

如果將「餘弦定律」改寫爲以下的形式：

$$\cos\angle C=\frac{a^2+b^2-c^2}{2ab} \quad ......〔5〕$$

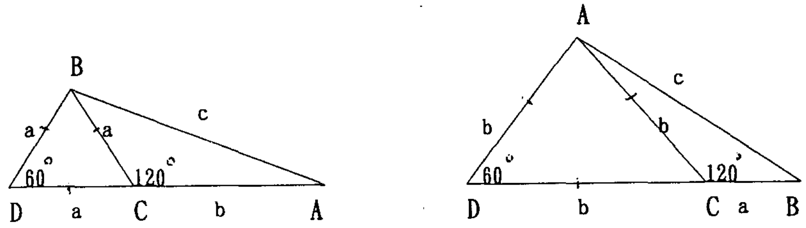
則可以發現：假若a、b及c爲整數，則cos∠C必定是有理數。而滿足cos∠C爲有理數且∠C爲整數角度的要求，就祇有∠C爲60°、90°和120°三個情況。所以我們除了可以找到「畢氏三元數組」和「120°三元數組」之外，就祇有「60°三元數組」了。

要尋找「60°三元數組」，當然可以改寫上文所提及



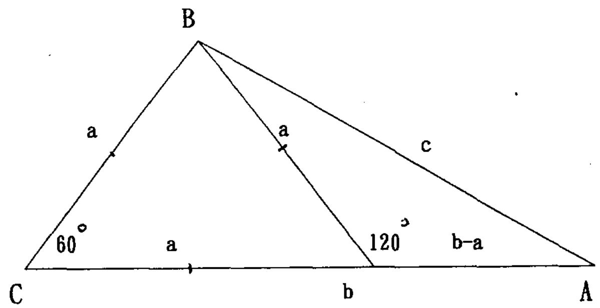
的電腦程式，去進行搜尋。但是，我們亦可以借助上文所提及的「120°三元數組」，再通過一些簡單的幾何性質去尋找。

首先，留意等邊三角形的每一隻角都是60°，所以對於任何整數n,(n,n,n)必定是一個「60°三元數組」。




圖三

另外，通過《圖三》，就可以明白，如果(a,b,c)爲一「120°三元數組」，則(a,a+b,c)和(b,a+b,c)都會是「60°三元數組」。相反，如果一個有60°角的非等邊三角形，我們祇要考慮該三角形的最短邊，就可以將該三角形分割成一個等邊三角形和一個有120°角的三角形。（見《圖四》）




圖四

因此，如果要求出「60°三元數組」，亦等價於求「120°三元數組」了。

【最後解答：圓滿解決】

今年年中，在另一個更偶然的機會下，無意中從某大學的圖書館內，發現了兩篇文章（即參考書目中的（二）和（三）），才知道原來我思考了多年的「120°三元數組」問題，早已有人提出了解答。現在就將他們的計算方法轉錄如下：

首先，將〔4〕式轉化為

$$ab = (a + b - c)(a + b + c) \cdots\cdots〔6〕$$

觀察〔6〕式，我們可以將它理解為將一個乘積，以兩種方法進行因式分解。假如它們的公共乘積為pqrs，我們不妨設

$a=pq$ ....（1）
$b = rs$ ....（2）
而 $a + b - c = pr$ .....（3）
和 $a + b + c = qs$ ....（4）

由（3）+（4）得 $2(a + b) = pr + qs$ ……（5）
由（4）-（3）得 $2c = qs - pr$ ……（6）

將（1）、（2）代入（5）得

$$2(pq + rs) = pr + qs$$



$\Longrightarrow p(2q - r) = s(q - 2r)$

$\Longrightarrow \dfrac{p}{q-2r} = \dfrac{s}{2q-r}$

假如 k 為上比例式的變數法常數且其值大於零，則

$$p = k(q - 2r), \qquad s = k(2q - r)$$

代入（1）得 $a = k(q^2 - 2qr)$
代入（2）得 $b = k(2qr - r^2)$
代入（6）得 $2c = k(2q^2 - qr) - k(qr - 2r^2)$
即 $c = k(q^2 - qr + r^2)$

因為 $a > 0 \Longrightarrow q^2 - 2qr > 0$
$\Longrightarrow q > 2r$ ……（7）

又因 $b > 0 \Longrightarrow 2qr - r^2 > 0$

$\Longrightarrow q > \frac{1}{2}r$ ……（8）

結合（7）和（8），則祇得 $q > 2r$。

總結以上的結果，並將 k 設定為1，我們就會有以下的結論：
假如 q、r 為正整數，$q > 2r$，而

$$\left.\begin{aligned} a &= q^2 - 2qr \\ b &= 2qr - r^2 \\ c &= q^2 - qr + r^2 \end{aligned}\right\} \cdots\cdots〔7〕$$

則 (a,b,c) 爲一個「120°三元數組」。

利用〔7〕式，就可以將「120°三元數組」列出。(見《表二》)

| r \ q | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| 1 | (3,5,7) | (8,7,13) | (15,9,21) | (24,11,31) | (35,13,43) |
| 2 | --- | --- | (5,16,19) | (12,20,28) | (21,24,39) |
| 3 | --- | --- | --- | --- | (7,33,37) |
| 4 | --- | --- | --- | --- | --- |

表二：120°三元數組

獲得《表二》和〔7〕式，可以算是大功告成了。但祇要細心觀察《表二》中的數據時，就不難發現，其實我們還有很多有趣問題，也是有待進一步研究的。

例如，由表內的數組所作出來的三角形，有不少是「相似」的。譬如 (3,5,7) 和 (15,9,21)。到底在甚麼情況下，由$(q_1,r_1)$ 和$(q_2,r_2)$而計算出的三元數組所作出的兩個三角形，是相似的呢？

在《表二》中，我們發現(3,5,7) 和(15,9,21)，但爲甚麼不見了(6,10,14) 或(10,6,14)？



另外，我們能否將上文所提及的計算方法推廣，令到它能應付，當cos∠C爲任何一個有理數時的一般情況呢？

還有，上文所提到的電腦程式，明顯是一個用來搜尋「丟番圖方程」整數解的其中一個方法，我們可否應用它來計算其他問題呢？

以上的每一個問題，都是有待大家去作進一步的探討和研究的！

【參考書目】


一) Pythagorean Numbers. School Mathematics Newletter Issue12 (1993), p.70 - p.71.
二) J.Gilder, Integer-sided triangles with an angle of 60°, Mathematical Gazatte Vol.66 (1982), p.261 - p.266.
三) K.Selkirk, Integer-sided triangles with an angle of 120°, Mathematical Gazatte Vol.67 (1983), p.251- p.255.

# 一題數學題目的聯想

郭家強
東華三院黃笏南中學

二千多年前有位希臘國王托勒密問當時最負盛名的數學家歐幾里得：「學習幾何（當時泛指數學）有何捷徑？」歐氏直率答道：「幾何無王者之途。」原來當時平民只能走普通的道路；而王族則另有專爲他們建造的寬坦大道，走來自是快捷多了。但在數學面前，人間權貴又算是什麼呢？

就教育觀點而言，歐氏的答覆指出了爲學必須認眞踏實，刻苦鑽研；尤其是數學，單作壁上觀，終究隔了一層，要眞正一窺數學殿堂之美及掌握數學神奇之用，則只有自己動腦動手才行。

很可惜，有時一些過量且刻板的作業，使大部分學



生覺得數學既枯燥乏味，又困難艱澀。結果很多學生只懂死背公式和硬套技巧罷了。

筆者曾經在一高中課堂給出以下習題：

試解 $\dfrac{1}{X-1}+\dfrac{1}{X+2}=\dfrac{2}{X^2-1}$

差不多所有學生（其他的當然是沒有嘗試便放棄了！）都是這樣開始的：

他們首先將左式通分母得 $\dfrac{(X+2)+(X-1)}{(X-1)(X+2)}=\dfrac{2}{X^2-1}$

即 $\dfrac{2X+1}{(X-1)(X+2)}=\dfrac{2}{X^2-1}$

部分學生將左右互乘得

$(2X+1)(X^2-1)=2(X-1)(X+2)$ …（1）

化簡得 $2X^3-X^2-4X+3=0$

當然，能運用餘式定理將以上的三次多項式分解，從而得出

答案的學生更少得可憐！

亦有部分學生發覺

$X^2-1=(X+1)(X-1)$，所以由（1）式得

$(2X+1)(X+1)(X-1)=2(X-1)(X+2)$

消去 $(X-1)$，得 $(2X+1)(X+1)=2(X+2)$
(註:甚少學生能解釋爲何可以消去$(X-1)$)。

化簡後 $2X^2+X-3=0$

將左式分解 $(2X+3)(X-1)=0$

$\therefore\ X=-\frac{3}{2}$ 或 $X=1$ (不適用)

所以 $X=-\frac{3}{2}$

事實上，這題目有以下的一個做法：

若方程成立，則 $X-1$，$X+2$ 及 $X+1$ 皆不等於零。



$$\frac{1}{X-1}+\frac{1}{X+2}=\frac{2}{X^2-1}$$

$$\frac{1}{X+2}=\frac{2}{X^2-1}-\frac{1}{X-1}$$

$$=\frac{2}{(X-1)(X+1)}-\frac{1}{X-1}$$

$$=\frac{2-(X+1)}{(X-1)(X+1)}$$

$$\frac{1}{X+2}=-\frac{X-1}{(X-1)(X+1)}$$

$$\therefore\frac{1}{X+2}=-\frac{1}{X+1}\qquad(\because X-1\neq 0)$$

$$X+1=-(X+2)$$

所以 $X = -\frac{3}{2}$ (註:這是一個初中學生也能明白的方法呢!)

政府不是鼓勵市民橫過馬路時要「先停步，看清楚嗎？」筆者認爲在數學解題時也有相似的道理。著名數學教育家波利亞(G.Polya)在怎樣解題(How to solve it)一書中不是鼓勵學習者在解題過程中先要想清楚解題的策略嗎？更重要的是他鼓勵學習者在解題後仍嘗試對題目作多方面的理解。就以上題目而言，老師若能在事後向學生作一簡短的回顧並加以分析，筆者相信這是可以加深學生對題目的理解的。我們作爲老師的實在應該多做此類工作，務求令學生靈活運用知識的本領得以充分發揮。



# 爲甚麼圓周＝$\pi$ ×直徑?

添

在小學數學的課程裡，介紹 $\pi$ 這比率的最佳方法是通過實際的量度活動，讓兒童比較不同圓形的直徑和圓周，從而找出兩者的關係。一般的小學生都樂於參與這活動，且享受從活動中得到的學習成果。

在處理這課題時，安排活動是有價值的。教師可因應兒童的學習能力來配合活動，從而引導他們歸納出圓周＝$\pi$ ×直徑這公式。

但是，我們可曾思考過以下的問題呢？

- 爲甚麼我們只探求圓周與直徑的關係？
- 爲甚麼圓周與直徑的比率會是 $\pi$？

要找出個中的道理，我們需要對圓周與直徑的關係有進一步的理解。事實上，度圓的活動，並沒有爲兩者的關係

作出解釋，就正如我們要探究爲甚麼椅子的高度多是40厘米左右一樣。如果我們只顧量度椅子，那麼我們只可以得到事實(即椅子的高度)而卻找不著問題的核心。我們應該想想是否這高度較配合一般人的身高呢？它是否一個可舒適地坐下的高度呢？

那麼，爲甚麼圓周等於 $\pi$ 乘直徑? 作爲數學教師，我們必須了解這是涉及相似(Similarity)的概念。

1. 所有圓皆是相似的圖形(即所有圓的形狀均相同)。

2. 凡相似的圖形，其對應的長度均有固定的比值。

根據著名教育心理學家布魯納(Bruner)的研究，兒童在十歲左右已可接受有關比例的概念。所以，對於一些理解力較強的學生，教師也可以考慮以下的提議，好讓他們可以較深入地探索圓周與直徑的關係。

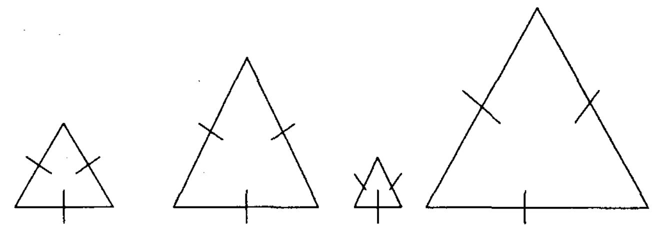


建議的學習過程:

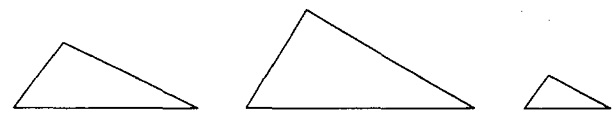
(一) 認識、辨別相似的圖形

讓兒童用直觀的方法辨別以下各組三角形的形狀。

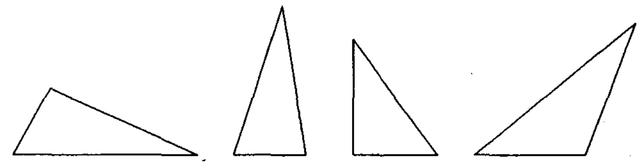
(a)



(b)



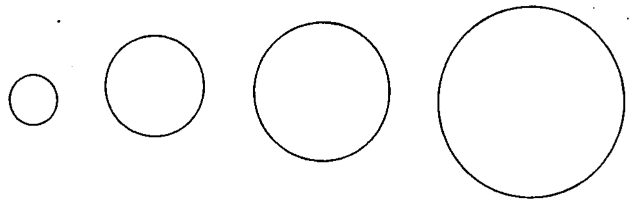
(c)



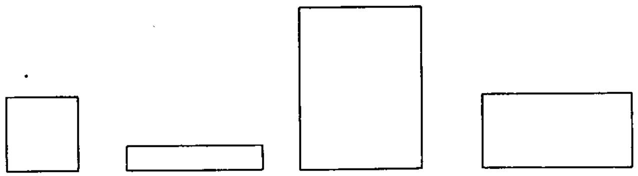
學生觀察後，教師可引導他們領悟到(a)組和(b)組中的三角形均有相同的形狀,只是大小不一而已。但(c)組中的三角形的形狀則各不相同。

同樣地，教師可讓學生從觀察以下的圓形總結出長方形和三角形的形狀是可以不相同的，但所有圓的形狀則盡是相同。

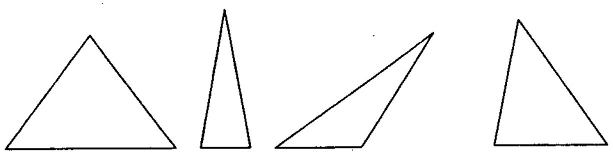
(a)



(b)



(c)

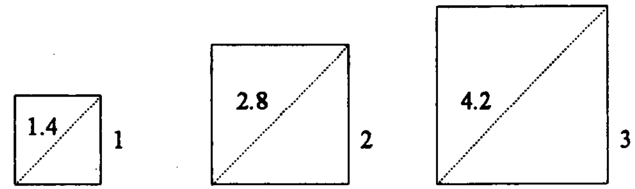


(二) 探索相似圖形周界與對角線的關係

教師可讓學生量度一些相似圖形的周界與最長的對角線，並找出它們的比值。

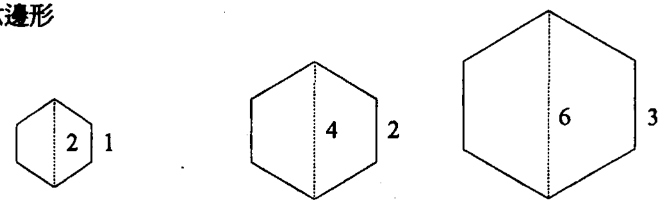


例如:正方形




$\frac{\text{周界}}{\text{對角線}} = \frac{4}{1.4} = 2.9$　　$\frac{\text{周界}}{\text{對角線}} = \frac{8}{2.8} = 2.9$　　$\frac{\text{周界}}{\text{對角線}} = \frac{12}{4.2} = 2.9$

正六邊形




$\frac{\text{周界}}{\text{對角線}} = \frac{6}{2} = 3$　　$\frac{\text{周界}}{\text{對角線}} = \frac{12}{4} = 3$　　$\frac{\text{周界}}{\text{對角線}} = \frac{18}{6} = 3$

如有需要，學生可再進一步觀察其他的多邊形如正八邊形、十邊形等，從而歸納出:

1. 相似圖形的周界與最長的對角線有固定的比值。

2. 該固定的比值約是3。

(三) 解釋圓形的周界與直徑的關係

由於所有圓盡是相似圖形，根據(二)的結論，圓形的周界與直徑必有一固定的比值。

再者，由觀察正方形、五邊形、六邊形、............,
學生不難發現多邊形的邊愈多，圖形愈趨近圓形，而周界與對角線的比值亦趨近一奇異的數($\pi$)了。

這比值，經世代的研究，是不能以一簡單分數來表示，所以人們便用一希臘字母$\pi$來代表。一般所用的22/7或3.14只是爲了方便計算而取的約數。

通過這學習過程和教師與學生反覆的討論，相信學生更能了解問題的核心。雖然沒有一種教學策略堪稱是最有效的，也沒有一種策略能放諸四海而皆準，但我們在教學過程中，若能多鼓勵學生發問及多提出「爲甚麼」、「怎樣」或「有多少不同方法」等問題，則必能引發學生的心智活動和促進思考。就如布魯納曾指出，多發問能提昇兒童的理解力，有助學問的重溫和知識的遷移。



# 概談小學數學教學法

佚名

個人一向認爲數學是一門既高深、且有趣，卻又易學難精的科目。所以，身爲一個小學數學教師，我們要怎樣施教才能將這個科目的知識有效地灌輸給年紀小小的學生呢？

在芸芸的研究中，學者們都認爲不可能找到一種特定的方法來教授數學。因爲，某位老師用一種方法去教授一班學生，結果十分成功及效果良好；可是，同樣的方法卻未必適用於另一位老師和他所面對的那班學生。因此，教學法必須與老師的經驗和學生的程度互相配合。此外，老師所教授的題材亦必須盡量與學生的日常生活有關，使他們可以學以致用，從而產生學習興趣！無論如何，在數學教學中，以下幾個步驟都是不可缺少的。

(一)老師講解：這是課堂授課的基本過程。透過講解，老師可和學生溝通對話。雖然，有些時候學生可能答非所問，但

是，只要老師能仔細聆聽他們的說話，往往能從學生的錯誤中了解及發現他們的弱點，從而進行輔導。

(二)討　　論：儘量給予學生討論的機會，讓他們嘗試解釋或討論從計算或活動過程中所得到的結果，從而幫助學生加深了解他們所學到的知識。

(三)堂課練習：每個學生都需要透過實習去鞏固他們在課堂上所學到的東西。不過，老師所選擇的堂課類型、數量及時間卻要因應不同程度的學生而改變，以其達到最好的效果。

(四)解決問題：這是學習數學的最終目的。學生首先要將問題化爲一些數學常用的詞彙，然後再利用他們所學的概念和技巧去解決問題。當然，這是一個非常困難和重要的步驟，老師需要從中給予學生一定的幫助。

(五)探究工作：某些老師可能認爲探究性的工作並不適用於小學生，而且，小學教材緊迫，亦沒有多餘的時間去進行探究。事實上，這也未必。探究往往可從學生的提問開始，他們通常都很喜歡探求問題的答案。其次無論是個人、組別或全班皆可一起參與探究的工作。這樣一來，我們不是在訓練他們鑽研的精神嗎？

我相信如果老師可以好好利用上述幾個步驟來教授數學，他們的學生一定得益不淺的。



# The 35th International Mathematical Olympiad (IMO)

TSANG Kin-wah
Secretary
IMO'94 Organizing Committee

The 35th IMO was held in Hong Kong during the period 8 to 20 July 1994. Hong Kong is the first South-east Asian city hosting this prestigious annual international mathematics competition. The Hong Kong Mathematical Society (HKMS) was responsible for running the 35th IMO. With the generous financial support of Sir Q.W. LEE of Hang Seng Bank Ltd., the Hong Kong Education Department and many other corporations and individuals, the HKMS accomplished this historical task with much compliments from participating countries.

An IMO '94 Organizing Committee was set up in early 1993 to take charge of the planning and actual running of the 35th IMO. Members of the Organizing Committee came from different tertiary institutions. There were also representatives from the Education Department sitting in the Organizing Committee serving as liaison persons between the Committee and various Government

Departments as support/assistance from these Government Departments were essential in organizing an international event. Many mathematics lecturers from tertiary institutions and over 50 secondary school teachers (mainly mathematics teachers) were also recruited to take charge of various activities of the 35th IMO. Basically, mathematics lecturers from tertiary institutions helped in the professional aspects of the contest, like moderating and selecting contest problems, conducting Jury Meetings, coordinating the scoring of contestants' solutions, etc.; while secondary school teachers assisted mainly in social and recreational activities, like running the two contestants' camps, looking after the daily routines of the student contestants, organizing tours and visits, etc.

Besides mathematics lecturers and secondary school teachers, over 80 students from tertiary institutions and international schools were recruited as student guides to the competing teams and student helpers in the camps. Each competing team of contestants was assigned a student guide who also served as an interpreter for the team. The student guides accompanied their respective competing teams day and night throughout the whole official period of the 35th IMO. There was difficulty in recruiting student guides who were able to speak Russian, Spanish, Vietnamese or Korean. However, later contacts with both contestants and student guides revealed that communication could often go beyond spoken languages and there was no report on breakdown of communication. Most student guides commented that it was a wonderful and unforgettable experience for them to live with fellow youngsters from other countries and showed them around Hong Kong. Long lasting friendships were established between some of the student guides and the contestants they looked after.



The Official Programme of the 35th IMO is briefly outlined below.

| | |
|---|---|
| 8 July 1994 | Arrival of Team Leaders, Observers and Accompanying Persons |
| 9 - 11 July 1994 | Jury Meetings to moderate and select contest problems |
| 11 July 1994 | Arrival of Deputy Team Leaders and Contestants |
| 12 July 1994 | Opening Ceremony at the Sha Tin Town Hall, officiated by the Governor of Hong Kong |
| 13 & 14 July 1994 | Day-1 & Day-2 Contest (8.45 a.m. - 1.15 p.m.) at the Chinese University of Hong Kong |
| 14 - 16 July 1994 | Coordination of scoring of contestants' solutions |
| 15 July 1994 | Official banquet hosted by the Director of Education |
| 17 July 1994 | Last Jury Meeting (determination of medalists) |

| | |
|---|---|
| 19 July 1994 | Closing Ceremony at the Sha Tin Town Hall, officiated by the Director of Education & Sir Q.W. Lee, Chairman of IMO '94 Host Committee |
| 20 July 1994 | Departure of IMO '94 participants |

To promote cross-cultural exchanges among IMO participants and to let them get to know Hong Kong better, many social and recreational activities were organized. These included visits to Space Museum, Science Museum, Sam Tung Uk Museum, Ocean Park, Ching Chung Koon, Wong Tai Sin Temple, Tiger Balm Gardens, etc. A ferry cruise round the harbour and the Hong Kong island was organized. IMO participants were also given time to tour around shopping areas. During all these visits/tours, student guides and teacher helpers were present to ensure the safety and well being of our overseas guests.

All adult IMO participants were accommodated in the Kowloon Panda Hotel in Tsuen Wan, whereas contestants together with student guides were accommodated in the Lady MacLehose Holiday Village and the Sai Kung Outdoor Recreation Centre. However, Deputy Team Leaders were required to live with their respective teams of contestants at the holiday camps before the end of the Day-2 Contest so that contestants could be better looked after by their trainers for the first few days in Hong Kong. In the holiday camps, various camp activities, e.g. different kinds of ball games, karaoke singing, movies showing, etc., were organized for contestants.



The IMO '94 Organizing Committee had made the best efforts to ensure that all IMO participants enjoyed their stay in Hong Kong and brought happy memories back home.

The actual IMO contest consists of 2 sessions, each of 4.5 hours. In each session, contestants have to solve 3 problems, i.e. contestants have to solve altogether 6 problems in the contest. 7 marks is allocated to each problem, with a total of 42 marks as the maximum score for each contestant. Each contestant's solutions are scored by his/her own Team Leader/Deputy Leader who then has to justify the score given in front of official coordinators. Medals are awarded on the merits of individual contestants and there is no champion team. However, competing teams can be ranked by the total scores of all their team members. Normally not more than half of the total number of contestants will be awarded medals, and the ratio of gold, silver and bronze medalists is 1:2:3. Honourable Mentions are awarded to contestants who are not medalists but have obtained a full score of 7 marks for one or more of the 6 problems. (Please see Appendix I for the six 35th IMO problems.)

The following are some statistical data concerning the 35th IMO.

Number of competing teams (countries/territories) : 69

(Please see Appendix II for the list of participating countries/territories.)

Number of contestants : 385
(Each participating country/territory can send a team of up to a maximum of 6 contestants.)

Number of countries/territories sending official observers : 4
(Please see Appendix II for the list of participating countries/territories.)

Number of awards : Gold Medals - 30
Silver Medals - 64
Bronze Medals - 98
Honourable Mentions - 93

Number of contestants with full score of 42 : 22

Hong Kong Team Results :
Number of contestants - 6
Number of medals won - 2 silver & 4 bronze
Total score of the Team - 162
Team rank order - 18 in 69

Total scores of top 5 Teams : USA - 252
China - 229
Russia - 224
Bulgaria - 223
Hungary - 221

The top 7 Teams in gold medals : USA - 6
Bulgaria - 3
China - 3
Russia - 3
Iran - 2
Poland - 2
UK - 2

To conduct an international event of such a scale, it was expected that the planning and actual running of the 35th IMO



would not be a perfectly smooth process without difficulties. In the initial stage of planning, there was difficulty in securing sufficient funds for holding the international event with reasonable programme quality. It took quite some time to confirm sponsorships, as a result, the official invitations to participating countries/territories were sent out later than what was initially intended. Communication with some of the participating countries/territories was extremely difficult, particularly with those former Soviet States. All forms of communication, e.g. fax, e-mail, postal mail and even contact through a neighbouring country, had been used. It was really wonderful that even Bosnia - Herzegovina Team finally made its way to Hong Kong in spite of the civil war going on there and the communication system broken-down. Translation and interpretation also posed a problem. Some of the incoming correspondence was in the native language of the participating countries/territories and there were a few competing teams with all the team members not knowing English. Russian was a particular headache. The Organizing Committee did manage to find two students from an international school who could speak Russian. As said earlier on, we found out later that communication could actually go beyond spoken languages.

The teachers who helped in the two holiday camps accommodating contestants did a marvellous job in keeping order and maintaining a harmonious atmosphere among participants. Due to a change of diet and also a change of climatic conditions, some of the contestants felt sick and some suffered from diarrhea. A few contestants had to be hospitalized. One contestant from Croatia even had to take the contest in hospital. Again teacher helpers showed great sense of responsibility, patience and kind heartedness in accompanying contestants in need of medical care to hospital, visiting them daily and cheering them up. I understand that some teacher helpers did not have sleep for over 30 hours looking after contestants' well being. The mathematics lecturers from tertiary institutions also showed great professionalism in moderating and selecting contest problems, coordinating the scoring of contestants' solutions, etc. It was a real challenge to discuss, debate and argue with top class mathematicians from all over the world.

The 35th IMO was over now and all participants had returned home, safe and with happy memories. All those who assisted in the running of the 35th IMO must agree that there were difficult, really difficult times and great efforts had been made. However, all these time and efforts were worthily spent on a good course. Members of the Organizing Committee, student guides, student helpers, teacher helpers and mathematics lecturers all had gone through an once-a-life time experience. I am sure we all will not forget this experience and should be proud of our contributions in making this international event a success.

Finally, the IMO '94 Organizing Committee would like to thank heartily all those corporations and individuals who had contributed to the successful running of the 35th IMO.



# Mersenne Numbers and Fermat Numbers

Peter

Mersenne Numbers

Numbers of the form $M_n = 2^n - 1$ (where n is a natural number) are called Mersenne Numbers. They were named after Mersenne Marin (1588-1648) for his introduction of Mersenne Numbers into number theory. Much effort has been made to find Mersenne primes - Mersenne numbers that are prime. Mersenne had asserted that $M_n$(for $n \leq 257$)were prime, but it was found incorrect. It is known that if $M_n = 2^n - 1$ is prime, then the number must be prime but the converse is not true. For example, when n=11, $M_{11} = 2^{11} - 1 = 23 \times 89$ is not a prime.

Fermat Numbers

Pierre de Fermat(1601-1665) gave a slight modification to Mersenne numbers and introduced Fermat numbers $F_n$ - numbers of the form $2^{2^n} + 1$, where n is a non-negative integer.

The first few Fermat numbers are

$F_0 = 3$ $\quad F_3 = 257$

$F_1 = 5$ $\quad F_4 = 65537$

$F_2 = 17$

The above 5 numbers are all prime. Fermat thought that all the Fermat numbers $F_n$ were prime. However, in 1732, Euler found that $F_5$(=4294967297) is divisible by 641.



$$\begin{aligned}
641 &= 5\times 2^7 + 1 \\
&= 5^4 + 2^4 \\
5^4 \times 2^{28} - 1 &= (5^2 \times 2^{14} + 1)(5^2 \times 2^{14} - 1) \\
&= (5^2 \times 2^{14} + 1)(5\times 2^7 + 1)(5\times 2^7 - 1) \\
&= 641(5\times 2^7 - 1)(5^2 \times 2^{14} + 1) \\
2^{32} + 1 &= (2^{32} + 5^4 \times 2^{28}) - (5^4 \times 2^{28} - 1) \\
&= 2^{28}(5^4 + 2^4) - (5^4 \times 2^{28} - 1) \\
&= 641 \times 2^{28} - 641(5\times 2^7 - 1)(5^2 \times 2^{14} + 1) \\
&\therefore 2^{32} + 1 \text{ is divisible by } 641.
\end{aligned}$$

Up to the present, no one knows, whether there are prime Fermat numbers after $F_4$.

In 1796, Gauss (at the age of 19) showed that Fermat numbers had a close connection with geometry. He discovered that a regular n-gon can be constructed with just an unmarked straight-edge and a pair of compasses if and only if n is of the form $2^s \cdot P_1^{r_1} \cdot P_2^{r_2}$......(where s is a non-negative integer, $P_1, P_2$,.....are distinct prime Fermat numbers and $r_1, r_2$,....are either 0 or 1).

Reference

Benjamin Bold. Famous Problems of Geometry and How to Solve Them. DOVER.

# A Way to enhance students' Problem Solving Ability

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Problem solving is often stressed in mathematics education. One of the methods that can enhance students' problem solving ability is showing them the possibility of having different approaches to a single problem. Furthermore, demonstration of the several approaches will probably lead to a lot of class discussion. "Which method is the shortest?","Which method is the most elegant?", "Which method can be easily generalized to other cases?" and "Is one method better than the others?Why?" are popular questions which can be discussed in the class. I have adopted such a method in some S4 and S6 lessons and the response is positive.

The following two problems (one from S4 and the other from S6) illustrate how it can be done.

Problem 1: Derive the quadratic formula.



The quadratic formula can be obtained by the following methods :

(1) By completing square

This is the method usually employed in textbooks.

(2) By substitution

Applying the transformation $x=y-h$ to the equation $ax^2+bx+c=0$, we have

$$a(y-h)^2+b(y-h)+c=0$$

i.e. $ay^2+(b-2ah)y+(ah^2-bh+c)=0$

Our aim is to eliminate the y-term, so we put $b-2ah=0$ or $h=b/2a$.

Thus, $ay^2+a(b/2a)^2-b(b/2a)+c=0$,

then $y=\dfrac{\pm\sqrt{b^2-4ac}}{2a}$

and finally

$$x=y-h=\{-b\pm\sqrt{b^2-4ac}\}/2a$$

[Note that such a technique can also be employed in solving a cubic equation.]

(3) Let p and q (p is greater than or equal to q) be the roots of $ax^2 + bx + c = 0$.

Then $ax^2+bx+c = a(x-p)(x-q)$ (Why?)

Expand the RHS and compare the coefficients with LHS (Caution: why can we do that?).

We get $p+q = -b/a$ and $pq = c/a$

Thus

$$\begin{aligned}(p-q)^2 &= (p+q)^2 - 4pq \\ &= (-b/a)^2 - 4(c/a) \\ &= (b^2 - 4ac)/a^2\end{aligned}$$

Hence $p-q = \frac{\sqrt{b^2-4ac}}{a}$. By solving this and $p+q = -b/a$, the result follows.

Problem 2: Prove the simplified version of Cauchy-Schwarz inequality in two variables.

If a,b,m and n are real numbers with $a^2+b^2=1$ and $m^2+n^2=1$, then $|am+bn| \le 1$

Proof:

(1) Since $1 = (a^2+b^2)(m^2+n^2)$



$$= (am+bn)^2 + (an-bm)^2$$

the result follows.

[We have used the Lagrange's identity which makes the generalization of this method quite easy.]

(2) Let **u**=(a,b) and **v**=(m,n) be unit vectors.

It follows that

$$|am+bn| = |u.v| = \big|\,|u|\,|v|\cos A\,\big| \le 1$$

where A is the angle between **u** and **v**. (This method gives the geometric interpretation of the inequality but it cannot be generalized. Rather, it is the general Cauchy-Schwarz inequality, i.e.

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right), \forall a_i, b_i \in \mathbf{R},$$

that generalizes the concept of angle between two vectors in n-dimensional inner product space.)

(3) Using AM-GM inequality we have

$|am| \le (a^2+m^2)/2$ and $|bn| \le (b^2+n^2)/2$

Thus

$$\begin{aligned}|am+bn| &\le |am| + |bn| \\ &\le (a^2+m^2+b^2+n^2)/2 \\ &= \frac{1}{2} \times 2 \\ &= 1\end{aligned}$$

[This method demonstrates the use of AM-GM inequality and can also be generalized.]

Since $x^2+2(am+bn)x+1=(ax+m)^2+(bx+n)^2$ is non-negative for all real numbers x, the discriminant of the equation $x^2+2(am+bn)x+1=0$ is non-positive. Hence the result follows.

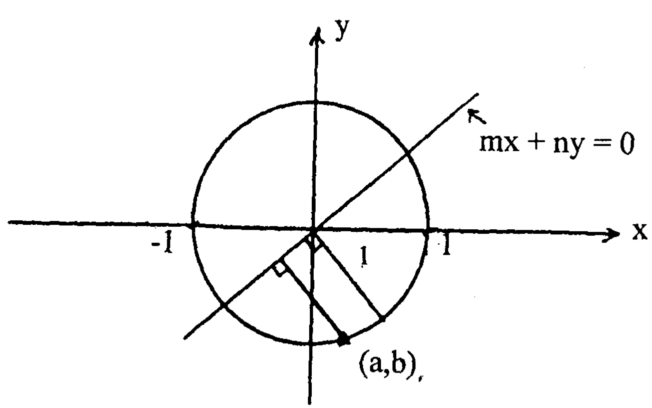
The following two methods are interesting and demonstrate the use of other branches of mathematics.

The fact that $a^2+b^2=1$ and $m^2+n^2=1$ is equivalent to (a,b) and (m,n) are points on a unit circle (the circle centred at origin with radius 1 unit) is useful in the proof of the inequality.

(5) If (a,b) and (m,n) are points on a unit circle, we can find P and Q in $[0°,360°)$ such that (a,b)=(cosP,sinP) and (m,n)=(cosQ,sinQ). Then

$$\begin{aligned}|am+bn| &= |\cos P\cos Q+\sin P\sin Q|\\ &= |\cos(P-Q)|\\ &\le 1\end{aligned}$$

(6) By the distance formula in coordinate geometry, $|am+bn|$ is the distance from the point (a,b) to the line mx+ny=0. Then the required inequality follows from the following figure.






# Use of Common Logarithms made Easy

George LUI

1. TRADITIONAL METHOD

Tables of logarithms and antilogarithms have long been compiled. The table of logarithms gives the mantissa of the logarithm of any number from 1 to 9.999 while the characteristic being 0. The table of antilogarithms is used for finding any number whose logarithm lies between 0.0001 and 0.9999. The traditional way of using common logarithms is illustrated by the following three examples.

Example 1 Find the value of $\dfrac{0.0123\times 56.78}{678.9}$.

Solution Let $N=\dfrac{0.01234\times 56.78}{678.9}$.

$$\log N = \log 0.01234 + \log 56.78 - \log 678.9$$
$$= \bar{2}.0913 + 1.7542 - 2.8318$$
$$= \bar{1}.8455 - 2.8318$$
$$= \bar{3}.0137$$
$$N = \text{anti}\log \bar{3}.0137$$
$$= 0.001032$$

In Example 1, it is noticed that the mantissa of the logarithm of a number is always kept positive while the characteristic may be positive or negative. Thus, if $\log N = \bar{3}.0137$ is expressed as logN=–2.9863, it is impossible to find N from the antilogarithm table.

<u>Example 2</u> Find $\sqrt{0.9876}$.

Solution Let $N = \sqrt{0.9876}$

$$\log N = \log \sqrt{0.9876}$$
$$= \log 0.9876^{\frac{1}{2}}$$
$$= \frac{1}{2}\log 0.9876$$
$$= \frac{1}{2} \times \bar{1}.9946$$
$$= \frac{1}{2} \times (\bar{2} + 1.9946)$$
$$= \bar{1}.9973$$
$$N = \text{anti}\log \bar{1}.9973$$
$$= 0.9938$$



<u>Example 3</u> Find the cube of 0.9876.

Solution Let $N = 0.9876^3$

$$\log N = \log 0.9876^3$$
$$= 3\log 0.9876$$
$$= 3 \times (\bar{1}.9946)$$
$$= \bar{3} + 2.9838$$
$$= \bar{1}.9838$$
$$N = \text{anti}\log \bar{1}.9838$$
$$= 0.9634$$

The calculations of $\frac{1}{2} \times (\bar{1}.9946)$ and $3 \times (\bar{1}.9946)$ are usually difficult to pupils. But the use of common logarithms can be made easier by the following new method.

2. <u>NEW METHOD</u> (Using the logarithm table together with the reciprocal table)

Let $N = 10^a$

$$\therefore \log N = a$$
$$\text{But } \frac{1}{N} = \frac{1}{10^a} = 10^{-a}$$
$$\therefore \log \frac{1}{N} = -a$$
$$\text{i.e.} \log \frac{1}{N} = -\log N$$

In the new method, we take both the characteristic and the mantissa as a whole, not as two separate parts. Then the whole logarithm can be positive or negative. For example, $\bar{2}.7539$ should always be written as $-1.2461$. Finally, in finding the antilogarithm, we must first change the negative logarithm into a positive one by making use of the rule $\log\frac{1}{N} = -\log N$ Details concerning the application of the new method are depicted in the examples below.

<u>Example 4</u> Find the value of $\frac{0.0567\times 198.7}{34.56}$.

Solution

$$\text{Let } N = \frac{0.0567\times 198.7}{34.56}$$

$$\begin{aligned}
\log N &= \log 0.0567 + \log 198.7 - \log 34.56 \\
&= \log(10^{-2}\times 5.674) + \log(100\times 1.987) \\
&\quad - \log(10\times 3.456) \\
&= (-2+0.7539) + (2+0.2982) - (1+0.5386) \\
&= -1.2461 + 2.2982 - 1.5386 \\
&= -0.4865
\end{aligned}$$

$$\frac{1}{N} = \text{anti}\log 0.4865 = 3.065$$

$$N = 0.3262$$

(from the table of reciprocals)



<u>Example 5</u> Find $\sqrt{0.4589}$.

Solution

$$\text{Let } N = \sqrt{0.4589}$$

$$\begin{aligned}
\log N &= \log 0.4589^{\frac{1}{2}} \\
&= \frac{1}{2}\log 0.4589 \\
&= \frac{1}{2}\log(10^{-1}\times 4.589) \\
&= \frac{1}{2}(-1+0.6617) \\
&= \frac{1}{2}(-0.3383) \\
&= -0.1692
\end{aligned}$$

$$\begin{aligned}
\log\frac{1}{N} &= -(-0.1692) \\
&= 0.1692
\end{aligned}$$

$$\begin{aligned}
\frac{1}{N} &= \text{anti}\log 0.1692 \\
&= 1.476
\end{aligned}$$

$$N = 0.6775$$

(from the table of reciprocals)

<u>Example 6</u> Find $\sqrt[5]{0.04589}$

Solution Let $N = 0.04589^{\frac{1}{5}}$

$$\log N = \frac{1}{5}\log 0.04589$$
$$= \frac{1}{5}(-1.3383)$$
$$= -0.2677$$
$$\log\frac{1}{N} = -\log N$$
$$= -(-0.2677)$$
$$= 0.2677$$
$$\frac{1}{N} = \text{anti}\log 0.2677$$
$$= 1.852$$
$$N = 0.5400$$
(from the table of reciprocals)

By using the new method, we can avoid all the bar notations (e.g. $\bar{1}.9876$) and hence minimize pupils' misunderstanding and difficulties. Obviously $-0.3383 \div 2$ and $-1.3383 \div 5$ can be manipulated much easier than $\bar{1}.6617 \div 2$ in Example 5 and $\bar{2}.6617 \div 5$ in Example 6.



# Sharing Corner

GM10000

When pupils are asked what their favourite subjects are, it is not surprising that Mathematics is one of their choices. Pupils have at least 11 years keeping in touch with Mathematics. There are times that pupils are much impressed when they spend days and nights on solving problems which are done by teachers in only a couple of minutes. On the other hand, Mathematics does not involve too much language and makes it easier for pupils to understand.

The writer has been teaching Mathematics in a secondary school since 1990. Thank SMN for giving me the opportunity to share some personal opinions on teaching Mathematics.

Although the passing rate of Mathematics in HKCEE is the highest in my school, it does not mean that pupils are able to appreciate the beauty of Mathematics. The teaching approach adopted is examination-oriented. Most of the pupils, particularly in the upper forms, are only interested in how to pass examinations.

It seems that passing examinations is their sole objective in learning Mathematics. Pupils will immediately lose interest to any teaching materials that are out of syllabus or not included in examinations.

Pupils are passive and lack of response in classroom. They are not willing to discuss questions or exchange their ideas in solving problems. Most of the pupils only sit still and wait for answers. Because pupils are self-centred, teaching becomes one-way only. Teachers know that one-way teaching is not good but sometimes they are forced to adopt it since there are no other alternatives. Lessons turn to be boring and there are no teacher-pupil interactions at all.

Pupils lack incentive to join extra-curricular activities which are related to Mathematics. As we know, it may be the best way that pupils learn Mathematics through activities since these activities are stimulating, arousing and impressive. They can also learn something that is not included in textbooks. Unfortunately, mass media always side-track their attentions. They are more occupied with TV viewing, movie-going and etc. Thus, many interesting topics prepared by hard-working teachers are not able to attract pupils' participation or attention. Interesting Mathematics books eventually are laid idle in the library.

Fields of Mathematics are wide and deep. It is definitely a very interesting subject. A huge treasure is waiting for pupils to discover. There is no doubt that pupils may have little difficulty in calculating but many of them are weak in the abilities of thinking. The writer would like to raise some questions that you may also face. How can teaching be relieved from the examination-oriented approach? How can teachers motivate pupils so that they are also involved in the teaching-learning process? How can teachers



encourage pupils to take part in activities? If you have any suggestions or comments,please let me have them via the Editor of the School Mathematics Newsletter .

# PASTIMES

1. The following questions provide practice with some of the important concepts in mathematics.

   What is (a) the square of the square of 4?
   (d) the double of the double of 7?
   (c) the opposite of the opposite of 5?
   (d) the reciprocal of the reciprocal of 2/3?
   (e) 50% of 50% of 40?
   (f) the square root of the square root of 81?
   (g) a multiple of a multiple of 5?
   (h) a factor of a factor of 36?
   (i) the absolute value of the absolute value of -8?
   (j) the complement of the complement of 30°?

2. Mr. and Mrs. Chan have seven children, born in alternate years. Mr. Chan is 5 years older than Mrs. Chan, and 30 years older than his eldest child. The sum of the ages of the whole family is 382 years. Find the ages in years of each member of the family.

3. Find all the 2-digit numbers which when reversed produce a number 75 percent greater?



4. There are two editions of a book, the wholesale prices of which are \$50 (paperback) and \$70 (cloth-bound).
   (a) A man plans to spend \$6000 exactly by purchasing not less than 20 copies of each edition. Show that the number of \$70 books must be a multiple of 5.

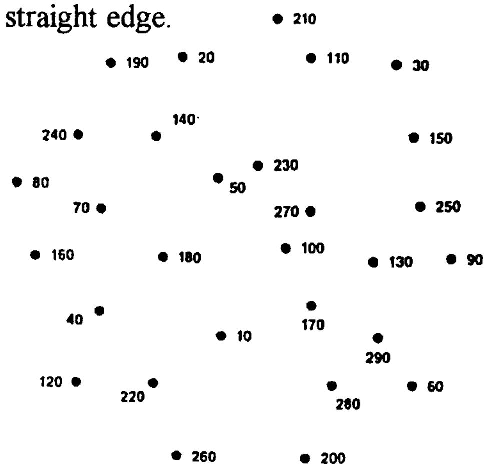
   (b) Show that the difference between the numbers of copies of each edition must be a multiple of 12.

5. (a) Round off each answer to the nearest ten.

| | | |
|---|---|---|
| (1) $12\times9=$ | (7) $245\div7=$ | (13) $5\times47=$ |
| (2) $21\times6=$ | (8) $34\times4=$ | (14) $3\times51=$ |
| (3) $472\div8=$ | (9) $162\div9=$ | (15) $22\times9=$ |
| (4) $828\div3=$ | (10) $13\times7=$ | (16) $628\div4=$ |
| (5) $446\div2=$ | (11) $5\times51=$ | (17) $784\div7=$ |
| (6) $31\times4=$ | (12) $536\div8=$ | |

   (b) Locate each "rounded off" dot.

   (c) Connect the dots in order of the question numbers in part (a), using a straight edge.



## *Suggested Solutions to PASTIMES*

1. (a) 256 (b) 28 (c) 5 (d) 2/3 (e) 10 (f) 3 (g) 5,10,15,20,... (h) 2,3,4,6,9,12,18,36 (i) 8 (j) 30°

2. If Mr. Chan is x years old, his wife is (x–5) years old, and the eldest child is (x–30) years old. The six other children are x–32, x–34, x–36, x–38, x–40 and x–42 years old.

   The seven children's ages form an A.P. with a sum of 7x–252; adding their parents ages, x+(x–5)+(7x–252) = 382.

   Hence $9x = 639$
   $x = 71$

   Therefore, Mr. Chan is 71; Mrs. Chan is 66 and their children are 41, 39, 37, 35, 33, 31, 29 years old.

3. If the unit and tens digits are y and x, then

$$10y + x = \frac{7(10x + y)}{4}$$



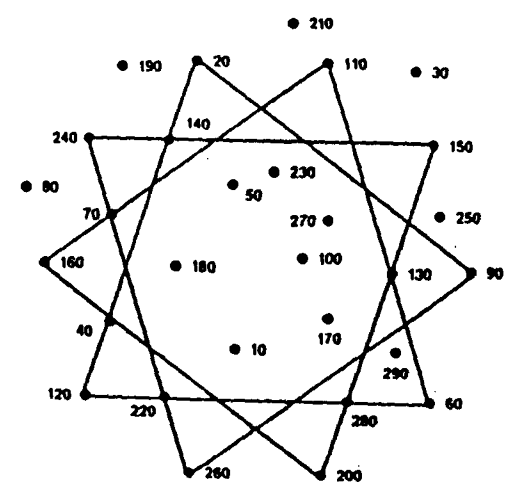
$$y = 2x$$

The four possible numbers are 12, 24, 36, 48.

4.(a) If the man buys x books at $70 each and y books at $50 each, then 70x+50y = 6000, i.e., 7x = 5(120–y) which shows that x must be a multiple of 5.

(b) Putting x = 5m, then y = 120–7m, and so x–y = 12m–120, which is a multiple of 12.

5.



# For Your Information

## 1. International Mathematical Olympiad (IMO)

The 35th IMO was held in Hong Kong during the period 8 to 20 July 1994. The contest was on 13 and 14 July 1994. Hong Kong is the first South-east Asian city hosting this prestigious annual international mathematics competition and the performance of Hong Kong Team was encouraging. It won two Silver and four Bronze Medals. Among the sixty-nine participating countries, Hong Kong Team was at the eighteenth place in the competition.

Other details like the programme of the IMO and the performance of contestants from other countries are divulged in the article "The 35th International Mathematical Olympiad (IMO)" by Mr. TSANG Kin-wah, Secretary of IMO '94 Organizing Committee, in this issue.

## 2. The Twelfth Hong Kong Mathematical Olympiad (HKMO)

173 secondary schools participated in the Twelfth HKMO. After the heat events, 40 schools were selected to enter the final event which was held on 28 January 1995 at the hall of the Northcote Campus, the Hong Kong Institute of Education.

The Prize Giving Ceremony was held after the final event. The Acting Assistant Director of Education(Chief Inspector of Schools), Mrs. C. WONG and the Campus Principal of the Northcote Campus, Mrs. W. N. MAK were the guests of honour and they presented the trophies and prizes to the winners.



The Champion of the competition was Tsuen Wan Public Ho Chuen Yiu Memorial College,the First Runner-up was St. Paul's College and the Second Runner-up was Heung To Middle School.

The Champion of the Poster Design Competition for the Twelfth HKMO was CHUI Hin-sing of Tsuen Wan Government Secondary School. The First Runner-up was KONG Lam of Queen's College Old Boys' Association Secondary School and the Second Runner-up was SHUM Chi-yuen of TWGHs Kap Yan Directors' College. The champion poster is shown in Appendix III.

A Mathematics Camp is scheduled to be held in April 1995 for the 40 teams participating in the Final Event of the Twelfth HKMO. It aims at providing the finalists recreational activities as well as enhancing their knowledge in Mathematics.

## 3. Mathematics Teaching Centre

The opening hours of the Mathematics Teaching Centres had been rescheduled as

| | |
|---|---|
| Monday | 9:00 a.m. - 12:30 p.m. |
| Wednesday | 2:00 p.m. - 5:00 p.m. |
| Saturday | 9:00 a.m. - 12:00 noon |

The TTRA Resource Corner in Mathematics in the Mathematic Teaching Centre was renamed as TOC Resource Corner in July 1993.

## From the Editor

I would like to express my gratitude to those who have contributed articles and also those who have given valuable comments and suggestions to the newsletter.

The SMN cannot survive without your contributions. You are, therefore, cordially invited to send in articles, puzzles, games, cartoons, etc for the next issue. Anything related to mathematics education will be welcomed. We particularly need articles on sharing teaching experiences, classroom ideas, teaching methodology on particular topics, organization of mathematics clubs and even the organization, administration and co-ordination of the mathematics panel. Please write to the SMN (with your contact address included) as soon as possible and the address is

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For information or verbal comments and suggestions, please contact the editor on 2892 6554.





## Countries/Territories Participating in the 35th International Mathematical Olympiad (IMO)

**Countries/Territories invited to send competing teams :**

1. Argentina
2. Armenia
3. Australia
4. Austria
5. Belarus
6. Belgium
7. Bosnia - Herzegovina
8. Brazil
9. Bulgaria
10. Canada
11. Chile
12. China
13. Chinese Taipei
14. Colombia
15. Croatia
16. Cuba
17. Cyprus
18. Czech Republic
19. Denmark
20. Estonia
21. Finland
22. Former Yugoslav Republic of Macedonia
23. France
24. Georgia
25. Germany
26. Greece
27. Hong Kong
28. Hungary
29. Iceland
30. India
31. Indonesia
32. Iran
33. Israel
34. Italy
35. Japan
36. Korea (South)
37. Kuwait
38. Kyrgyzstan
39. Latvia
40. Lithuania
41. Luxembourg
42. Macau

43. Mexico
44. Moldova
45. Mongolia
46. Morocco
47. Netherlands
48. New Zealand
49. Norway
50. Philippines
51 Poland
52. Portugal
53. Republic of Ireland
54. Romania
55. Russia
56. Singapore
57. Slovakia
58. Slovenia
59. South Africa
60. Spain
61. Sweden
62. Switzerland
63. Thailand
64. Trinidad & Tobago
65. Turkey
66. Ukraine
67. United Kingdom
68. United States of America
69.Vietnam

**Countries/Territories invited to send official observer(s) :**

1. Brunei
2. Malaysia
3. Papua New Guinea
4. Sri Lanka



# Appendix II

## 35th International Mathematical Olympiad (IMO) Contest Problems

Each question is worth 7 points.
Time Allowed for each paper is 4½ hours.

First Day

1. Let $m$ and $n$ be positive integers. Let $a_1, a_2, \ldots, a_m$ be distinct elements of $\{1, 2, \ldots, n\}$ such that whenever $a_i + a_j \le n$ for some $i, j, 1 \le i \le j \le m$, there exists $k, 1 \le k \le m$, with $a_i + a_j = a_k$. Prove that

$$\frac{a_1 + a_2 + \ldots + a_m}{m} \ge \frac{n+1}{2}$$

2. ABC is an isosceles triangle with AB=AC. Supose that
   (a) M is the midpoint of BC and O is the point on the line AM such that OB isperpendicular to AB;
   (b) Q is an arbitrary point on the segment BC different from B and C;
   (c) E lies on the line AB and F lies on the line AC such that E, Q and F aredistinct and collinear.

   Prove that OQ is perpendicular to EF if and only if QE=QF.

3. For any positive integer $k$, let $f(k)$ be the number of elements in the set $(k+1, k+2, \ldots, 2k)$ whose base 2 representation has precisely three 1s.
   (a) Prove that, for each positive integer $m$, there exists at least one positive integer $k$ such that $f(k)=m$.
   (b) Determine all positive integers $m$ for which there exists exactly one $k$ with $f(k)=m$.

Second Day

4. Determine all ordered pairs $(m,n)$ of positive integers such that

$$\frac{n^3+1}{mn-1}$$

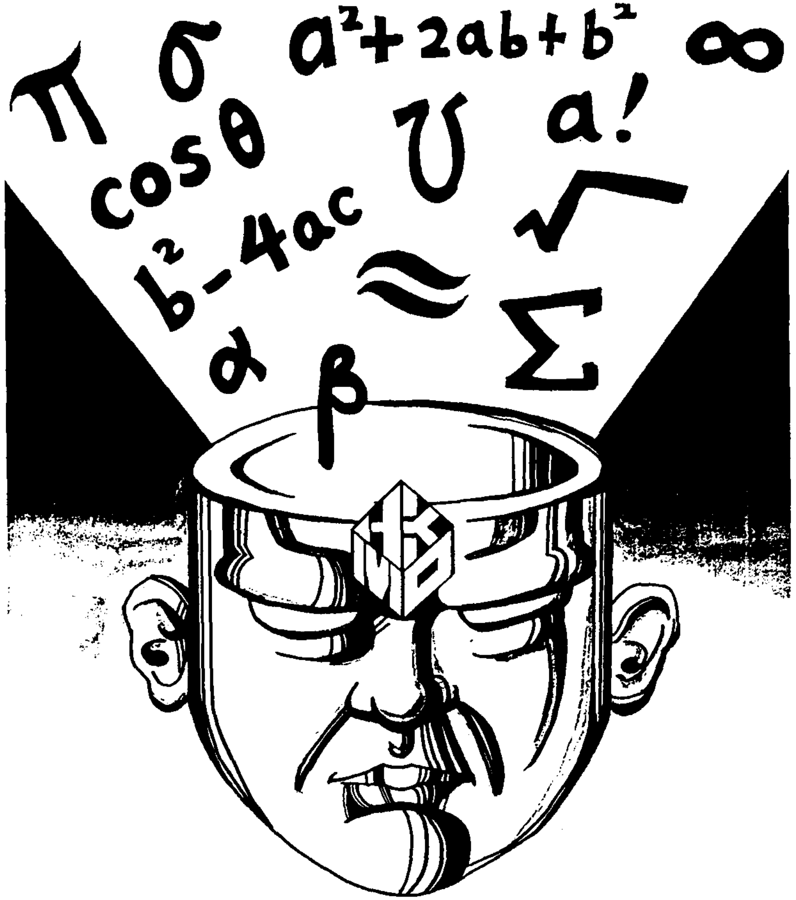
is an integer.

5. Let $S$ be the set of real numbers strictly greater than $-1$. Find all functions $f: S \rightarrow S$ satisfying the two conditions :
   (a) $f(x+f(y)+xf(y)) = y+f(x)+yf(x)$ for all $x$ and $y$ in $S$;
   (b) $\frac{f(x)}{x}$ is strictly increasing on each of the intervals $-1<x<0$ and $0<x$.

6. Show that there exists a set $A$ of positive integers with the following property : for any infinite set $S$ of primes there



exist two positive integers $m \in A$ and $n \notin A$ each of which is a product of $k$ distinct elements of $S$ for some $k \geq 2$.




# The Twelfth Hong Kong Mathematics Olympiad