SMN

學校數學通訊
School Mathematics Newsletter

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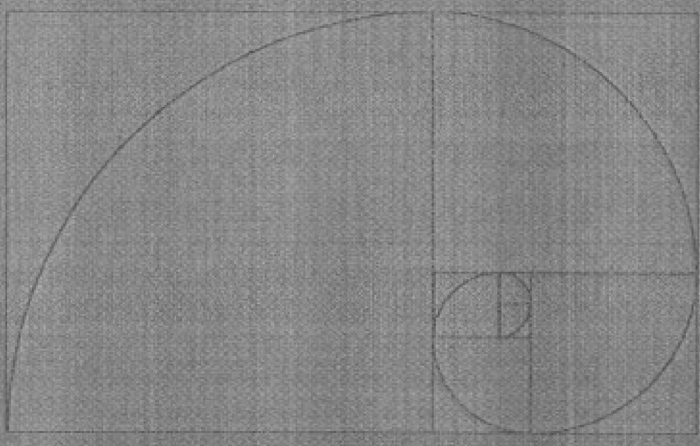

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《學校數學通訊》旨在爲香港數學教育界提供一個溝通渠道，故此懇請各校長將本通訊交給貴校所有數學科教師傳閱。

爲使本通訊能成爲教師的投稿公開園地，歡迎讀者提供任何與數學教育有關的文章。唯本通訊內所發表的意見，並不代表教育署的觀點。

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The School Mathematics Newsletter (SMN) aims at serving as a channel of communication for mathematics education in Hong Kong. School principals are therefore kindly requested to ensure that every member of their mathematics staff has an opportunity to read this Newsletter.

We welcome contributions in the form of articles on all aspects of mathematics education as the SMN is meant for an open forum for teachers of mathematics. However, the views expressed in the articles in the SMN are not necessarily those of the Education Department.

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FOREWORD

Welcome to the sixteenth issue of the School Mathematics Newsletter (SMN).

As in the past issues of the SMN, the articles in the present issue are contributed by professionals interested in mathematics education, many of whom have rendered uninterrupted support to the publication of the SMN in the past years. The Mathematics Section of the Advisory Inspectorate Division wishes to thank them sincerely for their contributions, without which the publication of this issue of the SMN will not be possible.

In the existing education system, mathematics teachers are faced with the tremendous challenge of teaching pupils of very different abilities, motivations and aspirations. To meet this challenge, mathematics teachers need to equip themselves with necessary mathematical skills and teaching strategies to cope with different teaching situations. To this end, the articles in this publication cover a variety of relevant topics, ranging from hot issues of mathematics teaching to daily applications. There are also some interesting puzzles to tap readers' mind. We do hope that all readers will find the content of this issue informative and stimulating.

The Editorial Board of SMN wishes to express again its gratitude to all contributors, and also to the fellow colleagues of the Mathematics Section who have made good efforts in producing the SMN Issue 16.

Mathematics Section
Advisory Inspectorate Division

教師、督學、講師的新伙伴關係

香港中文大學課程與教學學系
黃毅英博士

(一) 楔子

長久以來，教師，督學，講師之間可能是一個「既愛且恨」的「天仙配」。我們也許間中聽過以下的怨言:

「下星期又要全面視學了，他們只知叫人用顏色粉筆，給你一班第五組別學生，看督學怎麼教?」

「教育學院的講師只會說『皮亞諧』及『布魯姆』，我只想有人能幫我把學生的成績提高點、令他們上課時乖點。」

「教育學院只會教人寫『教學計劃』，講師們只知叫人用高映機，推銷自我發現法，爲何不去勸校長多置幾部高映機、叫教署將每班人數減少?」

「講師們在報章上太多話說了－『處士橫議』。」

「教師團體上星期辦的研討會真『正掂』，於其中認識了不少其他學校的教師，不止偷學了不少『板斧』，還收集了很多『評分計劃』！」

在以往（不局限於數學科也不局限於本港），課程大抵是一班人設計了交與教師執行的工作守則。由於當時社會與教育環境不同、亦無甚不妥，於是社會上需要一班人（督學）去監察課程的執行情況，再由另一班人（講師）去教導新入行者如何理解課程和如何執行。教師團體則扮演提供輔助資料以協助教師執行課程的角色。

（二）物換星移

以下列出的數則事件，正好標誌著教育的轉型：

一九七九年實施普及教育。

一九八二年發表國際顧問團報告書。

一九九二年應教統會四號報告書建議開展 TTRA 計劃。

一九九三年課程發展處成立並推行 TOC ，學者以「官民對峙」來形容。

一九九四年中小學課程進行改革。

一九九六年七位學者發表對 TOC 的意見，從新打破對話的僵局。

一九九七年全面檢討小學至預科數學課程特委會成立。

據筆者之分析，這些轉型涉及三種因素：

社會轉型：教育由精英轉向普及，舊有模式無法處理全民教育之個別差異；教育由量轉向質，教學無法不作出相應的改革。

政府之進步：轉向開放與問責。

來自教師之動力：邁向成熟與專業化。

所有對於新伙伴關係的討論亦無法不以這些轉變作基礎。

（三）教學專業化變成改革之核心

預先設定的課程當然有規範與指導作用，但顯然無法完全承接上述的轉型。在這新的情景下，只有教學上專業判斷才能根本地解決問題。各方面必須配合，好讓有識見的教師可以盡量發揮其教學心得，甚或由一些先行者帶頭

領導這新文化的締造。由這個新構思出發，教師，督學，講師無法不去建立新的伙伴關係 － 教師必須變成執行、詮釋、調適課程的主宰，利用其專業判斷方能達至眞正的因材施教。

除了基本技巧外，教師教育應由提供教學理論轉向培養教師將其教學經驗深化（theorising）的能力。如此，離開教育學院並不再是擁有「終身保用」之教學板斧，而得到的是由講師不斷促進反思，使教師向專業邁進的能力。

教署同工便順理成章的作「工作大隊」的「大後方」，實地了解前綫教師面對的問題而作出適當的支援。

教師團體亦應加以配合，其運作應以教師爲主導而講師則充當指導的角色。

三者之關係或許可以用教材實驗來體現。由教署同工領導和教育學院講師輔助下，教師進行一些教學嘗試，由小規模的做起，再透過教師團體將成果與所得經驗進一步與同行交流，漸漸形成新的文化。

當然周邊環境亦須配合這個轉型。課程應更有空間和靈活性，而工作環境（包括工作量、每班人數）等亦必須作大幅度改進，方能跳出「教師作爲教育機器看守者」的胡同。

極坐標法証 Simson 定理及其推廣

江蘇泰州橡膠總廠職工學校
于志洪

本文先給出垂足多邊形的定義，然後應用極坐標法對著名的西摩松(R. Simson, 1687 年 - 1786 年，英國數學家)定理及其推廣進行簡捷証明。

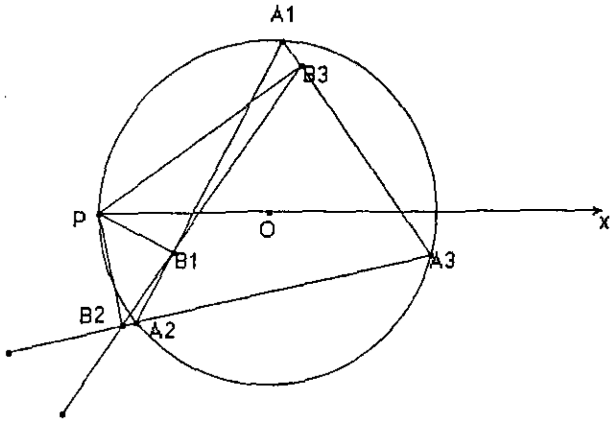
一. 垂足多邊形

由多邊形 $A_1A_2A_3 \ldots\ldots A_n$ 所在平面上的一點 P，向多邊形的各邊 $A_1A_2, A_2A_3, \ldots\ldots, A_nA_1$ 作垂綫，其垂足爲 $B_1, B_2, B_3, \ldots\ldots, B_n$，那麼多邊形 $B_1B_2B_3 \ldots\ldots B_n$ 便稱爲 P 點關於多邊形 $A_1A_2A_3 \ldots\ldots A_n$ 的一階垂足多邊形（或簡稱垂足多邊形）。

由 P 點再作 $B_1B_2B_3 \ldots\ldots B_n$ 各邊的垂綫，設垂足爲 $C_1, C_2, C_3, \ldots\ldots, C_n$，那麼多邊形 $C_1C_2C_3 \ldots\ldots C_n$ 便稱爲 P 點關於多邊形 $A_1A_2A_3 \ldots\ldots A_n$ 的二階垂足多邊形。由 P 點再作出 $C_1C_2C_3 \ldots\ldots C_n$ 的垂足多邊形，稱爲 P 點關於多邊形 $A_1A_2A_3 \ldots\ldots A_n$ 的三階垂足多邊形 …，依此類推，可定義 P 點關於多邊形 $A_1A_2A_3 \ldots\ldots A_n$ 的 n 階垂足多邊形。

二. Simson 定理的証明

設 P 爲$\Delta A_1A_2A_3$外接圓上任意一點，那麼 P 點關於$\Delta A_1A_2A_3$的一階垂足三角形的三個頂點在一條直綫上。



圖一

証明：如圖一，以 P 爲極點， PO 所在的半射綫爲極軸建立極坐標系。設$\Delta A_1A_2A_3$的外接圓直徑爲 d ，則以 O 爲圓心的圓方程爲$\rho=d\cos A$。設頂點爲$A_i(d\cos\theta_i，\theta_i)$， $i=1$ ， 2 ， 3 及 $\theta_i\in[0，2\pi]$。所以A_1A_2的兩點式方程爲

$$\frac{\sin(\theta_2-\theta_1)}{\rho}=\frac{\sin(\theta_2-\theta)}{d\cos\theta_1}+\frac{\sin(\theta-\theta_1)}{d\cos\theta_2}$$

$$\rho[\sin(\theta_2-\theta)\cos\theta_2+\sin(\theta-\theta_1)\cos\theta_1]$$
$$=d\sin(\theta_2-\theta_1)\cos\theta_1\cos\theta_2$$
$$\frac{1}{2}\rho[\sin(2\theta_2-\theta)+\sin(\theta-2\theta_1)]$$
$$=d\sin(\theta_2-\theta_1)\cos\theta_1\cos\theta_2$$
$$\rho[\sin(\theta_2-\theta_1)\cos(\theta-\theta_1-\theta_2)=d\sin(\theta_2-\theta_1)\cos\theta_1\cos\theta_2$$
$$\because\ \sin(\theta_2-\theta_1)\neq 0$$

$$\therefore\rho\cos(\theta-\theta_1-\theta_2)=d\cos\theta_1\cos\theta_2$$

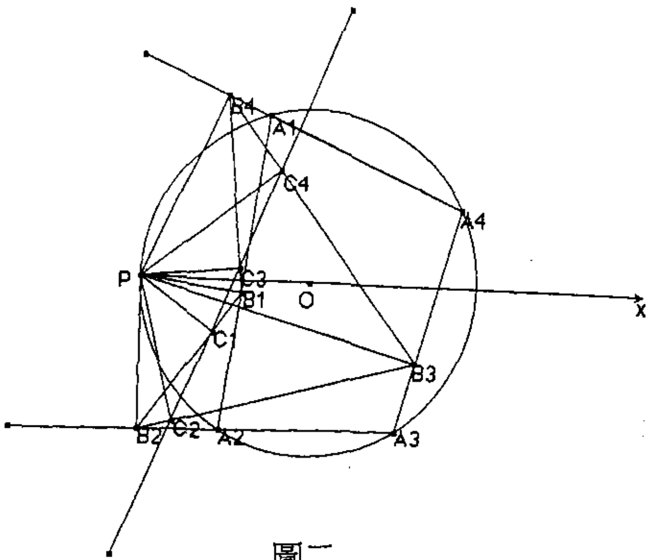
這是 A_1A_2 的法綫式方程，故知垂足 B_1 的坐標爲$(d\cos\theta_1\cos\theta_2，\theta_1+\theta_2)$。轉換三個頂點的坐標，得$B_2(d\cos\theta_2\cos\theta_3，\theta_1+\theta_3)$， $B_3(d\cos\theta_3\cos\theta_1，\theta_3+\theta_1)$。顯然 B_1、 B_2、 B_3 三點的坐標滿足方程

$$\rho\cos(\theta-\theta_1-\theta_2-\theta_3)=d\cos\theta_1\cos\theta_2\cos\theta_3$$

故 B_1、 B_2、 B_3 三點共綫。

三. 西摩松定理推廣的証明

1. 設 P 點與四邊形 $A_1A_2A_3A_4$ 的四個頂點同在一個圓周上。那麼， P 點關於四邊形 $A_1A_2A_3A_4$ 的二階垂足四邊形的四個頂點在一條直綫上。



圖二

証明：如圖二，以 P 爲極點， PO 所在的半射綫爲極軸建立極坐標系。設四邊形 $A_1A_2A_3A_4$ 的外接圓直徑爲 d ，則以 O 爲圓心的圓方程爲$\rho=d\cos\theta$ 。設頂點爲 $A_i(d\cos\theta_i，\theta_i)$， $i = 1,2,3,4$ 及 $\theta_i\in[0，2\pi]$，則由西摩松定理的証明知 A_2A_3 的方程爲

$$\rho\cos(\theta-\theta_2-\theta_3)= d\cos\theta_2\cos\theta_3$$

因此，垂足 B_2 的坐標爲 $B_2(d\cos\theta_2\cos\theta_3，\theta_2+\theta_3)$ 轉換四個頂點的坐標，得 $B_3(d\cos\theta_3\cos\theta_4，\theta_3+\theta_4)$、$B_4(d\cos\theta_4\cos\theta_1，\theta_4+\theta_1)$、 $B_1(d\cos\theta_1\cos\theta_2，\theta_1+\theta_2)$ 。故 B_2B_3 的兩點式方程爲

$$\frac{\sin(\theta_3+\theta_4-\theta_2-\theta_3)}{\rho}=\frac{\sin(\theta_3+\theta_4-\theta)}{d\cos\theta_2\cos\theta_3}+\frac{\sin(\theta-\theta_2-\theta_3)}{d\cos\theta_3\cos\theta_4}$$

$$\frac{\sin(\theta_4-\theta_2)}{\rho}=\frac{\sin(\theta_3+\theta_4-\theta)\cos\theta_4+\sin(\theta-\theta_2-\theta_3)\cos\theta_2}{d\cos\theta_2\cos\theta_3\cos\theta_4}$$

$$\because\ \sin(\theta_3+\theta_4-\theta)\cos\theta_4+\sin(\theta-\theta_2-\theta_3)\cos\theta_2$$
$$=\frac{1}{2}[\sin(\theta_3+2\theta_4-\theta)+\sin(\theta-2\theta_2-\theta_3)]$$
$$=\sin(\theta_4-\theta_2)\cos(\theta-\theta_2-\theta_3-\theta_4)$$

$$\therefore\ \frac{\sin(\theta_4-\theta_2)}{\rho}=\frac{\sin(\theta_4-\theta_2)\cos(\theta-\theta_2-\theta_3-\theta_4)}{d\cos\theta_2\cos\theta_3\cos\theta_4}$$

$$\because\ \sin(\theta_4-\theta_2)\neq 0$$

$$\therefore\ \rho\cos(\theta-\theta_2-\theta_3-\theta_4)=d\cos\theta_2\cos\theta_3\cos\theta_4$$

這是 B_2B_3 的法綫式方程，故知垂足 C_2 的坐標爲 $C_2(d\cos\theta_2\cos\theta_3\cos\theta_4，\theta_2+\theta_3+\theta_4)$。轉換 B_1 、 B_2 、 B_3 、 B_4 四個頂點的坐標，得垂足 C_3 、 C_4 、 C_1 的坐標分別爲

$$C_3(d\cos\theta_3\cos\theta_4\cos\theta_1,\ \theta_3+\theta_4+\theta_1)$$
$$C_4(d\cos\theta_4\cos\theta_1\cos\theta_2,\ \theta_4+\theta_1+\theta_2)$$
$$C_1(d\cos\theta_1\cos\theta_2\cos\theta_3,\ \theta_1+\theta_2+\theta_3)$$

顯然 C_1 、 C_2 、 C_3 、 C_4 的坐標滿足方程

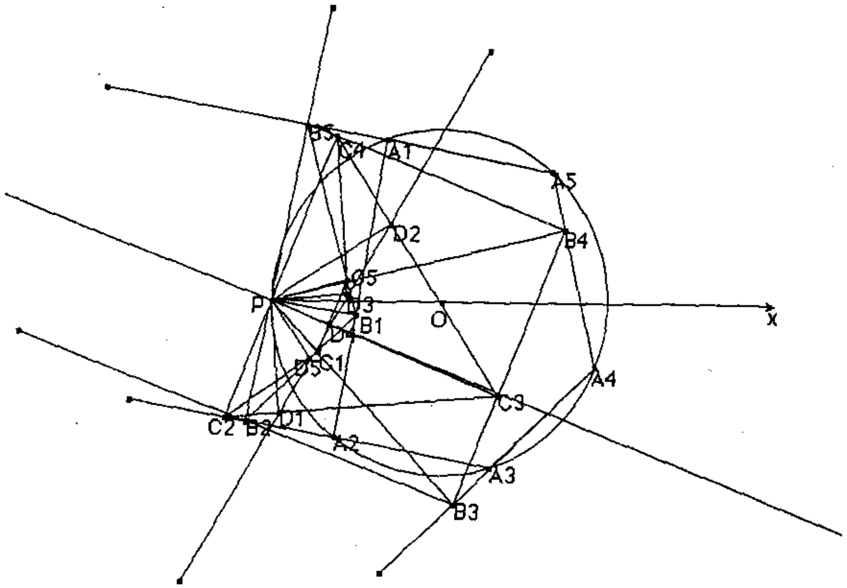
$$\rho\cos(\theta-\theta_1-\theta_2-\theta_3-\theta_4)=d\cos\theta_1\cos\theta_2\cos\theta_3\cos\theta_4$$

故 C_1 、 C_2 、 C_3 、 C_4 四點共綫。

2. 設 P 點與五邊形 $A_1A_2A_3A_4A_5$ 的五個頂點同在一個圓周上。那麼， P 點關於五邊形 $A_1A_2A_3A_4A_5$ 的三階垂足五邊形的五個頂點在一條直綫上。

証明：如圖三建立極坐標系。仿照上述証明，先求 B_1 、 B_2 、 B_3 、 B_4 、 B_5 的坐標，再求 C_1 、 C_2 、 C_3 、 C_4 、 C_5 的坐標，最後求 D_1 、 D_2 、 D_3 、 D_4 、

D_5的坐標分別爲



圖三

$D_1(d\cos\theta_1\cos\theta_2\cos\theta_3\cos\theta_4,\ \theta_1+\theta_2+\theta_3+\theta_4)$
$D_2(d\cos\theta_2\cos\theta_3\cos\theta_4\cos\theta_5,\ \theta_2+\theta_3+\theta_4+\theta_5)$
$D_3(d\cos\theta_3\cos\theta_4\cos\theta_5\cos\theta_1,\ \theta_3+\theta_4+\theta_5+\theta_1)$
$D_4(d\cos\theta_4\cos\theta_5\cos\theta_1\cos\theta_2,\ \theta_4+\theta_5+\theta_1+\theta_2)$
$D_5(d\cos\theta_5\cos\theta_1\cos\theta_2\cos\theta_3,\ \theta_5+\theta_1+\theta_2+\theta_3)$

顯然 D_1、D_2、D_3、D_4、D_5的坐標滿足方程

$$\rho\cos(\theta-\sum_{i=1}^{5}\theta_i)=d\prod_{i=1}^{5}\cos\theta_i$$

故 D_1、D_2、D_3、D_4、D_5五點共綫。

3. 設 P 點與 n 邊形 $A_1A_2...A_n$ 的所有頂點同在一個圓周上。那麼，P 點關於 n 邊形 $A_1A_2...A_n$ 的(n−2)階垂足 n 邊形的 n 個頂點都在一條直綫上。

證明：極坐標系的建立與上相仿，按照上述証明可知 P 點關於 n 邊形 $A_1A_2...A_n$的(n−2)階垂足 n 邊形的 n 個頂點的坐標滿足方程

$$\rho\cos(\theta-\sum_{i=1}^{n}\theta_i)=d\prod_{i=1}^{n}\cos\theta_i$$

故 P 關於 n 邊形 $A_1A_2...A_n$的(n−2) 階垂足 n 邊形的 n 個頂點都在一條直綫上。

從而得出：圓內接 n 邊形(n≥3)的(n−2) 階垂足共綫。

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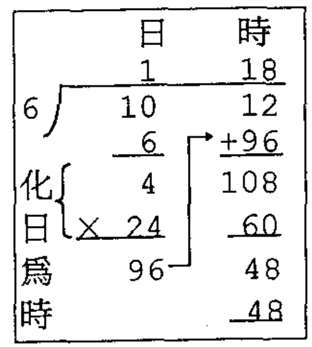
數學學習中之有意義和無意義

香港中文大學課程與教學學系
黃毅英博士

偶爾從小學五年級教科書中看到以下一道數學題：

10日12小時 ÷ 6。



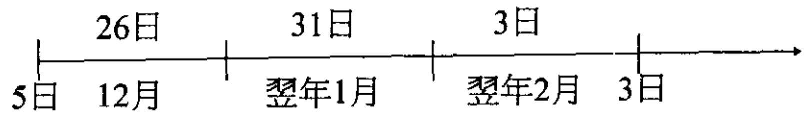
「標準」的做法是這樣的：「先除大數，若有餘數，要化爲下一個單位」(見右圖)。原來這屬於一類叫「時間單位乘除」的課題。此外，還有「時間單位的化聚」、「時間單位的加減」等，一共三個單元。如「學校學行攤位遊戲，由上午10時25分開始，一直到下午3時40分結束，問攤位遊戲共歷時多久？」、「12月5日是星期四，問翌年2月3日是星期幾？」都是屬於這一類題目。

一個自然的問題是：這類問題看似和生活甚有關係，但實質甚爲人工化，普通人恐怕一生都不會用到。既然如此，花這麼多時間學習，值得嗎？

又例如「分數的應用」這課題就可以有下面千奇百怪的數學題：

※ 今有橙汁350ml，小明喝了三分之一，問小明喝了多少？
※ 今有橙汁350ml，小明喝了三分之一，問還剩下多少？
※ 今有橙汁350ml，小明注入三分之一，問小明注入多少？
※ 今有橙汁350ml，小明注入三分之一，問現有多少？
※ 今有橙汁350ml，小明喝了三分之一，小珍喝了四分之一，問他們共喝了多少？
※ 今有橙汁350ml，小明喝了三分之一，小珍喝了四分之一，問小明比小珍多喝了多少？
※ 今有橙汁350ml，小明喝了三分之一，小珍喝了四分之一，問還剩下多少？
※ 今有橙汁350ml，小明喝了三分之一，小珍喝了餘下的四分之一，問小明比小珍多喝了多少？
※ 乙是甲的三分之一，丙是乙的四分之一，甲是300。求丙，並求三數之和。
※ 甲是乙的三分之一，丙是乙的四分之一，甲是300。求丙，並求三數之和。
※ 乙是甲的三分之一，乙是丙的四分之一，甲是300。求丙，並求三數之和。
※ 甲是乙的三分之一，乙是丙的四分之一，甲是300。求丙，並求三數之和。

我們還可以擬出更多類似的題目。從另一個角度看，這些題目都有帶出數學意味的可能。以「10日12小時 ÷ 6」爲例，這其實涉及不少解決問題的策略(爲何先除大數呢？爲何不可先把日化成時再除呢？)和綜合了不少曾經學習過的技巧。至於「12月5日是星期四，問翌年2月3日是星期幾？」一題，就如課本中的提示，可借助「時間線」去分析，這就帶出了一些數數(counting)的技巧。



其它如不少人所咎病的推理式問題，如

「1，1，2，2，3，4，3，5，6，4，7，8，？」

「小明現時面向北，向右轉三個直角，問左手的方向？」

筆者以爲，只要不斤斤計較於考核最終的答案，上面的問題均可變成有趣和極具啓發性的課堂活動。近閱已故何紫先生「童話數學」[(1)]一文，其中便談到如何透過不同的實物和輔助性活動，使數學變得如童話般有趣。學生可由此得到同一概念的不同的表象(representation)。這些前期的經驗實爲日後「高層次能力」的基礎。可惜從本地的研究[(2)]中顯示本地的師生均忽略一些較「不實質」的學習經驗，例如畫圖、作草圖(sketches)、畫非正式的表之類的活動等。

故此，花大量時間於某個課題是否值得看來端視我們能否讓學生在其中找到意義(making sense out of it)，或起碼讓學生知道自己正在做什麼，從而眞正經歷一段數學歷程。心理學家亦指出，不求甚解的操練(rote-learning)，與重複學習(repetitive learning)有所不同。要求學生記得一些法則沒所謂，但最重要的是記憶能否與理解相結合(memorisation with understanding)。

筆者相信，當課程在進度上有足夠的空間與彈性時，教師實有不少發揮的機會。

1. 「童話算術」，《何紫先生兒童小學系列：偏心》(山邊社，1977年初版，1997年修訂版)28-35頁，轉載於《數學教育》第5期19-53頁(1997)。
2. Wong, N.Y., Lam, C.C., Wong, K.M. (manuscript). Students' and teachers' conceptions of mathematics: A Hong Kong study.
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邊長爲整數且其中一角之餘弦爲有理數之三角形

K. L. LAW
佛教慧恩法師紀念中學

[前言:水戈木之 120°三元數組]

讀畢水氏在學校數學通訊第十三期發表的「 120° 三元數組」後，靈感驟至，因而寫下本文，希望與同工們共分享。

[問題]

水戈木君於「 120°三元數組」[1]一文中轉錄了 J.Gilder[2]及 K.Selkirk[3]在 Mathematical Gazatte 之文章的部份內容。他亦道出構作一三角形而其中一角爲 120°且邊長爲整數的具體方法。水氏並於文章末段要求讀者思考一個問題：當三角形之其中一角的餘弦爲有理數時，究竟我們是否仍可作出一個所有邊長皆爲整數的三角形呢？

[解答]

恆等式 $(a+b+c)^2 \equiv (a+b-c)^2 + 4bc + 4ac$ -------(*)

令 $b=u^2$ 及 $c=v^2$， $u,v \in N$

則 $4bc=(2uv)^2$

令 p=a+b+c , q=a+b−c , r=2uv 及

$2\frac{k}{m}rq = 4ac$ (m 爲負整數， k 爲整數，且 $\left|\frac{k}{m}\right|<1$) ----------(**)

則(*)式將化爲

$$p^2 = q^2 + r^2 + 2\frac{k}{m}rq \quad \text{-----------------(***)}$$

由(**)得出

$$a=\frac{(v^2-u^2)ku}{ku-mv}$$

設 $A=(ku-mv)a=(v^2-u^2)ku$

$B=(ku-mv)b=(ku-mv)u^2$

$C=(ku-mv)c=(ku-mv)v^2$

則

P=A+B+C, Q=A+B−C 及 R=2uv(ku−mv)

將滿足(***)。

因 P, Q, R 以 u 及 v 表出時皆有公因式 v ，故除去該公因式後便得

$$P'=\frac{P}{v}=2kuv-m(u^2+v^2)$$

$$Q'=\frac{Q}{v}=m(v^2-u^2) \quad \text{-----------------(****)}$$

$$R'=\frac{R}{v}=2u(ku-mv)$$

當 $u>v>\frac{ku}{m}$ 時， P'、 Q'及 R'皆爲正整數且滿足(***)。

至此，水氏之問題已獲完滿解答。

值得留意的是當 m= −2 及 k= −1 ，則除去因子 2 後得

$P''=\frac{P'}{2}=u^2+v^2-uv$

$Q''=\frac{Q'}{2}=u^2-v^2$

$R''=\frac{R'}{2}=u(2v-u)$

對應水氏所載，

$c=q^2-qr+r^2$

$b=2qr-r^2$ ---------- (*****)

$a=q^2-2qr$

除卻 P"與 c 在形式上一致外， Q"及 R"與 a 及 b 在形式上表面並不一樣。也許經些微之變數轉換後， P"、 Q"及 R"與 c 、 a 及 b 在形式上將達至一致。但作者對此等轉換無甚興趣。

若 u=3 及 v=2 ，則 (P",Q",R")=(7,5,3) 。

若於 (*****) 設 q =3 及 r =1 ，則 (c,b,a) = (7,5,3)；
而當 k=0 且 P'、 Q'及 R'皆除以−m 時，則由(****)得

$$P^* = \frac{P'}{-m} = u^2 + v^2$$

$$Q^* = \frac{Q'}{-m} = u^2 - v^2$$

$$R^* = \frac{R'}{-m} = 2uv$$

我們不單導出畢氏三元數組之具體求法，且在形式上與水氏所載的也是一致。

[後話]

因教務極度繁忙，本文的編排皆未經修飾。若因此令讀者不便，本人謹在此道歉。

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奇妙的「 72 」

柏

要多少時間才可「翻一番」？這是一個常見的投資問題。如果年利率是 r %，以複利每年結算一次計，需要多少時間才可將本金「翻一番」，使本利和為本金的兩倍呢？所需年期是可以利用下列公式概算出來的：

$$\text{所需年期} = \frac{72}{r} \qquad \text{---------------- (1)}$$

若年利率為 4%，則需要 18 年才可「翻一番」；若年利率為 9%，則需要 8 年。公式(1)是如何得到的呢？一般可以透過一連串的計算，列出年利率和所需時間從而觀察得到的。其次，我們亦可用下列方法導出公式(1)：

設 本金 = x 元
本利和 = $2x$ 元
年利率 = r%
每年結算次數 = k
時間(年) = n

以複利計算，可得下列公式：

$$2x = x\left(1+\frac{r}{100k}\right)^{nk}$$

$$2 = \left(1+\frac{r}{100k}\right)^{nk} \text{-------------------(2)}$$

我們可以利用 $\left(1+\frac{r}{100k}\right)^{\frac{100k}{r}}$ 的近似值來簡化公式(2)。$\frac{100k}{r}$ 的值愈大，$\left(1+\frac{r}{100k}\right)^{\frac{100k}{r}}$ 的值愈接近自然數 e (=2.7182818...)。若 k=2 ， r=10 ，則 $\left(1+\frac{r}{100k}\right)^{\frac{100k}{r}}$ 的值與 e 的相對誤差為 2.4%；若 k=12 ， r=8 ，則相對誤差降至 0.33%；若 k=365 (相當於複利每日結算一次)， r=8 ，則相對誤差僅 0.01%。故此，由(2)式我們可導出下列近似公式

$$\begin{aligned}
2 &= \left(1+\frac{r}{100k}\right)^{nk} \\
&= \left(1+\frac{r}{100k}\right)^{\frac{100k}{r}\times\frac{nr}{100}} \\
&= \left[\left(1+\frac{r}{100k}\right)^{\frac{100k}{r}}\right]^{\frac{nr}{100}}
\end{aligned}$$

$$\begin{aligned}
&\approx e^{\frac{nr}{100}} \\
\frac{nr}{100} &\approx \ln 2 \\
nr &\approx 100 \ln 2 \\
n &\approx \frac{100\ln 2}{r} \\
n &\approx \frac{69.3}{r} \text{ -----------------------(3)}
\end{aligned}$$

為方便概算起見，一般人多採用 72 來代替 69.3 (可能基於 72 與 69.3 接近而且是有較多因子的一個吧！)

下次要概算「翻一番」的時候，用神奇的「 72 」或「 70 」便可較快捷地得出所需時間，或是所需年利率了。

智性學習和慣性學習

立昭

暑假期間，在同事的介紹下，我看了《小學數學教育—智性學習》一書。作者史金在書中提及數學學習有兩類：一類爲智性學習，另一類爲慣性學習。

智性學習和慣性學習

智性學習是通過關聯式理解去學習，從而促進智力的增長。所謂關聯式的理解，根據史金的定義是：同學知道做什麼和爲什麼這樣做。同學有從特殊的法則或步驟中歸納出一般的數學關係去解決問題的能力。例如學習下列的數列：1、5、9、13、17......，如果學生發現了規律，他便能把整個數列推算出來。

慣性學習是透過機械式理解去學習。根據史金的定義，機械式理解是指學習者只按照規則做，但不知道理由的學習模式。同學在不明白法則來由的情況下，靠記憶一個或多個法則去解決

問題。例如減法的法則「借一當十」，分數除法的法則「顛倒除數的分子分母，然後將除號改爲乘號」，移項的法則「將數移過等號另一邊，然後改變符號」等。

慣性學習的例子

在一年級下學期，學生需要學「長度」這課題。假如老師一開始便向學生直接介紹厘米這單位，接著吩咐學生用厘米尺進行量度活動，取最接近的厘米爲答案。(例如量得擦膠長 6 厘米多些，長度就是 6 厘米；鉛筆差些少長 10 厘米，長度就算是 10 厘米。)這樣的學習便是慣性學習。學生雖然能依從老師的教導，準確量度出物件的長度，但他們根本未能掌握「長度」的概念，以及需要採用公認單位(厘米)的原因，這種形式的理解只屬機械式理解。

慣性學習的優劣

慣性學習的優點在於學生比較容易掌握學習的內容，並能夠較快及較有機會獲得正確答案，因此收穫是即時和明顯的。但是，學生遇到每個新問題時都必須聽候吩咐才能處理，故形成他們缺乏解決新難題的信心。這種學習方式也加強了行動和後果的對應作用，結果學習跟隨行動。事實上，透過慣性學習所學到的東西對隨後的學習是毫無幫助的。再者，由於慣性學習不能演繹出

數學觀念，學生因而缺乏適應能力去解決相關的問題，容易失去自信和自尊。

智性學習的例子

智性學習不在於記憶法則，而在於建立知識結構，即機略。機略必須由每一個學生在其腦海中自行建構，沒有任何人可以替他完成這個過程，但優質教學能夠提供學習情境以利進行機略建構。以學習「長度」這課題爲例，老師可按三個階段爲學生安排量度活動。首先是用直接比較的方法，即直接經驗，並幫助學生建立一個可驗証結果的心智模式。例如選擇長度較接近的兩件物件，讓學生先估計那一件物件較長，然後再作直接比較。跟著也可讓學生按長短次序排列數件物件，或讓學生互相比較手掌、手指、手臂等的長短。在完成直接比較的量度活動後，教師可利用實例，如「黑板和壁報板，何者較闊？」，引出用自訂的量度單位，如手掌、腳板、書本或木棒等的需要，讓學生先估計而後量度不同的物件。當各學生的量度結果因爲單位不相同而不能作比較時，就可踏入第三階段的量度活動，即通過意見交流和討論，引出公認單位(即厘米)的需要。透過上述的學習方法，同學們還可能發揮他們天生的創造力，如他們可找出手指、手臂、腳板的長度及指距、步距，作爲「永備尺」。以上的學習便是智性學習。因爲學生不但清楚掌握「長度」的概念，更能深切明白使用公認單位的需要，所以學生所理解的是關聯式理解。

智性學習的優劣

智性學習的優點在於使學生學習得更易和記憶得更牢固。智性學習是建基於理解的學習，故能使機略一天一天地擴展，進而提高適應力，培養學生有獨立的能力去處理新的情況。由於通過智性學習，機略的質量可自我成長，而知識本身也可作爲有效目標，同學不用依賴別人幫助，在固有的知識領域內建構，自己找出新方法解決問題，故能使同學產生內在的喜悅，而不需要依賴外在的獎勵或懲罰。

透過比較智性學習和慣性學習的優劣及以上兩個例子的說明，相信各位讀者都認同我的看法：在知識爆炸的今時今日，身爲數學老師的我們，不應再採用塡鴨式的教法，相反，啓導式的教學才能培養我們學生以智性學習的模式學習數學，發揮他們的潛能，讓他們有足夠的能力去面對這瞬息萬變的時代。

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電腦與小學數學的學習

祐

引言

近年科技的發展可謂一日千里，科技進步不但影響我們的生活，也使數學教學有了重大的轉變。這數年間，「教師以科技輔助教學」或簡稱 T3 (Teachers Teaching with Technology) 更成爲一個口號。大部份人都認爲電腦對學生有好的影響(Everybody Counts, 1989)。他們並不認爲電腦是「只求答案不求理解」的計算工具，而電腦也沒有「毀壞」傳統的數學學習。家長們更認爲學習電腦能使他們的子女保持在社會上的競爭能力。事實上，無論計算機或電腦，只要使用正確，而不被濫用或誤用，都能發揮數學教學的功能。故此我們不應把使用電腦視爲一個教學目標，而應把電腦視作一種教學資源，使它們能幫助學生加強對某些數學課題的理解。本文就如何利用電腦輔助小學數學教學作一討論，並說明如何在傳統教學法及運用新科技兩者之間作出平衡及取捨。

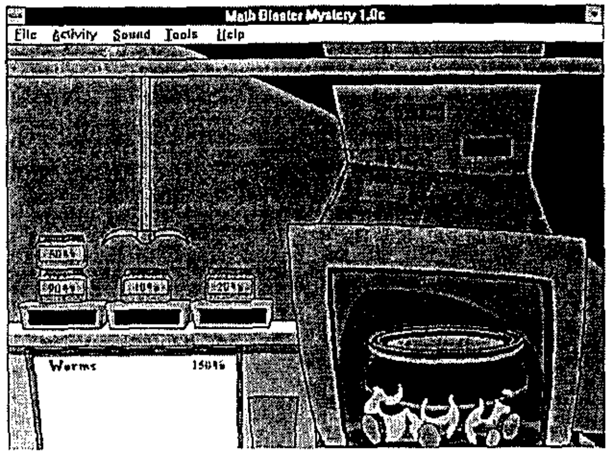
如以教學的性質來區分，我們可將電腦教學分爲三大模式，便是將電腦視作導師、學生及工具(Taylor, 1980)。每一個教學模式均有其本身的優點及缺點，而且與每個教學模式有關的電腦軟件亦有很大分別。下文便是這幾個教學模式及其有關之電腦軟件的簡介及討論。

導師模式

以電腦作導師的模式中，教師嘗試以電腦代替他們的工作。他們用一些特別爲數學而設計的軟件，嘗試爲學生提供一個能照顧個別差異的學習環境。現今大部份的電腦輔助學習或簡稱爲 CAI (Computer Assisted Instruction) 的軟件均屬於這一類。其優點是能對學生的回答─特別是錯誤的回答，作出即時的輔助，給予個別照顧。這些軟件更容許學生依自己的能力來調整學習速度。此外，軟件亦可顯示個別學生的進展情況，如教師發覺某個學生進展太慢，可給予額外的輔助。可惜人工智能的發展，還是差強人意。現今的電腦輔助學習軟件，均未能如教師般依學生的錯誤概念而自動給予選擇性的輔助。故在坊間所能購買到的軟件，大多只能夠模仿優越教師的教學方法而具人工智能的電腦輔助學習軟件，相信不會是一件易事。

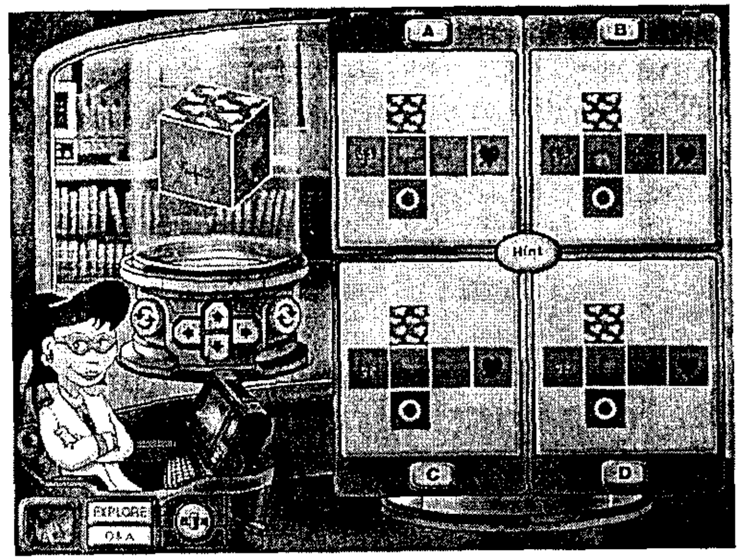
當然，除電腦輔助學習軟件外，坊間亦能購買到一些具遊戲性質的數學學習軟件。如 *Math Blaster Mystery* 其中的一個加法遊戲，旨在訓練學生加法運算的能力。在這遊戲中，電腦會給出一個隨機整數(例如 150)，而學生則需要從一些刻有不同數字的瓶子中找出三個，令瓶子上的數字的和是 150(例如刻有 20 、 40 及 90 的三個)。但只找出這三個瓶子並不足夠，學生還需依從梵天塔(Hanoi

Tower) 遊戲的規則，將這三個瓶子由大至小、由下至上順序放在預定的盤中(請參閱圖一)，才算完成整個遊戲。



圖一(取自 Math Blaster Mystery)

在另一數學教學軟件 *Mighty Math - Calculating Crew* 中，有一個名爲「三維空間探險」的遊戲。參與遊戲者需找出一立方體的對應平面圖，參與者更可將立體圖作上、下、左、右旋轉，以方便找出正確的平面圖(請參閱圖二)。這遊戲的目的是訓練學生的「空間感」。



圖二(取自 Mighty Math - Calculating Crew)

上述這兩個軟件雖具有一定的教學功能，但亦暫時未能取代教師的工作，它們只能用作鞏固學習和探究的訓練。

學生模式

以電腦作爲學生模式中，教師嘗試提供一個與導師模式剛好相反的學習環境。在這模式中，學生以「教師」的身份，編寫一些程式，執行指定的步驟。而這些步驟是經學生預先設計及構想好的。

在這些軟件中，圖龜電腦語言(LOGO)軟件是較爲突出的一種。在圖龜電腦語言中，學生只須輸入下列之簡短程式便可「指導」電腦畫出一個邊長五十單位的正方形。

```
TO SQUARE
    REPEAT 4 〔FORWARD 50 RIGHT 90〕
END
```

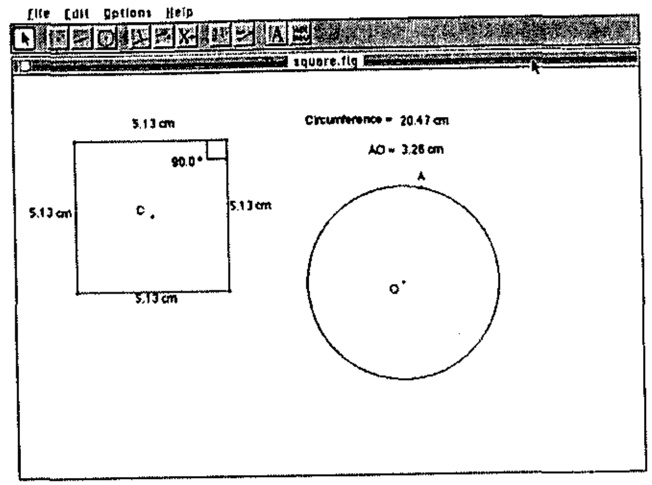
這模式的優點是能加深學生對圖形(例如上述的正方形)特性的了解，但其最大的弱點是學生需對圖龜電腦語言有一定的認識。學習這電腦語言，對小學生來說，並不容易，故這模式並不適合在小學應用。

工具模式

在以電腦作爲工具模式中，電腦軟件被視爲學習數學的工具。學生可利用各種軟件，加強或增加他們對某一數學範疇的認識。這模式包含了一系列不同功能的電腦軟件，例如電子試算表、文書處理軟件、圖表製作軟件及互動幾何軟件等。在這些軟件中，特別值得留意的是互動幾何軟件，如 *Cabri Geometry II* 及 *Geometer's Sketchpad* 等。它們能提供學生一個全新的學習環境，使學生能對幾何學上某些定理或幾何圖形的特性有更深刻的認識。

在傳統的幾何教學上，我們比較偏重理論知識的傳授，而較少涉及畫圖與探究幾何定理之關係(Laborde, 1995)。在學習正方形或圓形的幾何特性時，學生可用

Cabri Geometry II 的功能，去找出正方形邊長及角的特性或圓半徑與圓周的關係等(請參閱圖三)。



圖三(取自 *Cabri Geometry II*)

結語

從上述的討論中，我們發覺電腦並非只用作計算工具，也不是用來取代教師的教學工作。電腦的主要功能是輔助筆算及協助學生思考數學問題。更重要的是電腦軟件通常能產生多個相似而不相同的例子，以供學生學習及思考。如教師能善用如 *Cabri* 等的教學工具，必能啓發學生解決問題的能力。

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全面檢討數學課程

課程發展處

引言

數學科是香港的中、小學課程中的主要科目。現時的數學課程可根據不同的程度分爲七科：

小學程度 — 數學科（小一至小六）
中學程度 — 數學科（中一至中五）
附加數學科（中四至中五）
中六程度 — 數學及統計學科
（高級補充程度）
應用數學科（高級補充程度）
純粹數學科（高級程度）
應用數學科（高級程度）

各科目的課程綱要分別由四個科目委員會負責，每個委員會各自發展該程度課程的學習範圍。不少數學教育工作者及學術團體人士都曾提出需改善數學課程在層次上及內容上的相互配合，加強小學、中學及中六程度各數學課程的連

貫性。絕大部份科目委員會成員都同意要加強聯繫，藉此可以互相配合。

專責委員會

進行全面檢討數學科課程的提議於 1997 年 4 月 18 日得到課程發展議會 (CDC) 的批准，並於 1997 年 7 月成立全面檢討數學課程專責委員會。專責委員會的工作目標是基於穩妥的學術原則及實際需求，向課程發展議會提出建議，以改善和提高不同程度的數學課程的連貫性及課程之間的相關性。建議應包括如何協調修訂不同程度數學科課程綱要，及提供實施的方法和策略。專責委員會需於 1999 年底前向課程發展議會提交報告。

專責委員會的主要職權範圍是：
- 檢討由小一至中七年級的數學教育的宗旨和目標；
- 檢討不同程度的數學課程綱要，並特別關注課程的連貫性及一體性；
- 開展調查研究以支持全面檢討的工作；
- 提供改革各程度課程綱要之實施策略，並對教師的培訓需求及教材資源的供應提出建議；
- 向課程發展議會提交建議方案。

專責委員會由 13 位成員組成，其中包括中、小學校長及教師、院校代表、專業團體代表、工

商界人士和香港考試局、課程發展處及輔導視學處的代表。香港機場管理局成員黃景強博士被委任為委員會主席，課程發展處的總監為副主席，而秘書支援工作則由課程發展處提供。（全面檢討數學課程專責委員會的成員名單見附表一。）

全面檢討的工作範圍

當檢討不同程度的數學課程綱要時，專責委員會將首先考慮以下事項：
- 各數學科課程綱要的宗旨及目標；
- 各數學科課程綱要的設計是否合乎原先所定的宗旨及目標；和
- 各數學科課程綱要是否適當實施，從而達至所制定的宗旨及目標。

專責委員會在協調及統籌各級數學課程的發展工作中，應扮演領導的角式。專責委員會將統合各科目委員會的集體智慧，為未來的數學課程綱要的修訂定出一套完整的計劃方案。專責委員會亦將為其他學科的課程檢討工作開創一種新模式。

工作進展

直至現在，專責委員會已進行了多次會議，各委員主要關注學校數學教育的宗旨和目標。他們提議進行調查研究，以提供實質理論基礎支持

檢討工作。各委員均十分關注將來修訂課程綱要的實施策略，教師培訓的需求與及適當的教材資源提供。

專責委員會已完成了三項主要工作。首先、委員會搜集了不少有關香港及世界各地數學科的教學資料，並邀請了不同院校的學者專家親自向委員會發表意見，其中包括：

- 馮振業先生，黃家鳴先生及黃毅英博士分別詳細地介紹不同的國家及地區的數學課程的特性，包括澳洲、中國、德國、香港、紐西蘭、新加坡、台灣、英國及美國等。他們亦介紹如何設計一套完整及具一體性的數學課程，作用是可以有系統地提高學生的數學知識。
- 梁貫成博士向委員會簡介第三屆國際數學及科學研究 (TIMSS) 的調查報告，並參與有關香港學生在數學科方面的強處及弱點的討論。

其次、專責委員會致力於與各數學科目委員會的聯繫、溝通及交流意見。專責委員會成員曾多次與小學、中學及中六程度的科目委員會成員會面，就近期數學教育的發展進行多次深入的討論。主要討論範圍包括：

- 不同程度數學課程綱要的連貫性；
- 數學課程如何配合學生的不同的能力及志向；
- 發展及實施以目標爲本的中、小學數學科課程；

- 應用資訊科技於數學教學上（如計算機、電腦及互聯網絡等）。

第三、專責委員會決定進行專業的調查研究，以了解各界人士對學校數學教育的期望，以及數學教育的世界性趨勢。爲此，專責委員會制訂了兩個支援性的調查研究項目：

調查一：研究一些主要亞洲及西方國家的數學課程，將香港學生的能力及弱點與這些國家的學生作出比較。

調查二：調查不同範疇而與數學教育有關連人士的意見，諸如學生、家長、教師、僱主（人力資源方面）和高等院校學者等。藉以了解他們對現況的意見和對未來數學課程的期望。

各級數學課程內容的修訂和不同程度數學課程的連貫性

在全面檢討工作過程中，專責委員會十分著重與科目委員會成員間的聯繫溝通及意見交流。小學及中學數學科目委員會正分別修訂小一至小六及中一至中五兩個數學課程綱要，同時科目委員會亦開展了多層次和分段式的諮詢和磋商，獲得教師們及數學教育工作者提供多方面的意見，在全面課程發展的工作上吸收到寶貴的經驗，這方面的工作對全面檢討肯定十分有用。

關於中、小學數學科課程間的連繫，專責委

員會認為小學階段的數學課程應視為開啓學生對數學認識的基礎，拓展學生對數學的知識及技能。而中學數學科的課程將是小學課程的連續及伸延。

在中四至中五的數學課程，現時關注的重點在數學及附加數學兩科上。此階段的數學科目將會按學生不同的需求而重新制訂。

專責委員會亦了解到在 2001 年 9 月份，中學將實施經修訂後之中一至中五數學科課程，將目標為本的數學課程從小學伸展至中學階段。在目標為本數學課程的架構內，其中一個最受關注的地方是學習範疇的劃分。在小學程度的數學課程包含了五個學習範疇：（一）數、（二）代數、（三）數據處理、（四）度量、（五）圖形與空間。但在中學程度數學課程的學習範疇，則重組為三個：（一）數與代數、（二）數據處理、（三）度量、圖形與空間。專責委員會建議要小心處理學習範疇的劃分，要有適當的措施確保兩個課程之間的聯繫。

為不同能力及不同志向的學生而設計的數學課程

數學課程是需要適合學生的不同學習能力及興趣取向，故此必需著重研究正確的路向及方法，以提高學生在學習中的表現。數學是一門基本的科目，同時也是讓學生學習其他科目的有效

工具。學生可使用從數學科所學到的知識來解決實際困難。教授的方法是要讓學生發揮抽象思考分析以培養解決問題的能力。數學課程的設計可以包括一個核心課程，為大多數學生所採用；與此同時，亦需要為學術成績較遜的學生作出一些特別的安排，而對成績較佳的學生亦要提供增潤課程，以配合其學習能力。

高中之數學課程應為學生提供不同之學習途徑，部份學生可以專注於運算技巧及應用數學方面；而部份則可以著重理論及抽象數學思維。因此新的數學課程應容許彈性，為中一至中七的學生提供不同的選擇，以配合學生的不同需求與及不同的能力及志向。

資訊科技於數學教學的應用

全面檢討數學課程將包括研究使用現代化的資訊科技於數學上。專責委員會認同在數學教學上應盡量運用資訊科技，如計算機及電腦，兩者均可用作運算工具和數學學習工具。重點是怎樣才可以有效地運用資訊科技促進數學學習。

專責委員會同時亦探討在數學教學中採用沒有特定內容的電腦軟件，例如試算表 (spreadsheet) 及統計軟件。另一個使用資訊科技學習數學的例子，是老師及學生利用互聯網，從學校和高等院校的新聞小組取得資源，互相討論，以建立一個學校與院校間相互合作的學習環

境。

數學科之教學

專責委員會關注到是否所有的數學科教師均曾接受正統的數學教育訓練。在現實中，部分數學科教師並非受專職訓練的數學科教師，而被委派任教中、小學的數學科。因此需要設立更多的在職訓練課程，以提高這些教師的教學水平。而對一些新任職的老師，校內的持續培訓亦十分重要。邀請本港高等院校負責主持培訓課程，是保証課程的質量和水平的一個方法。

關於在學校中的數學教學，專責委員會希望教師能更多注意學生的學習態度和方法。教師應多花時間於學生的學習過程。另外，通常一般被教師忽視的是發展學生的數學感，故此要作出相應的措施以彌補這方面的缺陷。

關於培養學生解決問題的技能，專責委員會注意到在近年的公開考試中亦有測試學生在這方面的能力，某些教師對此改變可能仍未能適應，因此需要給予老師更多的培訓及支援，從而提高學生解決問題的能力。

未來的數學課程及數學課程綱要的設計

專責委員會傾向以簡易明瞭的「餐單」方式

列出數學課程綱要的內容。課程綱要的主要元素，如宗旨、目標和內容架構等均要清晰和簡潔地列明，而課程的更新可以透過補充性資料的印發。學習範疇則成爲數學課程的主要骨幹，可以定期爲教師印製各範疇的學習課業，爲教學提供資源上的支援。

數學教育的性質可分爲兩方面：數學科本身是一個獨立科目，故有其獨特性，而數學亦同時是學習其他科目（例如科學及經濟科等）的工具。未來數學課程的重點應著重培訓學生的邏輯思維及批判性的思考，與及發展學生的數學概念。

全面檢討數學課程將提出爲不同組別學生之需求而制訂一個包括基本部份和延伸部份的數學課程。其中一個設計方法是在每一個程度中加添數學增潤課題，好讓部份學生能有機會更深入和更廣泛探究數學。

專責委員會的目標是要制訂一個面向廿一世紀的數學課程，以培養學生的應變能力，使學生能面對香港未來的挑戰。一個理想數學課程應包含主要的課題，使所有的學生能在同一時間學習，亦同時能讓教師有彈性地安排教學。應使用學習目標及標準來描述數學課程的基本要求，同時亦可以讓個別學校基於學習目標及標準自行設計學校的課程。

每個學生都將會學習數學，而課題是採用循

步漸進式的學習模式。發展這個課程的目的是使學生能有系統地逐步加強對數學的認識和思考的能力。

新的數學課程應減少不同程度中重複的課題，騰出更多的時間讓教師可以加強培養學生的學習動機，幫助學生建立概念、深入了解、發展運算過程的能力和高層次的認知技能。在教導抽象的數學概念時，必須要照顧到學生是否已掌握了實質的學習經驗來支持。

前瞻

香港高等院校已被委任進行有關之調查研究以支持這次數學課程的全面檢討。專責委員會將同時進行討論主要課題和與各科目委員會繼續聯絡及交換意見。公眾的諮詢將在 1999 年 2 月份開展，調查研究報告亦將於 1999 年 5 月份完成。基於調查研究的結果，專責委員會將於 1999 年底前向課程發展議會提交最後的報告書，報告書將包括下列建議：

- 數學教育於不同程度的宗旨和目標；
- 各數學科目課程的設計和架構；
- 各數學科目課程綱要所需的修訂；
- 推荐有關的實施策略和計劃。

附表一

全面檢討數學課程專責委員成員名單（至 1998 年 6 月 30 日）

中、小學教師及校長	
陳國偉先生	大埔官立小學上午校教師
陳偉仲先生	荃灣聖方濟書院教師
何汝淳先生	英皇中學校長
李少鶴先生	聖公會始南小學上午校校長
邵忠良先生	樹仁中學教師
數學教育專業團體代表	
黃毅英博士	香港數學教育學會主席
高等院校代表	
鄭紹遠教授	香港科技大學數學系主任
馮志揚先生	香港教育學院數學系副主任
沈雪明博士	香港大學社會科學院院長
工商界人士	
黃景強博士（主席）	香港新機場管理局成員
香港考試局代表	
溫德榮先生	香港考試局數學科考試主任
教育署代表	
黎劉瑞娟女士（副主席）	課程發展處署理總監
曾健華先生	輔導視學處數學組首席督學

梁兆強先生 （秘書）	課程發展處數學及電腦教學組首席督學

Holistic Review of the Mathematics Curriculum in Hong Kong

Curriculum Development Institute

Introduction

Mathematics is a major subject in Hong Kong primary and secondary schools. The existing school mathematics curriculum comprises seven subjects at different levels of schooling. These subjects are :

Primary level - Mathematics (Primary 1 to 6);
Secondary level - Mathematics (Secondary 1 to 5), Additional Mathematics (Secondary 4 to 5);
Sixth form level - Mathematics and Statistics (Advanced Supplementary Level), Applied Mathematics (Advanced Supplementary Level), Pure Mathematics (Advanced Level), and Applied Mathematics (Advanced Level).

The subject syllabuses are looked after by four different Subject Committees, each responsible for their own level and scope of study. There have been frequent calls from different sectors of the mathematics education circle for better co-ordination of syllabus development among the mathematics subjects at primary, secondary and sixth form levels. These views are shared by members of various Subject Committees.

The Ad hoc Committee

The proposal of conducting a holistic review of the mathematics curriculum in Hong Kong was endorsed by the Curriculum Development Council (CDC) on 18 April 1997. The Ad hoc Committee for the Holistic Review of the Mathematics Curriculum was subsequently established in July 1997. The target of the holistic review is to make recommendations to the CDC on ways of enhancing continuity and intra-level coherence of the mathematics curriculum at various levels based on sound academic principles and practical demand. The implementation strategies of the recommendations through co-ordinated revision of mathematics syllabuses at various levels will also be reviewed. The Ad hoc Committee is expected to submit its final report to the CDC by the end of 1999.

The terms of reference of the Ad hoc Committee are:

- To review the aims and objectives of mathematics education from Primary One to Secondary Seven.
- To review the mathematics syllabuses at different levels with particular attention to curriculum

continuity and coherence.

- To initiate researches and surveys in support of the review.
- To propose the implementation strategies of the various syllabuses and to make recommendations on the need of teacher education and the provision of resources.
- To report the recommendations to the CDC.

The Ad hoc Committee comprises 13 members, including teachers and heads of primary and secondary schools, representatives of tertiary institutions, professional bodies and the commercial/industrial sector, and officers of the Curriculum Development Institute (CDI), Advisory Inspectorate and Hong Kong Examinations Authority. Dr. WONG King-keung of the Airport Authority was appointed as the Chairman. The Chief Executive of the CDI is the Vice-chairman and secretarial support is provided by the CDI. Table 1 is the membership list as at 31 March 1998.

Scope and Work of the Holistic Review

When reviewing the mathematics syllabuses at different levels, the Ad hoc Committee will examine:

- the aims and objectives of the mathematics syllabuses;
- whether the mathematics syllabuses are designed according to the aims and objectives; and
- whether the mathematics syllabuses are implemented to achieve the aims and objectives.

The Ad hoc Committee will play a leading role to steer curriculum development in mathematics. The Ad hoc Committee will co-ordinate the collective wisdom and concerted effort of Subject Committees of all levels, and work out an integrated plan for future syllabus revision for all the mathematics subjects. The Ad hoc Committee will also set a model for co-ordination of curriculum development work within a subject or subjects of a particular learning area.

The Progress of Holistic Review

The Ad hoc Committee held seven meetings so far. The members focused their attention on the aims and objectives of school mathematics education. They proposed to conduct researches and surveys to provide support and substantiate the review. The members also expressed keen concern in implementation strategies of future syllabus revisions, the need for teacher education and the provision of adequate resources.

The Ad hoc Committee have accomplished three major tasks. First, the Committee gathered general information about teaching and learning of mathematics in Hong Kong and overseas. Academics from education institutions were invited to make presentations in the meetings.

- Mr. C.I. Fung, Mr. K.M. Wong and Dr. N.Y. Wong made detailed presentations on the characteristics of mathematics curricula of various countries and regions, including Australia, China,

Germany, Hong Kong, Japan, New Zealand, Singapore, Taiwan, UK and USA. They also introduced ideas on the design of a co-ordinated mathematics curriculum to enhance students' cognitive development in the subject.

- Dr. K.S. Leung briefed the members on research findings of the Third International Mathematics and Science Study (TIMSS). Strengths and weaknesses of local students in various aspects of mathematics were discussed.

Second, the Ad hoc Committee put great emphasis on communicating and exchanging opinions with different Subject Committees. Long discussions on recent development of mathematics education in primary and secondary schools were held with members of the Subject Committees of the primary, secondary and sixth-form levels. Major issues discussed were :

- cross-level linkage of mathematics syllabuses;
- mathematics for students of different abilities and orientations;
- development and implementation of Target Oriented Curriculum (TOC) in primary and secondary school mathematics; and
- uses of IT in teaching and learning mathematics (such as calculators, computers and the Internet).

Third, the Ad hoc Committee agreed that supportive research would be needed to study views and expectation of different sectors on school mathematics as well as the world-wide trends of mathematics education. Two supportive research studies were designed.

- Research 1 - to study the mathematics curricula of some major Asian and western countries and the strengths and weaknesses of Hong Kong students in comparison to students of these countries.

- Research 2 - to study views of various concerned parties including students, parents, teachers, employers (from the human resources perspective), educators of tertiary institutions on the current mathematics curriculum and their expectations on the future mathematics curriculum.

Syllabus Revision at Various Levels and Cross-level Linkage of Syllabuses

The Ad hoc Committee stresses very much on communication and exchange of opinions with members of the Subject Committees during the process of holistic review. The Primary and Secondary Mathematics Subject Committees are revising the Primary 1 to 6 and Secondary 1 to 5 Mathematics syllabuses. Syllabus documents are under development and multi-stage consultation is conducted to solicit opinions of teachers and educators. Experiences gained from the current round of curriculum development work will be very useful to the holistic review.

Regarding the linkage of mathematics at primary and secondary levels, the Ad hoc Committee considers

primary mathematics a foundation stage for development of students' mathematics knowledge and skills. The secondary mathematics curriculum will serve as a continuation and extension of the primary mathematics curriculum.

Concerns about mathematics at the Secondary 4 to 5 level focus on the existence of 2 subjects : Mathematics and Additional Mathematics. Mathematics subjects for this level may have to be re-structured to cater for diversified needs of students.

Regarding the implementation of Target Oriented Curriculum (TOC) in mathematics, the secondary schools will implement the revised Secondary 1 to 5 Mathematics syllabus in September 2001 as a measure to extend TOC mathematics to secondary level. One area of concern is the number of learning dimensions in the TOC mathematics framework. For TOC mathematics at primary level, there are 5 learning dimensions : "Number", "Algebra", "Data Handling", "Measures" and "Shapes and Space". At secondary level, 3 combined dimensions : "Number and Algebra", "Data Handling" and "Measures, Shape and Space" are proposed. The Ad hoc Committee suggests to handle the combination with great care and measures should be taken to ensure cross-level linkage within dimensions.

Mathematics for Students of Different Abilities and Orientations

For the development of a mathematics curriculum to suit students of different abilities and orientations, emphasis will be placed on finding proper ways and means to enhance students' performance. Mathematics is a fundamental subject and it is also a tool to help students learn other subjects. Students should be able to apply their mathematics knowledge in problem solving. The subject should be taught in a way that students can develop the ability to solve problems which required abstract thinking. A core mathematics syllabus may be designed for the average students. And, at the same time, special arrangements will be made to help the academically low achievers and to provide enrichment to the more able students.

Students should be allowed to go through different tracks in the mathematics curriculum at senior secondary level or above. Some will spend more time in the technical aspects and applications of mathematics while others will have more training in theoretical and abstract parts of mathematics. Flexibility of introducing alternative mathematics syllabuses for Secondary 1 to 5 to cater for learner differences, such as different abilities and orientation will be studied.

Using Information Technology in Teaching and Learning Mathematics

The holistic review will examine the use of modern information technology (IT) in mathematics. The Ad hoc

Committee endorses the use of IT, such as calculators and computers, in teaching and learning mathematics. Calculators and computers are treated as calculating tools and also as tools for learning mathematics. Emphasis will be placed on how IT could be used efficiently and effectively to enhance learning in mathematics.

The use of content-free software, such as spreadsheets and statistical software packages will also be explored. Other examples of using IT in mathematics, such as teachers and students to acquire resources from the Internet and newsgroups for communication among schools and tertiary institutions, have been mentioned. This will probably provide a collaborative learning environment among the school community.

Teaching Mathematics in Schools

Regarding the training of mathematics teachers, the Ad hoc Committee is interested in whether all mathematics teachers in schools are properly trained. There may be teachers, who are not specialists in mathematics, but are assigned to teach primary and junior secondary mathematics. More in-service teacher training programmes will be required to raise the standard of these teachers. In-house training within schools are considered very important for new teachers. Commissioning teacher training programmes to academic institutions will be one of the measures to ensure the quality of the programmes.

Regarding the teaching of mathematics in schools, the Ad hoc Committee would like to see teachers paying more attention in the learning style and attitude of our students. Teachers should spend more time in students' learning process. Another area which is often neglected by teachers is the development of students' sense of mathematics. Remedial work has to be done to foster the notion.

Regarding the training of students' problem solving skills, the Ad hoc Committee notes that the public examination questions in recent years have been set to test students' ability in this area. Some teachers do not feel comfortable about these changes. More training and support in teaching problem solving in classrooms will be needed.

Design of the Mathematics Syllabuses and the Future Mathematics Curriculum

The Ad hoc Committee prefers to have mathematics syllabuses written in an easy-to-read "menu" format. The essential elements of the syllabuses, such as aims, objectives and content framework will be clearly elaborated. Supplementary information will be issued from time to time to update and refresh the syllabus contents. The learning dimensions will act as the backbone of the mathematics curriculum and learning tasks will be published regularly to provide resource support to teachers.

The nature of mathematics education has two fold. Mathematics is a discipline in itself and it is also a tool for learning of other subjects, such as Science and Economics. More emphasis will be put on training students' logical thinking and critical thinking, and on

developing students' mathematical concepts.

Issues such as foundation and extension topics for specific groups of students will be included as components of the holistic review. One of the strategies is to provide enrichment of mathematics topics for each level. Students will be given chances to develop a broader and deeper understanding of mathematics topics.

The Ad hoc Committee aims at proposing the development of a mathematics curriculum that meets the needs of students as individuals to face the challenges of Hong Kong in the 21st Century. An ideal mathematics curriculum should contain essential topics that all students should learn and at the same time, it should also allow teachers to have flexibility in teaching. Learning targets and standards could be set for delineating the basic requirements in the mathematics curriculum. Schools will be allowed to design their own school curriculum based on the learning targets and standards.

Mathematics should be studied by all students and its topics should be arranged into progressive stems or learning dimensions. The aim was to develop a curriculum that would enhance students' cognitive development in mathematics in a more co-ordinated way.

The mathematics curriculum should reduce duplicated topics at different levels to allow more time for motivation of learning, enhancing concept building, deep understanding and developing process abilities and high-level cognitive skills. Teaching of abstract mathematical ideas should be supported by students' concrete experiences at earlier stages.

Looking Ahead

Tertiary institutions are commissioned to conduct the support research studies. In the meantime, the Ad hoc Committee will tackle the major issues and further communicate with the Subject Committees. Public consultation will be conducted in February 1999. Reports of research studies will be ready by May 1999. Based on the research findings, the Ad hoc Committee will make recommendations in its final report to CDC by the end of 1999. The recommendations will cover:

- aims and objectives of mathematics education at various levels;
- design and framework of various mathematics subjects;
- necessary syllabus revisions in mathematics subjects; and
- strategic plans in implementing the recommendations.

Enquiry

For enquiry, please write to :

Secretary,
Ad hoc Committee for the Review of
Mathematics Curriculum
Curriculum Development Institute,
Education Department

Room 1308, Wu Chung House,
213 Queen's Road East,
Wan Chai, Hong Kong.
(Fax: 2573 5299 or E-mail :
edcdi102@hknet.com)

Table 1

CDC Ad Hoc Committee on Holistic Review of the Mathematics Curriculum Membership List (as at 31 March 1998)

School Principals and Teachers :

Mr. CHAN Kwok-wai
Teacher, Tai Po Government Primary School AM

Mr. CHAN Wai-chung
Teacher, St. Francis Xavier's School, Tsuen Wan

Mr. HO Yue-shun
Principal, King's College

Mr. LEE Siu-hok
Principal, SKH Chi Nam Primary School AM

Mr. SIU Chung-leung
Teacher, Shue Yan Secondary School

Member from Professional Bodies :

Dr. WONG Ngai-ying
Chairman, HK Association for Mathematics Education

Members from Tertiary Institutions :

Prof. CHENG Shiu-yuen
Head, Department of Mathematics,
Hong Kong University of Science & Technology

Mr. FUNG Chi-yeung
Deputy Head, Department of Mathematics,
The Hong Kong Institute of Education

Dr. SHEN Shir-ming
Dean, Faculty of Social Science,
The University of Hong Kong

Member from Commercial/Industrial Sector :

Dr. WONG King-keung
Member, Airport Authority (Chairman)

Member from Hong Kong Examinations Authority :

Mr. WAN Tak-wing
Subject Officer (Mathematics),
Hong Kong Examinations Authority

An Easy Method for Solving an Inequality of the Form $(x-a)(x-b)(x-c)\ldots\ldots(x-z)\geq 0$, >0, ≤ 0 or <0

George LUI
Chan Shu Kui memorial School

Let us consider $(x+1)(x-2)<0$.
When will the product $(x+1)(x-2)$ be less than zero?
It actually depends on whether $(x+1)$ and $(x-2)$ are positive or negative (i.e. on the signs of the factors).
If $(x+1)$ and $(x-2)$ have the same sign, then $(x+1)(x-2)>0$.
If $(x+1)$ and $(x-2)$ have different signs, then $(x+1)(x-2)<0$.
So, we must consider how the signs of $(x+1)$ and $(x-2)$ change.

		$x+1=0$		$x-2=0$	
$(x+1)$	−		+		+
$(x-2)$	−		−		+
		−1		2	
$(x+1)(x-2)$	+		−		+

Figure 1

From the bottom line in Figure 1, we see that $(x+1)(x-2)<0$ when $-1<x<2$.

Let us consider another inequality $(2x+1)(x-1)(2x-3)<0$

	D		C		B		A
$(2x+1)$	−		+		+		+
$(x-1)$	−		−		+		+
$(2x-3)$	−		−		−		+
		$-\frac{1}{2}$		1		$\frac{3}{2}$	
$(2x+1)(x-1)(2x-3)$	−		+		−		+

Figure 2

So, if $(2x+1)(x-1)(2x-3)<0$, then $1<x<\frac{3}{2}$ or $x<-\frac{1}{2}$.

It is easily seen that in the part above the horizontal line in Figure 2, there is no (−) sign in region A, one in region B; two in region C and three in region D. That is why in the row below the horizontal line, we have an alternation of (+) and (−) signs, starting with the (+) sign from the right. The above reasoning can be extended to an inequality of the form $(x-a)(x-b)$ .... ≥ 0 , >0 , ≤ 0 or <0 involving any number of factors.

Based on the findings above, we are able to solve the captioned form of inequalities easily. Details of the easy method of solving the inequalities are divulged in the examples that follow for your reference.

Example 1 Solve $(3x+1)(3x+2)(x-3)(2x-3)(x-1) \geq 0$.

Product	−		+		−		+		−		+
		$-\frac{2}{3}$		$-\frac{1}{3}$		1		$\frac{3}{2}$		3	

By easy method

Draw the number line first. Then, on the line mark the points corresponding to those x values making the factors zero, thus dividing the number line into regions. Finally, mark the regions with (+) and (−) signs alternately, starting with (+) sign from the right.

From the diagram, the solution is $x \geq 3$, $1 \leq x \leq \frac{3}{2}$ or $-\frac{2}{3} \leq x \leq -\frac{1}{3}$.

Remarks All the factors must be expressed in the form $(Ax+B)$ or $(Ax-B)$, where $A>0$ and $B>0$. Otherwise, the (+) and (−) signs would not occur alternately and the easy method cannot be applied.

Example 2 Solve (x−1)(2−x)(x+3)(−2x+3)<0 .

Solution

First of all, express (2−x) as −(x−2) and (−2x+3) as −(2x−3). Then, the inequality becomes

$$(x-1)[-(x-2)](x+3)[-(2x-3)]<0$$
$$\text{i.e.}\quad (x-1)(x-2)(x+3)(2x-3)<0$$

Product	+		−		+		−		+
		−3		1		$\frac{3}{2}$		2	

From the diagram, the solution is $\frac{3}{2}< x <2$ or $-3< x <1$.

Equation of Circle with Three Given Points

KWOK Ka-keung
T.W.G.Hs. Wong Fut Nam College

In most textbooks, it is a routine exercise in coordinate geometry to determine the equation of a circle through three given points. The method usually used is to let the equation of the circle be

$$x^2+y^2+Dx+Ey+F=0 .$$

Substitute the three given points into the equation to get three linear equations in unknowns D, E and F.

Here is another method using the idea of family of circles.
Let the three given points be $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$.
A circle passes through A and B is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

(This is the circle with A and B as the endpoints of a diameter.)
The straight line passes through A and B is

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

or $$(x - x_1)(y_1 - y_2) - (y - y_1)(x_1 - x_2) = 0$$

It follows that the family of circles through A and B is

$$\begin{aligned}&(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \\ &k[(x - x_1)(y_1 - y_2) - (y - y_1)(x_1 - x_2)] = 0\end{aligned} \quad \text{---(*)}$$

If (*) passes through C, then it should be satisfied by (x_3, y_3).
By substituting C into (*), k can be determined and hence the required equation may be obtained by the substitution of k in (*).

Example

Find the equation of the circle passing through the points A(3,6), B(8,1) and C(11,10).

Solution

The circle with AB as diameter is

$$(x-3)(x-8) + (y-6)(y-1) = 0$$

and the straight line through A and B is

$$y - 1 = \frac{6-1}{3-8}(x - 8) \quad \text{i.e. } x+y-9 = 0$$

The family of circles through A and B is

$$(x-3)(x-8) + (y-6)(y-1) + k(x+y-9) = 0 \text{ ------(*)}$$

If (*) passes through C(11,10), then

$$(11-3)(11-8) + (10-6)(10-1) + k(11+10-9) = 0$$

and so k = –5.

Substitute k = –5 into (*), the required equation is

$$(x-3)(x-8) + (y-6)(y-1) - 5(x+y-9) = 0$$

or $$x^2 + y^2 - 16x - 12y + 75 = 0$$

Is it a good method to find the equation of circle with three given points?

On the number π

Peter

It is almost impossible to find the exact value of π. However, through the procedures that follow, it is able to find a small enough interval containing the real number π.

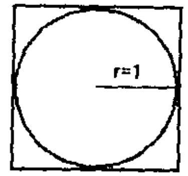
Definitions

C_n : the perimeter of a regular n-sided polygon which circumscribes a unit circle.

I_n : the perimeter of a regular n-sided polygon which is inscribed in a unit circle.

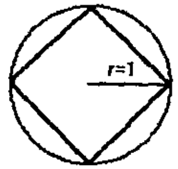
Examples

(1) Find C_4.



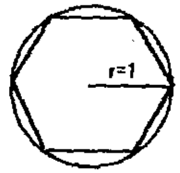
$C_4 = 8$

(2) Find I_4 .



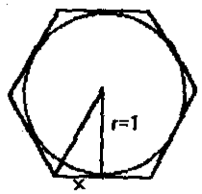
$I_4 = 4\sqrt{2}$

(3) Find I_6.



$I_6 = 6$

(4) Find C_6 .



$C_6 = 12x$

Since $x = \tan 30°$

$x = \frac{1}{\sqrt{3}}$

$$\therefore C_6 = \frac{12}{\sqrt{3}}$$
$$= 4\sqrt{3}$$

Definitions

$$A(a,b) = \frac{a+b}{2}$$
$$G(a,b) = \sqrt{ab}$$
$$H(a,b) = \frac{2ab}{a+b}$$

Examples

(5) $A(4,8) = \frac{4+8}{2}$
$= 6$

(6) $G(4,8) = \sqrt{32}$
$= 4\sqrt{2}$

(7) $H(4,8) = \frac{2 \times 4 \times 8}{12}$
$= \frac{16}{3}$
$= 5\frac{1}{3}$

Theorems (1) $C_{2n} = H(C_n, I_n)$
(2) $I_{2n} = G(I_n, C_{2n})$

Proof (1) $C_{2n} = 4nx$ where $x = \tan \frac{360^\circ}{4n}$

$$C_n = 2n \tan \frac{360^\circ}{2n}$$
$$C_{2n} = 4n \tan \frac{360^\circ}{4n}$$
$$I_n = 2n \sin \frac{360^\circ}{2n}$$
$$I_{2n} = 4n \sin \frac{360^\circ}{4n}$$

$$H(C_n, I_n) = \frac{8n^2 \tan \frac{360^\circ}{2n} \sin \frac{360^\circ}{2n}}{2n(\tan \frac{360^\circ}{2n} + \sin \frac{360^\circ}{2n})}$$

$$= 4n \frac{\tan \frac{360^\circ}{2n} \sin \frac{360^\circ}{2n}}{\tan \frac{360^\circ}{2n} + \sin \frac{360^\circ}{2n}}$$

Since $\frac{\tan 2\theta \sin 2\theta}{\tan 2\theta + \sin 2\theta} = \frac{\sin^2 2\theta}{\sin 2\theta + \sin 2\theta \cos 2\theta}$

$$= \frac{\sin 2\theta}{1+\cos 2\theta}$$

$$= \frac{2\sin\theta \cos\theta}{1+2\cos^2\theta - 1}$$

$$= \tan \theta$$

$$H(C_n, I_n) = 4n \tan\frac{360^\circ}{4n}$$

$$= C_{2n}$$

Proof (2)

$$G(I_n, C_{2n}) = \sqrt{8n^2 \tan\frac{360^\circ}{4n} \sin\frac{360^\circ}{2n}}$$

$$= \sqrt{8n^2 \tan\frac{360^\circ}{4n} \times 2\sin\frac{360^\circ}{4n} \cos\frac{360^\circ}{4n}}$$

$$= \sqrt{16n^2 \sin^2(\frac{360^\circ}{4n})}$$

$$= 4n\sin\frac{360^\circ}{4n}$$

$$= I_{2n}$$

Theorems (3) $C_n > C_{2n} > C_{4n} > \ldots\ldots\ldots$

(4) $I_n < I_{2n} < I_{4n} < \ldots\ldots\ldots \ldots$

for $n \geq 3$

Proof

Obviously $C_n > I_n$

Since $0^\circ \leq \frac{360^\circ}{2n} \leq 60^\circ$

$$0^\circ \leq \cos\frac{360^\circ}{2n} \leq 1$$

$$\frac{1}{\cos\frac{360^\circ}{2n}} > 1$$

or $\tan\frac{360^\circ}{2n} > \sin\frac{360^\circ}{2n}$

or $2n \tan\frac{360^\circ}{2n} > 2n \sin\frac{360^\circ}{2n}$

$$C_n > I_n$$

$$C_n(C_n + I_n) > 2C_n I_n$$

$$C_n > \frac{2C_n I_n}{C_n + I_n} = H(C_n, I_n)$$

$$C_n > C_{2n}$$

Moreover $C_{2n} > I_n$

Since $(C_n + I_n)I_n < (C_n + C_n)I_n$

$$(C_n + I_n)I_n < 2C_n I_n$$

$$I_n < H(C_n, I_n) = C_{2n}$$

$$\sqrt{I_n C_{2n}} > \sqrt{I_n I_n} = I_n$$
$$G(I_n, C_{2n}) > I_n$$
$$I_{2n} > I_n$$

Theorem (5)

$I_m < 2\pi$

$C_m > 2\pi$ for all m.

$$I_4 < \ldots\ldots < 2\pi < \ldots\ldots < C_4$$

Approximating π

n	I_n	C_n
4	$4\sqrt{2} = 5.656854249$	8=8
8	$G(I_4, C_8)$ $= \sqrt{I_4 C_8}$ $= 6.122934918$	$H(C_4, I_4)$ $= \dfrac{2 \times 5.656854249 \times 8}{5.656854249 + 8}$ $= 6.627416998$
16	$G(I_8, C_{16})$ $= 6.242890305$	$H(C_8, I_8)$ $= 6.365195756$
32	$G(I_{16}, C_{32})$ $= 6.273096981$	$H(C_{16}, I_{16})$ $= 6.303449815$
64	$G(I_{32}, C_{64})$ $= 6.280662314$	$H(C_{32}, I_{32})$ $= 6.28823677$

For n=64

$6.280662314 < 2\pi < 6.28823677$

$3.140331157 < \pi < 3.144118385$

n	I_n	C_n
6	6	$4\sqrt{3} = 6.928203238$
12	$G(I_6, C_{12})$ $= 6.211657082$	$H(C_6, I_6)$ $= 6.430780618$
24	$G(I_{12}, C_{24})$ $= 6.265257227$	$H(C_{12}, I_{12})$ $= 6.319319884$
48	$G(I_{24}, C_{48})$ $= 6.278700406$	$H(C_{24}, I_{24})$ $= 6.29217243$
96	$G(I_{48}, C_{96})$ $= 6.282063902$	$H(C_{48}, I_{48})$ $= 6.285429199$

For n=96

$6.282063902 < 2\pi < 6.285429199$

$3.141031951 < \pi < 3.1427146$

It is clear that the larger the number n, the better is the precision of the range of π.

A square is not a rectangle ?

Sze-Hau

The question given in the title must be familiar to mathematics teachers of junior forms. It illustrates very sharply the way in which teachers and learners may have different ideas about the meanings of words. Mathematicians attempt to remove ambiguities from the mathematics they write down and use words in technical ways to ensure this. It is therefore important for teachers to use these technical words correctly so that learners can acquire the meanings they have in mathematics.

One day a S1 student came and asked : " Sir, is a parallelogram a trapezium ? " Although my immediate answer was "Yes", I checked the correctness of my answer by referring to several reference books. Of course the key point is : What is a trapezium ? It surprised me that the answer varies from books to books ! Therefore, there may be different ideas about the meaning of some words among teachers (mathematicians) !

Few local textbooks have writen down a precise definition of a trapezium.

Some books, for example [1], [3], [5], say that a trapezium is a quadrilateral in which two opposite sides are parallel but the other sides are not parallel. Other books, for example [2] and [4], define trapezium as a quadrilateral with (at least) one pair of parallel sides.

In conclusion, I suggest that teachers should (1) keep alert to the ambiguities of words and symbols in the communication with students and (2) determine whether a learning difficulty is due to unfamiliarity with conventions, or is conceptual -- and take appropriate action.

References :

[1] Abbott, P. : Teach Yourself Geometry, ELBS.

[2] Clapham, C. : The Concise Oxford Dictionary of Mathematics, Oxford University Press.

[3] Daintith, J. and Nelson, R. D.(Eds) : The Penguin Dictionary of Mathematics, Penguin book.

[4] Gellert, W.; Gottwald, S.; Hellwich, M.; Kastner, H. and Kustner, H.(Eds) : The VNR Concise Encyclopedia of Mathematics, Van Nostrand Reinhold.

[5] Health, T.L.(Translated) : The Thirteen Books of Euclid's Elements, Dover.

The Fifteenth Hong Kong Mathematics Olympiad (HKMO)

Mathematics Section

The heat event of the 15th HKMO was held on 14 February 1998 in Belilios Public School, Tsuen Wan Government Secondary School and Sheung Shui Government Secondary School. A total of 188 schools participated in the event. The results of the heat event were as follows :

Hong Kong Region

Champion : Wah Yan College, Hong Kong
1st runner-up : King's College
2nd runner-up : Cheung Chuk Shan College

Kowloon Region

Champion : La Salle College
1st runner-up : Wah Yan College, Kowloon
2nd runner-up : Ying Wa College

New Territories Region

Champion : Tsuen Wan Public Ho Chuen Yiu Memorial College
1st runner-up : Tsuen Wan Government Secondary School
2nd runner-up : Tuen Mun Government Secondary School

After the heat event, 40 schools were selected to enter the final event which was held on 14 March 1998 at the hall of the Hong Kong Institute of Education (Town Centre). The 40 finalists were as follows :

Team No.	*Name of School*
1	Chan Sui Ki (La Salle) College
2	Cheung Chuk Shan College
3	Christian Alliance Cheng Wing Gee College
4	Christian Alliance S C Chan Mem College
5	Chuen Yuen College
6	Diocesan Boys' School
7	Heung To Middle School
8	Ho Lap College (Spon. By Sik Sik Yuen)
9	King's College
10	La Salle College
11	Ming Yin College
12	Munsang College
13	New Asia Middle School

14 Pui Ying College
15 Queen Elizabeth School
16 Queen's College
17 Raimondi College
18 San Wui Comm Society Chan Pak Sha School
19 Sha Tin Government Secondary School
20 Shatin Tsung Tsin Secondary School
21 Shau Kei Wan Government Secondary School
22 Shun Tak Frat Assn Lee Shau Kee College
23 Sing Yin Secondary School
24 SKH Bishop Baker Secondary School
25 SKH Bishop Mok Sau Tseng Sec School
26 St Joseph's College
27 St Mary's Canossian College
28 St Paul's College
29 The Bishop Hall Jubilee School
30 Tsuen Wan Government Secondary school
31 Tsuen Wan Public Ho Chuen Yiu Mem College
32 Tuen Mun Government Secondary School
33 TWGHs Wong Fut Nam College
34 TWGHs Yau Tze Tin Memorial College
35 TWGHs Kap Yan Directors' College
36 Valtorta College
37 Wah Yan College Hong Kong
38 Wah Yan College Kowloon

39 Ying Wa College
40 Ying Wa Girls' School

The prize giving ceremony was held after the final event. The Principal Inspector, Mathematics Section, Advisory Inspectorate Division of Education Department, Mr. TSANG Kin-wah and the Divisional Director, Division of Secondary, Technical & Special Education of the Hong Kong Institute of Education, Dr. John A. W. CALDWELL were the guests of honour and they presented the trophies and prizes to the winners.

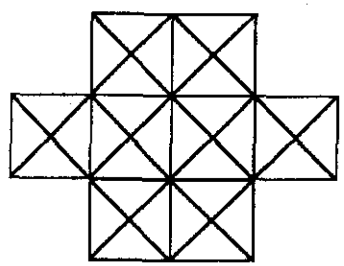
The champion of the final event was Diocesan Boys' School, the first runner-up and the second runner-up were respectively The Bishop Hall Jubilee School and Ying Wa College.

The champion of the poster design competition for the 15th HKMO was HO Wai-yee of Ju Ching Chu Secondary School (Yuen Long). The first runner-up was MAK Pui-yan of St. Simon's Lui Ming Choi Secondary Technical School and second runner-up was CHENG Mei-chun of Buddhist Chi Hong Chi Lam Memorial College. The champion poster is shown in Appendix I for your information.

A mathematics camp was held in May 1998 for the 40 teams participating in the final event of the 15th HKMO. It aimed at providing the finalists recreational activities as well as enhancing their knowledge in Mathematics. The 15th HKMO came to an end after the camp.

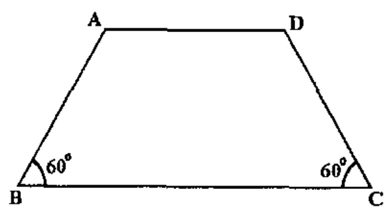
PASTIMES

1. How many squares are there in the figure on the right ?

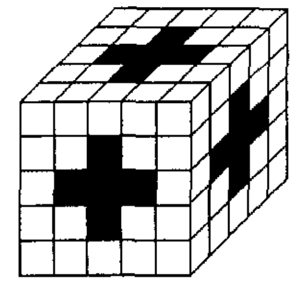


2. x is a four-digit number with the first two digits equal and the last two digits also equal (e.g. 1122). Find the value of x if it is also a square number.

3. ABCD is a trapezium with $\angle ABC = \angle DCB = 60^\circ$ and BC = 2AD. How can you cut the trapezium into four identical pieces ?



4. "+"–shaped holes are drilled through a cube of side 5cm long from all the six faces as shown in the figure on the right. The remained part of the cube is then dipped into paint and cut into cubes of side 1 cm long after dried. How many faces of all the small cubes are not painted ?



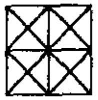
5. A, B, C, D, E and F are six different digits and

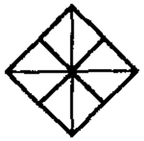
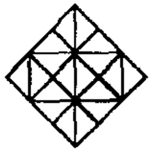
```
      AABC
  ×    DEF
 ----------
     □□□□□
    □□□□□
    □□□□
 ----------
    BBBBBB  .
```

What number does DEF stand for ?

Suggested Solutions to PASTIMES

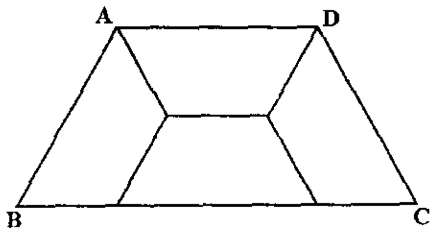
1. There are 24 squares :

8 , 2 , 9  or ,

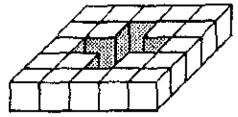
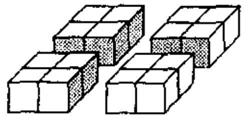
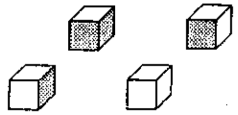
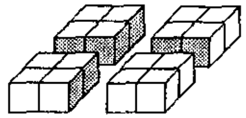
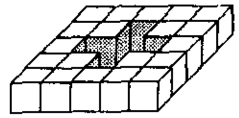
4  and 1 

2. Let the first two digits be $\boldsymbol{a}$ and the last two digits be $\boldsymbol{b}$. Obviously, $\boldsymbol{x}$ is divisible by 11 and the quotient is $100\boldsymbol{a}+\boldsymbol{b}$. Since $\boldsymbol{x}$ is a square number, $100\boldsymbol{a}+\boldsymbol{b}$ must also be a multiple of 11. So $100\boldsymbol{a}+\boldsymbol{b}$ can only be 209, 308, 407, ..., 902. When divided by 11, the quotients are 19, 28, 37, ..., 82. Among them, only 64 is a square number. So $\boldsymbol{x} = 11\times11\times64 = 7744$.

3.



4. There are 240 faces not painted.

Layer	No. of cubes in the layer	No. of painted faces
	20	20+20+12+4=56 (Top, Outer, Inner, Bottom)
	16	8×4+12=44 (Bottom)
	4	16
	16	8×4+12=44
	20	20+20+12+4=56
Total	76	216

$\therefore$ Number of faces not painted = 76×6−216 = 240.

5. DEF stands for 198.

- The product of AABC and E is a five-digit number while the product of AABC and D is a four digit-number. So D < E. (*)
- BBBBBB = B × 111111 = B×3×7×11×13×37. Obviously, AABC is not a multiple of 11. So DEF must be a multiple of 11.
- Since 3×37=111 and the difference between AAA×10 and AABC is a two-digit number, AABC is not a multiple of 111. Obviously, DEF is also not a multiple of 111. So, either 3 or 37 is a factor of DEF (but not both) and the other must be a factor of AABC.
- If 37 is a factor of DEF, then DEF must be either 407 (11×37=407), or 814. By (*), none of them are true. So 37 is factor of AABC and 3 (therefore, 33) is a factor of DEF.
- If 13 is a factor of DEF, then DEF must be either 429 (33×13=429), or 858. By (*), none of them are true. So 13 is factor of AABC.
- If 7 is a factor of DEF, then DEF must be either 231 (33×7), 462, 693 or 924. By (*), none of them are true. So 7 is factor of AABC.
- In conclusion, AABC is a multiple of 7, 13 and 37. So it must be 7×13×37 =3367 (6734 is rejected as the first two digits are unequal).
- DEF = 666666 ÷ 3367 = 198.

For Your Information

1. International Mathematical Olympaid (IMO)

(a) 38th IMO

The 38th IMO was held in Mar del Plata, Argentina from 18 to 31 July 1997 which attracted contestants from 82 countries/regions. Hong Kong had sent a team to participate in the Olympaid and the contestants of the team were

CHAN Chung-lam	(Bishop Hall Jubilee School)
CHEUNG Pok-man	(STFA Leung Kau Kui College)
LAU Lap-ming	(St. Paul's College)
LEUNG Wing-chung	(Queen Elizabeth School)
MOK Tze-tao	(Queen's College)
YU Ka-chun	(Queen's College)

The 5 top ranking countries in the Olympaid were :

Rank	Country
1	China
2	Hungary
3	Iran
4	Russia
4	USA

Note: Russia and USA got the same score in the Olympaid.

The performance of the Hong Kong Team was encouraging. It won 5 bronze medals and was ranked 31 among the 82 paricipating countries/regions.

(b) 39th IMO

The 39th IMO was held in Taipei, Taiwan from 10 to 21 July 1998 which attracted contestants from 76 countries/regions. Hong Kong had sent a team to participate in the Olympaid and the contestants of the team were

CHAN Kin-hang	(Bishop Hall Jubilee School)
CHEUNG Pok-man	(STFA Leung Kau Kui College)
CHOI Ming-cheung	(King's College)
LAU Lap-ming	(St. Paul's College)
LAW Ka-ho	(Queen Elizabeth School)
LEUNG Wing-chung	(Queen Elizabeth School)

The 5 top ranking countries in the Olympaid were :

Rank	Country
1	Iran
2	Hungary
3	Bulgaria
4	India
5	USA

Note: China did not participate in this IMO.

The performance of the Hong Kong Team was encouraging. It won 1 silver medal, 3 bronze medals and 1 honourable mention and was ranked 25 among the 76 paricipating countries/regions.

The 40th IMO will be held in Romania in July 1999.

2. Results of the Hong Kong Mathematics Olympaid (HKMO) and HKMO Poster Design Competition

Quite a number of teachers have expressed interest in knowing the results of the past HKMO and HKMO Poster Design Competitions on different occasions (e.g. during seminars/courses organized by the Mathematics Section). With the great effort of the editorial board, the results are found and divulged as follows.

Results of HKMO

First HKMO (1983-84)

Champion : Wong Tai Shan Memorial College
1st runner-up : SKH Lui Ming Choi Secondary School
2nd runner-up : CNEC Christian College

Second HKMO (1984-85)

Champion : Methodist College
1st runner-up : Ying Wa College
2nd runner-up : St. Louis School

Third HKMO (1985-86)

Champion : Ying Wa College
1st runner-up : Choi Hung Estate Catholic Secondary School
2nd runner-up : Ha Kwai Chung Government Secondary Technical School

Fourth HKMO (1986-87)

Champion : King's College
1st runner-up : St. Francis of Assisi's College
2nd runner-up : St. Joseph's College

Fifth HKMO (1987-88)

Champion : Ying Wa College
1st runner-up : Ying Wa Girls' School
2nd runner-up : Rosaryhill School

Sixth HKMO (1988-89)

Champion : King's College
1st runner-up : Ha Kwai Chung Government Secondary Technical School
2nd runner-up : Queen's College

Seventh HKMO (1989-90)

Champion : Clementi Secondary School
1st runner-up : Jockey Club Government Secondary Technical School
2nd runner-up : Pui Kiu Middle School

Eighth HKMO (1990-91)

Champion : Queen's College
1st runner-up : Ying Wa College
2nd runner-up : Chuen Yuen College

Ninth HKMO (1991-92)

Champion : NTHYK Yuen Long District Secondary School
1st runner-up : Ying Wa College
2nd runner-up : Heung To Middle School

Tenth HKMO (1992-93)

Champion : Clementi Secondary School
1st runner-up : Ying Wa College
2nd runner-up : Pui Kiu Middle School

Eleventh HKMO (1993-94)

Champion : Queen's College
1st runner-up : Shau Kei Wan Government Secondary School
2nd runner-up : Raimondi College

Twelveth HKMO (1994-95)

Champion : Tsuen Wan Public Ho Chuen Yiu Memorial School
1st runner-up : St. Paul's College
2nd runner-up : Heung To Middle School

Thirteenth HKMO (1995-96)

Champion : Mongkok Workers' Children School (Secondary Section)
1st runner-up : The Bishop Hall Jubilee School
2nd runner-up : Ying Wa College

Fourteenth HKMO (1996-97)

Champion : Queen's College
1st runner-up : Queen Elizabeth School
2nd runner-up : Mongkok Workers' Children School (Secondary Section)

Fifteenth HKMO (1997-98)

Champion : Diocesan Boys' School
1st runner-up : The Bishop Hall Jubilee School
2nd runner-up : Ying Wa College

Results of HKMO Poster Design Competition

1986-87

Champion	:	CHEUNG Chi-kan (Queen Elizabeth School)
1st runner-up	:	LAU Chun-yiu (St. Francis of Assisi's College)
2nd runner-up	:	CHIU Sze-wai (St. Francis of Assisi's College)

1987-88

Champion	:	CHEUNG Yin-har (Po Kok Girls' Middle School)
1st runner-up	:	CHAN Kwan-cheung (SKH Lam Kau Mow Secondary School)
2nd runner-up	:	YAM Yuen-fai (Raimondi College)

1988-89

Champion	:	TANG Sau-mei (Jockey Club Government Secondary Technical School)
1st runner-up	:	YONG May Koon-wan (St. Stephen's Girls' College)
2nd runner-up	:	KWAN Wing-kai (Queen Elizabeth School)

1989-90

Champion	:	NG Kwok-fai (STFA Seaward Woo College)
1st runner-up	:	SHUM Kwok-fung (CCC Rotary Prevocational School)
2nd runner-up	:	LEE Yan-ming (NTHYK Southern District Secondary School)

1990-91

Champion	:	WONG Tak-yee (Belilios Public School)
1st runner-up	:	LAI Shi-hang (St. Francis of Assisi's College)
2nd runner-up	:	HO Ada Chui-leung (Belilios Public School)

1991-92

Champion	:	MOK Hoi-yan (Tsuen Wan Government Secondary School)
1st runner-up	:	CHAN Jacqueline (Leung Shek Chee College)
2nd runner-up	:	WONG Yil-bun (St. Stephen's College)

1992-93

Champion : TSE Yuen-ling
(Queen's College Old Boys' Association Secondary School)

1st runner-up : LOU Ka-yan
(Queen's College Old Boys' Association Secondary School)

2nd runner-up : YU Man-ha
(Lung Kong World Federation School Limited Lau Wong Fat Secondary School)

1993-94

Champion : CHUI Hin-sing
(Tsuen Wan Government Secondary School)

1st runner-up : KONG Lam
(Queen's College Old Boys' Association Secondary School)

2nd runner-up : SHUM Chi-yuen
(TWGHs Kap Yan Directors' College)

1994-95

Champion : CHING Ka-fung
(Jockey Club Government Secondary Technical School)

1st runner-up : TANG Ka-man
(Jockey Club Ti-I College)

2nd runner-up : YU Man-ching
(Sacred Heart Canossian College)

1995-96

Champion : LEE Siu-chui
(Ju Ching Chu Secondary School (Tuen Mun))

1st runner-up : HO Hiu-yin
(Lok Sin Tong Yu Kan Hing School)

2nd runner-up : LAM Tsui-nga
(Po Leung Kuk Wu Chung College)

1996-97

Champion : HO Wai-yee
(Ju Ching Chu Secondary School (Yuen Long))

1st runner-up : MAK Pui-yan
(St. Simon's Lui Ming Choi Secondary Technical School)

2nd runner-up : CHENG Mei-chun
(Buddhist Chi Hong Chi Lam Memorial College)

1997-98

Champion : NG Sau-ha
(Buddhist Wong Fung Ling College)

1st runner-up : LEE Wai-huen
(St Mark's School)

2nd runner-up : CHAM Ngai-keung
(Lingnan Secondary School)

3. Mathematics Education Resources Centre

The centre was closed for renovation on 9 February 1998 and would probably be opened again in the 1998/99 school year. Details of the opening time of the centre would be announced through circular memorandum in due course. Please pay attention to it.

From the Editor

I would like to express my gratitude to those who have contributed articles and also those who have given valuable comments and suggestions to the Newsletter.

The SMN cannot survive without your contributions. You are, therefore, cordially invited to send in articles, puzzles, games, cartoons, etc for the next issue. Anything related to mathematics education will be welcome. We particularly need articles on sharing teaching experiences, classroom ideas, teaching methodology on particular topics, organization of mathematics clubs and even the organization, administration and co-ordination of the mathematics panel. Please write to the SMN (with your contact address included) as soon as possible and the address is

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The Fifteenth
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Mathematics
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