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sMN 學校數學通訊

School Mathematics Newsletter

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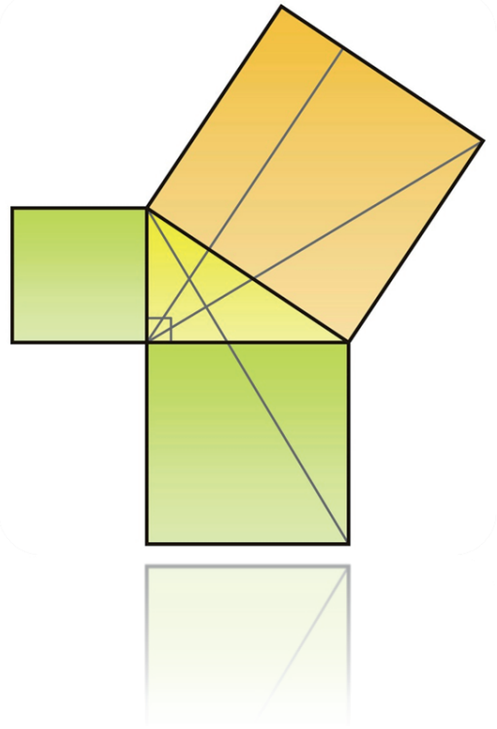

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教育局

School Mathematics Newsletter (SMN)

Foreword

It has long been our wish to inform teachers at large the development of Mathematics Education in Hong Kong, the organisation of our section, and many of the tasks we commit, e.g., forthcoming professional development courses, the Hong Kong Mathematics Olympiad, the School Mathematics Project Competition, etc., and to share with teachers some of our professional dialogues – in the form of a booklet called the School Mathematics Newsletter (in short, the SMN).

We hope that the SMN could serve as "news" in the wider sense. Reading it could enhance our understanding of information about the development of the Mathematics curriculum, informed practical ideas as well as intellectual stimulation about mathematics, its nature, its learning, its teaching and its assessment.

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黃毅英、張僑平[1]
香港中文大學課程與教學學系

改善教學

作為專業化的教師，我們都想改善教學，不過「教」與「學」有所不同，甚至有人認為「教得愈多學的愈少」。無論如何，若談改善教學，縱然近年不少人在探討何謂「善」的教學（effective mathematics teaching），但究竟所謂「善」的意何所指，仍是需要深究的。

箇中的討論源於不滿足於學習的「量」（即是學了多少），轉而追求學習的「質」。但這「質」是些甚麼呢？籠統而言，就是學生學過後，不只填滿了一大堆知識，而是真的「聰明」了。在數學而言，學生不只會做數（我們刻意保留「做數」這通俗用語，有別於含有更廣泛意義的「問題解決」），而是擁有確切的理解。但所謂「理解」又是甚麼呢？

做數、概念與理解

數十年來，數學教育界出現一個口號，就是「不只要教做數，而且要教概念」。口號歸口號，我們又是否真的理解這些想法的實質呢？我們先要攪清楚幾個問題。

我們常說學生只懂做數，不懂概念。其實，懂做數又有何不

[1] 本文得香港教育學院梁景信教授的寶貴意見，謹此致謝。

妥？我們不是說「問題解決」是學習數學的主要原因嗎？

縱使（只說縱使，詳見後）某學生只會做數，但若他對某個課題的不同題目無堅不摧，你能說他不懂這個課題嗎？例如：一個學生對於種種類型的分數相加，如 $\frac{3}{4}+\frac{5}{7}$，$\frac{-8}{1.5}+\frac{6.8}{\frac{7}{9}}$，$1\frac{8}{\sqrt{2}}+\frac{\sqrt{2}}{9}$，$\frac{b-c}{b+c}+\frac{b+c}{b-c}$ 等都會做，而他沒有（包括不能和未有）解釋怎樣做，你能說他懂或不懂分數加法嗎？就如 16 世紀時，Fior 向 Tartaglia 挑戰解三次方程，由於保密理由，Tartaglia 不把公式公開，但成功解了 Fior 提出 30 條三次方程，我們不就相信 Tartaglia 已掌握解三次方程的技巧嗎？

你可以說，掌握是掌握了，但還未算完全理解——世界上有「完全理解」這回事嗎？

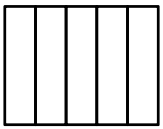
話說回頭，我們常說「問題解決」是學習數學的主要原因，但我們對一般性問題解決能力與策略（諸如類比、推廣、驗證等）和**數學**問題解決的關係應有進一步的反思。例如 1980 年代十分有影響力的 Cockcroft 報告書[2]指出，問題解決的訓練並非數學所獨有，故此，如果說學習數學只為了培養問題解決能力，並不能顯示數學這學科在學校課程中的不可取代性。

[2] Cockcroft, W. H. (1982) (chairperson). *Mathematics counts (Report of the Committee of Inquiry into the Teaching of Mathematics in Schools)*. London, U.K.: H.M.S.O.

不求甚解——誰人的「解」？

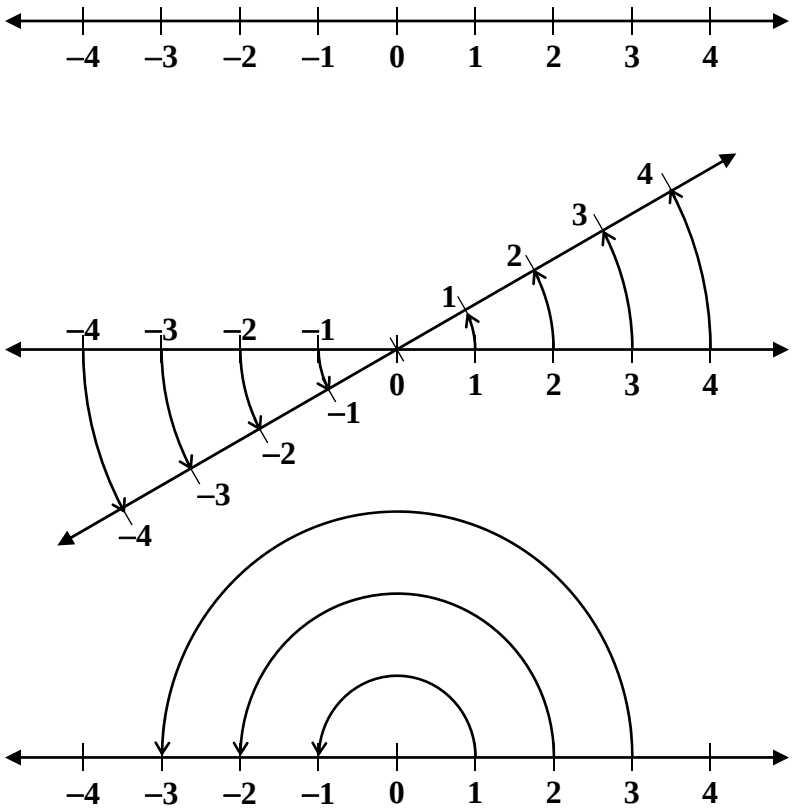
承接上面的討論，可以說，我們不是反對做數，只是反對學生不求甚解地做數。但我們可再細想一層，所謂不求甚解，究竟是誰人的「解」呢？其實，每個人完成一個工序，多多少少都有其「解」。只是他的「解」和「你」（老師、課本）的「解」不同罷了。例如，一名學生在做 $(ab)^2 = a^2b^2$ 時，他可能心中所想的是：快餐店有贈品，a 和 b 同往，當然「贈品」「2」會同時派給 a 和 b 兩人，所以不就有 a^2b^2 嗎？你可能會笑他這並非數學化之理解，但這種理解顯然「管用」（work），亦可推廣至 $(a_1a_2...a_n)^m$ 等。你又可能再質疑當算式變成 $(a+b)^2$ 時他便會碰壁，但學生可能在面對 $(a+b)^2$ 又會拿出另外一套方法來解釋相關工序。你也會再進一步質疑：這樣便沒有統一貫串的方式，會帶來混亂。不過，現時分數教學一時用分餅，一時用分割矩形；負數加法用欠單解釋，負數乘法又用數線依原點作反射（reflection）來解釋[3]——情況不也是類似嗎？

分餅



分割矩形

[3] 黃毅英（2010）。〈數學課堂的陷阱與盲點〉。《朗文教育專訊》19期，3-6。



負數乘法用數線依原點作反射來解釋

概念卻無法幫助做數！

現時普遍存在的想法是：做數＝程式性理解＝操練＝不理解（或只 know how 而未到「高層次」的 know why）。對於這種誤解，近年不同學者不斷開展出了更廣泛的視野，戮破上述的迷思。無論如何，由於對「教概念」（或「先有概念後做數」）的重視，教師花了很多時間在這方面，卻發現得不到預期的效果。讓我們先撇開既定的想法，試想想，一

般人（包括數學工作者）在做簡單計算，如分數加法時，顯然都是依照運演算法則去處理，不會也無須牽動分數的理解，無論這「理解」是指涉及分數的定義、圖像或現實情境。

除非遇到一些慣常工序無法處理的情況（如 $\frac{1}{\sqrt{2}}+\frac{\sqrt{2}}{7}$ 、$\frac{3+6i}{8-9i}+\frac{2i+7}{(3-8i)^2}$……），我們才會返回到最根本，問究竟分數是什麼？除法又是什麼？等等……

因此在學校環境中，就往往出現這樣一種現象：教師（及課程）一腔熱誠地「教」概念，但最終發現概念與做數是兩碼子事。學生概念掌握了，做數不一定靈了多少，學生做數時也不會一定要透過概念去處理。於是大家便自圓其說，說我們總不能不求甚解，既要做數也要懂得概念。於是不再理會概念是否是促進做數的東西，而把它變成另一種學習目的，把概念與問題解決二分對待[4]。

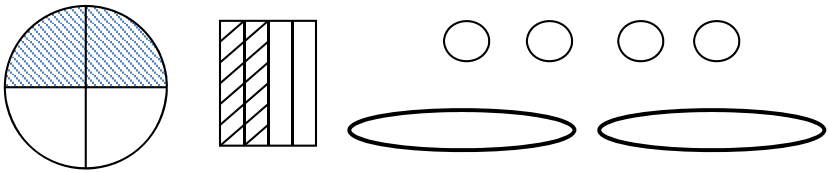
概念又是甚麼？

沿著這個想法，既要重視做數，亦要重視概念，本身並無不妥。於評考而言，就是不只考學生是否懂得做數，得到正確答案，也要考察他是否理解做數的過程，背後的理據和所牽涉的概念，例如考考他們會否畫圖等。然而問題是「概念」又是甚麼呢？

概念就是圖像表示和現實情境嗎？算式何以不能是概

[4] 黃毅英（2007）。〈數學化過程與數學理解〉。《數學教育》25 期，2-18。

念的一部分？以除法為例，除法＝等分（分物）、包含嗎？（究竟是等分還是包含，還是兩者合起來才是分數？）除法＝圖示嗎？哪種圖示？如果說「除＝乘的逆」又或「$\frac{a}{b}$即求$bx=a(b\neq 0)$的解」有何不妥？



4÷2　　4÷2：四塊餅分到兩隻碟

除法的幾種圖示

如果我們認為低年級學生無法理解「$\frac{a}{b}$是$bx=a$的解」，而又確認它是「數學」的「終極」概念，那我們是否應最終讓學生理解「除＝乘的逆」呢？換言之，我們是否要把整個（橫跨中小學）教程佈置到這一目標上來呢？

下面我會再談一個數學物體（object）有哪些概念這一問題。在此之前，讓我們先看看概念和定義間的關係。

概念＝定義？

以上的討論好像引導了讀者得出這樣一個結論：概念與定義就算不相等亦有著密切關係。當然，我們絕對承認定義的重要性，但上面的想法有一個隱含的假設，就是每一個數學概念（或名詞）都有一個清楚、統一、公認的定義，而且

透過定義就可協助我們理解概念（或推導出各定理）。這兩者顯然都與事實有一段距離。歷史上，一些定義是經過許多世紀慢慢發展開來(evolve)，中間經過不少修訂(諸如函數、極限等)，亦不排除將來再會有改變。我們不能說，在「終極」定義出現之前，(在歷史上)該方面的探究會停滯不前。換言之，沒有定義不一定不能做數，也不一定對該概念就沒有理解。在歷史上如是，在學校裏也如是。事實上，中小學階段的大部分重要概念，如自然數，上面所說的除法……都無法作出數學上「嚴謹」的定義，不是嗎？[5]

怎樣教概念？

談過「何謂概念」之後，「怎樣教概念」這個問題就更清楚了。如果我們接受圖示呀、情境呀、甚至算式都（只）是概念的不同表現（或面相），我們就要反問自己：我們常說教概念，我們是在教哪一種概念？我們是否假設了每一數學物體都有一套公認的概念讓我們可以去教？除法的例子前面說過了，乘法「×」又是一例。現時開始有更多人「唱反調」說「×」不是連加(最少連加不一定是「×」的所謂「官方概念」)。面積（之所以「×」的結果叫做**積**）是更準確和豐富的概念。例如日本數學教育早在 1970 年代已提出要把離散量和連續量的學習分開，從離散量學＋、－，連續量學×、÷，以速度等引入連續量而非透過連加。

我們不想牽涉入「×是否連加」這個別問題的討論，但這卻讓我們更進一步的思考。我們常說數學化學習或數學化過程，就是要帶領學生從周遭經驗邁向統一的數學世界，大

[5] 黃毅英（2012）。〈追尋定義之路〉。《數學教育》。待刊。

體來說這是對的。但如前所述，其中假定了有一套統一的、牢固的「數學」作為我們的最終目的地。從上面看到，（不同）數學其實是可以有著大同中的微細差異。愈來愈多人提出（傳統）紙筆數學、IT 環境中的數學、動手數學……，無論所涉及的技能甚至概念都有著微細的不同。若說進行「數學化」教學，就要問是朝向哪種數學？若說「教概念」，也不得不問，縱使對於同一數學物體，要教同一數學物體的哪一個概念（因為可以有多種）？

概念能教嗎？

傳統上，我們不會說「教概念」而說（協助）學生的概念**形成**（concept formation），這不只是一種咬文嚼字，除了說概念要靠學生自己形成而不是由教師灌輸之外，亦指向建構主義的核心思想（雖然現時不少人對建構主義有種種誤解而帶負面印象），就是不同人對同一概念的理解可以不同，也不可能相同，亦不必相同。

這給出一個印象，好像概念的形成十分隨便。每個人都可以（對一特定數學物體）形成其個人化（personalize）的概念。如此這般地概念就算形成了。事實上，這確是心理學上的一些想法，甚至有人認為（不只認為，包括實證研究發現）數字（如「2」）的概念（concept，或「觀」）可包含顏色等[6]。對於有些人，數字可以是有顏色的（例如「2」是綠色），有味道的，甚至有愛憎的（「2」是美好的，「17」是「巉巖」的——高斯 Gauss 可能不這麼想！）。我們可以說，雖然不排

[6] Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

除在個別人的腦袋中，數字確實有可能聯繫到這些東西來，但應將數學的觀念和非數學的觀念分野開。不過這又返回先前的問題了。怎樣和誰人界定哪些觀念屬於數學的概念？圖示算不算數學概念？……在歷史上（如古希臘人）確實有間歇出現過把情感拉進數字，如**完全**數等。這些「觀」憑甚麼一定要排拒於數學之外？

不過心理學也不是這麼隨便的，縱然每人都可以有權形成其個人化的概念，但概念有強弱（powerful）之分。心理學考究概念是否夠豐富，包括有多少維度和概念間聯繫的連通性（connectedness）及這些聯繫有多強，亦可考慮這些概念（或個人化的「觀」）的可持續發展性。例如，維恩圖（Venn diagram）十分形象化亦十分方便，但若涉及 6-7 個集合就變得複雜了，坐標圖有助我們將代數和幾何聯繫起來，但在高維空間亦出現問題。故此，縱然每個人對於同一數學物體可以有不同的思維網絡（「觀」），但漸漸他會發現有些「觀」（如 2 是美好的）用不著，不管用，也許會就此漸漸「萎縮」了。從此亦可看到，大部分圖像表示縱然有一定的數學作用，卻不能把之等同於數學概念。

人能教的嗎？

好了，就算不說「教概念」而說形成概念，我們又怎樣協助學生形成概念呢？在探討這問題之前，讓我們先帶入另一個更有趣的問題，就是有「教」這回事嗎？是否所有東西都「teachable」、「learnable」？就算既「teachable」也「learnable」，是否能有一套清楚的教法呢？

也許近年流行規劃（及規範化之風尚）的緣故，大家都

想尋找一套理想的課程和有效（甚或最新）的教學法，追求課程與教學設計是理所當然的(尤以我們受薪於**課程**與**教學**學系——一笑)。不過最終這些都是大人世界在估計兒童的學習軌跡（所謂 hypothetical learning trajectory)，而且這種軌跡往往傾向於學科知識結構而非兒童實際的學習進路。這種估計除了不一定準確外（這還算次要，因為大人畢竟經過兒童階段，憑著豐富的教學經驗估計不一定錯到那裡)，在從上面談到的由教學科知識技能轉而教(無論「教」還是「培養」）概念時，因概念是這麼個人化的事情，所形成的概念又會並且有權萬別千差，用傳統「以教材教概念」的思維恐怕有甚多需要調整的空間。說句笑話，我們用九牛二虎之力，包括利用畫圖、引入「倒數」、現實情境、分物、分階段($\frac{1}{b}\div c$、$a\div\frac{1}{d}$ 、$\frac{a}{b}\div c$ ……）引入等等方法，來教分數除法，對（一些）學生來說可能更覺是架床疊屋，學生甚或可能透過整數情況的類比很快和很自然地掌握了 $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$，然後再透過反復的應用（運算）慢慢加深瞭解。

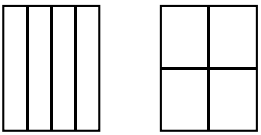
嗯！你可能不同意這觀點，我雖不能證明我的想法，但你也沒法否定我的想法——因為我們也許皆建基於對學生的臆測，直至我們真的(大量地)對學生進行診斷性的探討，一切都會是上面所說的：大人一廂情願地認為這樣學，兒童會學好。無論如何，教程（包括課程與教學設計）——教師（作為教程的執行者)——實際教學——學生——學習之間的關係是要仔細釐清的，不能簡單的認為有了設計精密的教

程，有專業的老師，有用心的學生，學習成果就會如期出現。這顯然與現實相距甚遠。

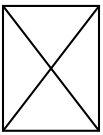
如何協助學生形成（數學）概念？

我們要做的可能就是要鋪排一條由粗糙到完整，由含糊到嚴密，由學生世界到數學內部（所謂「內行人的數學」：esoteric mathematics[7]，就是抽離現實情境的數學）之路，這似乎是老生常談，但箇中十分微妙，亦是教學精髓之所在。

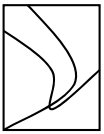
以分物為例，這是學生實實在在會遇到的問題，隨便的分物，其中會出現很多「沙石」、不整潔的地方，如沒有強調等分，或者把數學上的等分與將長方形紙張分成數個全等（congruent）圖形混淆等。老師不是要去否定或更正他們的不完整想法，因為這些「實作」均是形成所謂「正式」（formal，有別如標準 standard——因標準有著「規定」的意味）概念的寶貴經驗，作為老師正好是要巧妙地在適當的點不斷修正（rephrase，而不是 replace）他們的想法和用詞，不知不覺地將之挪移到「正式」的做法（formulation）（即我們常認為的數學內部的做法），包括概念和用詞。



面積相等且全等的 4 份



只是面積相等，未嘗不是 4 等



如果上圖四份面積相等，未嘗不是 4 等分！

[7] Cooper, B., & Dunne, M. (1998). Anyone for tennis? Social class differences in children's responses to National Curriculum Mathematics Testing. *The Sociological Review, 46*(1), 115-148.

這裡仍以向學生講授分物為例，教師引導學生看每一份是否均等，學生說要「平均分開」，就巧妙到引入「等分」這專有名詞（術語），再由透過數數（counting）或一件一件的分，到瞭解分物過程的思維過程（mental process），進而整理這個思維過程導出法則，然後再修正其格式，單位元和表達方式等。如此，師生合手塑造（co-shape）出有關的概念與法則。

這種進路不只協助學生形成特定的概念，亦協助他們瞭解一般的概念形成（包括建模）過程。在這個進路中，教師需要一般性的教學技巧，如掌握授課的節奏、流暢度、緩急，在適當地方著重強調（highlight）及總結，能順著學生思維的趨勢，透過一次或多次地把整件事情正式化（formalize）出來。

由於當中「概念」緊扣著「處理問題」，就不會存在兩者格格不入，也就很難出現教師教了「概念」而最終學生發現無助其做數的情況。

出題與題意

既然「教」了概念就理應考概念了，看學生的概念有否形成。如果大家接受上面的觀點，學生形成的概念不會亦不必統一，考概念又變成了只不過是考畫圖等等的玩意，與「真正」理念不盡相同。另一個環節自然是看看他們做數解題是否知其所以然了。亦即從以往只看答案是否正確，到更重視解題步驟。然而上面的問題又出現了，得到同一答案可以有不同的**正確**思路和方法，有時亦很難分優劣。就如蔡金法教授著名的「售帽題」（知前 3 周售帽量，知 4 周平均售帽量，

求第 4 周售帽量），用代數法、算術法、圖解法……均各領風騷，我們卻不能說步驟少的方法就好些，用「蠻力」（brute force）的就不可取。

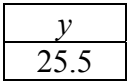
早期擬題者控制了問題的多解性（「正常」學生都不可能有另解）到現在不斷鼓吹一題多解。理論上說，只要是每一步的推導正確，整個解題方案能導致所求的答案就應算學生做對。但若欠缺上面的觀點，當中仍會出現問題。

例如，有這樣一道題目：小明原有 y 元，媽媽再給他 25.5 元，他便有 40 元，問 y 是多少？有學生答 $40 - y = 25.5$。「標準答案」是 $25.5 + y = 40$。有老師認為 $40 - y = 25.5$ 不合題意，應算錯。但什麼是題意呢？

首先，用上面的準則

$$40 - y = 25.5$$

沒有錯。此式亦足以得出所需答案，有何不妥呢？我們只可以說這個學生的解題（設方程）方式與擬題者不同。用上面的觀點來看，為何要相同呢？若要相同，無疑是規範學生的思維方式，沒有必要。思維方式（縱使局限於解題思路而言）可以萬別千差，況且學生若有對＋、－的充分理解（天曉得沒有？我們又沒問過他），如用圖示，



40

他會很自然地知道

$$25.5 + y = 40$$

$$40 - y = 25.5$$

$$40 - 25.5 = y$$

是「三而一，一而三」的事。老實說，正是因為課程的需要，我們才要如此這般的擬題。「正常」的解題者，理應一眼便看出答案是「40 – 25.5」，根本不用設未知數立方程！不過無奈的是，設未知數立方程的威力在後面的學習階段（中學）才能展露，但又不能不先以簡單的情況作練習，硬要學生設未知數立方程實在是殺雞牛刀（所謂「為了用拐杖打斷腿」）。無論如何，這再一次顯現課程（不只說現行的課程而是先天性之問題）未必一定能體現學生的思維和學習方式。

再看「題意」

學生要求填答（當然我們假設之前有**實際**的分物活動）

（a）10 是 2 的__倍

（b）10 包含 5 個__

（c）把 10 塊餅平分給 2 人，每人有_塊餅，即 $\frac{10}{2}$=__

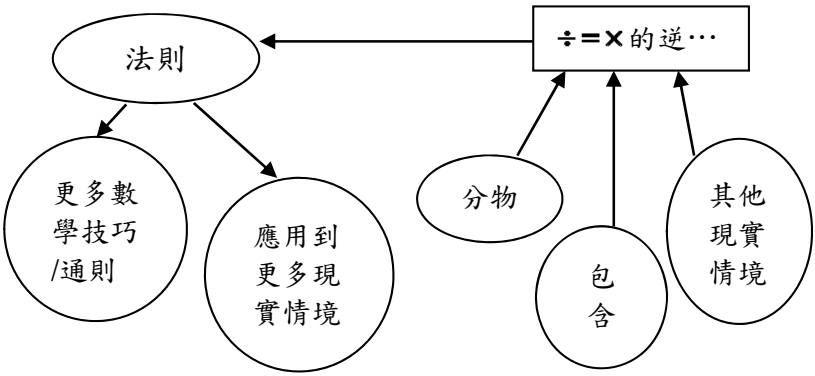
（d）10 的 $\frac{1}{2}$ 是__，即 10×__=__

（e）10÷__=__

對於（e），有學生寫 10÷5=2，這與上面的其他例子有點不同，這顯然是不合「題意」（除非學生能給出個人化而又合理的詮釋，如 10 塊餅要每 5 塊一分，就只能平分給 2

個人），然而有趣的是 10÷5=2 又是關於 10、5、2 **正確**的數學關係。從上面可見，我們不只要學生懂相關的數學，而且要正確地用數學解釋特定的現實情境及反過來用現實情境詮釋數學關係。所以這些學習對象都是關於數學與現實情境的關係而不只是數學本身。要建立這些關係（如「×」就是「的」：8×2 是 8 的 2 倍，「÷」是「等分」或「包含」……）是需要學生確切對現實情境的處理（**實際**的分物、**實際**的切餅……）來完成這種建模，而不只是形式化的填寫上述答案，要填寫上述的答案，實可以透過盲目地模仿（blind imitation）來完成。以上應該是一些動作（如分物……——起碼是 mentally）的總結，不斷用數學解讀（make sense），亦用現實情境來明白一些數學內容。畫圖只不過是其中的一個手段。

所以，以上的概念建立未必對傳統的「做數」如計算 $1\frac{1}{2}\div 3\frac{8}{9}$，直接有幫助，而其威力將會在文字題（應用題）中發生，因為學生已習以為常地穿梭於情境、數學、詮釋、建模，以數學解釋現實問題之間。對於新的情境，就容易套進合適的數學加以解決。



有多少個概念？

以上面（a）—（e）為例，一個自然的問題是：單一除法有多少個概念要教？這命題本身也許與上面的心理分析相違背：概念可以萬別千差，很個人化，也不應標準化，但現實上在特定的時空對於每一數學對象確實有一組常用的「概念群」（其實不是概念，而是相連的關係性理解）。我們可以看成是一種課程問題，透過共識就可訂定這樣的一種課程要求。不過縱說共識，既然是一種課程要求，就得問學習目標：（規定）學習某個數學的某個表徵的意義何在。承接上面所說的，這些表徵的「可持續發展性」仍是需要考慮的。以現象圖式學 (phenomenography) 來解釋，縱然不同人對於「2」可以建構出不同的思維網絡，其中可能包括序、量、甚至顏色等，但作為教育任務（課程要求），我們必須（透過專家審定）訂定哪些屬於「關鍵（數學）特徵」 (critical aspect)[8]，哪些不是（明顯地，顏色就不是），其中一個考慮點自然是它與高等數學的連繫，就是上面所說的「可持續發展性」。

那末，甚麼是數學「內部」？

上面談到除法，它與分物、包含等又是甚麼樣的關係呢？分物是除法套用到現實情境的應用（即分物建模為除法來解決實際問題）、還是人類祖先透過分物等活動提煉出「÷」這觀念？又或者兩者兼有呢？無論如何，當數學學習由現實物

[8] 馬飛龍（Ference Marton）（2005）。〈論學習的必要條件〉。載祈永華、謝錫金、岑紹基（編）。《變易理論與學習空間》（頁 19-32）。香港：香港大學出版社。

慢慢提升到「人做物」時，我們再無可能做太多這類「呼應」。譬如 4^2 、$(2.8)^9$ 、$\left(\frac{1}{3}\right)^{\frac{1}{4}}$ 、$(6)^{-8}$……到 π^e 已經不可以用「自乘」去理解冪。無論怎樣，用現實情境提煉出數學也好，把數學應用回現實情境也好，這些多多少少屬於數學「內部」與數學外部（姑且這麼暫作二分）的橋樑。無意貶低其重要性，我們可進一步問，數學「內部」又是些什麼？不難發現，在處理「內部」數學時，我們仍是需要按照一些法則（無論是代數算式或非代數的）進行演算（manipulation）或推導。這樣看來，算式、做數並不是一些等而下之的東西，仍是數學學習中重要一環。這亦是後階段學習（包括中學及高等數學）的學習重點。在初段學習如何為此作準備仍是需要考慮的。

從類比進行抽象化

從上所述，當數學學習愈來愈抽象，上述透過情境瞭解／詮釋數學對象已愈不可能（縱非不可能亦漸難）。以乘法為例，如何瞭解 7 的 π 倍？怎樣理解乘法和積分間的關係，不連續函數的積分又如何？……我們無可避免地在某個點要多多少少脫離現實情境去操作，就算不說 π 等，我們可以叫學生用分餅去操作 $9\div 8$、用 $\frac{1}{2}l$ 容量的杯倒 $9l$ 的果汁去理解 $9\div\frac{1}{2}$ ，……但 $158\div\frac{1}{267}$ 又如何？當然不是不可以，但費時失事。我們總要在某處跳出情境，要透過一些法則上的「類

比」推進，如「整數÷整數」→「$\frac{1}{n}$÷整數」→「整數÷$\frac{1}{n}$」→「整數÷$\frac{m}{n}$」……。當然，在什麼點（時間）「轉軌」和是否可以在不同點作多次「轉軌」又是另一回事[9]。

結論

上面好像談了一籃子錯綜複雜、互相交織的理念，但其實想帶出一點，作為專業老師，一方面我們不要害怕理論，這些理論往往確都能夠把各種學習現象整理得條理分明，但與此同時，不同理論只能解釋學習現象的一個面相。故此在另一方面，我們亦不應迷信理論（尤其這些理論被簡化成一些口號）。保持反思與批判性，這便是「學養教師」的根本想法。

[9] 黃毅英、林智中、孫旭花（2006）。《變式教學課程設計原理：數學課程改革的可能出路》。香港：香港中文大學教育學院香港教育研究所。

2. Sum of Two Squares

Leung Chi-kit[10]

Methodist College

According to the *Mathematics Curriculum and Assessment Guide (S4–6)*, students are requested to perform addition, subtraction, multiplication and division of complex numbers. Some teachers might think that this requirement is neither enough nor interesting as not many things can be done with just the 4 operations of complex numbers. In this article, I would like to recommend an activity which involves the operations of complex numbers that might interest our students.

First, students are asked to select two integers and find the sum of their squares. For example, we select 12 and 17. Then the sum of squares is equal to $12^2 + 17^2 = 433$. Then they are asked to select another pair of integers and find their sum of squares, too. If we select 10 and 6, then the sum of squares = $10^2 + 6^2 = 136$.

Finally, I calculate the product of the two sums, i.e. $433 \times 136 = 58888$, and claim that the product is also a sum of two squares. The question is: how can we find these two integers of which the sum of the squares is equal to 58888? (I suppose this

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is not easy to answer if we do not know the secret behind.)

In fact, this can be done easily if we use complex numbers — to be more exact, if we factorize the sum of two squares by using complex numbers.

We note that $433 = 12^2 + 17^2$

$= (12 + 17i) \times (12 - 17i)$

and $136 = 10^2 + 6^2$

$= (10 + 6i) \times (10 - 6i)$.

Since $(12 + 17i) \times (10 + 6i) = 18 + 242i$

and $(12 - 17i) \times (10 - 6i) = 18 - 242i$,

$58888 = 433 \times 136$

$= (12 + 17i) \times (12 - 17i) \times (10 + 6i) \times (10 - 6i)$

$= (12 + 17i) \times (10 + 6i) \times (12 - 17i) \times (10 - 6i)$

$= (18 + 242i) \times (18 - 242i)$

$= 18^2 + 242^2$.

So, we know that 58888 is the sum of the squares of 18 and 242 ! (In fact, 58888 can also be expressed as $222^2 + 98^2$ if we consider $(12 + 17i) \times (10 - 6i)$.)

I think this activity is good. All computations can be done by using calculators. It demonstrates how complex numbers can

be used to solve problems in number theory. This activity can be further developed by asking students to solve this problem abstractly.

That is

$$
\begin{aligned}
&(a^2 + b^2) \times (c^2 + d^2) \\
&= (a + bi)(a - bi)(c + di)(c - di) \\
&= (a + bi)(c + di)(a - bi)(c - di) \\
&= ((ac - bd) + (ad + bc)i)((ac - bd) - (ad + bc)i) \\
&= (ac - bd)^2 + (ad + bc)^2
\end{aligned}
$$

Besides, the above proof also illustrates a very important property of complex conjugates, which is $\overline{(a+bi)\times(c+di)} = \overline{(a+bi)} \times \overline{(c+di)}$.

To conclude the discussion, students can be challenged to find *all* possible ways to express any integer as a sum of two squares. Teachers should note that for any prime number p greater than 2, p can be expressed as a sum of two squares if and only if $p = 4k + 1$, where k is an integer, according to a result in Elementary Number Theory. Therefore, a positive integer N is the sum of two squares if and only if each prime factor p of N such that $p = 4k + 3$, where k is an integer, occurs to an even power in the prime factorization of N.

3.「專制的」數學知識

馮志揚
前課程發展主任（數學）

課堂軼事

讓我們先了解一下學生和教師在數學課堂中的一些經常遇到的事情。

軼事 1

課堂上，學生詢問教師：「請問正方形有多少條邊？」
教師回答：「四。」
學生：「為甚麼？」(Hewitt, 2002, p. 48）

軼事 2

學生：「為甚麼 0 是偶數？」
教師：「因為它是可以被 2 整除的整數。」 (例如，九章，1995，頁 5）

軼事 3

學生：「請問符號 $a \mid b$ 表示甚麼？」
教師：「符號 $a \mid b$ 表示 a 整除 b。」(陳景潤，1978，頁 2）

軼事 4

教師：「有禮貌的數（polite numbers）可以寫成兩個或以上連續正整數的和。」(Huckstep, 2002, p. 101）
學生：「為甚麼叫做有禮貌？」

軼事 5

學生：「甚麼是點？」

教師：（用顏色筆在白板上點上一點「●」）「這是點。我們以大寫英文字母，例如，P、Q 來做它的名稱。」（文耀光、梁志強、吳銳堅，1997，頁 148）

學生：「那麼，請問究竟甚麼是點呢？」

「專制的」數學知識

以上向壁虛構的課堂軼事指出代表數學概念的數學詞彙（例如，正方形、偶數、有禮貌的數、點）、符號、簡寫和標籤等（例如，四、0、2、$a|b$、P、Q）往往充斥着每一個數學課堂。這些數學用語是由特別的文化、數學家社群（a particular culture / community of mathematicians）接受的和被認為是「正確的」（Hewitt, 2002, p. 49）。

事實上，數學所用的詞彙、符號、簡寫和規定，及它們的定義（如果有的話）都是數學家經過漫長的創造過程，和積極的討論之後 – 由好奇心的驅使、或由需要所使然、或將複雜事件簡化、由未知到已知、由歸納驗證和抽象的經驗、以至發現、發明、確認和接受 – 用來表達數學概念的用語（例如，Crease, 2008, p. 13-20）。但是，數學家所接受的未必能為其他的人所理解和認同。我們當然可以對這些協定作出懷疑和挑戰，但是 Hewitt（2002, p. 49）建議我們倒不如接受了這些「專制的」（arbitrary [11]）數學知識，入鄉隨俗，設法學好

[11] 陸谷孫（1992，頁 82-83）對「arbitrary」作出多於一種解釋；我認為將「arbitrary」譯作「專制的」較為合適。

這些代表數學概念的語言，進一步享受「做數學」的樂趣。

同時，由於這些名字和標籤是數學界的選擇，不是數學界的人士，例如一般的學生，未必能夠知悉這些特定的名稱和符號，所以，Hewitt（2000, p. 49; p. 55; p. 57）更建議數學教師有必要將這些特殊用語告訴有意學習數學的人士。

以下的討論是基於接受了數學界對概念所作的協定之後，嘗試對這「專制的」數學知識，組織可行的教學策略。

詞彙、符號等的定義

根據李國凡、李天舟（2000），規定一個詞 [和符號] 的意義的句子叫做這個詞 [和符號] 的定義（頁 59），而每一個定義都具有「雙向」功能（頁 99）。以下是平行四邊形定義的例子。

平行四邊形定義

李國凡、李天舟（2000，頁 99）定義***平行四邊形***這個詞如下：

兩組對邊分別平行的***四邊形***叫做***平行四邊形***

以上的句子告訴我們：

1° 首先，研究的對象是***四邊形***。

2° 其中一些四邊形有某種共同的、可抽象的性質或特徵，例如，兩組對邊分別平行。

3° 如果四邊形的***兩組對邊分別平行***，那麼，這個四邊形叫做 / 是 ***平行四邊形***。

4° 反過來，如果四邊形是***平行四邊形***，那麼，這個四邊形的***兩組對邊***必定***分別平行***。

另類定義

根據以上 3° 和 4° 的「雙向」分析，我們可以用簡單（或殊不簡單？）的句子來定義平行四邊形：

四邊形是***平行四邊形***

當且僅當（if and only if 或 iff）

四邊形的***兩組對邊分別平行***。

一般來說，可能為了較流暢的表達方式和語言的運用，很少數學課本會利用「當且僅當」（if and only if 或 iff）來定義數學概念。例外的情況可參考例如 Leung and Chen (1967) 頁 25。

表列定義

以下，我們嘗試用表列的方式定義平行四邊形：

表 1：平行四邊形的定義

研究的對象	詞彙 / 符號	當且僅當	性質 / 特徵
四邊形	平行四邊形	當且僅當	兩組對邊分別平行

練習 1

學生在**軼事 1** 至**軼事 4** 中提出一些數學的詞彙和符號。請嘗試利用表列方法定義這些詞彙和符號。

（請參考附錄中的建議答案。）

無定義的詞

要了解特殊的數學詞彙、符號和約定，用文字的解釋似乎是較好的方法。但是，「我們不能為 ……每一個概念都給與一個（不循環）定義」（文耀光等，1997，頁 147）。**軼事 5** 中提出的***點***便是一個不定義的詞（undefined term）（文耀光等，1997，頁 148）。但是，要討論不定義的詞的意義和性質，我們仍然可以依照一般的教學策略：利用一組不同的、適當的例子，對這詞彙所代表的概念作出讀、寫和解說，讓學生自我連繫舊知識和建構新知識（e.g., Skemp, 1989, pp. 62-67）。若果學生能夠從一組新的例子中分辨這個詞所代表的概念，和能自擬和解釋新的一組例子，學生已掌握這個概念（例如，陳澤民譯，1995，頁 80-81）。我們在下一節將會對教授「專制的」數學知識有較詳細的討論。

教學策略

要學生認識「專制的」數學知識，我們都須要對每一個詞或一個符號和學生作出恰當的介紹和解釋。不同的、適當的例子能夠幫助學生建構新的知識。為了找尋和設計這些有關某詞或符號的例子，我們首先要分析這「約定的」數學知識，而分析的方法可以像上一節處理平行四邊形定義的方式一樣。這裏，讓我們再看看另一個例子。

偶數

軼事 2 指出：可以被 2 整除的整數叫做偶數。這個定義讓我們知道：

1° 首先，研究的對象是***整數***。而整數的例子可以是：–3，–2，–1，0，1，2，3 等；整數有正、有負，整數亦可以是零！

2° 有一些整數有某種共同的、可抽象的性質或特徵，例如，可以被 2 整除。將這類***整數***（包括 2 本身）被***整數*** 2 ***去除***的時候，結果是：商必定是一個***整數***，並且，餘數必然是 ***0***。

3° 如果整數 ***可以被 2 整除***，那麼，這個整數 叫做 / 是 ***偶數***。

4° 反過來，如果整數是***偶數***，那麼，這個整數必定 ***可以被 2 整除***。

偶數的例子

根據以上 1° 至 4°，我們首先設計第一組較簡單的偶數和不是偶數的例子，利用它們讓學生了解偶數的意義。以小學高年級為例，第一組例子可以是：

6，8，24，2，0 等　　（偶數的例子）

和

3，7，1，$\frac{1}{2}$，1.4 等　　（***不是***偶數的例子）

教師先讓學生利用幾分鐘的時間，嘗試將 2 去除以上的數，指出哪些數字能滿足被 2 整除的要求，將這些數以條件：

能被 2 整除

分類，讓***個別學生***重溫整除的概念，自行判斷哪些數能被 2 整除，求同存異，作出批判性的思考。同一時間，教師可以介入學生的計算和分類過程，協助學生回顧整數、0、分數、小數、除數、被除數、餘數和整除的概念。

教師隨着要求小組討論和個人匯報，進一步以舊的和新的例子釐清整除的概念，繼而介紹偶數的名稱（讀、寫、定義），分辨不同的偶數和不是偶數的例子。

教師隨後派發新的一組包含偶數和不是偶數的工作紙，讓個別學生分辨、讀、寫和解釋。在這過程中，學生利用初建構的知識，分辨和解釋（口頭和/或書面）哪是偶數和不是偶數

的特徵，作出批判思考，創意解決問題。同時，教師（和學得較快的學生）亦可評估同學學習偶數概念的成果，協助同學修訂暫時建構了的概念。

教師可繼續要求個別學生自擬和解釋新的偶數例子和***非例子***（即不是偶數的例子）。這時，學生要重看和利用已建構的知識，選擇可以被2整除的整數，作出批判性的思考，創造教師要求的偶數，解決教師要求的問題。同時，學生在與同學和教師的互動中，更可修正可能未必完善的偶數概念。

「表現未如理想的」學生可重溫舊的例子，重複以上的建構過程，或者再研習新的、教師或同學給與的例子，鞏固所思所想。

「學習速度較快的」學生可以考慮解決進一步的問題，並且自擬例子作出解說，支持自己的猜想：

(a) 兩個偶數的和是否仍然是偶數？（即：現有兩個偶數，將它們加在一起，那麼，結果是否一個偶數呢？）

(b) 現在有一個偶數，我們知道這個偶數是兩個整數的和。那麼，這兩個整數是否一定是偶數？（即：先有一個偶數，然後將這個偶數分解為兩個整數，再分辨分解出來的整數是否偶數。）

等問題。

教師可以在稍後適當的課節中，例如，第一次學習偶數之後的下一個課節、兩三日後、兩三星期後等，向全班學生提出類似以上（a）和（b）的問題，讓學生自擬或再自擬（對已

經接觸過這類問題的學生而言）例子、探究猜想、歸納結論。教師除了讓學生接觸不同類型的偶數問題，學習了解題目，找出已知的條件和未知的結果，構思和寫出解決的方法，培養解決問題的技巧之外，更重要的目的是讓學生回憶偶數的定義，重複思考有關的特徵和性質，從多方面應用偶數，鞏固建構了的偶數概念和名稱。

如果可能的話，教師可讓學生自擬新的問題，例如，

(c) 任何兩個偶數的積是否必定是偶數？兩個偶數的商又是否偶數？

(d) 兩個偶數的差（大的一個減去小的一個）是否偶數？

(e) 兩個***不是偶數***的數的和是否仍然***不是偶數***？

(f) 現在有一個***不是偶數***的整數，我們知道這個整數是兩個整數的和。那麼，這兩個整數是否一定***不是偶數***呢？

自擬例子、推斷、歸納、驗證猜想，從中培養學生批判性思考、溝通和創意解決問題的能力。

在以上的過程中，教師應留意給與***每位學生***充分的機會進行解決和自擬問題、探究、及與教師（expert）和對偶數概念學得較好的同學（a more knowledgeable peer）（例如，谷瑞勉譯，1999，頁 34）就偶數例子和非例子的特徵/性質討論，判斷自己的學習成果，修訂和內化已建構的知識。

練習 2

分析**軼事 4** 中***有禮貌的數***（polite numbers）的定義，回答以下問題：

(a) 9 是否***有禮貌的數***？

(b) 30 是否***有禮貌的數***？

(c) 有否***不是有禮貌的數***？如有的話，請舉例說明。

(d) 有否決定有禮貌的數的方法？

（Huckstep, 2002, p. 101）

小結

年輕時，數學極好的同學對我說：「要學好數學要先學好概念」。我相信這位同學所指的概念並非一般的數學知識而是數學概念的定義。先了解概念的定義有助學好數學；數學概念是學習數學的基礎。但是，學習數學並非演繹地由公理和定義出發，而是由好的例子入手。Krantz（1993）認為：

> It is of paramount importance, epistemologically speaking, for us as scholars to know that mathematics can be developed *deductively* from certain axioms.… However mathematics, as most other subjects, is not learned deductively: it is learned *inductively*. We learn by beginning with simple examples and working from them to general principles. Even when you give a colloquium lecture to seasoned mathematicians, you should motivate your ideas with good examples. The principle applies even more assuredly to classes of freshmen and sophomores (Krantz, 1993, pp. 16-17).

在大學的數學教學如是，何況在小學呢？

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附錄

軼事	研究的對象	詞彙 / 符號	當且僅當	性質 / 特徵
1	平行四邊形	正方形	當且僅當	有一組鄰邊相等，並且有一個角是直角
2	整數	偶數	當且僅當	可以被 2 整除
3	整數 a 和 b (相同或相異)	$a \mid b$	當且僅當	$a \neq 0$，並且 b 可以被 a 整除
4	正整數	有禮貌的數	當且僅當	可以寫成為兩個或以上連續正整數的和

4. 種籽計畫反思

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為配合課程改革，課程發展處於二零零一至零二學年開始推行協作研究及發展（「種籽」）計畫，至今已超過十年。在2011/12 學年我透過教師借調計畫參與了數學教育組的種籽計畫，確實讓我獲得很多寶貴的經驗。

2011/12 年度的種籽計畫以「探討及發展圖形與空間範疇的學與教的有效策略」為主題，這計畫的宗旨是為了配合小學數學科課程推行，與參加的「種籽」學校協作研究及發展「圖形與空間」範疇的有效學與教策略，以提升學生的學習興趣及發展他們的共通能力。數學教育組的課程發展主任和我一起到不同的「種籽」學校，與教師一同討論並設計切合學校需要的教學及評估策略。當完成觀課後，我們分別與教師和學生進行面談，進一步了解有關策略的成效。

回想我參與計畫的過程，有三方面最令我印象深刻和值得反思，包括教學流程設計、課堂實作活動和工作紙設計。這些經驗對我日後的教學有著很大的啟發。

教學流程設計

現時大部分的小學都有同儕備課的安排，讓教師共同商議某學習單位或課題的教學流程。然而礙於時間和資源所限，一

般只會以教科書的鋪排作為討論的根據。我發現數學教育組課程發展主任與教師的討論，卻是從《數學課程指引（小一至小六）》[12] 中的學習重點出發，同時考慮以往學生在學習該課題時會出現的學習難點，有策略地設計教學流程。

例如：有關 4S1 四邊形（三）的學習重點，「課程指引」的描述如下：

單位	學習重點	建議節數
4S1 四邊形(三)	1. 認識梯形及菱形的簡單特性。 2. 比較各種四邊形的特性。 3. 用不同的方法製作四邊形。	10

一般教科書只會把這學習單位的教學內容以認識及製作四邊形分成兩個不同的課題。根據教師的經驗，他們通常只安排 6 至 7 教節完成教科書的內容，所以他們對如何有效地分配及運用建議的 10 教節來完成「課程指引」列出的三個學習重點感到困惑。透過數學教育組課程發展主任參與的共同備課會議後，教師在設計學習單位 4S1 四邊形（三）的教學流程時，嘗試從課程的橫向、縱向和連貫性各方面考慮，不再被教科書的內容所局限。他們先考慮學生在一至三年級時的有關知識及經驗，例如：學生曾認識哪些四邊形？學生是否掌握了四邊形的概念？學生是如何學習四邊形的？學生認識了這些四邊形的甚麼特性？二年級時有關梯形及菱形的教學內容與四年級的有何不同？應如何配合？此外，教師

[12]下稱「課程指引」

須釐定「課程指引」中「簡單特性」的含意，以及各四邊形之間有什麼異同的特性可作比較。他們亦關顧到製作甚麼實物以配合相關圖形的教學，每個學習重點都經過細緻的考慮，從而設計出一個校本的教學流程。

課堂實作活動

課堂活動是教學重要的一環，如何將活動結合教學內容，引導學生從參與活動的過程中邁向教學目標，都是教師花很多時間思考的問題。透過參與是次「種籽」計畫，我深刻體會到在教授「圖形與空間」範疇的課題時，讓學生觀察和接觸實物的課堂實作活動十分重要。學生經過實際的操作，能更深入認識和掌握幾何圖形的特性，比起教師只拿著一個教具在課室中作展示為佳。此外，教師能從觀察學生在活動過程中的表現，更清楚地了解學生對該學習內容掌握的程度。

以其中一個課堂實作活動為例，教師教授學習單位 4S1 四邊形（三）時，設計了多個課堂實作的活動，希望學生利用量度和摺紙等方法，找出圖形的特性，而學生亦對這些活動甚感興趣。

圖 1：找出邊長

圖 2： 找出「對角線」是否相交成直角

圖 3： 透過摺紙方法找出對邊長度是否相等

圖 1 和圖 3 顯示，學生能掌握如何利用工具或摺紙的方法找出圖形的特性，但圖 2 的學生明顯對「對角線」的概念有誤解[13]。

另一個課堂實作活動是要求學生透過量度圖形對邊之間的垂直距離，藉以找出這對對邊是否平行。

圖 4

圖 5

[13]對角線的概念並不是「課程指引」的要求，教師可因應學生的能力作出調適。

圖 4 及圖 5 顯示學生 A 及學生 B 的學習成果，兩人經量度後均認為手上的圖形對邊是平行的。若觀察學生 B 的量度方法，教師便可評估出他/她對「對邊垂直距離」的概念有所誤解。這個誤解可能引致該學生日後學習及計算平行四邊形面積時，未能正確地判別平行四邊形對邊的垂直距離，即該圖形的高。

上述的例子可指出教師可從課堂實作活動的過程中即時了解學生對有關概念掌握的程度。如發現學生在學習的過程中遇到問題，可即時糾正，給予回饋，避免學生只懂背誦圖形的特性，而未能正確地掌握相關的概念或解決有關問題。

工作紙設計

工作紙的設計能否配合教學活動亦是一大學問。一般活動工作紙具備記錄和總結的作用，但如何設計一份合適的工作紙，既有清晰的活動指引，亦能引導學生思考和探究，真是十分值得考究。

在其中一次與教師討論工作紙設計的過程中，教師表示希望學生能將實際操作的過程記錄下來，並透過記錄的結果比較正方形和菱形的特性，他們根據教科書的建議，製作了以下的工作紙 1：

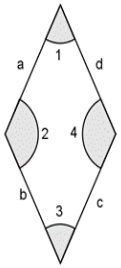
比較正方形和菱形的特性，並把相關結果以「✓」記錄於下列方格內。

	正方形	菱形
四邊相等		
四角相等		
兩組對邊相等		
兩組對邊平行		
兩組對角相等		
只有一組對邊平行		
對角線相等		
對角線相交構成直角		

工作紙 1

教師指出雖然大部分學生能在工作紙 1 上以「✓」表示正確的答案，但從實物操作的過程中卻發現學生的量度技巧其實十分粗疏，部分學生不能正確說出如何從量度得出工作紙 1 的結果，他們只是記著有關四邊形的特性，然後選取答案，並沒有依照活動指引的要求，從量度的結果驗證有關圖形的特性。

數學教育組課程發展主任因應這情況，向教師提供了意見，改善了工作紙 1 的設計，以針對四邊形中有關對邊及對角的特性，詳列如下：



菱形	對邊的垂直距離		比較對角的大小	
	a 至 *c* 的距離	*b* 至 *d* 的距離	角 1 和角 3	角 2 和角 4
A	_____ cm	_____ cm	角 1 ＜ 角 3	角 2 ＜ 角 4
	_____ cm	_____ cm	角 1 ＝ 角 3	角 2 ＝ 角 4
	_____ cm	_____ cm	角 1 ＞ 角 3	角 2 ＞ 角 4
B	_____ cm	_____ cm	角 1 ＜ 角 3	角 2 ＜ 角 4
	_____ cm	_____ cm	角 1 ＝ 角 3	角 2 ＝ 角 4
	_____ cm	_____ cm	角 1 ＞ 角 3	角 2 ＞ 角 4
C	_____ cm	_____ cm	角 1 ＜ 角 3	角 2 ＜ 角 4
	_____ cm	_____ cm	角 1 ＝ 角 3	角 2 ＝ 角 4
	_____ cm	_____ cm	角 1 ＞ 角 3	角 2 ＞ 角 4

菱形特性	**對邊**	*a* 和 *c* 是/不是 平行 *b* 和 *d* 是/不是 平行	一對/兩對 對邊平行
	對角	角 1 和角 3 相等/不相等 角 2 和角 4 相等/不相等	一對/兩對 對角相等

工作紙 2

教師要求學生將量度的答案填寫在工作紙 2 上，而學生須根據自己量度的結果，去判斷菱形的兩組對邊是否平行及兩組對角是否相等。雖然活動的內容沒有改變，只是工作紙 2 的記錄方式改變了，學生的活動表現便大為改善。工作紙的問題鋪排，就如一個啞老師，它在活動過程中給與學生指引，帶領學生去學習和探究。

總括來說，今年借調到數學教育組參與「種籽」計畫，讓我對「圖形與空間」範疇的學與教有更深入的了解，由協

助教師設計教學計畫和教案，到學校觀課，與教師和學生面談，及分享各校的教學成果，都是難得的體驗。以往的教學經驗主要來自自己的學校，能有機會到不同的學校觀摩，體驗不同的教學模式，確實對我作為教師的專業成長提升不少，希望能將今年所學的經驗帶返校園，幫助推動學校數學課程的發展。

參考資料：

香港課程發展議會(2000)。《數學教育學習領域 數學課程指引（小一至小六）》。香港：政府印務局。

5. Reflection on secondment

Ng Ka Lok
Wah Yan College Kowloon

Introduction

It is a precious learning opportunity for me to be seconded to the Mathematics Education Section of Education Bureau. Throughout this year, I read plenty of mathematics books, browsed journals, and most importantly, I had the chance to visit schools and conduct lesson observations. Furthermore, because of preparing talks for seminars, I needed to probe deeper in some particular topics (e.g. similar solids, and Archimedes' way to prove the volume of a sphere formula). All these helped me reflect on a deeper level understanding of how a teacher can facilitate students to learn better in classroom.

Experience gained in participation in "Seed" Project

For me, my participation in "Seed" project granted me the advantage to visit schools and had meetings with teachers of other schools. As I had been teaching in my own school for more than 15 years, I was especially curious to understand how the mathematics panels of other schools operated. Accompanied by the curriculum development officers of the Mathematics Education Section, I visited three seed schools. I found that it was a very interesting and pleasurable experience to learn from other schools.

In the meetings with teachers of seed schools, we discussed and prepared warm up activities to address students' learning difficulties. Moreover, a series of well-structured worksheets were designed to build up students' conceptual understanding. I was quite impressed by the worksheets on "Similar quadrilaterals" and "Similar solids" as these topics were seldom mentioned in textbooks.

(a) Mind map quiz

I recalled an impressive lesson during a lesson observation of a seed school. An experienced mathematics teacher divided the students into groups of four. As the students had some experience in drawing 3-D figures from what was taught in worksheet, the teacher further challenged them to create their own ways of drawing 3-D figures. I was quite impressed by the students' works and their motivation in learning. At the end of the lesson, the teacher asked the students to write a mind map (Figure 1) as part of the assessment. I found that this was a very good activity to encourage students to reflect on what they had learnt. Students were enthusiastic in jotting down their thoughts and recalled what they learnt during the lesson. Although some students apparently did not learn a lot, it did provide them a chance to reflect on their learning. The mind map quiz provided a useful platform for the teacher to understand students' learning difficulties and their misconceptions. I found that using a mind map quiz was a very good roundup

activity after teaching a topic.

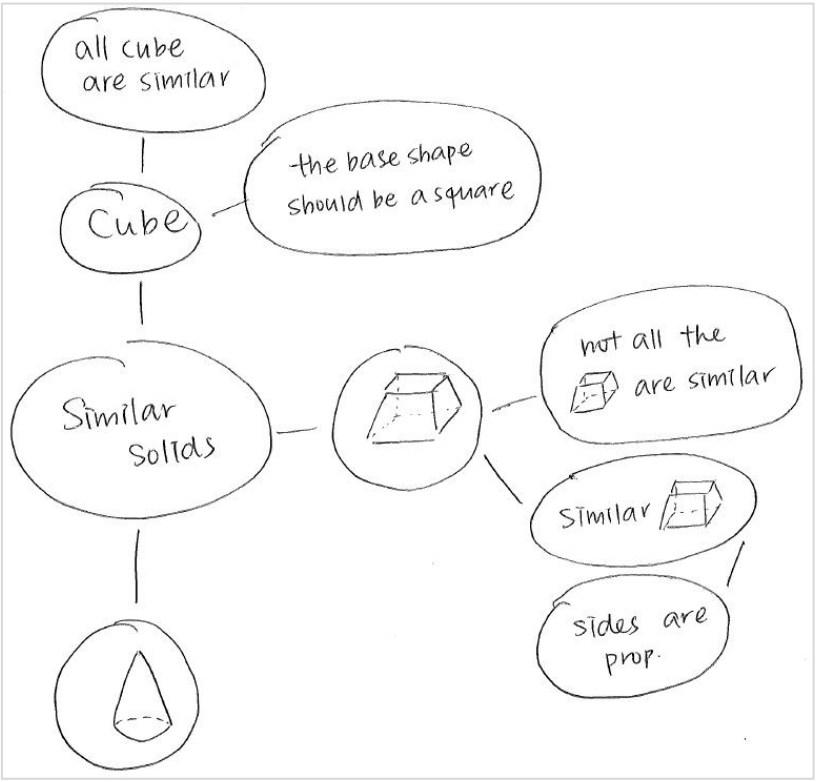


Figure 1

(b) Designing Multiple Choice (MC) Questions and Worksheets

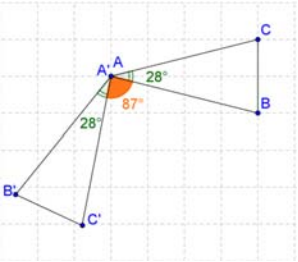
I also had chances to draft and revise the worksheets for each subtopic of "Similar figures" and "3-D figures". In the past, I never had time to make a worksheet on my own. This year I had a chance to focus on the process of creating a purposeful worksheet to diagnose students' learning

difficulties and I got valuable feedback from the officers of the Mathematics Education Section during discussion. In the process of drafting and revising worksheets, I benefited a lot in how to make a meaningful worksheet and improve it based on students' learning difficulties.

The idea of creating distractors in MC questions was also new to me. It was an eye-opening experience for me to get involved in designing appropriate MC distractors in pre-tests or post-tests to diagnose and verify students' conceptual understanding. After each pre-test and post-test, meetings were held to discuss and analyse students' errors. The discussion on the reasons behind students' poor performance enhanced my understanding of students' learning difficulties in some particular topics and was very beneficial to my future teaching in mathematics.

Below is an MC question with some MC distractors to test whether the students really understand the angle of rotation of a rotational transformation:

$\triangle ABC$ is rotated clockwise about point A to form the image $\triangle A'B'C'$ where A' is the image of point A and so on. The angle of rotation is



A. 87°.

*B. 115°.

C. 135°.

D. 143°.

(c) Hands-on activity

The use of different teaching tools for students to explore was very impressive to me too. In teaching the topic "3-D figures" and their properties, the Mathematics Education Section provided learning resources for tryout (Figure 2). It seemed that this learning resource was very useful in explaining the concept of dilation with reference to a point of projection.

Use of the learning resource to explain the dilation of 3-D figures

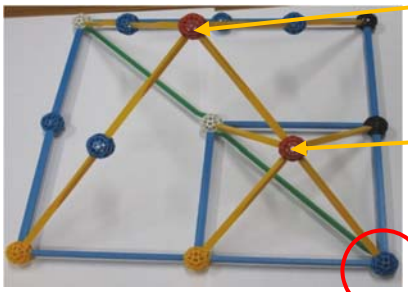


Figure 2

Students were asked to construct the images of a right square pyramid (as the right-bottom pyramid in Figure 2) with scale factor 2. Students would find it not difficult to form another right square pyramid.

In addition, a learning resource with different types of regular polygons (Figure 3 - 6) was very useful when teaching the topic "Regular polyhedra". An exploratory activity using this learning resource was arranged for students to explore whether they could form regular polyhedra using different types of regular polygons. Some follow-up questions, such as what kind of regular polygons could make up a regular polyhedron or whether solids constructed by regular polygons must be regular polyhedrons, could be drawn in the process of playing with these regular polygons. Yet, to make full use of these two kinds of tool kits,

besides having good class control, it was important that the teachers could conduct the activities systematically and also conclude students' observation in precise words at the end of the activity. Otherwise the students could not get the conclusion and be enlightened by the activity.

Figure 3

It is a regular polyhedron (Dodecahedron) as it is formed by regular pentagons with the same number of faces attached to each vertex.

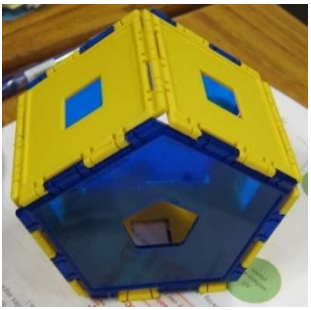


Figure 4

It is not a regular polyhedron as it is not formed by a single type of regular polygons even the same number of faces are attached to each vertex.

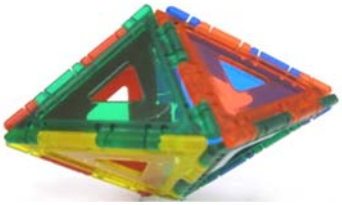


Figure 5

It is not a regular polyhedron as the number of faces attached to different vertices are not the same even it is formed by the same type of regular polygons (Equilaterial triangles).

Figure 6

It is a regular polyhedron (Octahedron) as it is formed by equilateral triangles with the same number of faces attached to each vertex.

Enrichment in Knowledge of Mathematics

(a) Similar Solids

I was assigned to give a talk to share with other teachers the effective learning and teaching strategies related to the learning topic "3-D figures". In the preparation process, I began to learn more about different definitions of similar solids and their uses in identifying similar solids. In the past, I did not notice the confusion in using "two solids with the same shape" as the definition for similar solids. Under this definition, students would easily regard 2 frustums with square bases as similar figures. Now I realised the

importance of learning from different perspectives in viewing a definition or a particular mathematical problem.

Traditional definition[14]:
If 2 solids have the same number of faces, and that the faces of one solid are correspondingly similar to those of the other, then they are called similar solids.

One counterexample of this definition is illustrated in Figure 7. The top vertex of the solid A is pressed down to form solid B. Nevertheless, according to the traditional definition, Solid A is similar to Solid B, which is problematic.

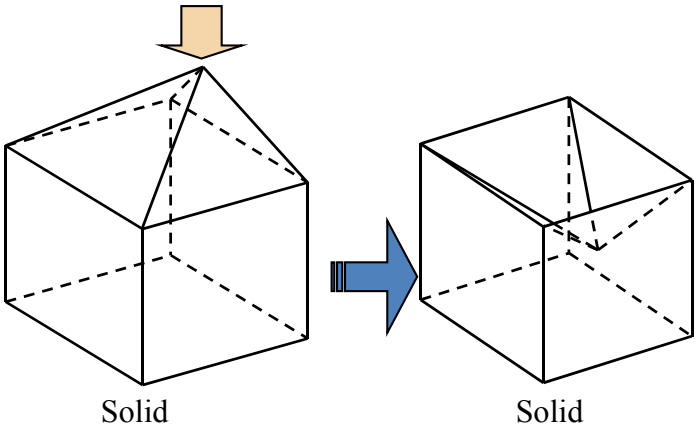


Figure 7

[14] Quoted from the definition 9 in Book XI of Euclid's *Elements*

New definition:
2 solids are similar if and only if they can coincide after a finite number of reflection, rotation, translation and dilation transformation.

Most students, participated in the "Seed" project, seemed more readily to understand the meaning of similar solids by using this new definition. They could visualise the images of a cube and square pyramid after dilation. Students could more easily observe the relation between the ratios of corresponding lengths, areas of corresponding surfaces and volumes of two similar figures. Besides, this new definition can extend the discussion to all 3-D figures whereas the traditional definition covers only polyhedrons.

(b) Compound Angle Formulae

In delivering talks during seminars, I became more familiar with the subject content and also the depth and breadth of the curriculum requirements. For example, in giving a talk on the seminar of the Extended Part Module 2, I learnt that there were many different ways to prove the compound angle formulae (Figure 8).

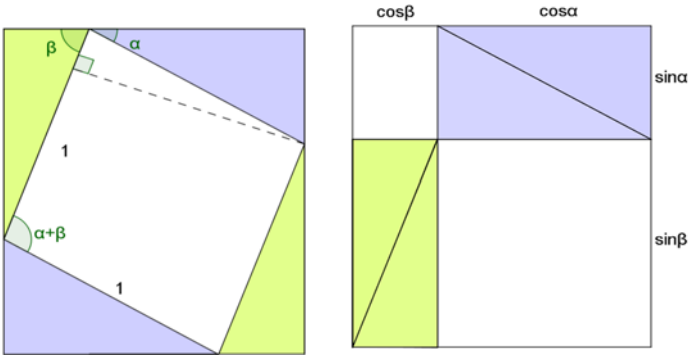


Figure 8

To compare the areas of the white regions in Figure 8, it is not difficult to deduce the compound angle formula:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$

(c) Sharing in the Mathematics Education Section

In the Mathematics Education Section, there was internal professional development sharing session among colleagues in nearly every month. I had the opportunity to learn from other colleagues in tackling a mathematical problem and ways to inspire students to learn. At one time, I was impressed by Dr YK Ng's presentation about mathematical reasoning in learning and teaching. The point rather struck me was on the teaching of algorithm. Dr Ng remarked that the teaching of algorithm without explaining the rationale behind (or thinking about the rationale) was not constructive to good mathematics learning.

Enrichment of Information Technology (IT) skills

In addition, my IT skills related to mathematics were enhanced, because I needed to deliver a talk on the use of GeoGebra 4. Before secondment, I hardly knew how to use GeoGebra 4. Since I needed to teach this software, I spent quite some time to learn by myself how to draw regular polygons, circles, vectors, etc. By now I can declare that I am rather proficient in the use of this software, and it may facilitate my teaching of mathematics in future.

Last but not least, my PowerPoint skills and word processing skills improved significantly during this year. From time to time I was assigned to use computers to amend worksheets, pre-tests or post-tests. In the past, I did not know how to draw 3-D figures in Microsoft Word and PowerPoint. But now I am quite competent in drawing 3-D figures in these environments.

Involvement in Mathematics Competitions

It was my honour to be an adjudicator in several Mathematics Competitions in Hong Kong, namely Mathematics Project Competition, Mathematics Book Report Competition and Statistics Creative-Writing Competition. As a result, I had to read more mathematics books and study more about some mathematical concepts. My personal growth in the knowledge of mathematics in different areas was thus developed. I learnt more about the beauty and diversity of mathematics knowledge.

I realised that as a teacher, I needed to update my mathematics knowledge regularly.

To conclude, the experience I gained this year in the Mathematics Education Section as a seconded teacher was fruitful and had a good impact on my perspectives on learning and teaching of mathematics. It also gave me many precious opportunities to study different branches of mathematics. I do hope that when I go back to school, I can make good use of the teaching resources and the ideas learnt in this year to become a good and competent mathematics teacher.

6. Application of geometric transformations to solving construction problems

NG, Yui-kin

In this short article, we will use a few examples to illustrate how geometric transformations may lead to elegant solutions of construction problems. The first problem is:

> Suppose that the quadrilateral $ABCD$ is a square. Construct a quadrilateral such that the four points A, B, C and D are the vertices of right-angled isosceles triangles constructed on the sides of the desired quadrilateral and the angles at A, B, C and D are right angles.

Assuming that the problem has a solution and $PQRS$ is the desired quadrilateral (see Figure 1).

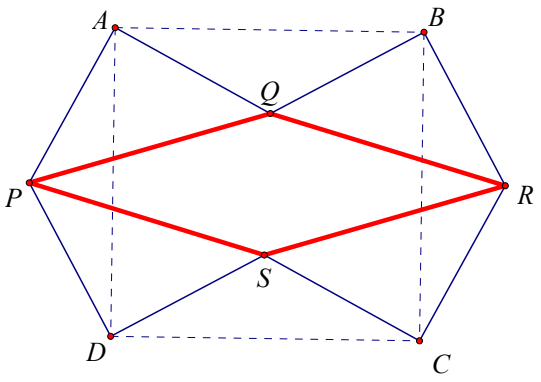


Figure 1

Obviously, the quadrilateral can be constructed once we can locate a vertex of it. For example, if we are able to construct the point P, the point Q will be obtained by rotating P about A through 90°,[15] then R be obtained by rotating Q about B through 90°, and finally S be obtained by rotating R about C through 90°. In this connection, the problem is reduced to the problem of locating a vertex of the quadrilateral.

For any point P' in the plane, from these three rotations, we can still obtain three consecutive points Q', R' and S' such that A, B and C are the vertices of right-angled isosceles triangles constructed respectively on the line segments $P'Q'$, $Q'R'$ and $R'S'$. But $P'Q'R'S'$ will not be the desired quadrilateral unless $\Delta P'DS'$ is a right-angled isosceles triangle. If one constructs an arbitrary point P' and the three consecutive points Q', R' and S' in a dynamic geometry platform, by dragging the point P' around the screen, one will, sooner or later, observe that $\Delta P'DS'$ seems to be *always* a right-angled isosceles triangle. Is it really true that any point in the plane can be the vertex of the desired quadrilateral? Moreover, what would be the solution if the quadrilateral $ABCD$ is not a square? Before going into the detail of the answers, two points have to be noted here:

1. The right-angled isosceles triangles constructed on

[15] In this article, all angles of rotations are directed angles and the angles associated with anti-clockwise rotations are taken to be positive.

the sides of the quadrilateral are not necessarily lying towards the quadrilateral's exterior (see Figure 2).

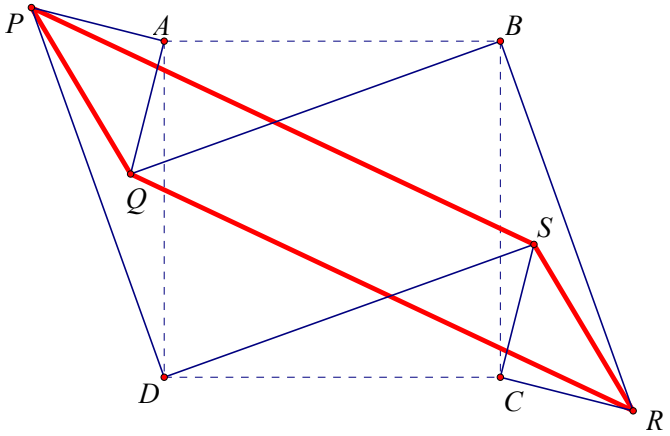


Figure 2

2. When *P* coincides with a vertex of the given square, the quadrilateral is the square *ABCD* and the triangles are degenerate.

Theorem (Rotation). Let $\Re$ be the sum (or product) of n rotations about n points through angles $\theta_1, \theta_2, \ldots, \theta_n$ and $\phi = \theta_1 + \theta_2 + \ldots + \theta_n$. $\Re$ is a rotation through the angle ϕ provided that ϕ is not a multiple of 360°, and is a translation otherwise.[16]

[16] The proof, without using coordinates nor vectors, can be found elsewhere.

In our first problem, consider the sum of four rotations ($\mathfrak{R}_1$), each through 90°, about the points A, B, C and D. Since the sum of angles is 360°, $\mathfrak{R}_1$ is a translation by Theorem (Rotation). As $ABCD$ is a square, the first rotation carries A to A, the second carries A to C, the third carries C to C and the fourth carries C to A. That is to say, $\mathfrak{R}_1$ carries A to A. A translation will not carry a point to itself unless the translation is the identity transformation. Hence, $\mathfrak{R}_1$ is the identity transformation and carries any point P in the plane to itself. That is to say, the point obtained by rotating S about D through 90° must be P and $\triangle PDS$ is always a right-angled isosceles triangle. As a result, no matter where P is, P and the points Q, R and S constructed by the rotations about B, C and D through 90° will form the desired quadrilateral.

If $ABCD$ is not a square, but the sum of the four rotations with centres A, B, C and D and angles $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle CDA$ respectively is still a translation that carries A to A (for example, when $ABCD$ is a rhombus)[17], then any point in the

For example, Modenov & Parkhomenko, 1965; Yaglom, 1975.

[17] There exist other sequences of rotations, apart from the one we discuss, whose sum can generate the desired quadrilateral, especially when we accept that the desired quadrilateral can be non-simple. For example, when $ABCD$ is a non-square rectangle, although the sum of four rotations, each through 90°, about the points A, B, C and D is not the identity transformation, the sum of rotations, first rotate about A through 90°, about B through -90°, about C through 90°, and finally about D through -90°, is the identity transformation and is thus able to generate the desired quadrilateral (see Figure 3).

plane can be the vertex of the desired quadrilateral. That is to say, the problem has infinitely many solutions. On the other hand, if, for any such sequence of rotations[18], the sum is a non-identity translation, then the problem will have no solution.

With the use of Theorem (Rotation), we can easily solve the following seemingly hard construction problem (Yaglom, 1975, p.37):

> Construct an n-gon, given the n points that are the vertices of isosceles triangles constructed on the sides of the n-gon, with the angles $\alpha_1, \alpha_2, \ldots, \alpha_n$ at the outer vertices.

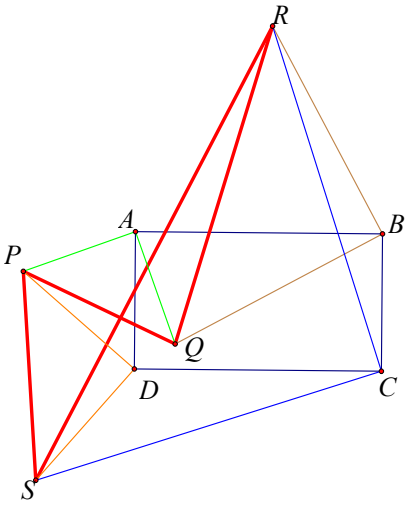


Figure 3

[18] Angles of rotations may be positive or negative.

Let $A_1, A_2, \ldots, A_n$ be the given n points and the n-gon be $P_1P_2 \ldots P_n$, where $\Delta P_1A_1P_2, \Delta P_2A_2P_3, \ldots, \Delta P_nA_nP_1$ are isosceles triangles with the angles $\alpha_1, \alpha_2, \ldots, \alpha_n$ at $A_1, A_2, \ldots, A_n$, and $\Re_2$ be the sum of rotations about the n points $A_1, A_2, \ldots, A_n$ through the angles $\alpha_1, \alpha_2, \ldots, \alpha_n$ respectively (i.e. first about A_1 through an angle α_1, then about A_2 through an angle α_2, … , and finally about A_n through an angle α_n) . If $\alpha_1 + \alpha_2 + \ldots + \alpha_n$ is not a multiple of 360°, then $\Re_2$ is a rotation (by Theorem (Rotation)) and the centre of rotation is P_1 as $\Re_2$ carries P_1 to P_1. Construct two arbitrary points M and N, distinct from the given n points, in the plane. Following the sequence of rotations, we will obtain points M' and N' respectively. The point of intersection of the perpendicular bisectors to MM' and NN' will then be P_1. Having constructed P_1, we obtain P_2 by rotating P_1 about A_1 through an angle α_1. Similarly, we obtain P_3 by rotating P_2 about A_2 through an angle α_2, and so on. The n-gon can then be constructed.

If $\alpha_1 + \alpha_2 + \ldots + \alpha_n$ is a multiple of 360°, the analysis is similar to that in solving the first problem and we won't repeat the

discussion here.

Having solved the second problem, the following three well-known construction problems (Yaglom, 1975, p.12) will become very *simple*!

> Construct a triangle, given the three points in the plane that are the outer vertices of equilateral triangles constructed outward on the sides of the desired triangle.
>
> Construct a triangle, given the three points in the plane that are the centres of squares constructed outward on the sides of the desired triangle.
>
> Construct a heptagon, given the seven points that are the mid-points of its sides.

References:

Modenov, P.S., & Parkhomenko, A.S. (1965). *Geometric transformations Volume 1: Euclidean and affine transformation*. New York: Academic Press.

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7. 香港數學競賽回顧

第 30 屆

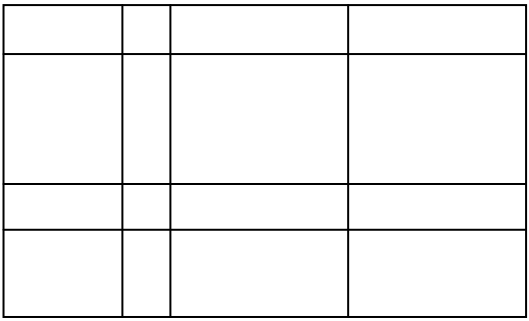
第 30 屆香港數學競賽已於 2013 年 4 月 20 日順利進行。而喇沙書院、聖言中學及伊利沙伯中學則分別奪得比賽的總冠軍、亞軍及季軍。

頒獎典禮亦緊接著比賽後舉行，大會邀請得香港數學競賽第 1 屆籌備委員會副主席 蕭卓平 女士、香港數學競賽第 10 屆籌備委員會主席 馬兆權 先生、香港數學競賽第 20 屆籌備委員會主席 衛之賚 女士、香港教育學院數學與資訊科技學系系主任 江紹祥 博士、數學與資訊科技學系副系主任 文耀光 博士、教育局總課程發展主任（數學）衛國強 先生及教育局課程發展主任（數學）鄭仕文 先生擔任主禮嘉賓。

當天的比賽題目及答案現輯錄如下，以方便各位老師及同學參考及作為未來培訓參賽隊伍之用。

個人項目（一）

1. 圖 1 共有 a 個長方形。求 a 的數值。
   Figure 1 has a rectangles. Find the value of a.



圖一
Figure 1

2. 已知 111111 能被 7 整除。若 b 為 $\underbrace{111111...111111}_{a\text{ 個}}$ 除以 7 的餘數，求 b 的數值。
   Given that 7 divides 111111. If b is the remainder of $\underbrace{111111...111111}_{a\ times}$ divided by 7, find the value of b.

3. 若 c 為 $\left[(b-2)^{4b^2}+(b-1)^{2b^2}+b^{b^2}\right]$ 除以 3 的餘數，求 c 的數值。

If c is the remainder of $\left[(b-2)^{4b^2}+(b-1)^{2b^2}+b^{b^2}\right]$ divided by 3, find the value of c.

4. 若 $|x+1|+|y-1|+|z|=c$，求 $d=x^2+y^2+z^2$ 的最大可能數值。
If $|x+1|+|y-1|+|z|=c$, find the maximum possible value of $d=x^2+y^2+z^2$.

個人項目（二）

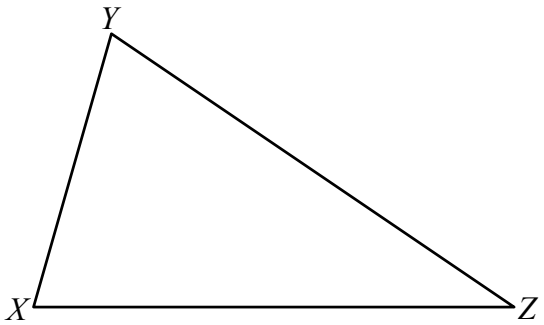
1. 已知函數 $f(x)=x^2+rx+s$ 和 $g(x)=x^2-9x+6$ 有以下特性：$f(x)$ 的根之和是 $g(x)$ 的根之積，且 $f(x)$ 的根之積是 $g(x)$ 的根之和。若 $f(x)$ 的最小值取值於 $x=a$，求 a 的值。
Given that functions $f(x)=x^2+rx+s$ and $g(x)=x^2-9x+6$ have the properties that the sum of roots of $f(x)$ is the product of the roots of $g(x)$ and the product of roots of $f(x)$ is the sum of the roots of $g(x)$. If $f(x)$ attains its minimum at $x=a$, find the value of a.

2. 一正方體的表面積是 b cm^2。若它每一條邊的長度增加 3 cm，它的體積隨之增加 $(2b-a)$ cm^3，求 b 的值。

The surface area of a cube is b cm^2. If the length of each side is increased by 3 cm, its volume is increased by $(2b-a)$ cm^3, find the value of b.

3. 設 $f(1)=3$，$f(2)=5$ 且對所有正整數 n，$f(n+2)=f(n+1)+f(n)$。當 $f(b)$ 除以 3 的餘數是 c，求 c 的值。
Let $f(1)=3$, $f(2)=5$ and $f(n+2)=f(n+1)+f(n)$ for positive integers n. If c is the remainder of $f(b)$ divided by 3, find the value of c.

4. 如圖 2，三角形 XYZ 的角度滿足 $\angle Z \le \angle Y \le \angle X$ 且 $c\cdot\angle X = 6\cdot\angle Z$。若 $\angle Z$ 的最大可能值是 $d°$，求 d 的值。
In Figure 2, the angles of triangle XYZ satisfy $\angle Z \le \angle Y \le \angle X$ and $c\cdot\angle X = 6\cdot\angle Z$. If the maximum possible value of $\angle Z$ is $d°$, find the value of d.



圖二
Figure 2

個人項目（三）

1. 若 $a=\dfrac{(7+4\sqrt{3})^{\frac{1}{2}}-(7-4\sqrt{3})^{\frac{1}{2}}}{\sqrt{3}}$，求 a 的整數值。

If $a=\dfrac{(7+4\sqrt{3})^{\frac{1}{2}}-(7-4\sqrt{3})^{\frac{1}{2}}}{\sqrt{3}}$, find the integer value of a.

2. 設 $f(x)=x-a$ 及 $F(x,y)=y^2+x$。如 $b=F(3,f(4))$，求 b 的值。
Suppose $f(x)=x-a$ and $F(x,y)=y^2+x$. If $b=F(3,f(4))$, find the value of b.

3. 已知 392 除以一個兩位正整數的餘數是 b，符合這個條件的兩位正整數共有 c 個, 求 c 的值。
The remainder of 392 divided by a 2-digit positive integer is b. If c is the number of such 2-digit positive integers, find the value of c.

4. 若 x 為實數及 d 為函數 $y=\dfrac{3x^2+3x+c}{x^2+x+1}$ 的最大值，求 d 的值。
If x is a real number and d is the maximum value of the function $y=\dfrac{3x^2+3x+c}{x^2+x+1}$, find the value of d.

個人項目（四）

1. 設實函數$f(x)$對於所有實數 x及y滿足 $f(xy)=f(x)f(y)$，且$f(0)\neq 0$。求$a=f(1)$的值。
Let $f(x)$ be a real function that satisfies $f(xy)=f(x)f(y)$ for all real numbers x and y, and $f(0)\neq 0$. Find the value of $a=f(1)$.

2. 設函數 $F(n)$ 滿足 $F(1)=F(2)=F(3)=a$ 及 $F(n+1)=\dfrac{F(n)\cdot F(n-1)+1}{F(n-2)}$，其中 $n\geq 3$ 為正整數。求 $b=F(6)$ 的值。

Let $F(n)$ be a function with $F(1)=F(2)=F(3)=a$ and $F(n+1)=\dfrac{F(n)\cdot F(n-1)+1}{F(n-2)}$ for positive integers $n\geq 3$. Find the value of $b=F(6)$.

3. 若 $b-6$、$b-5$、$b-4$ 為方程 $x^4+rx^2+sx+t=0$ 的根，求 $c=r+t$ 的值。

If $b-6$, $b-5$, $b-4$ are three roots of the equation $x^4+rx^2+sx+t=0$, find the value of $c=r+t$.

4. 設 (x_0,y_0) 是以下方程組的一個解：

$$\begin{cases} xy=6 \\ x^2y+yx^2+x+y+c=2 \end{cases}$$

求 $d=x^2{}_0+y_0^2$ 的值。

Suppose that (x_0,y_0) is a solution of the system:

$$\begin{cases} xy=6 \\ x^2y+yx^2+x+y+c=2 \end{cases}$$

Find the value of $d=x^2{}_0+y_0^2$.

團體項目（一）

1. 求 $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$ 的個位數字。

Find the unit digit of $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$.

2. 求 $16 \div (0.40 + 0.41 + 0.42 + ... + 0.59)$ 的值的整數部分。
Find the integer part of $16 \div (0.40 + 0.41 + 0.42 + ... + 0.59)$.

3. 從 1、2、4、6、7 中選三個數字組成三位數。這些三位數有多少個能被 3 整除？
Choose three digits from 1, 2, 4, 6, 7 to construct three-digit numbers. Of these three-digit numbers how many of them are divisible by 3?

4. 用 1、2、3、4、5、6 組成一個 6 位數：*ABCDEF*，使得：*A* 能被 1 整除，*AB* 能被 2 整除，*ABC* 能被 3 整除，*ABCD* 能被 4 整除，*ABCDE* 能被 5 整除，及 *ABCDEF* 能被 6 整除。求 *A* 的最大值。

Using numbers: 1, 2, 3, 4, 5, 6 to form a six-digit number: *ABCDEF* such that *A* is divisible by 1, *AB* is divisible by 2, *ABC* is divisible by 3, *ABCD* is divisible by 4, *ABCDE* is divisible by 5 and *ABCDEF* is divisible by 6. Find the greatest value of *A*.

團體項目（二）

1. 若 $4^3 + 4^r + 4^4$ 是一平方數，其中 r 是正整數，求 r 的最小值。
If $4^3 + 4^r + 4^4$ is a perfect square and r is a positive integer, find the minimum value of r.

2. 三男 B_1、B_2、B_3 和三女 G_1、G_2、G_3 就坐一排座

位，並滿足以下兩個條件:

1) 一男不會坐在另一男旁邊及一女不會坐在另一女旁邊

2) B_1 必須坐在 G_1 旁邊

若 s 是這樣就坐的排列數量，求 s 的值。

Three boys B_1, B_2, B_3 and three girls G_1, G_2, G_3 are to be seated in a row according to the following rules:

1) A boy will not sit next to another boy and a girl will not sit next to another girl

2) Boy B_1 must sit next to girl G_1

If s is the number of different such seating arrangements, find the value of s.

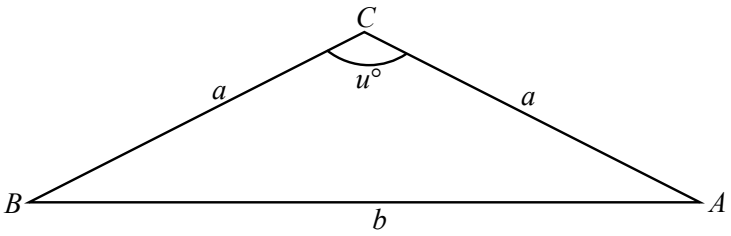
3. 設 $f(x)=\dfrac{x+a}{x^2+\dfrac{1}{2}}$，$x$為實數且$f(x)$ 的最大值和最小值分別是 $\dfrac{1}{2}$ 和 -1。若$t=f(0)$,求 t 的值。

Let $f(x)=\dfrac{x+a}{x^2+\dfrac{1}{2}}$, where x is a real number and the maximum value of $f(x)$ is $\dfrac{1}{2}$ and the minimum value of $f(x)$ is -1. If $t=f(0)$, find the value of t.

4. 如圖一，ABC 是一等腰三角形，其中 $\angle ABC=u^\circ$，$AB=\mathrm{BC}=a$ 和 $AC=b$。若二次方程 $ax^2-\sqrt{2}\cdot bx+a=0$ 有兩個實根，它們的絕對差為 $\sqrt{2}$ ，求u的值。

In Figure 1, ABC is an isosceles triangle with $\angle ABC = u^\circ$, AB = BC = a and $AC = b$.

If the quadratic equation $ax^2 - \sqrt{2} \cdot bx + a = 0$ has two real roots, whose absolute difference is $\sqrt{2}$, find the value of u.



圖一

Figure 1

團體項目（三）

1. 若 m 和 n 是正整數且 $m^2 - n^2 = 43$，求 $m^3 - n^3$ 的值。

   If m and n are positive integers with $m^2 - n^2 = 43$, find the value of $m^3 - n^3$.

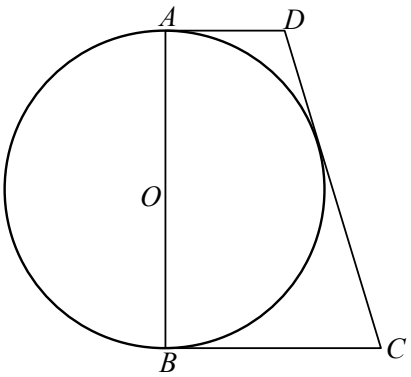
2. 設 x_1、x_2、…、x_{10} 為非零整數，且滿足 $-1 \le x_i \le 2$ 其中 i = 1、2、…、10。若 $x_1 + x_2 + ... + x_{10} = 11$，求 $x_1^2 + x_2^2 + ... + x_{10}^2$ 的最大可能值。

   Let $x_1, x_2, \ldots, x_{10}$ be non-zero integers satisfying $-1 \le x_i \le 2$ for $i = 1, 2, \ldots, 10$. If $x_1 + x_2 + \ldots + x_{10} = 11$,

find the maximum possible value for $x_1^2+x_2^2+\ldots+x_{10}^2$.

3. 若 $f(n)=a^n+b^n$，其中 n 是正整數且 $f(3)=[f(1)]^3+f(1)$，求 $a\cdot b$ 的值。
If $f(n)=a^n+b^n$, where n is a positive integer and $f(3)=\left[f(1)\right]^3+f(1)$, find the value of $a\cdot b$.

4. 如圖二，AD，BC 和 CD 是以 O 作圓心且直徑 $AB=12$ 的圓的切線。若 $AD=4$，求 BC 的值。
In Figure 2, AD, BC and CD are tangents to the circle with centre at O and diameter $AB=12$. If $AD=4$, find the value of BC.



團體項目（四）

1. 若 P 為整數 3,659,893,456,789,325,678 與 342,973,489,379,256 的乘積，求 P 的位數。
If P be the product of 3,659,893,456,789,325,678 and 342,973,489,379,256, find the number of digits of P.

2. 若 $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$，求 $1872+48\times\left(\frac{2013x}{x+2013}\right)$ 的值。

If $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$, find the value of $1872+48\times\left(\frac{2013x}{x+2013}\right)$.

3. 有一個正整數被 10 除，餘數為 9；被 9 除，餘數為 8；被 8 除，餘數為 7；等等直至被 2 除，餘數為 1。求此正整數的最小值。

The remainders of a positive integer when divided by 10, 9, 8, …, 2 are 9, 8, 7, …, 1, respectively. Find the smallest such positive integer.

4. 如圖三，A、B、C、D、E 代表不同的個位數字。求 $A+B+C+D+E$ 的值。

In Figure 3, A, B, C, D, E represent different digits. Find the value of $A+B+C+D+E$.

$$\begin{array}{r} ABCDE \\ \times \qquad 9 \\ \hline 1AAA0E \end{array}$$

圖三

Figure 3

第 31 屆

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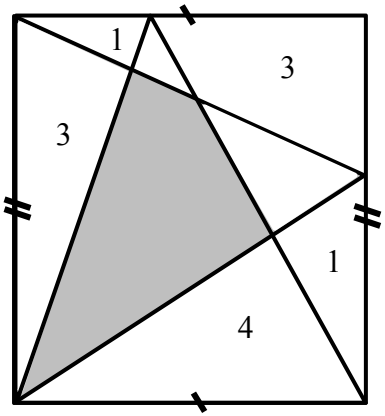
個人項目（一）

1. 求下圖中陰影部分的面積 α。
   Determine the area of the shaded region, α , in the figure below.



圖一
Figure 1

2. 如果 10 個不同的正整數的平均值是 2α，求這 10 個數中，最大的一個數 β 的最大可能值。
   If the average of 10 distinct positive integers is 2α, what is the largest possible value of the largest integer, β , of the ten integers?

3. 考慮兩組由正整數組成的有限數列：1, 3, 5, 7,…, β 和 1, 6, 11, 16, … , $\beta + 1$。求它們之間相同數字的數目 γ。
   Given that 1, 3, 5, 7, … , β and 1, 6, 11, 16, … , $\beta + 1$ are

two finite sequences of positive integers. Determine γ, the numbers of positive integers common to both sequences.

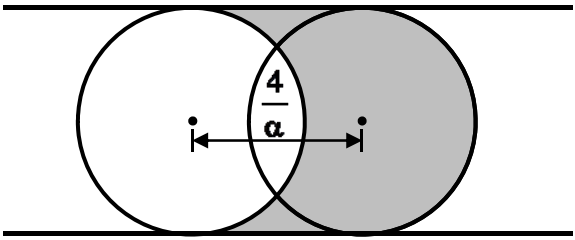
4. 若 $\log_2 a + \log_2 b \geq \gamma$ ，求 $a + b$ 的最小值 δ。

If $\log_2 a + \log_2 b \geq \gamma$, determine the smallest positive value δ for $a + b$.

個人項目（二）

1. 求方程 $\sqrt{(x+\sqrt{x})} - \sqrt{(x-\sqrt{x})} = \sqrt{x}$ 的正實根 α。
Determine the positive real root, α , of $\sqrt{(x+\sqrt{x})} - \sqrt{(x-\sqrt{x})} = \sqrt{x}$.

2. 下圖為兩個半徑為 4 的圓，其圓心相隔 $\frac{4}{\alpha}$ 。求陰影部分的面積 β。
In the figure below, two circles of radius 4 with their centres placed apart by $\frac{4}{\alpha}$. Determine the area, β, of the shaded region.



3. 求正整數 γ 的最小值，以使得方程 $\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ 對 x 有正整數解。

Determine the smallest positive integer γ such that the equation $\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ has an integer solution in x.

4. 求 $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$ 的個位數 δ。

Determine the unit digit, δ, of $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$.

個人項目（三）

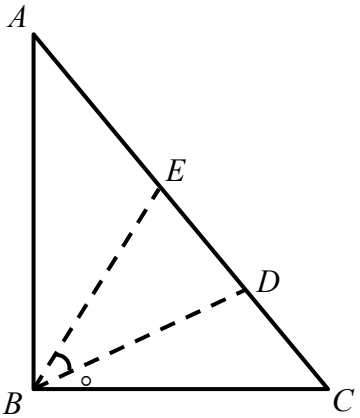
1. 若數列 $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \cdots, 10^{\frac{\alpha}{11}}$ 中所有數字的乘積為 1 000 000，求正整數 α 的值。

If the product of numbers in the sequence $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \ldots, 10^{\frac{\alpha}{11}}$ is 1 000 000, determine the value of the positive integer α.

2. 若 $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$，求 β 的值。

Determine the value of β if $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$.

3. 在下圖的三角形 ABC 中，$\angle ABC = 2\beta°$，$AB = AD$ 及 $CB = CE$。設 $\gamma° = \angle DBE$，求 γ 的值。
In the figure below, triangle ABC has $\angle ABC = 2\beta°$, $AB = AD$ and $CB = CE$. If $\angle DBE = \gamma°$,determine the value of γ.



4. 考慮數列 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, …，求首 γ 項的和 δ。
For the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, … , determine the sum δ of the first γ terms.

個人項目（四）

1. 若 $\dfrac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 3\sqrt{\alpha} - 6$，求 α 的值。
If $\dfrac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 3\sqrt{\alpha} - 6$, determine the value of α.

2. 考慮形如 $\frac{n}{n+1}$ 的分數，當中 n 是一個正整數。若同時把該分數的分子和分母減去 1，得出的分數是小 $\frac{\alpha}{7}$，且大於 0，求這樣的分數的數目 β。

Consider fractions of the form $\frac{n}{n+1}$, where n is a positive integer. If 1 is subtracted from both the numerator and the denominator, and the resultant fraction remains positive and is strictly less than $\frac{\alpha}{7}$, determine, β, the number of these fractions.

3. 一個等邊三角形和一個正六邊形的周長相同。若該等邊三角形的面積為 β 平方單位，求正六邊形的面積 γ (平方單位)。

The perimeters of an equilateral triangle and a regular hexagon are equal. If the area of the triangle is β square units, determine the area, γ, of the hexagon in square units.

4. 求 $\delta = \frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \frac{33}{32} + \frac{65}{64} - \gamma$ 的值。

Determine the value of $\delta = \frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \frac{33}{32} + \frac{65}{64} - \gamma$.

團體項目（一）

1. 若一個等腰三角形對應底邊（不是兩條等腰邊）的高是 8，且周長是 32，求該三角形的面積。
   If an isosceles triangle has height 8 from the base, not the legs, and perimeters 32, determine the area of the triangle.

2. 若 $f(x)=\dfrac{\left(x+\dfrac{1}{x}\right)^6-\left(x^6+\dfrac{1}{x^6}\right)-2}{\left(x+\dfrac{1}{x}\right)^3+\left(x^3+\dfrac{1}{x^3}\right)}$ 當中 x 是一個正實數，求 $f(x)$ 的最小值。
   If $f(x)=\dfrac{\left(x+\dfrac{1}{x}\right)^6-\left(x^6+\dfrac{1}{x^6}\right)-2}{\left(x+\dfrac{1}{x}\right)^3+\left(x^3+\dfrac{1}{x^3}\right)}$ where x is a positive real number, determine the minimum value of $f(x)$.

3. 求 81 位數 $\overline{111\ldots1}$ 除以 81 的餘數。
   Determine the remainder of the 81-digit integer $\overline{111\ldots1}$ divided by 81.

4. 給定一實數數列 $a_1, a_2, a_3, \ldots$，它滿足
   1) $a_1=\dfrac{1}{2}$，及
   2) 對 $k\geq 2$，有 $a_1+a_2+\cdots+a_k=k^2a_k$。
   求 a_{100} 的值。

Given a sequence of real numbers a_1, a_2, a_3, … that satisfy

1) $a_1 = \frac{1}{2}$, and

2) $a_1 + a_2 + \cdots + a_k = k^2 a_k$, for $k \geq 2$.

Determine the value of a_{100}.

團體項目（二）

1. 若在 $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$ 中刪去若干項後剩 1，求刪去各項的乘積。

By removing certain terms from the sum, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$, we can get 1. What is the product of the removed term(s)?

2. 若 $S_n = 1-2+3-4+\cdots+(-1)^{n-1}n$，當中 n 是正整數，求 $S_{17}+S_{33}+S_{50}$ 的值。

If $S_n = 1-2+3-4+\cdots+(-1)^{n-1}n$, where n is a positive integer, determine the value of $S_{17}+S_{33}+S_{50}$.

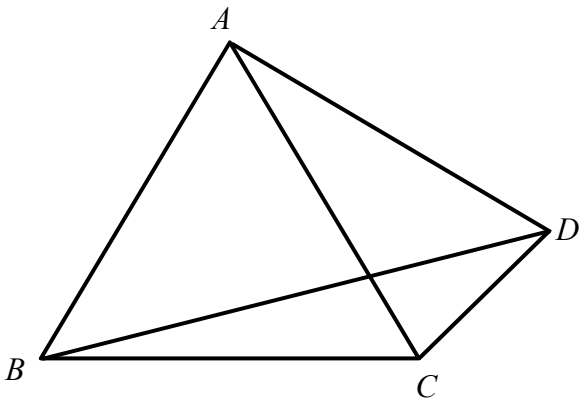
3. A, B, C, D, E 和 F 六人根據英文字母的順序輪班工作。A 在第一個星期日當值，然後 B 在星期一當值，

如此類推。A 於第 50 個星期的哪一天當值？（答案以數字 0 代表星期日，數字 1 代表星期一，……，數字 6 代表星期六）。

Six people A, B, C, D, E and F are to rotate for night shifts in alphabetical order with A serving on the first Sunday, B on the first Monday and so on. In the fiftieth week, Which day does A serve on? (Represent Sunday by 0, Monday by 1, …, Saturday by 6 in your answer.)

4. 在下圖中，D 以直線連接著等邊三角形 ABC 的頂點，當中 $AB = AD$。設 $\angle BDC = \alpha°$，求 α 的值。

In the figure below, vertices of equilateral triangle ABC are connected to D in straight line segments with $AB = AD$. If $\angle BDC = \alpha°$, determine the value of α.



團體項目（三）

1. 求乘積 $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$ 的值。

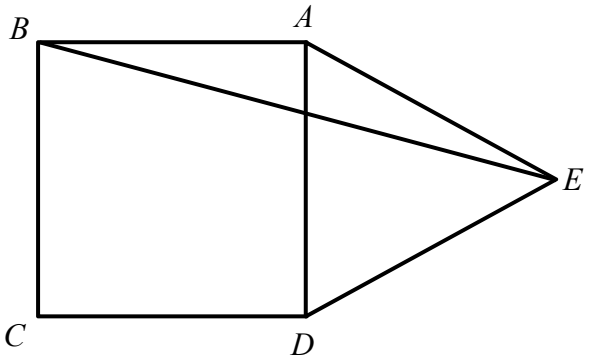
Determine the value of the product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$.

2. 求和 $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$ 的值，當中 $100!=100\times 99\times 98\times\ldots\times 3\times 2\times 1$。

Determine the value of the sum $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$, where $100!=100\times 99\times 98\times\ldots\times 3\times 2\times 1$.

3. 在下圖中，*ABCD* 是一個正方形，*ADE* 是一個等邊三角形，且 *E* 是正方形 *ABCD* 外的一點。設 $\angle AEB = \alpha°$，求 α 的值。

In the figure below, *ABCD* is a square, *ADE* is an equilateral triangle and *E* is a point outside of the square *ABCD*. If $\angle AEB = \alpha°$, determine the value of α.



4. 把不同的非零個位數填進下表白色的正方格內，使所有橫、直的等式均成立。求 α。

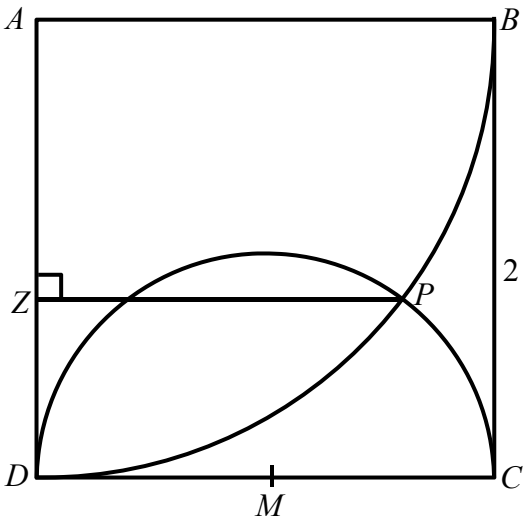
Fill the white squares in the figure below with distinct non-zero digits so that the arithmetical expressions, read both horizontally and vertically, are correct. What is α?

	÷		=	
+		×		
	+		=	α
=		=		

團體項目（四）

1. 在下圖中，*ABCD* 是一個邊長為 2 的正方形。先以 *A* 為圓心畫出弧 *BD*，再以 *CD* 的中點 *M* 為圓心從 *C* 到 *D* 畫出一個半圓。弧 *BD* 和弧 *DC* 相交於 *P*。求 *P* 與 *AD* 的最短距離，即 *PZ* 的長度。

   In the figure below, *ABCD* is a square of side length 2. A circular arc with centre at *A* is drawn from *B* to *D*. A semicircle with centre at *M*, the midpoint of *CD*, is drawn from *C* to *D* and sits inside the square. Determine the shortest distance from *P*, the intersection of the two arcs, to side *AD*, that is, the length of *PZ*.



2. 若 $x=\dfrac{\sqrt{5}+1}{2}$ 及 $y=\dfrac{\sqrt{5}-1}{2}$，求 $x^3y+2x^2y^2+xy^3$ 的值。

If $x=\dfrac{\sqrt{5}+1}{2}$ and $y=\dfrac{\sqrt{5}-1}{2}$, determine the value of $x^3y+2x^2y^2+xy^3$.

3. 若 a, b, c 及 d 是不同的個位數，且

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array}$$

求 d 的值。

If a, b, c and d are distinct digits and

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array},$$

determine the value of d.

4. 求方程 $x^4+(x-4)^4=32$ 所有實根的乘積。

Determine the product of all real roots of the equation $x^4+(x-4)^4=32$.

8. Mathematics Project Competition for Secondary Schools

Project learning is one of the Four Key Tasks advocated in the curriculum reform. It is a powerful learning and teaching strategy to promote students' capabilities for self-directed learning within and across Key Learning Areas. It allows students to have more space for learning and provides an alternative learning experience.

In this regard, the Mathematics Education Section of the Education Bureau initiated the Mathematics Project competition (MPC) for Secondary Schools since 2001. The competition aims at promoting the interest of students in learning mathematics and developing students' generic skills through project learning.

The numbers of participants in MPC 2012/13 and MPC 2013/14 are as follows:

School Year	No. of Projects Submitted	No. of Schools Participated	No. of Students Participated
2012/13	75	28	287
2013/14	76	27	315

The Prize-giving Ceremony of Mathematics Project and Mathematics Book Report Competition for Secondary Schools (2013/14) cum Exhibition of the Winning Entries was smoothly

held on 9 July 2014 at Lecture Theatre, 4/F, West Block, Education Bureau Kowloon Tong Education Services Centre, 19 Suffolk Road, Kowloon Tong, Kowloon. 106 winners of MPC from 24 schools, 38 teachers and 11 members of the Organising Committee of MPC (2013/14) joined the Ceremony. The winners of the MPC presented their projects with the audience. Chairman from the Organising Committee of MPC (2013/14) gave a brief account of the entries. The trophies, medals and certificates of merit were presented to the winners in the Ceremony. The winning entries of the MPC were also exhibited at the venue of the Ceremony.

The next page shows the results of MPC in 2012/13 and 2013/14.

Results of Mathematics Project Competition for Secondary Schools (2012/13)

Result	School	Title of Project
Champion	HKSYC & IA Wong Tai Shan Memorial College	九巴行車距離和票價的直線關係
1st runner-up	Pui Ching Middle School	Sicherman Dice 的進一步探究
2nd runner-up	Elegantia College (Sponsored By Education Convergence)	小販煩惱的轉角問題

Award for Best Presentation

School	Title of Project
Wong Shiu Chi Secondary School	拍照的秘密－搞笑諾貝爾獎的深入探究

Teams of Outstanding Performance and Their Winning Projects

School (in alphabetic order)	Title of Project
Fanling Kau Yan College	全等三角形條件與三角形面積
St Paul's Co-educational College	The Art Gallery Problem
Tsuen Wan Government Secondary School	航行與數學的微妙關係
Wong Shiu Chi Secondary School	陰影線・三角演義
Wong Shiu Chi Secondary School	拍照的秘密－搞笑諾貝爾獎的深入探究

Teams of Good Performance and Their Winning Projects

School (in alphabetic order)	Title of Project
Christian Alliance S C Chan Memorial College	麥記我愛你
Christian Alliance S C Chan Memorial College	分？蛋？糕？
Christian Alliance S C Chan Memorial College	神秘的記數系統
CNEC Lau Wing Sang Secondary School	數讀你心
CNEC Lau Wing Sang Secondary School	幻方
CNEC Lau Wing Sang Secondary School	The Secret of Amazing Origami_info

School (in alphabetic order)	Title of Project
Diocesan Girls' School	Fated to Win
Elegantia College (Sponsored By Education Convergence)	破解密碼
Fanling Kau Yan College	紙杯的秘密
NLSI Peace Evangelical Secondary School	「圓」來自你
SKH Li Ping Secondary School	三角規
SKH Li Ping Secondary School	快速中心定位尺～可能係世上最快捷的工具
St Paul's Co-Educational College	The Mathematics of Copy and Paste
STFA Tam Pak Yu College	用數學揀靚橙
STFA Yung Yau College	密密麻麻
STFA Yung Yau College	火災危險警告訊號新指標
STFA Yung Yau College	倒錢入海
STFA Yung Yau College	平面鏡與圖形變換
Wong Shiu Chi Secondary School	New Doomsday 新末日論
Yuen Long Public Secondary School	六合彩的背後

Results of Mathematics Project Competition for Secondary Schools (2013/14)

Result	School	Title of Project
Champion	Pui Ching Middle School	中線垂直三角形的探究
1st runner-up	St Paul's Co-educational College	Park n' Space
2nd runner-up	Tsuen Wan Government Secondary School	強積金的真面目

Award for Best Presentation

School	Title of Project
St Paul's Co-educational College	Park n' Space

Teams of Outstanding Performance and Their Winning Projects

School (in alphabetic order)	Title of Project
Christian Alliance S C Chan Memorial College	繡出圓周率
St Paul's Co-educational College	The Secret to my Planned Day
Wong Shiu Chi Secondary School	望天打掛
Wong Shiu Chi Secondary School	逃出 2D

Teams of Good Performance and Their Winning Projects

School (in alphabetic order)	Title of Project
Elegantia College (Sponsored By Education Convergence)	如何製作最好的貨櫃
Fanling Kau Yan College	質數的故事
Good Hope School	Circle Packing Problem in Cookies Manufacturing
HKSYCIA Wong Tai Shan Memorial College	港鐵觀塘綫票價怎樣計?
Kiangsu-Chekiang College (Kwai Chung)	沙發可以有多大？
La Salle College	To Queue Or Not To Queue
Pui Ching Middle School	蒙提霍爾悖論
Pui Ching Middle School	關關難過過三關
Queen Elizabeth School	$6 - 4 - 0! = 1$
Queen Elizabeth School	Method for Calculating the Area of Polygon
SKH Lam Woo Memorial Secondary School	Fermat Equation
St Paul's Co-educational College	A General Formula to Find the Area of Regular Polygons
St Paul's Secondary School	Turn the time
STFA Yung Yau College	語言的難題

School (in alphabetic order)	Title of Project
STFA Yung Yau College	識還息
Tsuen Wan Government Secondary School	拿破崙三角形與幾何學微妙的關係
Wong Shiu Chi Secondary School	居心不良的餐廳總管

9. Mathematics Book Report Competition for Secondary Schools

"Reading to learn" is one of the four key tasks advocated in the curriculum reform. It helps students develop self-directed learning capabilities within and across different Key Learning Areas more readily. Reading widens students' exposure and knowledge and helps schools achieve the learning targets of the school curriculum.

In view of this, the Mathematics Education Section organised the Mathematics Book Report Competition (MBRC) for Secondary Schools since 2007. The competition aims at promoting students' interest in learning mathematics through reading. At the same time, it serves as a platform for students to share their experience in writing Mathematics book reports.

The numbers of participants in 2012/13 and 2013/14 are as follows:

School Year	No. of Schools Participated	No. of Students Participated
2012/13	63	146
2013/14	57	140

The Prize-giving Ceremony of Mathematics Project Competition and Mathematics Book Report Competition for Secondary Schools (2013/14) cum Exhibition of the Winning

Entries was held on 9 July 2014 (Wednesday) from 2:30 p.m. to 5:10 p.m. in Lecture Theatre, 4/F, West Block, EDB Kowloon Tong Education Services Centre, 19 Suffolk Road, Kowloon Tong, Kowloon. Twenty eight students and sixteen teachers from twenty schools, and ten members of the Mathematics Education Section and the Organising Committee members of MBRC participated in the Ceremony. The representative from the adjudication panel gave a brief report on the selection criteria and the features of the winning entries. The trophies and certificates were presented to the winners in the Ceremony. The reports received the First Class Prize were also exhibited at the venue of the Ceremony.

The next page shows the results of MBRC in 2012/13 and 2013/14.

Results of Mathematics Book Report Competition for Secondary Schools (2012/13)

First Class Prize Winners
(Not arranged in order of merit)

School	Participant	Title of Book Report
Diocesan Girls' School	CHENG Wai-chung	給青年數學家的信
Diocesan Girls' School	TANG Ka-yan, Tiffany	How to Cut a Cake: And Other Mathematical Conundrums
Good Hope School	CHENG Cheuk-ying	從月曆學數學 （阿草的曆史故事）
Good Hope School	HO Lok-yan	Math Wonders to Inspire Teachers and Students
Pui Ching Middle School	SI Yang	幾隻襪子湊一雙
St Paul's Convent School	KWOK Kit-wing	How to Cut a Cake: And Other Mathematical Conundrums
Tsuen Wan Government Secondary School	LEE Tsan-ho	數學恩仇錄： 數學史上的十大爭端
Wong Shiu Chi Secondary School	AU Tsz-nga	數學可以救羅馬？！20 個數學世界裡的奇妙謎題

Second Class Prize Winners
(Not arranged in order of merit)

School	Participant	Title of Book Report
Canossa College	CHAN Choi-yi	幾隻襪子湊一雙
Chinese YMCA College	YU Pui-shuen	數學奧林匹克特訓班的一年：從奧林匹克競賽看資優生特質與數學之美
Diocesan Girls' School	CHOW Yui-ji, Candice	How to Cut a Cake: And Other Mathematical Conundrums
Elegantia College (Sponsored By Education Convergence)	LIU Hoi-ling	數學恩仇錄： 數學史上的十大爭端
Elegantia College (Sponsored By Education Convergence)	TSANG Ue-ki	數學可以救羅馬？！20個數學世界裡的奇妙謎題
Fanling Rhenish Church Secondary School	AU Mei-ling	數字的祕密生命：頂尖數學家如何工作和思考的50則有趣故事
Good Hope School	CHAN Lok-yee	How to Cut a Cake: And Other Mathematical Conundrums
Heep Yunn School	LEE Hoi-ting, Athena	數學可以救羅馬？！20個數學世界裡的奇妙謎題
Heep Yunn School	LUK Hiu-ching	別讓統計圖表唬弄你

School	Participant	Title of Book Report
HKCWC Fung Yiu King Memorial Secondary School	TSOI Ho-yu	幾隻襪子湊一雙
HKCWC Fung Yiu King Memorial Secondary School	LEE Kwok-siu	數學可以救羅馬？！20 個數學世界裡的奇妙謎題
HKCWC Fung Yiu King Memorial Secondary School	NG Ka-yu	像統計學家一樣思考
Homantin Government Secondary School	CHUNG Fung-siu, Frankie	從生活學數學（阿草的數學聖杯）
Homantin Government Secondary School	LEUNG Tak-shing	數學可以救羅馬？！20 個數學世界裡的奇妙謎題
Ju Ching Chu Secondary School (Yuen Long)	LEUNG Po-man	從天文地理學數學（阿草的數學天地）
La Salle College	CHONG Chi-hin	數學恩仇錄：數學史上的十大爭端
Ling Liang Church M H Lau Secondary School	AU YEUNG Tsz-chung	給青年數學家的信
PLK Wai Yin College	CHEN Zhi-wen	幾隻襪子湊一雙
Pui Ching Middle School	LU Wen-yu	數學可以救羅馬？！20 個數學世界裡的奇妙謎題

School	Participant	Title of Book Report
Pui Ching Middle School	TONG Kin-fung	神奇數學 117
Queen Elizabeth School	HUNG Yui-chi	幾隻襪子湊一雙
Queen Elizabeth School Old Students' Association Secondary School	LAM Tsz-ki	從旅遊學數學
Sha Tin Methodist College	NG Yuen-lam	How Many Socks Make a Pair?: Surprisingly Interesting Everyday Maths
St. Clare's Girls' School	WONG Wai-kei, Winky	費馬最後定理
St Francis of Assisi's College	HUANG Yan-yu	從旅遊學數學
St Francis of Assisi's College	TSENG Man-ching	從天文地理學數學（阿草的數學天地）
St Mark's School	LIU Man-yi	當數學遇見文化
St Paul's Co-educational College	TSANG Long-kiu	幾隻襪子湊一雙
St Paul's Co-educational College	TUNG Yat-ying, Irma	The Numbers Behind NUMB3RS: Solving Crime with Mathematics

School	Participant	Title of Book Report
St Paul's Convent School	ASAI Karin	How to Cut a Cake: And Other Mathematical Conundrums
St Paul's Convent School	YIU Eunice	Math Wonders to Inspire Teachers and Students
Stewards Pooi Kei College	NG Sen-fung	陶哲軒教你聰明解數學
STFA Yung Yau College	LAM Ki-yan	從旅遊學數學
TWGH Kwok Yat Wai College	YEUNG Cheuk-yuen	數學女孩：費馬最後定理
TWGH Kwok Yat Wai College	KWOK Sin-ting	數字的祕密生命：頂尖數學家如何工作和思考的 50 則有趣故事
TWGH Sc Gaw Memorial College	CHONG Lai-ching	幾隻襪子湊一雙
TWGH Sc Gaw Memorial College	WONG Tsz-ching	數學沒什麼好怕的
Wong Shiu Chi Secondary School	CHOI Man-sze	當數學遇見文化
YWCA Hioe Tjo Yoeng College	WONG Wing-man	神奇數學 117

Results of Mathematics Book Report Competition for Secondary Schools (2013/14)

First Class Prize Winners
(Not arranged in order of merit)

School	Participant	Title of Book Report
Diocesan Girls' School	LEUNG Wing-sze, Winnie	The Housekeeper and the Professor
Diocesan Girls' School	POON Ho-kiu, Allie	The Housekeeper and the Professor
St Paul's Convent School	Ellie TO	Mathematics for the Imagination
Wong Shiu Chi Secondary School	WONG Shin-ying	生活數學故事

Second Class Prize Winners
(Not arranged in order of merit)

School	Participant	Title of Book Report
Elegantia College (Sponsored by Education Convergence)	TSANG Ue-ki	從生活學數學（阿草的數學聖杯）

School	Participant	Title of Book Report
Elegantia College (Sponsored by Education Convergence)	WONG Ka-lai	數字的祕密生命：頂尖數學家如何工作和思考的 50 則有趣故事
Heep Yunn School	Dawn CHEUNG	別讓統計圖表唬弄你
HKCWC Fung Yiu King Memorial Secondary School	WOO Ka-lam	別讓統計圖表唬弄你
Ju Ching Chu Secondary School (Yuen Long)	LEUNG Po-man	The Secret Life of Numbers: 50 Easy Pieces on How Mathematicians Work and Think
Kiangsu-Chekiang College (Kwai Chung)	LI Ka-kuen	從旅遊學數學
La Salle College	CHAN Chak-fu	The Secret Life of Numbers: 50 Easy Pieces on How Mathematicians Work and Think
PLK Celine Ho Yam Tong College	TONG Wai-leung	另類數學教室
PLK Wai Yin College	CHEN Zhiwen	從生活學數學（阿草的數學聖杯）
Po Leung Kuk Lo Kit Sing (1983) College	SHEK Hiu-ching	澡堂裡遇見阿基米德
Pui Ching Middle School	LAM Yiu-hin	別讓統計圖表唬弄你

School	Participant	Title of Book Report
Pui Ching Middle School	CHAU Oliver	從生活學數學（阿草的數學聖杯）
Pui Ching Middle School	WONG Man-hin	從生活學數學（阿草的數學聖杯）
Queen Elizabeth School	HUNG Yui-chi	數學女孩：費馬最後定理
Queen Elizabeth School	YIM Yuen-yan	數學沒什麼好怕的
Queen's College	NG Yan-lik, Samuel	How to Lie With Charts
Sha Tin Government Secondary School	TANG Wing-hin	陶哲軒教你聰明解數學
Sha Tin Government Secondary School	LEUNG Wai-chin	從生活學數學（阿草的數學聖杯）
Sha Tin Methodist College	LEUNG Leo	典雅的幾何
Sha Tin Methodist College	MAK Pik-tung	數字的祕密生命：頂尖數學家如何工作和思考的 50 則有趣故事
Sha Tin Methodist College	TSOI Hoi-lam	從旅遊學數學

School	Participant	Title of Book Report
Sheng Kung Hui St Benedict's School	TING Sum-yi	數學恩仇錄： 數學史上的十大爭端
St Paul's Co-educational College	HO Tsi-lok	澡堂裡遇見阿基米德
St Paul's Convent School	CHUNG U-qing, Qianna	How to Lie With Charts
St Teresa Secondary School	NG Hiu-wai, Vivian	從生活學數學 （阿草的數學聖杯）
Stewards Pooi Kei College	LAM Yun-ka, Kitty	How to Lie With Charts
STFA Tam Pak Yu College	CHOW Tse-yin	當數學遇見文化
STFA Yung Yau College	ZHOU Zhiyu	給青年數學家的信
Tsuen Wan Government Secondary School	CHAN Ngai-chiu	數學的語言
Tsuen Wan Government Secondary School	CHENG Kei-kei	數學的語言
TWGH Kwok Yat Wai College	KUNG Ka-ming	黃金比例： 1.61803…的祕密

School	Participant	Title of Book Report
TWGH Kwok Yat Wai College	YEUNG Ngo-hin	從生活學數學（阿草的數學聖杯）
Yan Oi Tong Tin Ka Ping Secondary School	CHAN Hiu-ching	澡堂裡遇見阿基米德
Yan Oi Tong Tin Ka Ping Secondary School	LEUNG Cham-fai	別讓統計圖表唬弄你
YWCA Hioe Tjo Yoeng College	WONG Ka-man	從生活學數學（阿草的數學聖杯）

10. Statistical Project Competition for Secondary School Students

2012/13

The 2012/13 Statistical Project Competition (SPC) for Secondary School Students, which was organised by the Hong Kong Statistical Society, co-organised by Education Bureau and sponsored by the Hang Seng Indexes Company Limited, has been completed successfully. The prize presentation ceremony was held at Function Room N201, Hong Kong Convention and Exhibition Centre (HKCEC), 1 Expo Drive, Wanchai, Hong Kong on 28 August 2013. Officiating guests for the Prize Presentation Ceremony included Mrs OU-YANG FONG Lily, Commissioner for Census and Statistics, and Mr NG Siu-kai, Permanent Education Officer of Education Bureau.

During the Ceremony, Prof K.W. NG, President of Hong Kong Statistical Society, gave the opening address. Mr Daniel WONG, Head of Research and Development and Executive Vice President and Miss Viviane HO, Assistant Vice President of the Hang Seng Indexes Company Limited, the sponsor of the Competition, gave a talk on the use of the HSI Volatility Index to participants.

The SPC has been an annual event of the Hong Kong Statistical Society and the 2012/13 SPC was its 27th round. The aim of the Competition is to promote a sense of civic awareness and encourage students to understand the local community in a scientific and objective manner through the proper use of statistics. Participants are requested to select, analyse and interpret official data on any social and economic issues in Hong Kong.

The adjudication panel comprises 32 statistics practitioners from local post-secondary institutions and the Census and Statistics Department. Professor WONG Heung of the Hong Kong Polytechnic University is the Chief Adjudicator.

The Competition was divided into two Sections, namely Junior Section for Secondary 1 to 3 students and Senior Section for Secondary 4 to 6 students. A total of 79 statistical projects from 324 students of 45 secondary schools were received this year. The projects covered a wide variety of themes, focusing on various social and economic aspects of Hong Kong. Contemporary issues studied by participating students include housing, environmental protection, medical and health services and Hong Kong's economic development, etc.

得獎習作名單 **List of Winning Projects**

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
初級組 **Junior Section**		
恒生指數有限公司初級組冠軍 Hang Seng Indexes Company Limited First Prize for the Junior Section	新山火危險警告訊號	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College
恒生指數有限公司初級組亞軍 Hang Seng Indexes Company Limited Second Prize for the Junior Section	「青」貧如洗	趙聿修紀念中學 Chiu Lut Sau Memorial Secondary School
恒生指數有限公司初級組季軍 Hang Seng Indexes Company Limited Third Prize for the Junior Section	Renewable Energy for Power Generation in Hong Kong, From A Statistical Perspective	沙田循道衛理中學 Sha Tin Methodist College

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
香港城市大學管理科學系初級組最佳統計圖表獎 Department of Management Sciences, the City University of Hong Kong Prize for the Best Graphical Presentation of Statistics for the Junior Section	Renewable Energy for Power Generation in Hong Kong, From A Statistical Perspective	沙田循道衛理中學 Sha Tin Methodist College
優異獎 Distinguished Prize	SOS! Our Youngsters need Your help	拔萃男書院 Diocesan Boys' School
優異獎 Distinguished Prize	Performance of Male and Female Students in the New Senior Secondary Curriculum	聖士提反女子中學 St. Stephen's Girls' College
優異獎 Distinguished Prize	香港人破壞環境嚴重嗎？	文理書院(香港) Cognitio College (Hong Kong)

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
恒生指數有限公司初級組冠軍 Hang Seng Indexes Company Limited First Prize for the Junior Section	新山火危險警告訊號	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College

得獎習作名單 **List of Winning Projects**

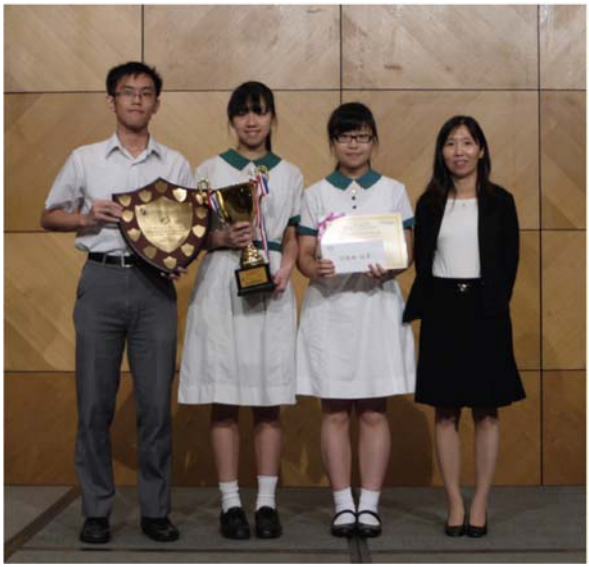
獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
高級組 **Senior Section**		
恒生指數有限公司高級組冠軍 Hang Seng Indexes Company Limited First Prize for the Senior Section	港・生活 ―香港適合居住程度評估	中華基督教會銘賢書院 The Church of Christ in China Ming Yin College
恒生指數有限公司高級組亞軍 Hang Seng Indexes Company Limited Second Prize for the Senior Section	Ageing Population in Hong Kong	中華基督教會銘賢書院 The Church of Christ in China Ming Yin College
恒生指數有限公司高級組季軍 Hang Seng Indexes Company Limited Third Prize for the Senior Section	A Statistical Study of Solid Waste Disposal in Hong Kong	德望學校 Good Hope School
香港城市大學管理科學系高級組最佳統計圖表獎	CO-LABOUR-ATION	拔萃男書院 Diocesan Boys' School

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
Department of Management Sciences, the City University of Hong Kong Prize for the best Graphical Presentation of Statistics for the Senior Section		
恒生指數有限公司高級組最佳指數應用獎 Hang Seng Indexes Company Limited Prize for the Best Index Application for the Senior Section	A Statistical Insight into the Statutory Minimum Wage in Hong Kong	聖保祿中學 St. Paul's Secondary School
優異獎 Distinguished Prize	置業夢	順利天主教中學 Shun Lee Catholic Secondary School

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
優異獎 Distinguished Prize	Story behind Ashes-Comparison between the priority of building incinerator or improving waste recovery in Hong Kong	英皇書院 King's College

(Reference: http://www.hkss.org.hk/SPC/winlist/wl-y1213.htm)

Champion of Junior Section with officiating guests



Champion of Senior Section with Vice-president of Hong Kong Statistical Society

2013/14

The 2013/14 Statistical Project Competition (SPC) for Secondary School Students, which was jointly organised by the Hong Kong Statistical Society and the Education Bureau and sponsored by the Hang Seng Indexes Company Limited, has been completed successfully. A prize presentation ceremony was held on 26 April 2014 at Cultural Activities Hall, Sha Tin Town Hall. Officiating guests for the Prize Presentation Ceremony included Mr Leslie TANG, Deputy Commissioner for Census and Statistics, and Dr Catherine CHAN, Deputy Secretary for Education.

The SPC has been an annual event of the Hong Kong Statistical Society since 1986 and the 2013/14 SPC was its 28th round. The aim of the Competition is to promote a sense of civic awareness and encourage students to understand the local community in a scientific and objective manner through the proper use of statistics. Participants are requested to select, analyse and interpret official data on any social and economic issues in Hong Kong.

The Competition was divided into two Sections, namely Junior Section for Secondary 1 to 3 students and Senior Section for Secondary 4 to 6 students. A total of 102 statistical projects

from 349 students of 37 secondary schools were received. The projects covered a wide variety of themes, focusing on various social and economic aspects of Hong Kong.

In 2013/14 SPC, participating teams in junior section were required to design and submit statistical posters for the competition instead of writing reports. More participating teams were involved in the competition. The requirements of the competition would remain unchanged for 2014/15 SPC.

得獎習作名單 **List of Winning Projects**

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
初級組 **Junior Section**		
恒生指數有限公司初級組冠軍 Hang Seng Indexes Company Limited First Prize for the Junior Section	探討 35-39 歲及 40-44 歲年齡組別女性生育趨勢	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College
恒生指數有限公司初級組亞軍 Hang Seng Indexes Company Limited Second Prize for the Junior Section	Unaffordable Housing in Hong Kong	香港神託會培基書院 Stewards Pooi Kei College
恒生指數有限公司初級組季軍 Hang Seng Indexes Company Limited Third Prize for the Junior Section	Has the recovery of Municipal Solid Waste Improved?	德望學校 Good Hope School

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
香港城市大學管理科學系初級組最佳統計圖表獎 Department of Management Sciences, the City University of Hong Kong Prize for the Best Graphical Presentation of Statistics for the Junior Section	Unaffordable Housing in Hong Kong	香港神託會培基書院 Stewards Pooi Kei College
恒生指數有限公司初級組最佳指數應用獎 Hang Seng Indexes Company Limited Prize for the Best Index Application for the Junior Section	探討 35-39 歲及 40-44 歲年齡組別女性生育趨勢	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College
優異獎 Distinguished Prize	香港變「臭」港?	天主教崇德英文書院

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
		Shung Tak Catholic English College
優異獎 Distinguished Prize	分擔? 負擔?	皇仁書院 Queen's College
優異獎 Distinguished Prize	香港棄置及回收固體廢物的情況	觀塘官立中學 Kwun Tong Government Secondary School
優異獎 Distinguished Prize	Low Birth Rate in Hong Kong and Problems it brings	德望學校 Good Hope School
優異獎 Distinguished Prize	環保? 還[保]郊野公園!	趙聿修紀念中學 Chiu Lut Sau Memorial Secondary School

得獎習作名單 **List of Winning Projects**

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
高級組 **Senior Section**		
恒生指數有限公司高級組冠軍 Hang Seng Indexes Company Limited First Prize for the Senior Section	Feasibility of the implementation of Universal Retirement Protection Scheme in Hong Kong	聖保祿中學 St. Paul's Secondary School
恒生指數有限公司高級組亞軍 Hang Seng Indexes Company Limited Second Prize for the Senior Section	Gender Disparity in Hong Kong's Education System	沙田循道衛理中學 Sha Tin Methodist College
恒生指數有限公司高級組季軍 Hang Seng Indexes Company Limited Third Prize for the Senior Section	New Beach Water Quality Index	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
香港城市大學管理科學系高級組最佳統計圖表獎 Department of Management Sciences, the City University of Hong Kong Prize for the best Graphical Presentation of Statistics for the Senior Section	Gender Disparity in Hong Kong's Education System	沙田循道衛理中學 Sha Tin Methodist College
恒生指數有限公司高級組最佳指數應用獎 Hang Seng Indexes Company Limited Prize for the Best Index Application for the Senior Section	New Beach Water Quality Index	順德聯誼總會翁祐中學 Shun Tak Fraternal Association Yung Yau College

獎項 **Prize**	習作題目 **Project Title**	學校名稱 **Name of School**
優異獎 Distinguished Prize	Parlous Pollution Perish Public Pacification	聖士提反女子中學 St. Stephen's Girls' College
優異獎 Distinguished Prize	一枝之棲	趙聿修紀念中學 Chiu Lut Sau Memorial Secondary School
優異獎 Distinguished Prize	Further Beyond Death's Veil	德望學校 Good Hope School
優異獎 Distinguished Prize	A Retrospective Analysis – How Did Socio-Economic Events and Government Policies Affect Housing Affordability?	英皇書院 King's College

(Reference: http://www.hkss.org.hk/SPC/winlist/wl-y1314.htm)

Champion of the Junior Section with officiating guests

Champion of the Senior Section with officiating guests

11. Statistics Creative-Writing Competition for Secondary School Students

2012/13

Statistics Creative-Writing Competition for Secondary School Students has been held every year since 2009. The Competition was in its 4th year. The aims of the Competition are to raise the interest of students in statistics and its application; and to encourage them to creatively express in words the daily application of statistical concepts or put statistical concepts into a story in a scientific and objective manner.

The 2012/13 Statistics Creative-Writing Competition for Secondary School Students (SCC) were co-organised by the Hong Kong Statistical Society and Education Bureau, and sponsored by the Department of Statistics and Actuarial Science, the University of Hong Kong.

The 2012/13 Statistics Creative-writing Competition for Secondary School Students, which commenced in October 2012, attracted 121 participating students from 23 schools with a total of 47 submissions. After a series of adjudication and interviews, 8 awarded entries were eventually selected. The prize presentation ceremony was held at Function Room N201, Hong

Kong Convention and Exhibition Centre (HKCEC), 1 Expo Drive, Wanchai, Hong Kong on 28 August 2013.

We were honoured to invite Mr Leslie TANG Wai-kong, Deputy Commissioner for Census and Statistics, Mr LEE Sha-lun, Sheridan, Principal Education Officer, Education Bureau and Ms Annie CHAN Yuen-wai, Vice-President of Hong Kong Statistical Society, as the guests of the Ceremony. The teams winning the first three prizes and Department of Statistics & Actuarial Science, The University of Hong Kong Prize for the Best Thematic Writing: "Outliers" were invited to give presentations on their creative writings/stories and to share the joys with us. All award-winning entries would be printed in the publication issued by the organisers.

得獎習作名單 List of Winning Projects

獎項 Prize	習作題目 Project Title	學校名稱 Name of School
冠軍 Champion	Mysteries in Ecology - An Astonishing Truth or just a Game of Numbers?	筲箕灣官立中學 Shau Kei Wan Government Secondary School
亞軍 First runner-up	如何運用統計學種神奇豌豆	荃灣官立中學 Tsuen Wan Government Secondary School
季軍 Second runner -up	得籃板，得天下？	順德聯誼總會李兆基中學 STFA Lee Shau Kee College
優異獎 Distinguished Prize	Detective • Banker	德望學校 Good Hope School
優異獎 Distinguished Prize	估市？股市 – 從過去估計將來	香港道教聯合會圓玄學院第三中學 HKTA The Yuen Yuen Institute No.3 Secondary School

獎項 Prize	習作題目 Project Title	學校名稱 Name of School
優異獎 Distinguished Prize	移宮換羽	循道中學 Methodist College
優異獎 Distinguished Prize	NBA 勝利因素	聖公會呂明才中學 SKH Lui Ming Choi Secondary School
優異獎 Distinguished Prize	一血定生死	香港神託會培基書院 Stewards Pooi Kei College
香港大學統計及精算學系最佳專題寫作獎 Department of Statistics & Actuarial Science, The University of Hong Kong Prize for the Best Thematic Writing	如何運用統計學種神奇豌豆	荃灣官立中學 Tsuen Wan Government Secondary School

(Reference: http://www.hkss.org.hk/SCC/WinList/SCCwinEntry.htm)

Winning teams with officiating guests

Presentation of the Champion

2013/14

Statistics Creative-Writing Competition for Secondary School Students has been held every year since 2009. The Competition was in its 5th year. The aims of the Competition are to raise the interest of students in statistics and its application; encourage them to creatively express in words the daily application of statistical concepts or put statistical concepts into a story in a scientific and objective manner.

The 2013/14 Statistics Creative-Writing Competition (SCC) for Secondary School Students, which was jointly organised by the Hong Kong Statistical Society and the Education Bureau and sponsored by the Department of Statistics & Actuarial Science, The University of Hong Kong, has been completed successfully. A prize presentation ceremony was held on 28 June 2014 at the auditorium of Christian Family Service Centre in Kwun Tong. Officiating guests for the Prize Presentation Ceremony included Mr Leslie TANG, Commissioner for Census and Statistics, and Mr Stephen YIP Yam-wing, Principal Assistant Secretary (Curriculum Development).

The 2013/14 Statistics Creative-writing Competition for Secondary School Students, which commenced in October 2013, attracted 115 participating students from 21 schools with a total of 53 submissions. To attract more students participate in the competition, the prize of "selected entries" was arranged in this year. The articles were innovative and interesting. We are gratified to note that many students participating in the Competition could effectively apply what they have learnt about statistics and probability.

得獎習作名單 List of Winning Projects

獎項 Prize	習作題目 Project Title	學校名稱 Name of School
冠軍 Champion	跆拳武鬥會，人多好辦事？	宣道會鄭榮之中學 Christian Alliance Cheng Wing Gee College
亞軍 First runner up	缺值生的分數捍衛戰	香港神託會培基書院 Stewards Pooi Kei College
季軍 Second runner up	赤壁前傳	順利天主教中學 Shun Lee Catholic Secondary School
優異獎 Distinguished prize	別讓港鐵的複雜數字欺騙到你 ── 票價研究	香港培正中學 Pui Ching Middle School
優異獎 Distinguished prize	Please help me to find the quantity!	德望學校 Good Hope School
優異獎 Distinguished prize	假波風雲	東華三院吳祥川紀念中學 TWGHS S.C. Gaw Memorial College

獎項 Prize	習作題目 Project Title	學校名稱 Name of School
優異獎 Distinguished prize	Data Missing - Health Missing - Job Missing	聖保羅男女中學 St. Paul's Co-educational College
優異獎 Distinguished prize	換樂無窮	香港四邑商工總會黃棣珊紀念中學 HKSYC&IA Wong Tai Shan Memorial College
香港大學統計及精算學系 最佳專題寫作獎 Department of Statistics & Actuarial Science, The University of Hong Kong Prize for the Best Thematic Writing	缺值生的分數捍衛戰	香港神託會培基書院 Stewards Pooi Kei College
入選作品 Selected entry	學以致用	福建中學 (小西灣) Fukien Secondary School (Siu Sai Wan)

獎項 Prize	習作題目 Project Title	學校名稱 Name of School
入選作品 Selected entry	Curious Incidents of Wendy	德望學校 Good Hope School
入選作品 Selected entry	Inspector Goole and the Missing Data	德望學校 Good Hope School
入選作品 Selected entry	The Banana Inc. Incident	德望學校 Good Hope School
入選作品 Selected entry	The Mystery of Album Sales	德望學校 Good Hope School
入選作品 Selected entry	The Hidden Truth one false move must lose the game?	筲箕灣官立中學 Shau Kei Wan Government Secondary School
入選作品 Selected entry	補習的盲點	香港真光中學 True Light Middle School of Hong Kong

(Reference: http://www.hkss.org.hk/SCC/WinList/SCCwinEntry.htm)

All the winning teams with the officiating guests

Presentation of winning team for Department of Statistics & Actuarial Science, The University of Hong Kong Prize for the Best Thematic Writing

12. Learning and Teaching Resources for Secondary Mathematics

With the greater emphasis on inquiry learning in Mathematics Education, teachers have to make use of a wide range of learning resources: textbooks; concrete materials such as geometric models; audio visual materials; written resources such as journals; and IT resources such as dynamic geometry software and computer algebra systems.

Education Bureau hence has produced a **Learning and Teaching Resources for Secondary Mathematics (as at Mar 2014)** and uploaded to the website for teachers' reference. The web-link is as follows:

http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/js/ltresources%20(e).pdf

The table contains learning and teaching resources and reference materials prepared by Education Bureau and Hong Kong Examinations and Assessment Authority. Teachers can efficiently get the learning and teaching resources of secondary mathematics by the hyperlink in the table.

Extracts of the resource table is listed below:

Learning and Teaching Resources for Secondary Mathematics
(as at Mar 2014)

	Learning and Teaching Resources	Categories	Brief Descriptions & Internet Links
1.	Mathematics Curriculum Guide (P1-P6) Year of publication: 2000	Curriculum	Current Mathematics curriculum for primary education http://www.edb.gov.hk/en/curriculum-development/kla/ma/curr/pri-math-2000.html
2.	Syllabus for Secondary Schools: Mathematics (Secondary 1-5) Year of publication: 1999	Curriculum	Current Mathematics curriculum for junior secondary education http://www.edb.gov.hk/en/curriculum-development/kla/ma/curr/sec-math-1999.html
3.	The New Senior Secondary Mathematics Curriculum and Assessment Guide (Secondary 4 - 6) Year of publication: 2007	Curriculum	Current Mathematics curriculum for senior secondary education http://www.edb.gov.hk/en/curriculum-development/kla/ma/curr/ss-math-2007.html
4.	Explanatory Notes to New Senior Secondary Mathematics Curriculum – Compulsory Part Year of publication: 2009	Explanatory notes of curriculum	The explanatory notes in the booklet further explicate the requirements of the learning objectives of the Compulsory Part (CP), the strategies suggested for the teaching of the CP, the connections and structures among different learning units of the CP, the context of development from different key stages, such as Key Stage 3, to the CP, and the curriculum articulation between the CP and the Extended Part. http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/explancp_e.pdf

13. Teachers' Professional Development Programmes in 2012/13 and 2013/14

Mathematics Education Section conducted a series of teachers' Professional Development Programmes for (a) primary school teachers and (b) secondary school teachers in 2012/13 and 2013/14.

The following yearly training calendars show information of the planned courses for the last two years. For details of courses, please refer to the Training Calendar System (TCS).

Apart from that, the Mathematics Education Section will also conduct some ad-hoc seminars and workshops for the learning and teaching of Mathematics; teachers may refer to the circulars for details, browse the related webpage and enroll for seminars and workshops.

Lists of Teachers' Professional Development Programmes 2012/13

(a) Primary

Month	Title
October 2012	Primary Mathematics Curriculum Induction Series - (1) New Mathematics Teachers (Re-run)
November 2012	Primary Mathematics Curriculum Induction Series - (2) Mathematics Panel Chairpersons (Re-run)
	Primary Mathematics Curriculum Interface Series - (2) The Primary and Secondary Levels – Selected Topics (New)
December 2012	Primary Mathematics Curriculum Assessment for Learning Series - (1) Diversified Modes of Assessment (Re-freshed)
	Primary Mathematics Curriculum Learning and Teaching Series - (1) Interpreting the Primary Mathematics Curriculum and Designing the School-based Curriculum (Re-run)
January 2013	Primary Mathematics Curriculum Learning and Teaching Series - (2) Catering for Learning Diversity (Re-run)

Month	Title
	Primary Mathematics Curriculum Selected Topics Series - (1) Money, Time (Re-run)
February 2013	Primary Mathematics Curriculum Learning and Teaching Series - (3) Use of e-Learning & e-Resources to Enhance the Learning and Teaching of Primary Mathematics (New)
	Primary Mathematics Curriculum Selected Topics Series - (2) Four Operations and Algebra (Re-run)
March 2013	Primary Mathematics Curriculum Selected Topics Series - (3) Fractions (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series - (4) Solving Word Problems (Re-run)
April 2013	Primary Mathematics Curriculum Enriching Knowledge Series - (1) Inquiry on “Numbers” – Multiples and Factors (Re-run)
	Primary Mathematics Curriculum Enriching Knowledge Series - (2) Inquiry on "Measures" – Area (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series - (5) Reading to Learn and Moral, Civic and National Education

Month	Title
	(Re-freshed)
May 2013	Primary Mathematics Curriculum Learning and Teaching Series - (6) Nurturing Students' Creativity and Critical Thinking (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series - (7) Effective Use of Quality Learning and Teaching Resources in Primary Mathematics (New)
	Primary Mathematics Curriculum Interface Series - (1) The Primary and Pre-primary Levels (Re-run)
	Primary Mathematics Curriculum Assessment for Learning Series - (2) Making Use of Assessment Data to Enhance the Learning and Teaching of Mathematics (Re-run)
June 2013	Primary Mathematics Curriculum Interface Series - (3) The Primary and Secondary Levels – 2-D Shapes and Fractions (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series - (8) Nurturing Students' Number Sense (Re-run)

(b) Secondary

Month	Title
October 2012	Induction on the Learning and Teaching of Mathematics for New Teachers (Re-run)
	Induction Course for New Teachers of Senior Secondary Mathematics (New)
November 2012	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (2) Further Applications (Re-run)
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (1) Applications of Mathematics (Refreshed)
	Interface between Primary and Secondary Mathematics on Area and Algebra (New)
	Curriculum Leadership for Mathematics Education (Re-run)
	Understanding and Interpreting the Senior Secondary Mathematics Curriculum (Refreshed)
	15th Anniversary of the Establishment of the HKSAR - Reflections of some statistical indicators
December 2012	Assessing Student Learning for the Mathematics Curriculum: Diversified Modes of Assessment in KS3 (Re-run)

Month	Title
	Smart application of statistics!
	Enriching Knowledge for the Mathematics Curriculum Series: (1) Mathematics Projects for Secondary Schools (Re-run)
	Enriching Knowledge for the Mathematics Curriculum Series: (2) Geometric Construction (Re-run)
January 2013	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (4) Module 1 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (5) Module 2 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (6) Inquiry and Investigation (Re-run)
February 2013	Assessing Student Learning for the Senior Secondary Mathematics Curriculum (Refreshed)
	Learning and Teaching Strategies for 3D Figures in Key Stage 3 (Re-run)
	Use of Language for Learning and Teaching Mathematics (Re-run)

Month	Title
March 2013	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (1) Applications of Mathematics (Refreshed)
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (2) Curve Sketching (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (2) Promoting Creativity (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (4) Gifted Education in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (8) Interface between Junior and Senior Secondary Mathematics (Re-run)
April 2013	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (3) Use of IT (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (4) Gifted Education in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (6) Promoting Critical Thinking Skills (Re-run)

Month	Title
	Learning and Teaching Strategies for Mathematics Curriculum Series: (3) Remedial Teaching in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (6) Promoting Critical Thinking Skills (Re-run)
	Effective Use of Resources for e-learning in Mathematics (New)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (9) Measure, Shape and Space Strand in Compulsory Part (New)
May 2013	Learning and Teaching Strategies for Mathematics Curriculum Series: (5) Promoting Moral, Civic and National Education (Re-run)
	Assessing Student Learning for the Senior Secondary Mathematics Curriculum (Refreshed)
	Subject Knowledge Required for Primary and Secondary Mathematics Teachers
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (3) History of Mathematics (Re-run)

Month	Title
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (1) Data Handling in Compulsory Part (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (10) Learning and Teaching of Permutation and Combination (Refreshed)
	Use of Transformations in Geometric Proofs, Part 1 (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (1) Reading to Learn (Re-run)
	Use of Transformations in Geometric Proofs, Part 2 (Re-run)
	Learning and Teaching of Geometry in Key Stage 3 (New)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (7) Selected topics (Re-run)
	Learning and Teaching of Geometry in Key Stage 3 (New)
	Learning and Teaching of Geometry in Key Stage 3 (New)
June 2013	Use of Language for Learning and Teaching Mathematics (Re-run)

Month	Title
	Learning and Teaching Strategies for Selected Topics in Key Stage 3 (Re-run)
	Interface between Primary and Secondary Mathematics on 2D shapes and Fractions (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (10) Learning and Teaching of Permutation and Combination (Refreshed)
	Use of IT in the Preparation of Mathematics Examination Papers (Re-run)

2013/14

(a) Primary

Month	Title
October 2013	Primary Mathematics Curriculum Induction Series : (1) New Mathematics Teachers (Re-run)
November 2013	Primary Mathematics Curriculum Induction Series : (2) Mathematics Panel Chairpersons (Re-run)

Month	Title
November 2013	Primary Mathematics Curriculum Interface Series : (2) The Primary and Secondary Levels – Area and Algebra (Re-run)
December 2013	Primary Mathematics Curriculum Assessment for Learning Series : (1) Diversified Modes of Assessment (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series : (1) Catering for Learning Diversity (Re-run)
January 2014	Primary Mathematics Curriculum Learning and Teaching Series : (2) Interpreting the Primary Mathematics Curriculum and Designing the School-based Curriculum (Re-run)
	Primary Mathematics Curriculum Selected Topics Series : (1) Money, Time (Re-run)
February 2014	Primary Mathematics Curriculum Assessment for Learning Series : (2) Making Use of Assessment Data to Enhance the Learning and Teaching of Mathematics (Refreshed)
	Primary Mathematics Curriculum Learning and Teaching Series : (3) Use of e-Learning to Enhance the Learning and Teaching of Primary Mathematics (Refreshed)

Month	Title
February 2014	Primary Mathematics Curriculum Learning and Teaching Series : (4) Nurturing Students' Number Sense (Re-run)
March 2014	Primary Mathematics Curriculum Learning and Teaching Series : (5) Nurturing Students' Creativity and Critical Thinking (Re-run)
	Primary Mathematics Curriculum Selected Topics Series : (2) Fractions (Re-run)
April 2014	Primary Mathematics Curriculum Enriching Knowledge Series : (1) Inquiry on "Numbers" – Multiples and Factors (Re-run)
	Primary Mathematics Curriculum Enriching Knowledge Series : (2) Inquiry on "Measures" – Area (Re-run)
May 2014	Primary Mathematics Curriculum Learning and Teaching Series : (6) Reading to Learn and Moral and Civic Education (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series : (7) Solving Word Problems (Re-run)
	Primary Mathematics Curriculum Learning and Teaching Series : (8) Effective Use of Quality Learning and Teaching Resources in Primary Mathematics (Re-run)

Month	Title
June 2014	Primary Mathematics Curriculum Interface Series : (1) The Primary and Pre-primary Levels (Re-run)
	Primary Mathematics Curriculum Interface Series : (3) The Primary and Secondary Levels – 2-D Shapes and Fractions (Re-run)
	Primary Mathematics Curriculum Selected Topics Series : (3) Percentage and Algebra (Refreshed)

(b) Secondary

Month	Title
October 2013	Enriching Knowledge for the Mathematics Curriculum Series: (2) Statistical Projects for Secondary Schools (Re-run)
	Enriching Knowledge for the Mathematics Curriculum Series: (3) Statistics Creative Writings for Secondary Schools (Re-run)
	Enriching Knowledge for the Mathematics Curriculum Series: (6) Geometric Construction (Re-run)
	Induction Course for Senior Secondary Mathematics Teachers (Re-run)

Month	Title
October 2013	Induction on the Learning and Teaching of Mathematics for New Teachers (Re-run)
	Use of IT in the Preparation of Mathematics Examination Papers (Re-run)
November 2013	Curriculum Leadership and Curriculum Planning for Mathematics Teachers (Re-run)
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (1) Applications of Mathematics (Refreshed)
	Interface between Primary and Secondary Mathematics on Areas and Algebra (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (1) Reading to Learn (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (5) Promoting Moral and Civic Education (Re-run)
	Understanding and Interpreting the Senior Secondary Mathematics Curriculum (Re-run)
December 2013	Assessing Student Learning for the Mathematics Curriculum: Diversified Modes of Assessment in KS3 (Re-run)
	Enriching Knowledge for the Mathematics Curriculum Series: (1) Mathematics Projects for Secondary Schools (Re-run)

Month	Title
January 2014	Learning and Teaching of Geometry in KS3 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (10) Learning and Teaching of Permutation and Combination (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (5) Module 1 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (6) Module 2 (Re-run)
February 2014	Assessing Student Learning for the Senior Secondary Mathematics Curriculum (Re-run)
	Inquiry, Investigation and Problem Solving in Secondary Mathematics (New)
	Learning and Teaching Strategies for 3D Figures in Key Stage 3 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (1) Learning and Teaching of Probability (New)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (2) Use of Geogebra (Introductory Level) (Refreshed)

Month	Title
February 2014	Use of Language for Learning and Teaching Mathematics (Re-run)
March 2014	Effective use of resources for e-learning in Mathematics (Re-run)
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (1) Applications of Mathematics (Refreshed)
	Learning and Teaching of Geometry in KS3 (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (2) Promoting Creativity (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (4) Gifted Education in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (2) Use of Geogebra (Introductory Level) (Refreshed)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (4) Use of IT (Refreshed)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (8) Interface between Junior Secondary and Senior

Month	Title
	Secondary Mathematics (Re-run)
April 2014	Effective use of resources for e-learning in Mathematics (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (3) Remedial Teaching in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (4) Gifted Education in Secondary Mathematics (Re-run)
	Learning and Teaching Strategies for Mathematics Curriculum Series: (6) Promoting Critical Thinking Skills (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (2) Use of Geogebra (Introductory Level) (Refreshed)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (3) Use of Geogebra (Advanced Level) (Refreshed)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (6) Module 2 (Re-run)
	Use of IT in the Preparation of Mathematics

Month	Title
	Examination Papers (Re-run)
May 2014	Enriching Knowledge for the Mathematics Curriculum Series: (4) History of Mathematics (New)
	Enriching Knowledge for the Mathematics Curriculum Series: (5) Number Systems (New)
	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (2) Curve Sketching (Re-run)
	Learning and Teaching of Geometry in KS3 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (3) Use of Geogebra (Advanced Level) (Refreshed)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (5) Module 1 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (7) Selected topics (Re-run)
June 2014	Enriching Knowledge for the Senior Secondary Mathematics Curriculum Series: (2) Curve Sketching (Re-run)

Month	Title
June 2014	Interface between Primary and Secondary Mathematics on 2D Shapes and Fractions (Re-run)
	Learning and Teaching Strategies for Selected Topics in Key Stage 3 (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (10) Learning and Teaching of Permutation and Combination (Re-run)
	Learning and Teaching Strategies for the Senior Secondary Mathematics Curriculum Series: (9) Measures, Shape and Space Strand in Compulsory Part (Re-run)
	Use of Language for Learning and Teaching Mathematics (Re-run)