Paradoxes

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1. What are paradoxes?

- A scenario that involves an **argument** that
  - begins with premises that seem to be true
  - proceeds with reasoning that seems to be valid
  - arrives at a conclusion that is unacceptable (such as a falsehood, a contradiction, or an absurdity)

  (See, for example, Cook (2013))
- Paradoxes vs fallacies (sophisms) vs mistakes

2. Some introductory examples of paradox

- **Pinocchio paradox**

  Pinocchio says, "My nose will grow now".
- **Barber paradox**

  There is a village in which there is a *barber* named "Bertrand". Bertrand shaves all the men in the village who do not shave themselves, And Bertrand shaves none of the men in the village who do shave themselves. Does Bertrand shave Bertrand, or not?
- **Liar paradox**

  This sentence is false.
- **Strengthened liar paradox**

  This sentence is *not true*.

  (See, for example, Rieger (2001))
- **Multi-sentence liar paradox**

  :   (A): Sentence (B) is true

  (B): Sentence (A) is false
- **Yablo paradox**

  (A_1): For all $m > 1$, (A_m) is false

  (A_2): For all $m > 2$, (A_m) is false

     …

     …

  (A_n): For all $m > n$, (A_m) is false

     …

  (See, for example, Yablo (1985); Cook (2014))

3. Some paradoxes related to mathematics

- Five proofs of $1 = 2$
- Two equations whose numbers of roots seem to be unacceptable
- Centre of gravity paradox
- Area of a circle and area of a sphere
- Sorites paradox, surprise examination paradox and a paradox related to MI
- Unfair subway paradox
- Two-envelopes paradox
- Doomsday argument
- Hilbert's hotel paradox

4. Are Gödel incompleteness theorems paradoxical?

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