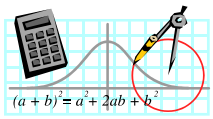


Exemplar 1:
Basic Ideas of Functions (1)

Objectives : 1. Relate the concept of 'input-process-output' on number patterns learnt at junior secondary levels to the meaning of dependent and independent variables
2. Understand the basic ideas of function from the tabular, symbolic and graphical representations of a function

Key Stage : 4

Learning Unit : Functions and Graphs

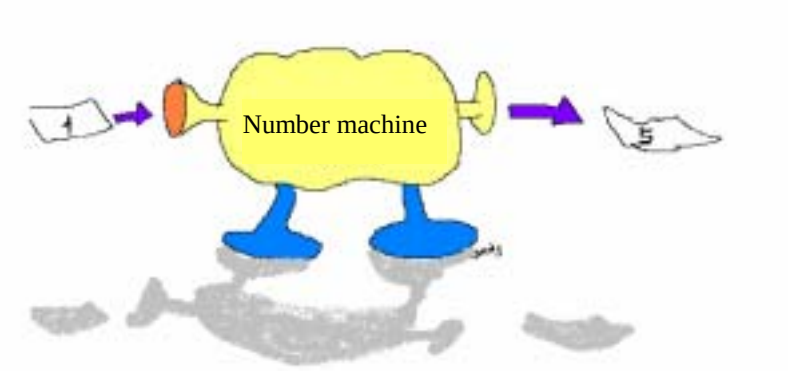
Materials required : Worksheets

Prerequisite Knowledge :
1. The meaning of the term of a sequence and the term number
2. Locate a point on the Cartesian coordinate plane from given coordinates
3. Understand algebraic languages and understand the method of substitution

Description of the activity:

1. The teacher may revise with students the concept of sequence. The teacher and asks students to observe the pattern of the sequence shown in Question 1 on Worksheet and to complete Question 2.

2. The teacher discusses with students the answers of Questions 2(a) and 2(b) and asks them their problem-solving strategies and how they get their answers. Hence, the relation between "the number of bricks" and "the figure number" is obtained. The teacher may then use a 'number machine' to introduce the terms of independent and dependent variables.



3. After completing Question 3 of the Worksheet, the teacher discusses the answers with students. From the graph, students can observe that each value of x corresponds to a unique value of y. The concept of a function is introduced and the concept "the values of independent variable determine the values of dependent variable" is discussed. Moreover, the teacher may ask students to determine whether "the number of bricks" is the function of "the figure number". Students are asked to use an algebraic expression to represent the relation between "the number of bricks" and "the figure number".

4. The teacher introduces the symbol '$f(x)$' and the meanings of $f(1)$ and $f(2)$, etc. Students are asked to use these symbols to verify these values with those listed in Table 1.

5. The teacher emphasizes the following two points to students:
   (a) The letter x represents a variable. It is neither a constant nor an unknown;
   (b) We may use variables other than x to represent the input value. For example, we may use z to represent the input value and the relation between "the number of bricks" and "the figure number" becomes $f(z) = 4z + 1$.

6. The teacher should remind students that the input values of number sequences at the junior secondary levels and that in this exemplar are natural numbers. However, in general, the independent variables of a function are not confined to natural numbers. For secondary levels, the domain of a function is always the real numbers. Meanwhile, the teacher should point out that the graph in Question 3 is 8 discrete points and not a straight line. Nevertheless, if the input values are extended to real numbers, the graph of $f(x) = 4x + 1$ will be a continuous straight line.

Worksheet: Basic ideas of Functions

1. A worker lays bricks according to the pattern below.

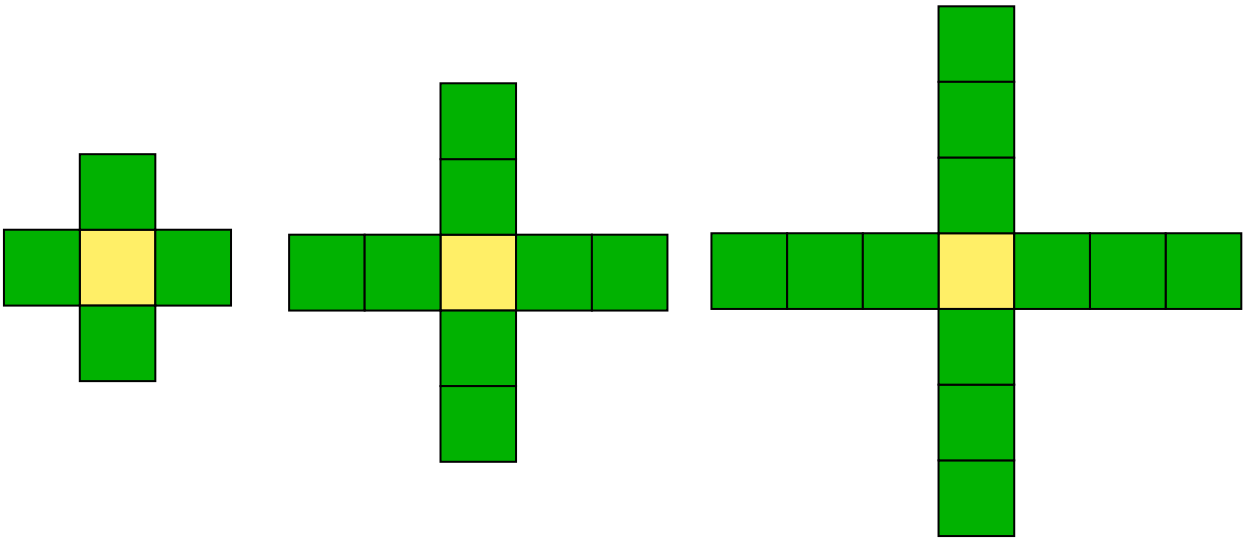


Figure 1 Figure 2 Figure 3

2. (a) According to the above pattern, complete the following table.

Figure No.	No. of brick(s)	Figure No.	No. of brick(s)
1	5	5	
2		6	
3		7	
4		8	

Table 1

(b) How many pieces of brick are required to construct Figure 12?

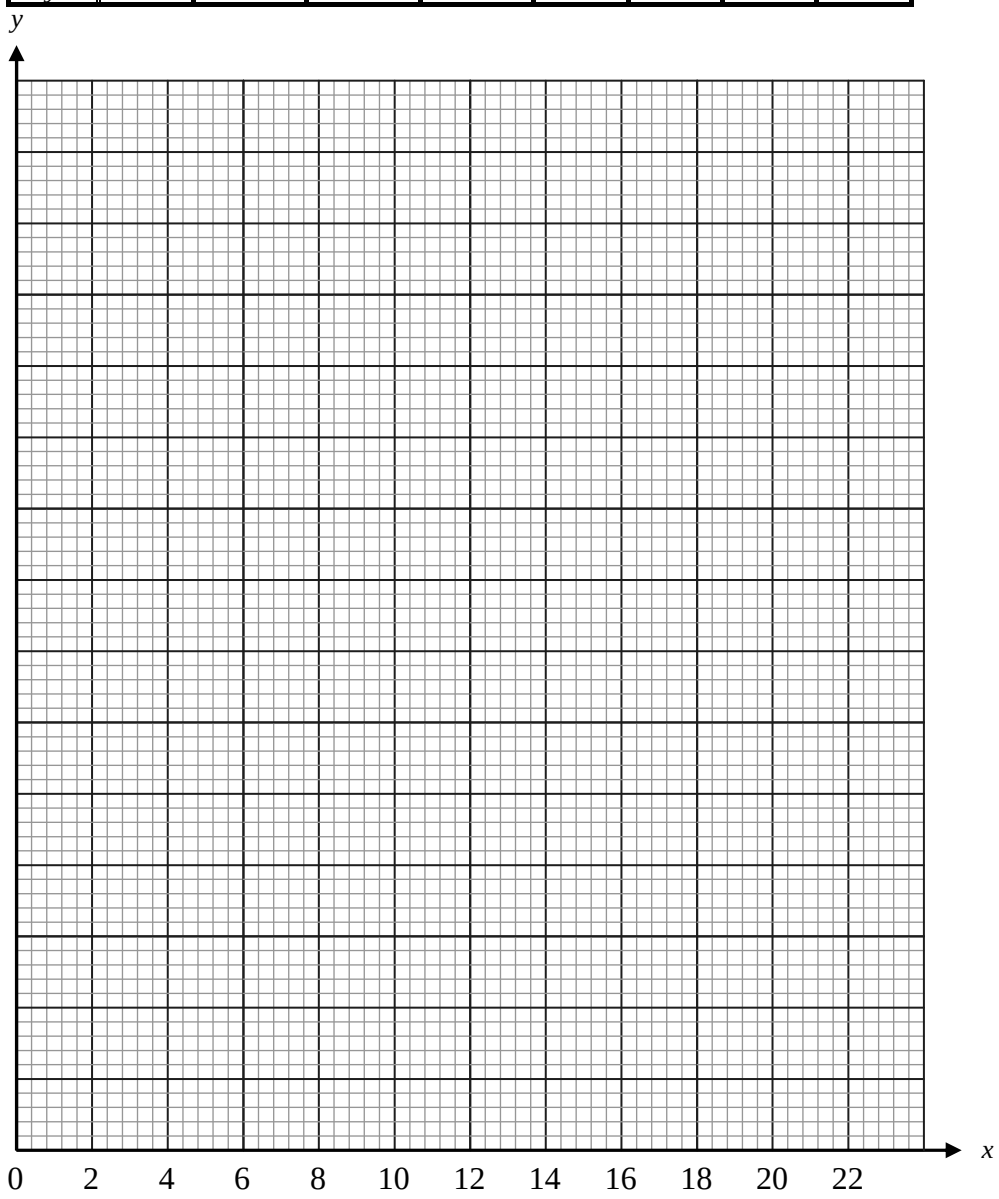
_____

(c) How many pieces of brick are required to construct Figure *n*?

_____

3. Let x be the figure number and y be the number of bricks. Using the data in table 1, plot the graph of y against x in the rectangular coordinate plane below.

x	1	2	3	4	5	6	7	8
y								



(a) Describe the feature of the above graph.

(b) From the trends of the graph, guess the corresponding values of y when $x = 12$ and $x = 16$ graphically.

(c) From the graph, will every value of x give exactly one and only one corresponding value of y?

Notes for teacher:

1. The time required for the activities of this exemplar is approximately 20 to 30 minutes.

2. When the teacher introduces the Chinese translation of the term ‘function’, he/she may introduce the Chinese mathematician “Li Shanlan”. More information of the background of Li can be referred to the following websites:

   http://www.dyu.edu.tw/~mfht206/history/19/china.htm

   For details, the teacher may read the Chinese version of this exemplar.

3. This exemplar considers the basic idea of sequence learnt at junior secondary levels as a starting point to develop the idea that output values are determined by input values. The teacher consolidates the concept through the tabular (Q2a), symbolic (Q2c) and graphical representation of function (Q3). More discussions on different representations of a function can be found in exemplar 4.

4. In discussing the concept of function, the teacher may introduce daily life examples such as soft drink vending machines. These exemplars can be found in Exemplars 2 and 3.

5. The suggested answers for the worksheet are as below:

2. (a)

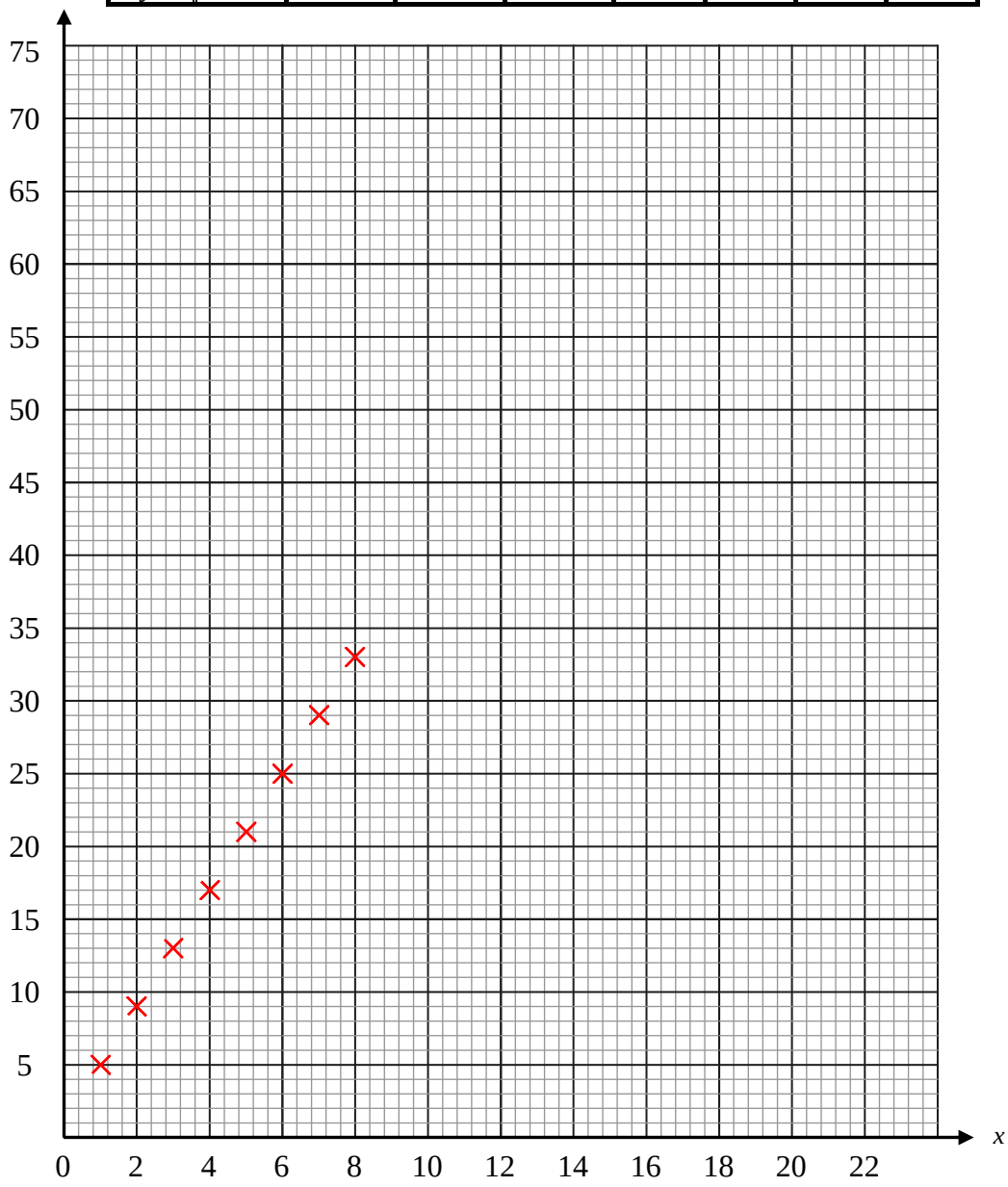
Figure No.	No. of brick(s)	Figure No.	No. of brick(s)
1	5	5	21
2	9	6	25
3	13	7	29
4	17	8	33

(b) 49

(c) $4n+1$

3.

x	1	2	3	4	5	6	7	8
y	5	9	13	17	21	25	29	33



(a) The 8 points lie on a straight line with the y-intercept 1.

(b) When $x = 12$, $y = 49$; and when $x = 16$, $y = 65$.

(c) yes。

Reference book:

Kleiner, I. (1989). Evolution of the function concept. *The college Mathematics Journal*, *20*(4), 282-300.

Markovits, Z., Eylon, B., & Bruckheimer, M. (1986). Functions today and yesterday. *For the Learning of Mathematics*, *6*(2), 18-28.

劉福增（2003）.《邏輯思考》. 台北：心理出版社。