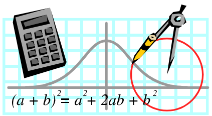



# Exemplar 2:
# Basic Ideas of Functions (2)

**Objectives** : 1. Relate the idea of Input-Process-Output to the meaning of dependent and independent variables
2. Understand the basic ideas of function from given examples and counter-examples
3. From the tabular and algebraic representations of function to understand the basic ideas of function and the concept of dummy variable of function

**Key Stage** : 4

**Learning Unit** : Functions and Graphs

**Materials required** : Worksheets and computers accessible to the Internet

**Prerequisite Knowledge** : 1. Use symbols to represent numbers
2. Understand the language of algebra and be able to use the method of substitution

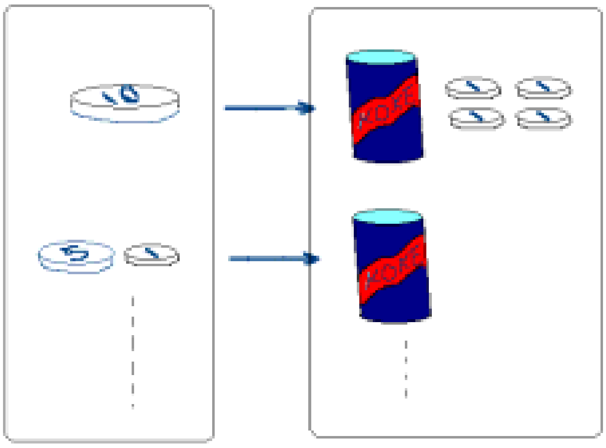
**Description of the activity:**

1.(a) The teacher uses daily life examples[2] to discuss the idea of a function on "independent variables determine the values of dependent variables" or through the idea of "Input-Process-Output" to bring out the idea "input determines the output". Nevertheless, it is not a must to introduce the terms 'dependent variable' or 'independent variable'.

Example 1－Soft Drink Vending Machine (Suppose the vending machines sell only one type of soft drink costing \$6 and it accepts only coins with denominations greater than or equal to \$1 and the change are \$1 coins.)

Examples are used to explain that, when 'the inserted coins' are decided, 'the outcomes' are fixed. For example, when a \$10 coin is inserted, a can of soft drink and four \$1 coins come out. Also, when a \$5 coin and a \$2 coin are inserted, a can of soft drink and a \$1 coin come out.

The teacher uses figures to demonstrate the relationship between the input and the output. For example,



(b) The teacher uses the meaning of the word 'function'(3) (a special activity of a thing) to explain the idea of "Input-Process-Output". This special activity makes sure that no different outputs will come out with the same input. In this regard, as the 'outcome' of the vending machine is completely determined by the coins inserted, 'the outcomes' of the vending machine may be said to be a function of 'the coins inserted'. If $f$ is used to represent the function, the above examples can be expressed as:

$f$(a \$10 coin) = a can of soft drink and four \$1 coins

$f$(a \$5 coin and a \$1 coin) = a can of soft drink

Problems for discussions include:

(i) What are the outcomes when a \$5 and a \$2 coins are inserted? Use the representation of function to represent the above relationship.

(ii) Should only a \$10 coin be inserted to give the outcome of a can of soft and four \$1 coins?

(iii) In real life situations, is 'the outcome' of a soft drink vending machine(4) (for example: the vending machine in schools) a function of 'the inserted coins'?

(c) Example 2－Date and Day of the Week

The data (day, month and year) of an event can determine the day of the week of the event. In other words, the day of the week is a function of the date.

Problems for discussions include:

(i) Is the date a function of the day of the week? Why?

(ii) Is the day of the week of an event determined if the month and the day without the year of the event are given?

(d) Example 3－The Name and the Class Number of a Student

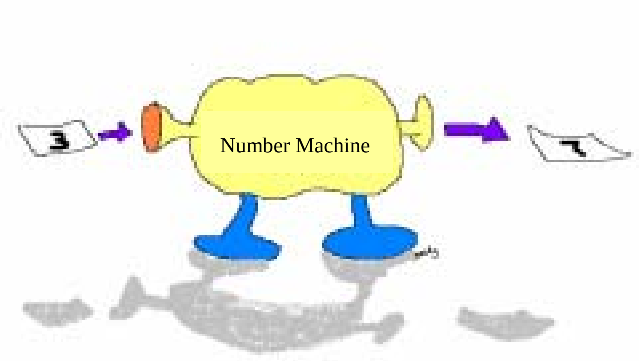
In a class, although there are students having identical names, all students may be specified only by their class numbers. The name of a student is a function of the class number.

Problems for discussions include:

(i) Is the class number a function of the name of a student? Why?

(ii) Is a person a function of a name? Why?

2. The teacher introduces the idea of the number machine (related figures can be shown on the blackboard) and explains the basic operation of the machine. When a paper with a number written on it is input into the machine, a paper with a number printed will be output and this number is totally determined by the input number(5). In other words, when the numbers input are identical, the numbers output will not be different. The teacher may use examples (e.g. the number machine with the operation "output number = input number + 4") to explain this special activity and ask students to record the input and output in a table.




The teacher distributes Worksheet to students and asks them to complete Questions 1 and 2 by themselves.

3. The teacher discusses with students the answers(6) of Questions 1 and 2, algebraic representation of the special activity of number machine 1 and the relation between the input and output values. For example, for number machine 1, if we use $x$ to represent the input value, the output value is $x + 5$. On the other

hand, if $f$ is used to represent the machine, the output value is $f(x)$. Therefore,

$$f(x) = x + 5.$$

Teachers should emphasize two points:

(a) Since the input is not restricted to a fixed number, $x$ represents a variable but not a fixed number or an unknown;

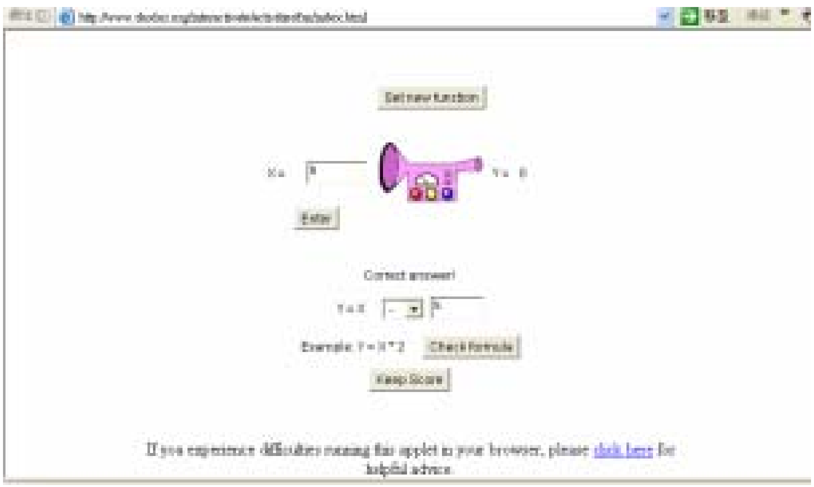
(b) ‘$x$’ is not the only representation of input. For example, we may use ‘$y$’ to represent input as well. In this case, the mechanism is represented by

$$f(y) = y + 5 .$$

The teacher instructs students to use an algebraic expression to represent the special activity of number machine 2 in Worksheet 1.

4. Students are asked to divide into groups of two. Firstly, each student writes down an arithmetic operation of a number machine which contains at most 2 steps. Students tell a number to their groupmates alternatively. Their groupmates should treat the number as input to their number machines and tell the outputs. The first one who can guess the correct arithmetic operation of the machine of his/her counterpart wins the game. The teacher may also use the number machine in the following web site to play the game with the whole class.

[http://score.kings.k12.ca.us/lessons/functions/machine.html](http://score.kings.k12.ca.us/lessons/functions/machine.html)



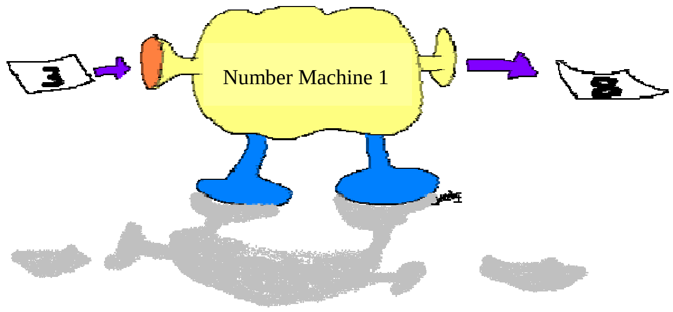
5. Students are asked to discuss Questions 3 and 4 in Worksheet with their groupmates.

# *Worksheet : Basic Ideas of Functions*

**Questions：**

1. The arithmetic operation of number machine 1 is as follows:

   Output = Input + 5




(a) Fill the inputs and outputs of the number machine 1 in the following table.

| Input | Output |
|---|---|
| 3 | 8 |
| 4 | |
| 6 | |
| | 20 |
| | 4 |

(b) Explain why the output of number machine 1 is a function of the input. If we use *f* to represent the function, use this symbol to express the relation of the input and output. (e.g. $f(3) = 8$)。

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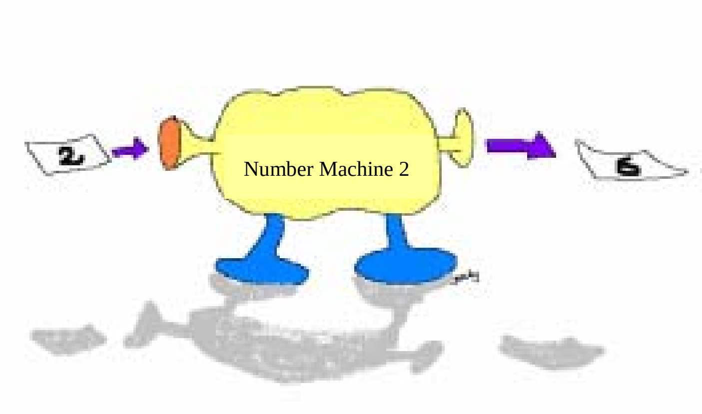
2. The following table shows the input and output of number machine 2.

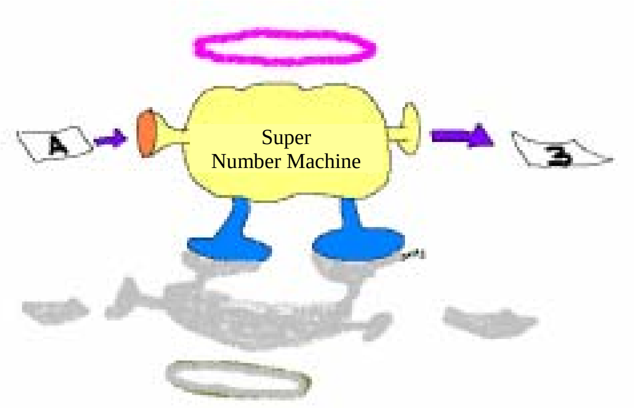



| Input | Output |
|---|---|
| 2 | 6 |
| 4 | 12 |
| 0 | 0 |
| –3 | –9 |
| –12 | –36 |

Guess the arithmetic operation of number machine 2.

If we use *g* to represent the function, use this symbol to express the relation of the input and output. (e.g. $g(2) = 6$)。

3. A super number machine like a number machine gives a paper when a paper is input. However, the word on the input paper is not restricted to numbers only and it is not known whether the word on the output paper is totally determined by the input paper. The following tables are inputs and outputs of three super number machines. Determine which super number machine gives outputs which are NOT function of the inputs and explain why.




Super number machine 1

| Input | Output |
|---|---|
| –4 | –4 |
| –2 | –2 |
| 0 | 0 |
| 2 | 2 |
| 4 | 4 |

Super number machine 2

| Input | Output |
|---|---|
| –4 | A |
| –2 | A |
| 0 | A |
| 2 | A |
| 4 | A |

Super number machine 3

| Input | Output |
|---|---|
| 1 | A |
| 1 | B |
| 1 | A |
| 1 | B |
| 1 | A |

4. In number machine 3, if the output is a function $h$ of the input, $x$, and

$$h(x) = x^2 + 1$$

(a) Fill in the outputs of number machine 3 in the following table.

| Input | Output |
|---|---|
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |

(b) If the output is 10, can we find the corresponding input? Why?

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(c) Is the input of number machine 3 a function of the output? Why?

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**Notes for Teachers:**

1. The time required for this activity is about 35－45 minutes.

2. In the first part of the lesson, the teacher may, according to students' background knowledge and their abilities, choose other examples to explain the idea of function. For example, the name and the HKID card number, the weight of a parcel and the postage, a person and the date of birth or any examples posed by students. More detailed discussion can be found in exemplar 3.

3. The website below provides more information on the Chinese translation of function.

   http://www.dyu.edu.tw/~mfht206/history/19/china.htm

   The content of the piece of information is in Chinese only.

4. For soft drink vending machines in daily life, pressing button to choose a desired soft drink is needed in addition to inserting coins. Outcomes cannot be solely determined by the coins inserted. In this case, 'the outcome' is not a function of 'the coins inserted'.

5. It is not advisable, at this stage, to describe outputs as a result of calculation on inputs. Otherwise, students may have the misconception that outputs of a function must involve certain calculations on the inputs. This will hinder their understanding on the constant function. (Teachers may refer to Markovits, Eylon, & Bruckheimer for difficulties of learning the constant function)

6. Answers to the worksheet:

1(a)

| Input | Output |
|---|---|
| 3 | 8 |
| 4 | 9 |
| 6 | 11 |
| 15 | 20 |
| –1 | 4 |

(b) The same inputs do not give different output.

$f(4) = 9$

$f(6) = 11$

$f(15) = 20$

$f(-1) = 4$

2. e.g.: Output = 3 × Input。

$g(4) = 12$

$g(0) = 0$

$g(-3) = -9$

$g(-12) = -36$

3. The outputs of super number machine 3 are not function of its inputs because outputs are not uniquely determined by the inputs.

4.(a)

| Input | Output |
|---|---|
| −2 | 5 |
| −1 | 2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 5 |

(b) No, it is because both inputs 3 and −3 give the same output 10.

(c) No. From the result of (b), it is clear that the "input" is not uniquely determined by the "output".

Reference:

Kleiner, I. (1989). Evolution of the function concept. *The college Mathematics Journal*, *20*(4), 282-300.

Markovits, Z., Eylon, B., & Bruckheimer, M. (1986). Functions today and yesterday. *For the Learning of Mathematics*, *6*(2), 18-28.

劉福增（2003）.《邏輯思考》. 台北：心理出版社。