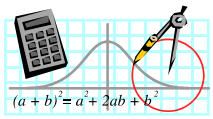


Exemplar 3:
Basic Ideas of Functions (3)

Objective : Understand the basic ideas of functions through daily–life examples and counter-examples

Key Stage : 4

Learning Unit : Functions and Graphs

Materials required : Worksheets

Prerequisite Knowledge : Formulas of the perimeter and area of a rectangle

Description of the activity:

1. The teacher introduces the objective of the lesson. Students are asked to complete Questions 1, 2 and 3 of the Worksheet.

2. After the students completed the questions, they are asked to compare their answers with classmates sitting next to each other. In Question 2, if students have no ideas in matching the Chinese Zodiac with "year of birth", the teacher may give hints like "I know that a teacher is Snake and she was born at 1977."

3. The teacher discusses with students the answers of Questions 1, 2 and 3. The teacher then discusses Parts (a) to (c) of Question 6 to introduce the definition of a function. For each value of the independent variable, there is exactly one value of the dependent variable corresponding to it. According to this definition, the area of a square is a function of its perimeter in Question 1; 'year of birth' is not a function of Chinese Zodiac in Question 2; the name is a function of the I.D. card number in Question 3.

4. The teacher may consider to further discuss with students the following depending on the availability of time and students' interest. (That is, in Question 7, Parts (a) and (b), (c) and (d); (e) and (f))

For example:

(a) It is known that the area of a square is a function of its perimeter. Is the perimeter of a square also a function of its area? In other words, if the area of a square is given, can its perimeter be determined?

(b) It is known that "year of birth" is not a function of one's Chinese Zodiac. Is Chinese Zodiac a function of "year of birth"? In other words, if the "year of birth" of a man is given, can his/her Chinese Zodiac be determined?

(c) It is known that the name is a function of the I.D. card number. Is the I.D. card number also a function of the name? In other words, if a name of a person is given, can his/her I.D. card number be determined?

5. If time allows, students will complete Questions 4 and 5 of the Worksheet and then the whole Worksheet. The teacher checks the answers with students and conducts further discussion on the examples provided by the Worksheet..

   For example:

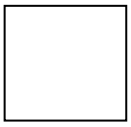
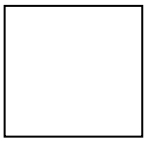
(a) It is known that the number of matches is a function of the number of squares. Is the number of squares a function of the number of matches? If the number of matches is given, can the number of squares be determined?

(b) It is known that the area of a rectangle is not a function of its perimeter. Is the perimeter of a rectangle also a function of its area? If the area of a rectangle is given, can its perimeter be determined?

6. The teacher concludes that "if Y is a function of X, X is not necessarily a function of Y".

Worksheet : Basic Ideas of Functions

1. The first column of the following table gives perimeters of different squares. Draw possible square(s). Label the lengths of each side of these squares as shown in the example.

Perimeter/cm	Possible corresponding squares
4	 1cm
12	
24	

Write down the areas of the squares above.

Perimeter/cm	**4**	**12**	**24**
Area/cm^2			

(a) Is your result the same as your classmates' results?

_____

(b) How many possible value(s) of area(s) does a square of perimeter 24cm correspond to?

_____

(c) Can the area of a square be determined if its perimeter is given?

_____

2. Write down your “year of birth” and your Chinese Zodiac.（If you want to keep these data secret, you may leave them in your mind.）

Year of Birth：	Chinese Zodiac：

The order of the twelve animals in the Chinese Zodiac is Rat, Ox, Tiger, Rabbit, Dragon, Snake, Horse, Ram, Monkey, Rooster, Dog and Pig. The following table lists four of them. Deduce, from your own “year of birth” and Chinese Zodiac, the “year of birth” of the teachers and students in your school.

Chinese Zodiac	 Rabbit	 Dragon	 Snake	 Horse
Year of Birth				

(a) Is your result the same as your classmates’ results?

_____

(b) How many possible year(s) of birth does one who was born in the Year of Dragon correspond to?

_____

(c) Are two teachers born in the same year if they are both “Snake”?

_____

(d) Can the year of birth be determined if one’s Chinese Zodiac is given?

_____

3. The information below is about 15 members in a youth centre:

					
I.D. Number	A123456(7)	K654321(1)	G456456(2)	Z555553(1)	B123445(5)
Name	Peter Chan	Mary Lee	Robert Cheung	Ben Wong	Paul Cheung
					
I.D. Number	Z238238(8)	D765432(1)	K828282(8)	E432132(1)	H741852(9)
Name	Albert Lau	Matthew Lam	Rose Fong	Kenny Ho	Thomas Yip
					
I.D. Number	F987654(3)	Z789456(0)	K963852(7)	D258369(1)	Z369123(4)
Name	Paul Cheung	Iris Leung	Steven Tang	Peter Chan	Bernice Chow

(a) The I.D. card numbers of active members in the youth centre are listed below. Please write down the names of the corresponding members.

			
I.D. Number	A123456(7)	K828282(8)	F987654(3)
Name			
			
I.D. Number	D258369(1)	H741852(9)	Z555553(1)
Name			

(b) If the I.D. card number is given, can the name be determined?

(c) Is Paul Cheung an active member in the youth centre? Give reasons.

(d) It is known that Peter Chan is a volunteer worker in the youth centre. Can you give his I.D. card number? Why?

(If the name is given, can his/her I.D. card number be determined?)

4. The following diagrams show different number of squares formed by matches:

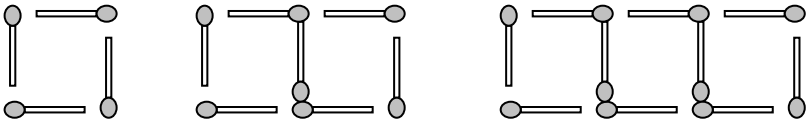


Diagram (1) Diagram (2) Diagram (3)

(a) According to the pattern of the above diagrams, please fill in the number of matches in the corresponding diagrams.

Diagram	(1)	(2)	(3)	(4)	(5)
No. of matches					

(b) Are your results in (a) the same as your classmates' results?

_____

(c) From the results in (a) and (b), what is the number of matches used if ten squares (i.e. Diagram (10)) are formed by matches in the above way?

_____

(d) Can the number of matches used be determined if *n* squares are formed? How many matches are used?

_____

5. The first column of the following table gives perimeters of different rectangles. Draw possible corresponding rectangle(s). Label the lengths and widths of these rectangles.

Perimeter/cm	Possible corresponding rectangles
10	
14	
26	

Write down the areas corresponding to the rectangles above.

Perimeter/cm	**10**	**14**	**26**
Area/cm^2			

(a) Is your result the same as your classmates' results?

_____

(b) Does a rectangle with perimeter 10 cm correspond to more than one area?

_____

(c) Can the area of a rectangle be determined if its perimeter is given?

_____

6. In Questions 1 to 5, we find that:

   (a) for squares, a given perimeter corresponds to _____(one/ more than one) value(s) of area.

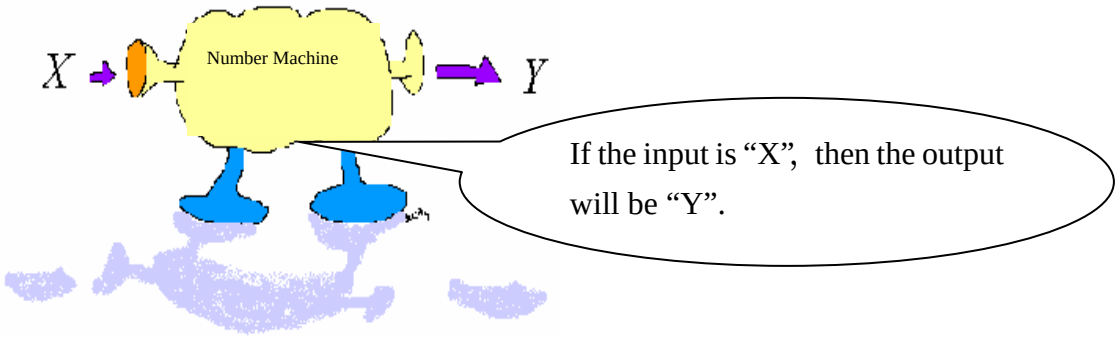
   (b) a given Chinese Zodiac corresponds to _____(one/ more than one) year(s) of birth.

   (c) a given name corresponds to _____(one/ more than one) I.D. card number.

   (d) a given number of squares formed corresponds to _____(one/ more than one) value(s) on the number of matches used.

   (e) for rectangles, a given perimeter corresponds to _____(one/ more than one) value(s) of area.

7. If, for any given *X*, there is one and only one corresponding value of *Y*, then *Y* is called a function of *X*.



According to the above definition, which one of the following is correct?

(Put a ✓in ☐ if the statement is correct.)

(a) ☐ "The area of a square" is a function of "the perimeter of the square".

(b) ☐ "The perimeter of a square" is a function of "the area of the square". (i.e. the perimeter of a square can be determined by its area)

(c) ☐ "Year of birth" is a function of "Chinese Zodiac".

(d) ☐ "Chinese Zodiac" is a function of "Year of birth". (i.e. the Chinese Zodiac can be determined by the year of birth)

(e) ☐ "I.D. card number" is a function of "the name".

(f) ☐ "The name" is a function of "I.D. card number". (i.e. the name can be determined by the ID card number)

(g) ☐ "The number of matches used" is a function of "number of squares formed".

(h) ☐ "The number of squares formed" is a function of "the number of matches used". (i.e. the number of squares is fixed by the number of matches used)

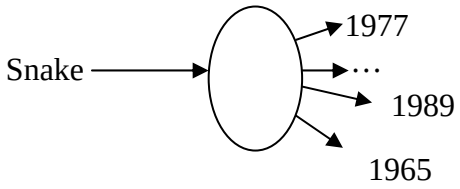
(i) ☐ "The area of a rectangle" is a function of "the perimeter of the rectangle".

(j) ☐ "The perimeter of a rectangle" is a function of "the area of the rectangle". (i.e. the perimeter of a rectangle can be determined by its area)

Notes for Teachers:

1. The time required for this activity is about 40－60 minutes.

2. Students may find difficulties in doing Question 6. The teacher may use diagrams to explain the strategies. For example, a specific Chinese Zodiac input may give several “years of birth” as output.



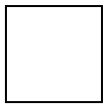
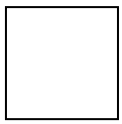
3. In Question 2, “Year of birth” is not a function of “Chinese Zodiac”. It is because different “Year of birth” can have the same Chinese Zodiac. On the other hand, “Chinese Zodiac” is also not a function of “Year of birth”. Students may query that one’s Chinese Zodiac should correspond to a specific year of birth and conclude a relation between two variables. The teacher should remind students that there are two Chinese Zodiac every year.

4. The I.D. card number in Question 3 is a fake number. It is not worked out according to the coding mechanism of HKID card numbers. This is intended to minimize the clashing of the proposed ID card numbers and those of individual students. The teacher can introduce how to get the number in the bracket of the I.D. card number. Further details can be found in 《生活的數學》羅浩源(1997) 第 13 頁香港：香港教育圖書公司。

5. The teacher should remind the seriousness of mathematics languages. Thus, it is inappropriate to say “There is a function relation between x and y”. It is more precise to say whether “Y is a function of X” and/or “X is a function of Y”. There are different meanings of the latter 2 statements.

6. Suggested answers for the worksheet are as follows:

1.

Perimeter/cm	Possible corresponding squares
4	 1cm 1cm
12	 3cm 3cm
24	 6cm 6cm

Perimeter/cm	**10**	**14**	**26**
Area/cm^2	1	9	36

(a) Exactly the same.

(b) One.

(c) Yes.

2.

Chinese Zodiac				
Year of Birth	1952 1953 1964 1965 1976 1977 1988 1989 2000 2001	1953 1954 1965 1966 1977 1978 1989 1990 2001 2002	1954 1955 1966 1967 1978 1979 1990 1991 2002 2003	1955 1956 1967 1968 1979 1980 1991 1992 2003 2004

(a) Not the same.

(b) Two.

(c) Not certain.

(d) No.

3.(a)

			
I.D number.	A123456(7)	K828282(8)	F987654(3)
Name	Peter Chan	Rose Fong	Paul Cheung
			
I.D number	D258369(1)	H741852(9)	Z555553(1)
Name	Peter Chan	Thomas Yip	Ben Wong

(b) Yes.

(c) Certainly not. There are two members with the same name called “Paul Cheung”. We do not know who is the active member.

(d) No, there are two “Peter Chan” in the youth centre.

4.(a)

Diagram	(1)	(2)	(3)	(4)	(5)
No. of matches	4	7	10	13	16

(b) Exactly the same.

(c) 31 matches.

(d) Yes, the number of matches = $3n+1$.

5.

Perimeter/cm	Possible corresponding rectangles
10	 2cm × 3cm (or other possible answers)
14	 2cm × 5cm (or other possible answers)
26	 3cm × 10cm (or other possible answers)

Perimeter/cm	**10**	**14**	**26**
Area/cm^2	6 (or other possible answers such as 4, 6.25 etc)	10 (or other possible answers)	30 (or other possible answers)

(a) Not exactly the same.
(b) Yes.
(c) No.

6. (a) One.
(b) More than one.
(c) One.
(d) One.
(e) More than one.

7.

(a) ☑ "The area of a square" is a function of "the perimeter of the square".

(b) ☑ "The perimeter of a square" is a function of "the area of the square". (i.e. the perimeter of a square can be determined by its area)

(c) ☐ "Year of birth" is a function of "Chinese Zodiac".

(d) ☐ "Chinese Zodiac" is a function of "Year of birth". (i.e. the Chinese Zodiac can be determined by the year of birth)

(e) ☐ "I.D. card number" is a function of "the name".

(f) ☑ "The name" is a function of "I.D. card number". (i.e. the name can be determined by the ID card number)

(g) ☑ "The number of matches used" is a function of "number of squares formed".

(h) ☑ "The number of squares formed" is a function of "the number of matches used". (i.e. the number of squares is fixed by the number of matches used)

(i) ☐ "The area of a rectangle" is a function of "the perimeter of the rectangle".

(j) ☐ "The perimeter of a rectangle" is a function of "the area of the rectangle". (i.e. the perimeter of a rectangle can be determined by its area)