Exemplar 4:

The tabular, graphical and algebraic representations of functions (1)

Objectives	**:**	1. Explore number patterns
		2. Relate the concept of 'input-process-output' learnt from number patters at KS3 to the relation between dependent and independent variables
		3. Further understand the basic ideas of functions from the tabular, graphical and algebraic representations of functions
Key Stage	**:**	4
Learning Unit	**:**	Functions and Graphs
Materials required	**:**	Worksheets and computers accessible to the Internet
Prerequisite Knowledge	**:**	1. Use algebraic symbols to represent numbers
		2. Understand algebraic languages and the method of substitution
		3. Sketch the graph of a function in the Cartesian coordinate plane
		4. Understand the basic ideas of functions

Description of the activity:

1. The teacher revises the concept of sequence and distributes Worksheet. Students are asked to attempt Question 1 on the Worksheet.

2. After completing Question 1, the teacher may use the 'Number Machine' to elaborate the relation between the term number and the term itself and then the idea of "Input-Process-Output". Hence, the teacher revises the concept of function and further discusses the sequence in Question 1.
   For example:
   (a)   Is the value of a term a function of the term number?

      (b)   What is the algebraic expression of the function?

      (c)   Are there any restrictions on the input value?

3.   From the discussions on the restrictions of the input values for the number sequence, the teacher points out that most functions encountered at senior secondary levels have real numbers as the input values. The teacher asks students to complete Questions 2(a)-(c) and 3(a)-(c). The teacher finally checks the answers with students.

4.   The teacher explains to students that the relation between 2 quantities (i.e. x, y in Questions 2 and 3) can be presented in tabular, graphical and algebraic representations.

      (a)   The tabular form only shows part of the values of x and y.

      (b)   The graphical form covers more values and gives continuous values of x and y.

      (c)   The algebraic form can present a comprehensive profile of the function.

The teacher may point out that the functions $f(x)$ and $T(n)$ are used to represent the functions of the independent variables x and n respectively. In drawing the conclusion for Question 1(d), the teacher may further reiterate the concept of using n as a variable and its corresponding meaning in $T(3)$, $T(10)$ and $T(99)$.

5.   After completing Questions 2(d), 3(c) and (d), students consolidate the notation of a function. For more able students, the teacher can further explain the concept of dummy variables in $f(a)$, $f(a+1)$.

Worksheet :

The tabular, graphical and algebraic representations of functions

Question:

1. Consider the sequence 4   8   12   16   …

   (a) Estimate the possible value(s) of the 5th and 6th terms.

   The value of the 5th term

   The value of the 6th term

   (b) Write down the possible value(s) of the 5th to 10th terms of the sequence in the following table.

n^{th} term	1	2	3	4	5	6	7	8	9	10
value of nth term	4	8	12	16						

   (c) Let *n* be the term number. Discuss with your classmates to find an algebraic expression to represent the above sequence, where

   $T(n)$.

   Use this formula to find:

   The value of the 3rd term $T(3)$,

   The value of the 10th term $T(10)$,

   The value of the 99th term $T(99)$.

   (d) If the term number is given, can the value of the term be determined?

   (e) Is the value of a term of a sequence a function of the term number?

2.   Below is a 'Number Machine'. For each input, there is one and only one output (In other words, the same value of input will not give different values of output).
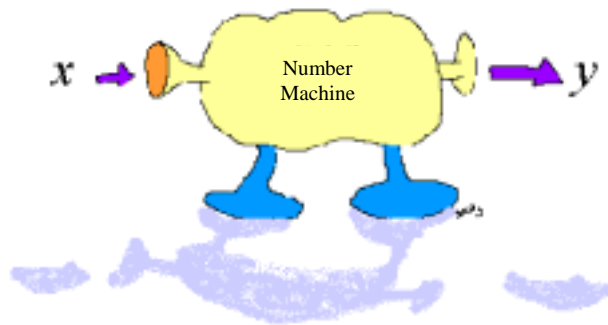

Below is a set of input and output values of the machine

Input x	5	4	0	1	3	4.2
Output y	15	12	0	3	9	12.6

(a)   Work with your team members to guess the operating mechanism of the machine and form an algebraic expression to represent the function. (i.e. the relation between the input and the output values, such as $y = 2x + 3$.)

(b)   In Part (a), we work out the operating mechanism of the machine according to several given input and output values. Assuming that the expression is correct, it is easy to find the output value from an input value. Find the output value when the input is 1.5.

(c)(i) Use the data provided and the algebraic expression formed in Part (a) to sketch the corresponding graph in the Cartesian coordinate plane.
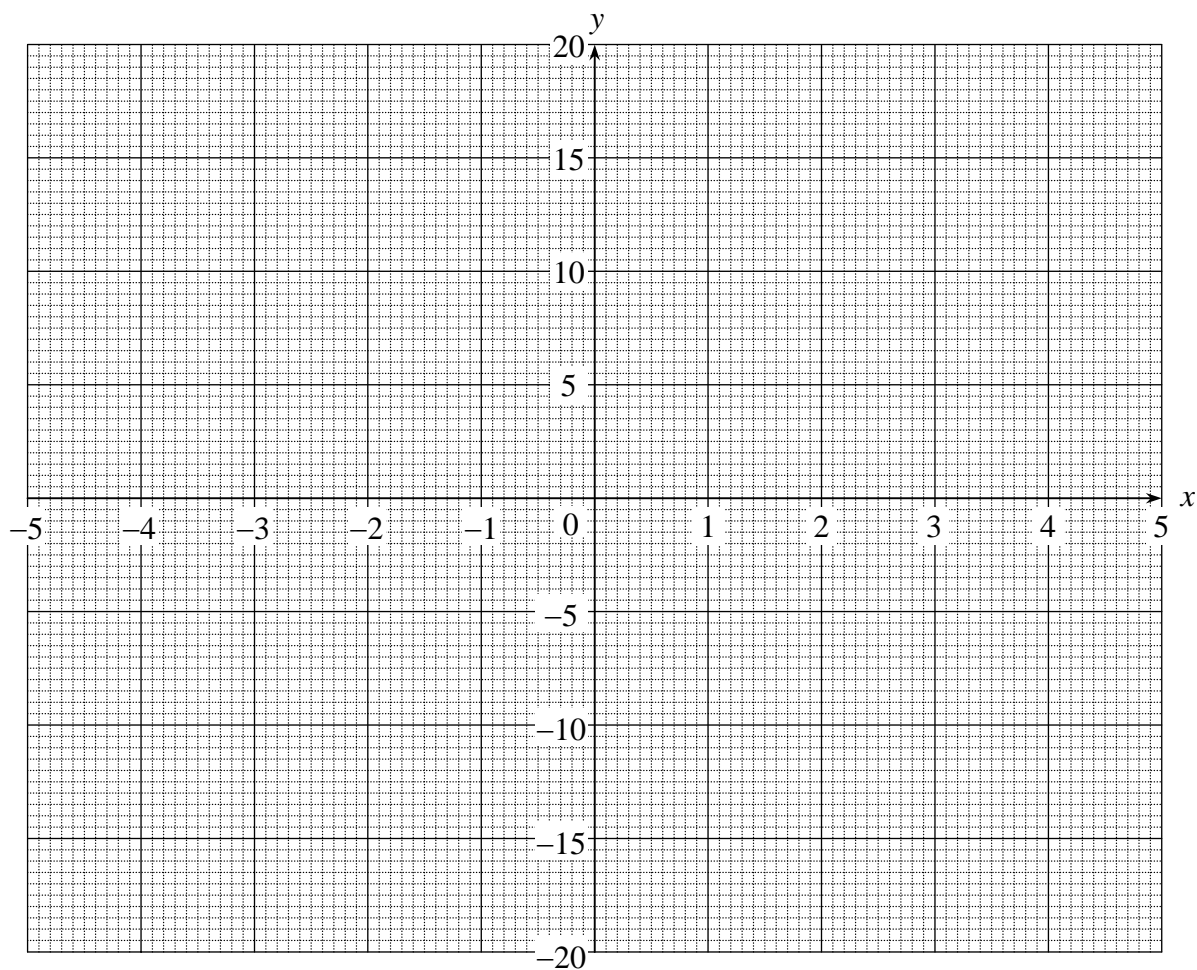
(ii)   From the graph, find the output value when the input value is 4.5.

_____

(iii) How many outputs (y) does an input(x) correspond to?

_____

(iv) Is the output(y) a function of the input (x)?

_____

(d) (i) Assuming that the algebraic expression formed in Part (a) is $y = f(x)$,

then $f(x) =$ _____

(ii) Use the above notation, complete the following table.

x	$f(x)=$_____	
-1000	$f($ $) = $ _____	$= $ _____
-0.5	$f($ $) = $ _____	$= $ _____
11	$f($ $) = $ _____	$= $ _____
73.4	$f($ $) = $ _____	$= $ _____
x	$f($ $) = $ _____	$= $ _____
a	$f($ $) = $ _____	$= $ _____
$a+1$	$f($ $) = $ _____	$= $ _____

3.   Below is the set of input and output values of another number machine.

Input x	−4	−3	−2	−1	0	1	2	3	4
Output y	16	9	4	1	0	1	4	9	16

(a)   Work with your team members to guess the operating mechanism of the machine and form an algebraic expression to represent the function (i.e. the relation between the input value x and output value y such as $y = 2x + 3$.)

_____

(b)(i)   Use the given set of input and output values and the algebraic expression formed in Part (a) to sketch the corresponding graph in the Cartesian coordinate plane.
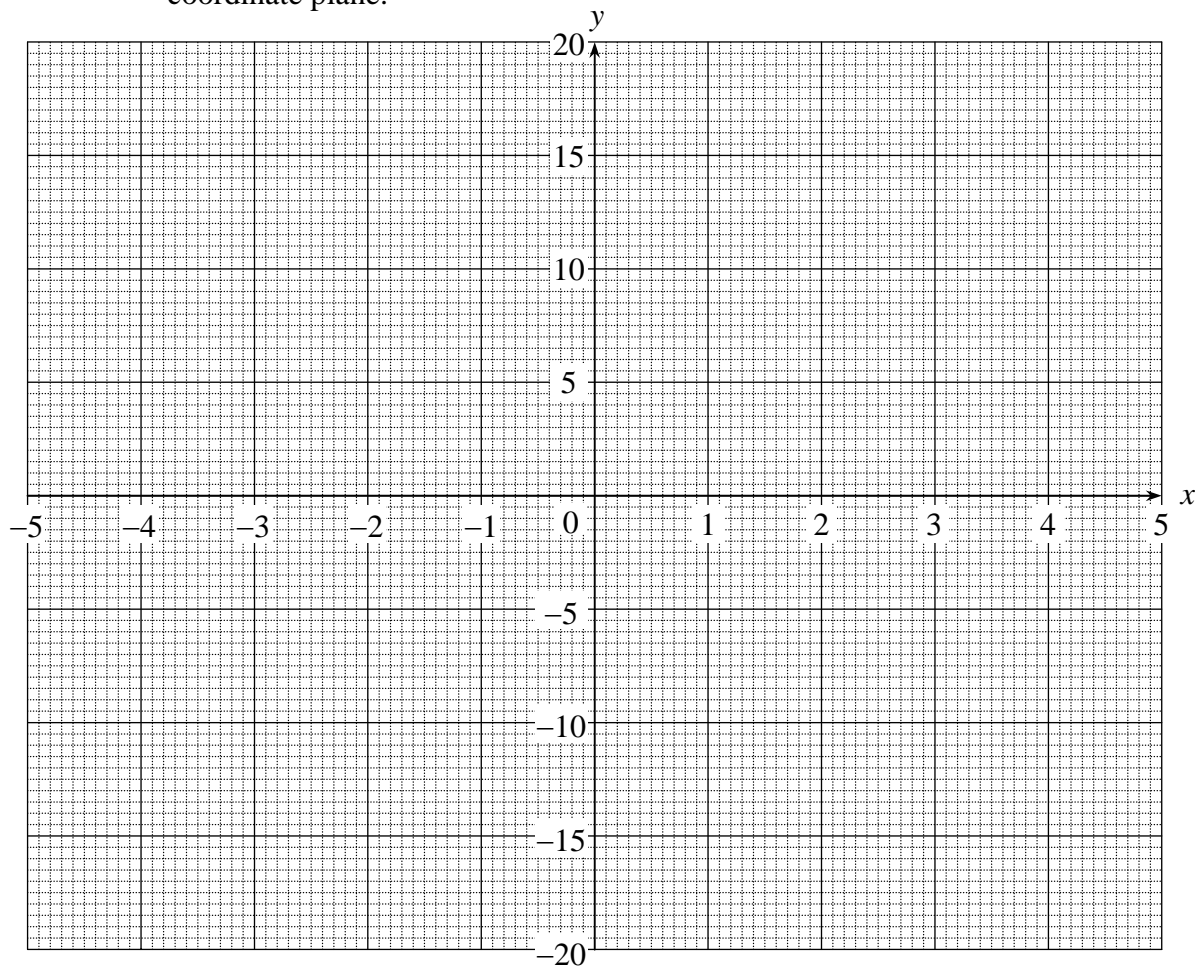
(ii)   How many outputs(y) does an input(x) correspond to?

_____

(iii) From the graph or the values in the table, is the output(y) a function of the input(x)?

_____

(iv) If $y = g(x)$, then $g(x) =$ _____

(c) According to the function in Part (b), complete the following table.

x	$g(x) =$ _____		
−100	$g($ _____ $) =$ _____	$=$ _____
−2.3	$g($ _____ $) =$ _____	$=$ _____
3.15	$g($ _____ $) =$ _____	$=$ _____
1050	$g($ _____ $) =$ _____	$=$ _____

Notes to teacher:

1. The time required for this activity is about 60 minutes.

2. Some students may find difficulties in sketching the graph. The teacher may give some hints while they are doing Question 2(c).

3. Some students may feel confused about the notation of functions. For example, students have confusion like " $f(x) = f \cdot x$ ". This exemplar introduces the functional notation $f(x)$ through concrete cases instead of direct introduction of the definition. The teacher may use the symbols such as $T(3)$, $T(10)$ in Worksheet 1 and $f(3)$, $f(11)$ in Worksheet 2 to consolidate the meaning of the notation $f(x)$. The teacher may also use Exemplar 6 "Common Confusion on the Concepts of Function" to further discuss the concept of function such as whether $f(a + b)$ is equal to $f(a) + f(b)$.

4. When the teacher discusses with students that the different representations of functions, it is better to explore the restrictions of each form of representations. For example,
   (a) The tabular form only shows discrete values.
   (b) The graphical form can cover more values of x and y and the values are continuous. However, we have no ideas of the function for the values outside the given range.
   (c) The algebraic form can give a comprehensive relation between the dependent and independent variable. Nevertheless, algebraic relations of quantities in our real life problems are not always given. It requires a number of tools to find the algebraic relation.

5. The suggested answer for the worksheet:

   1. (a) The value of 5th term   20
          The value of 6th term   24

      (b)

No. of term	1	2	3	4	5	6	7	8	9	10
The value of term	4	8	12	16	20	24	28	32	36	40

      (c) $T(n)$ $4n$

The value of the 3rd term $T(3)$ 12,

The value of the 10th term $T(10)$ 40,

The value of the 99th term $T(99)$ 396.
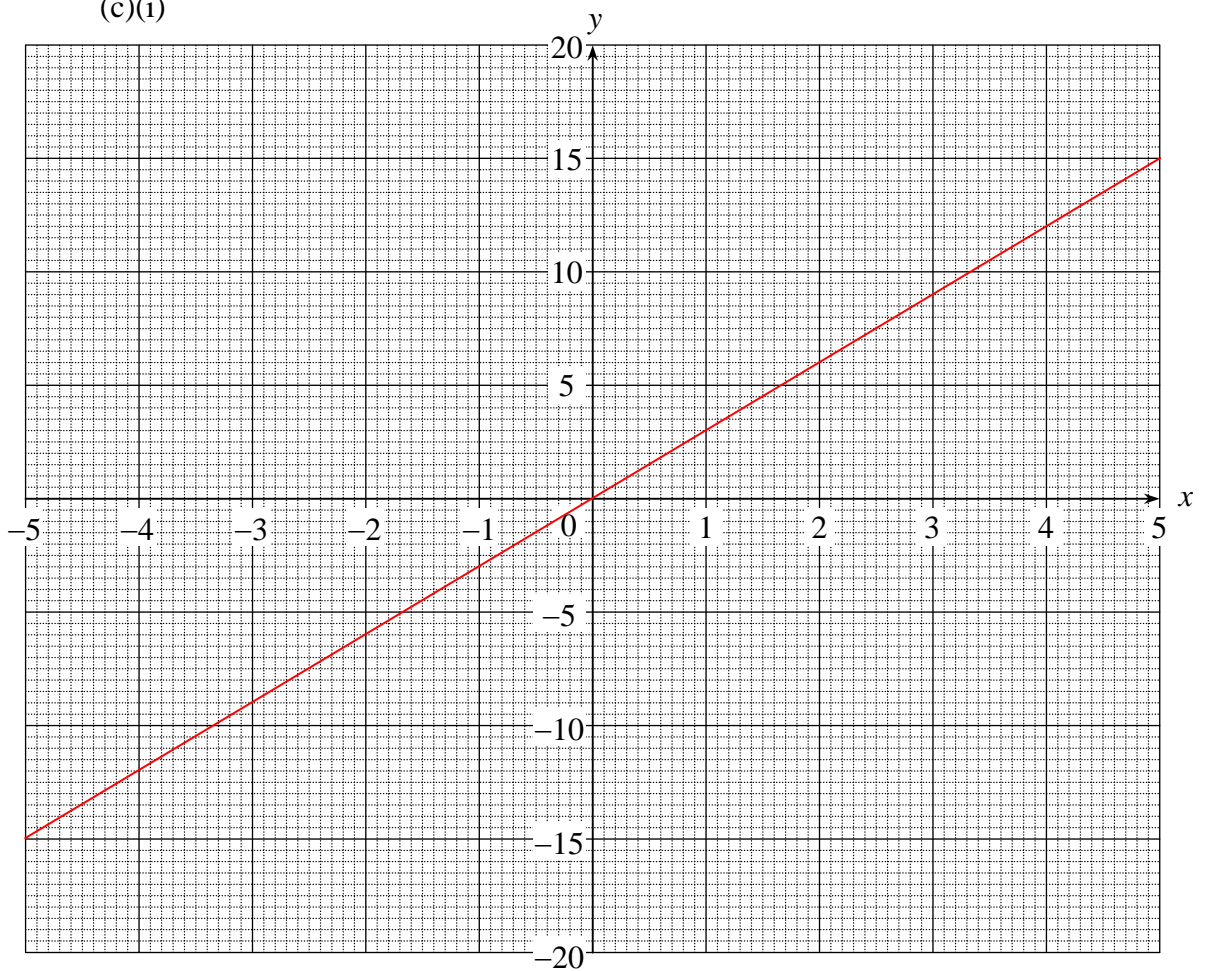
(d) Yes

(e) Yes

2.   (a) $y = 3x$

(b) The output value $= 3(1.5) = 4.5$

(c)(i)

(ii) 13.5.

(iii) One.

(iv) Yes.

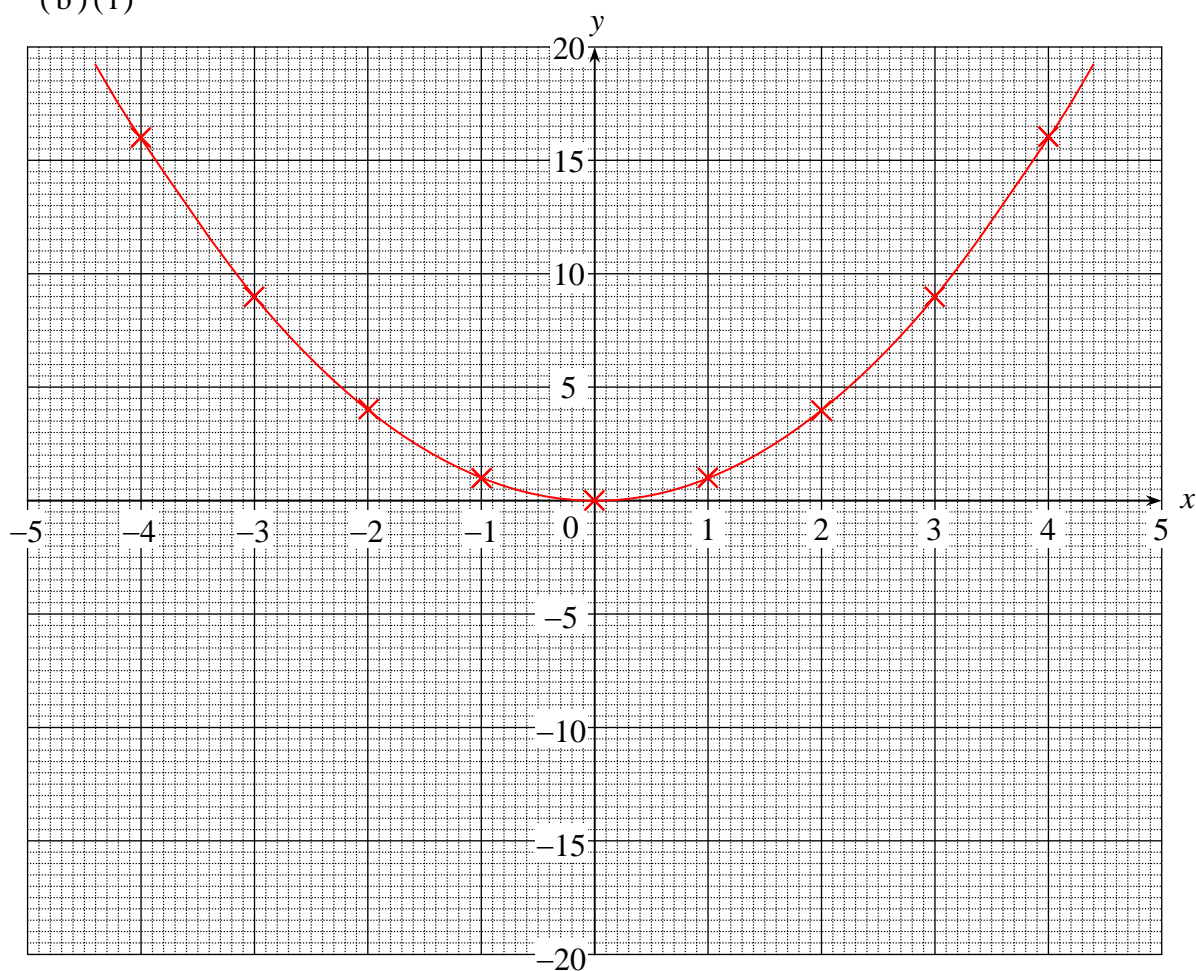(d)(i) $f(x) = 3x$

(ii)

x	$f(x) = \ 3\,x$
-1000	$f(-1000) = 3 \times (-1000) = -3\,000$
-0.5	$f(-0.5) = 3 \times (-0.5) = -1.5$
11	$f(11) = 3 \times (11) = 33$
73.4	$f(73.4) = 3 \times (73.4) = 220.2$
x	$f(x) = 3 \times (x) = 3x$
a	$f(a) = 3 \times (a) = 3a$
$a+1$	$f(a+1) = 3 \times (a+1) = 3a + 3$

3. (a) $y = x^2$

(b)(i)

(ii) One.

(iii) Yes.

(iv) If $y = g(x)$, then $g(x) = x^2$.

(c)

x	$g(x) = x^2$
-100	$g(-100) = (-100)^2 = 10\ 000$
-2.3	$g(-2.3) = (-2.3)^2 = 5.29$
3.15	$g(3.15) = (3.15)^2 = 9.9225$
1050	$g(1050) = (1050)^2 = 1\ 102\ 500$