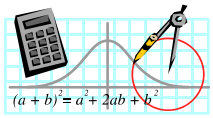


Exemplar 8: The effect of transformation on the graph of functions (1)

Objective : Understand the upward or downward translations on the graph of functions and the corresponding change on the algebraic expression

Key Stage : 4

Learning Unit : Functions and Graphs

Materials required :
1. Spreadsheet software such as *Microsoft Excel* (*graph.xls*), *Graphmatica* and worksheets
2. The transparencies printed with the graphs of functions $y = x^2$ and $y = x^3$ and graph papers

Prerequisite Knowledge :
1. Recognize the graph of function $f(x) = x^2$.
2. Plot graphs

Description of the activity:

1. The teacher revises with students the graph of function $y = x^2$ and the relation between the points on the graph and the algebraic expression.

2. The teacher distributes the Worksheet 1 to students and asks them to calculate the value of y from the corresponding value of x of the given function (C_2) $y = x^2 + 3$. From these values, students are asked to plot the graph of the function $y = x^2 + 3$.

3. The teacher discusses the answers with students and asks them, from graphs of the functions $y = x^2$ and $y = x^2 + 3$, to explore the relation of the values of y points with the same x coordinate. In addition, they need to compare the relations of these functions from the data in tables.

4. The teacher further discusses with students on the changes on the graph $y = x^2$ if the equation of C_2 changes to $y = x^2 + 5$. The teacher may use a transparency to illustrate the graphs of functions $y = x^2$ and $y = x^2 + 5$. Hence, it is expected that students can conclude that, from the graphs of C_1 and C_2, the graph of $y = x^2 + 5$ is to translate upwards by 5 units of the graph $y = x^2$.

5. The teacher distributes Worksheet 2 to students and asks them to complete Questions 1 and 2. The teacher then concludes the relations between the graphs of functions $y = x^2$ and (C_3) $y = x^2 - 6$.

6. The teacher invites students to guess the relations between the graphs of functions $y = x^2 - 10$ and $y = x^2$. The teacher may use a transparency to show the graph of the function $y = x^2 - 10$ and request students to explain their conjecture.

7. The teacher arranges students in groups to further discuss the relations between the following pairs of graphs of C_1 and C_2.

   (a) C_1 : $y = x^2 + 5$ and C_2 : $y = x^2 + 10$
   (b) C_1 : $y = x^3$ and C_2 : $y = x^3 + 4$

   The teacher invites students using transparencies to show their graphs and explain their answers.

8. The teacher uses the *Excel file*(*graph.xls*) to check the answers with students and then explains the effect on the points of graphs of functions corresponding to the change on algebraic expressions.

9. The teacher asks students the possible changes on the algebraic expression if the graph of the function $y = f(x)$:
   (a) $f(x) = x^2 + 2x$; or
   (b) $f(x) = x^2 + 2x + 4$; or
   (c) $f(x) = x^3 - 2x$; or
   (d) $f(x) = x^3 - 2x + 4$.
   is translated upwards or downwards by 3 units. After the discussion, the teacher uses the *Excel file*(*graph.xls*) to demonstrate the answers.

10. The teacher distributes Worksheet 3 to students in which they may consolidate

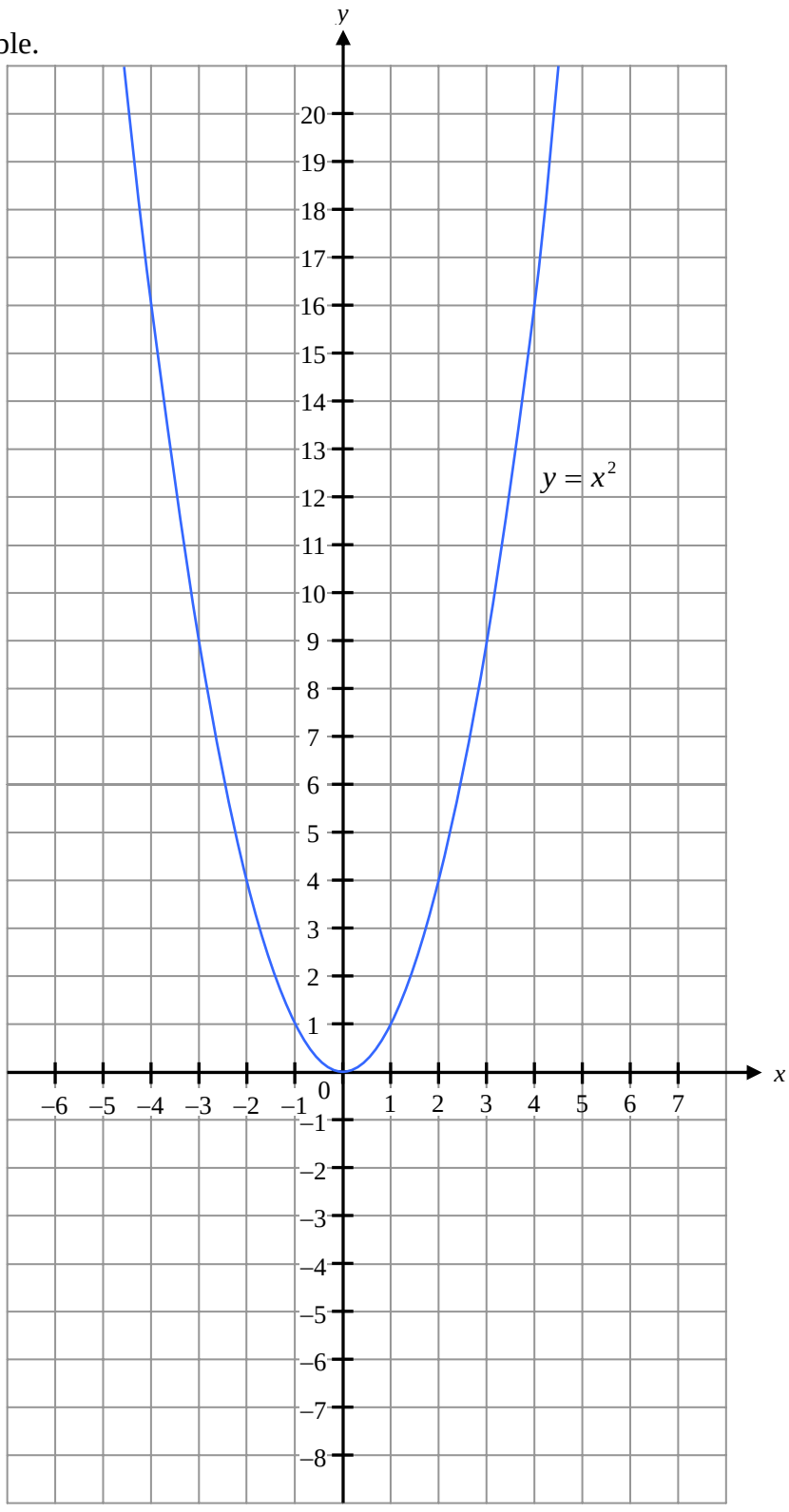
their understanding on the effect on algebraic expressions when the graphs of functions are translated upwards or downwards.

11. The teacher uses the software *Graphmatica* to discuss the effect on upward and downward translations of graphs of other functions such as $y = x^4$ and $y = \sin x$. The teacher then concludes that:
    (a) If the graph of the function $y = f(x)$ is translated upwards by k units, the new function will be $y = f(x) + k$.
    (b) If the graph of the function $y = f(x)$ is translated downwards by k units, the new function will be $y = f(x) - k$.

Worksheet 1

1. Complete the following table.

x	C_1 $y = x^2$	C_2 $y = x^2+3$
–4	16	19
–3	9	
–2	4	
–1	1	4
0	0	
1	1	
2	4	7
3	9	12
4	16	

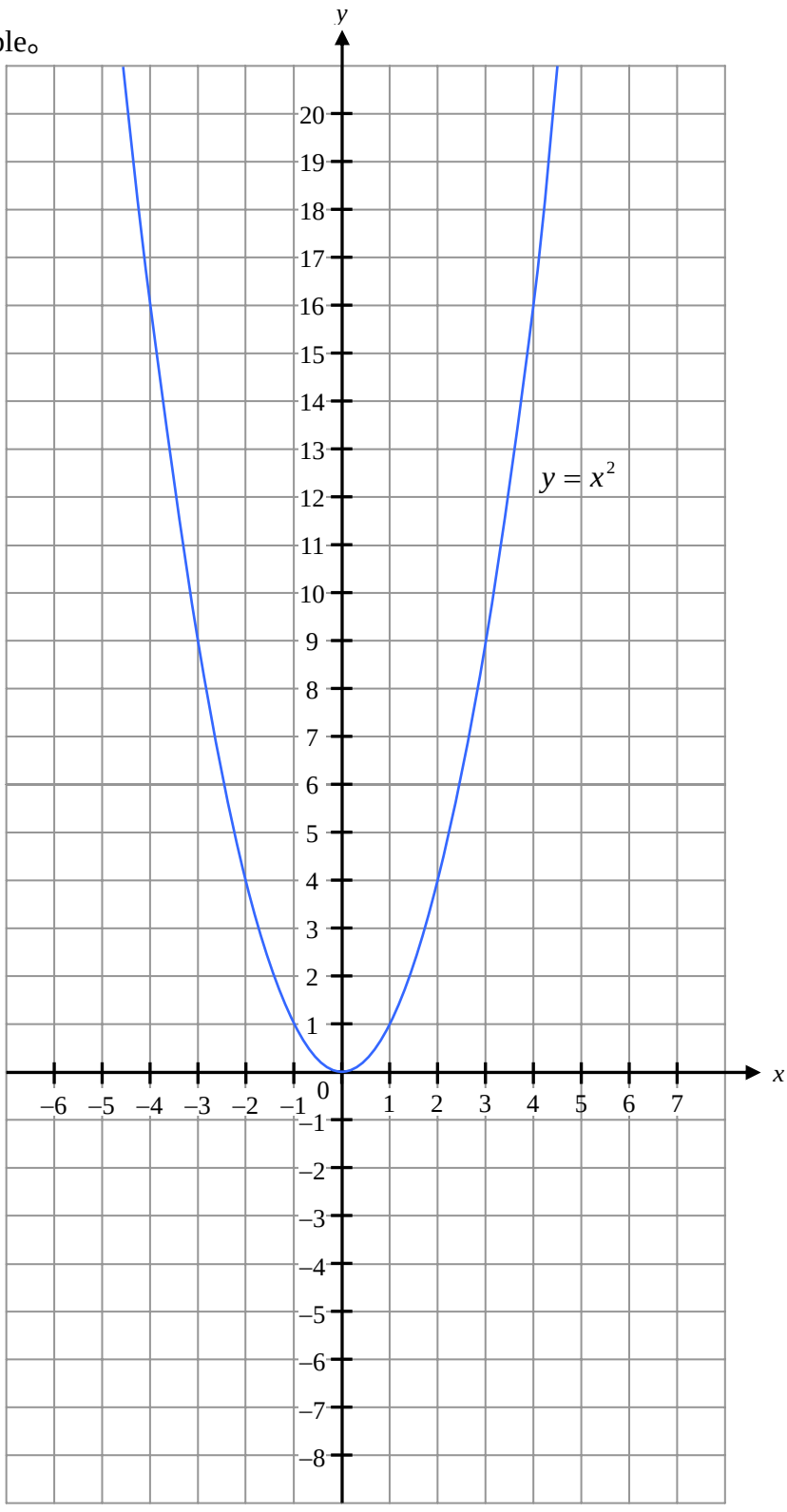


2. Plot the graph of C_2 on the above graph paper.

Worksheet 2

1. Complete the following table.

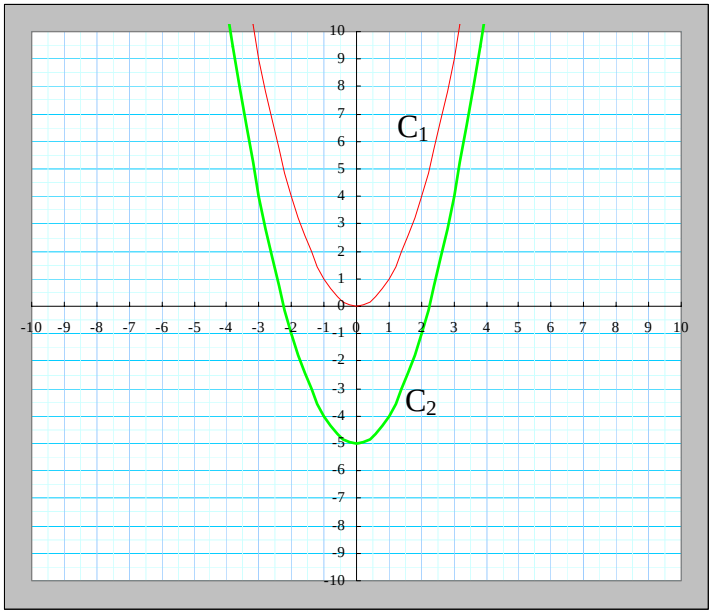
x	C_1 $y = x^2$	C_3 $y = x^2 - 6$
–4	16	10
–3	9	3
–2	4	–2
–1	1	
0	0	–6
1	1	–5
2	4	
3	9	
4	16	10



2. Plot the graph of C_3 on the above graph paper.

Worksheet 3

1.

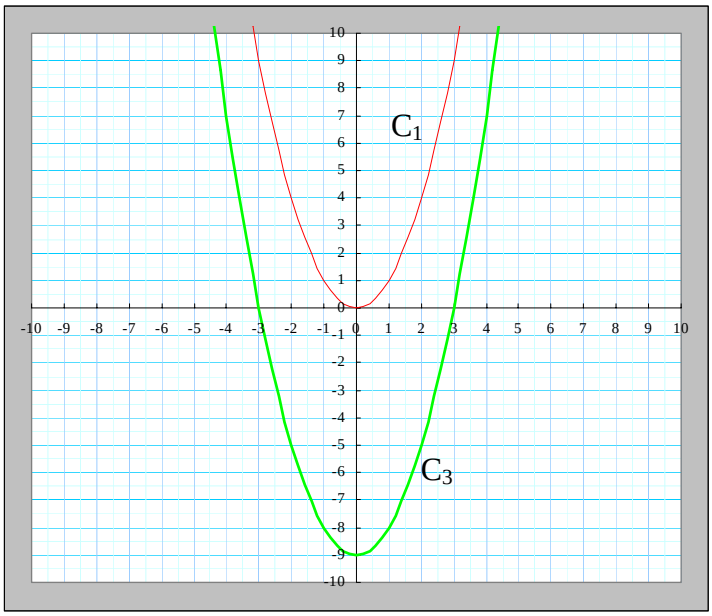


The figure shows the graphs of quadratic functions: C_1: $y = x^2$ and C_2.

(a) The graph of the function C_2 is translated C_1 (upwards / downwards) by_____ units.

(b) The equation of C_2 is _____.

2.

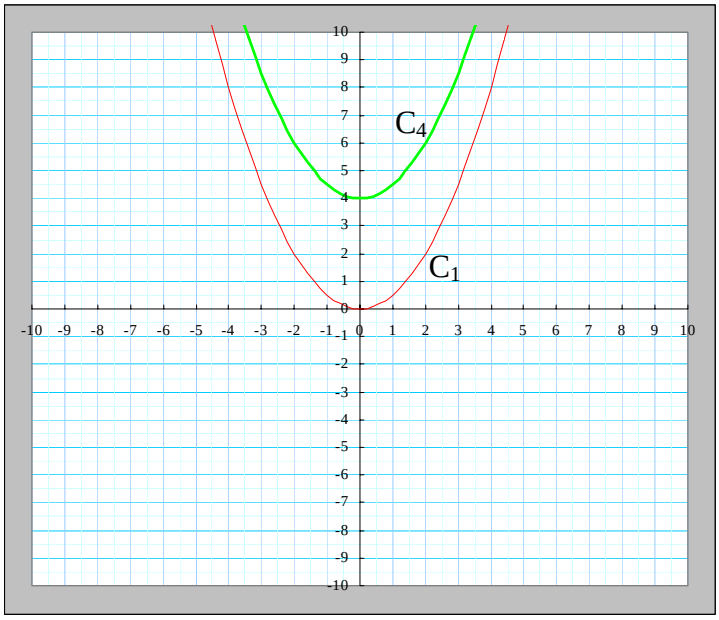


The figure shows the graphs of quadratic functions: C_1: $y = x^2$ and C_3.

(a) The graph of the function C_3 is translated C_1 (upwards / downwards) by_____ units.

(b) The equation of C_3 is _____.

3.

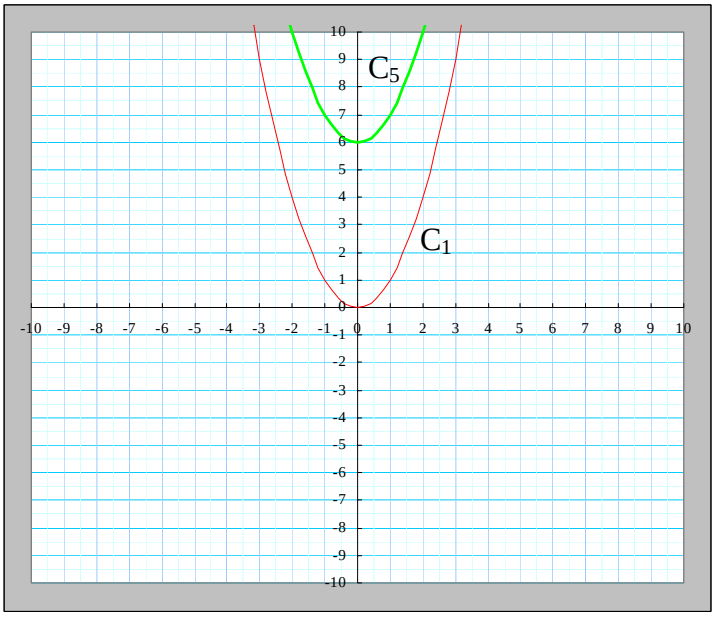


The figure shows the graphs of quadratic functions: C_1: $y = x^2$ and C_4.

(a) The graph of the function C_4 is translated C_1 (upwards / downwards) by_____ units.

(b) The equation of C_4 is _____.

4.



The figure shows the graphs of quadratic functions: C_1: $y = x^2$ and C_5.

(a) The graph of the function C_5 is translated C_1 (upwards / downwards) by_____ units.

(b) The equation of C_5 is _____.

5. Write down the effect on the graphs of the following transformations.

	Equation of C_1	Translated upwards/downwards	Equation of C_2 after translation
(a)	$y = x^2 + 1$	Translated *upwards/downwards by ____units	$y = x^2 + 3$
(b)	$y = x^2 + 5$	Translated *upwards/downwards by ____units	$y = x^2 + 2$
(c)	$y = x^2 + x$	Translated *upwards/downwards by ____units	$y = x^2 + x + 1$
(d)	$y = x^2 + 2x + 1$	Translated *upwards/downwards by ____units	$y = x^2 + 2x + 3$
(e)	$y = x^3 + 1$	Translated *upwards/downwards by ____units	$y = x^3 + 4$
(f)	$y = f(x)$	Translated *upwards/downwards by ____units	$y = f(x) + k$
(g)	$y = x^2$	Translated *upwards/downwards by ____units	$y = x^2 - 4$
(h)	$y = x^2 + 2$	Translated *upwards/downwards by ____units	$y = x^2 - 3$
(i)	$y = x^2 - 3$	Translated *upwards/downwards by ____units	$y = x^2 - 8$
(j)	$y = x^2 - 5$	Translated *upwards/downwards by ____units	$y = x^2 - 7$
(k)	$y = x^2 + 3x - 1$	Translated *upwards/downwards by ____units	$y = x^2 + 3x + 5$
(l)	$y = x^3 - 4$	Translated *upwards/downwards by ____units	$y = x^3 - 5$
(m)	$y = f(x)$	Translated *upwards/downwards by ____units	$y = f(x) - k$

* Circle whichever is applicable.

6. Write down the algebraic form of the new function after the given transformation.

	Equation of C_1	Translated upwards/downwards	Equation of C_2 after translation
(a)	$y = x^2 + 1$	Translated upwards by 8 units	
(b)	$y = x^2 - 1$	Translated upwards by 9 units	
(c)	$y = x^2 + 2x + 3$	Translated upwards by 10 units	
(d)	$y = 3x^2 - x + 1$	Translated upwards by 7 units	
(e)	$y = x^3 + 2$	Translated upwards by 2 units	
(f)	$y = x^3 - 2$	Translated upwards by 6 units	
(g)	$y = f(x)$	Translated upwards by k units	
(h)	$y = x^2 - 4$	Translated downwards by 12 units	
(i)	$y = x^2 + 1$	Translated downwards by 4 units	
(j)	$y = x^2 + x$	Translated downwards by 3 units	
(k)	$y = x^2 + 7x - 3$	Translated downwards by 1 unit	
(l)	$y = x^3 - 1$	Translated downwards by 2 units	
(m)	$y = f(x)$	Translated downwards by k units	

-

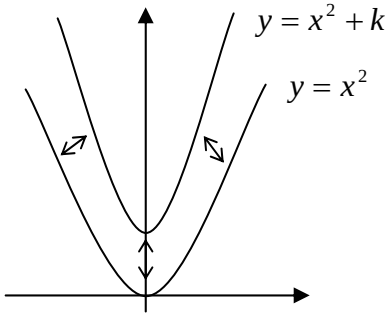
Notes for teachers:

1. The time required for the activities is approximately 30 to 40 minutes

2. When students are asked to calculate values of coordinates in tables of Worksheets 1 and 2, the teacher may not provide any values of *y* and let students calculate all the values. In order to shorten the time for calculation, some of the values of *y* are provided in this exemplar. It is much easier for students to understand the relationship among the algebraic, tabular and their graphical representations when students engage in calculating the values and plotting the graph of function. Meanwhile, some students may also find that the corresponding values of *y* of function C_2 in Question 1 can be obtained by adding three units of the values of *y* in C_1 by observing the pattern in the table.

3. It is time-consuming for all students to plot both graphs of functions $y = x^2 + 3$ or $y = x^2 - 6$. Thus, teacher may instruct half class of students to plot the graph of the function $y = x^2 + 3$ and the remained other half to plot the graph of the function $y = x^2 - 6$. Nevertheless, the teacher should illustrate the correct graphs.

4. Some students have a misconception that when the function $y = x^2$ is translated upwards / downwards by *k* units, the graph will change as follow:



The teacher should remind students that when comparing the same *x*-coordinates of points in the 2 graphs, the corresponding *y*-coordinates will be differ in the same number of units. They should notice that the 2 graphs should not be in the same distance as above. The teacher may use transparencies and moves upwards the transparencies to demonstrate the upward translation of the graph. Then students understand the misconception easily.

5. When students find the new algebraic expression for the function after transformation, they often have a misconception that if the function $y = x^2 + x$ is translated upwards by 3 units, the corresponding function will change to

$y = (x^2 + 3) + (x + 3)$. The teacher may use the software to illustrate the graphs of function $y = x^2 + x$ and $y = (x^2 + 3) + (x + 3)$ and then consolidate students on the understanding of corresponding changes on algebraic expressions.

6. If students have no idea on how to compare the graphs of functions C_1: $y = x^2 - 1$ and C_2: $y = x^2 - 3$, the teacher may suggest students to change the function of C_2: $y = x^2 - 3$ to $y = (x^2 - 1) - 2$. Then students can determine that the function is translated downwards by 2 units.

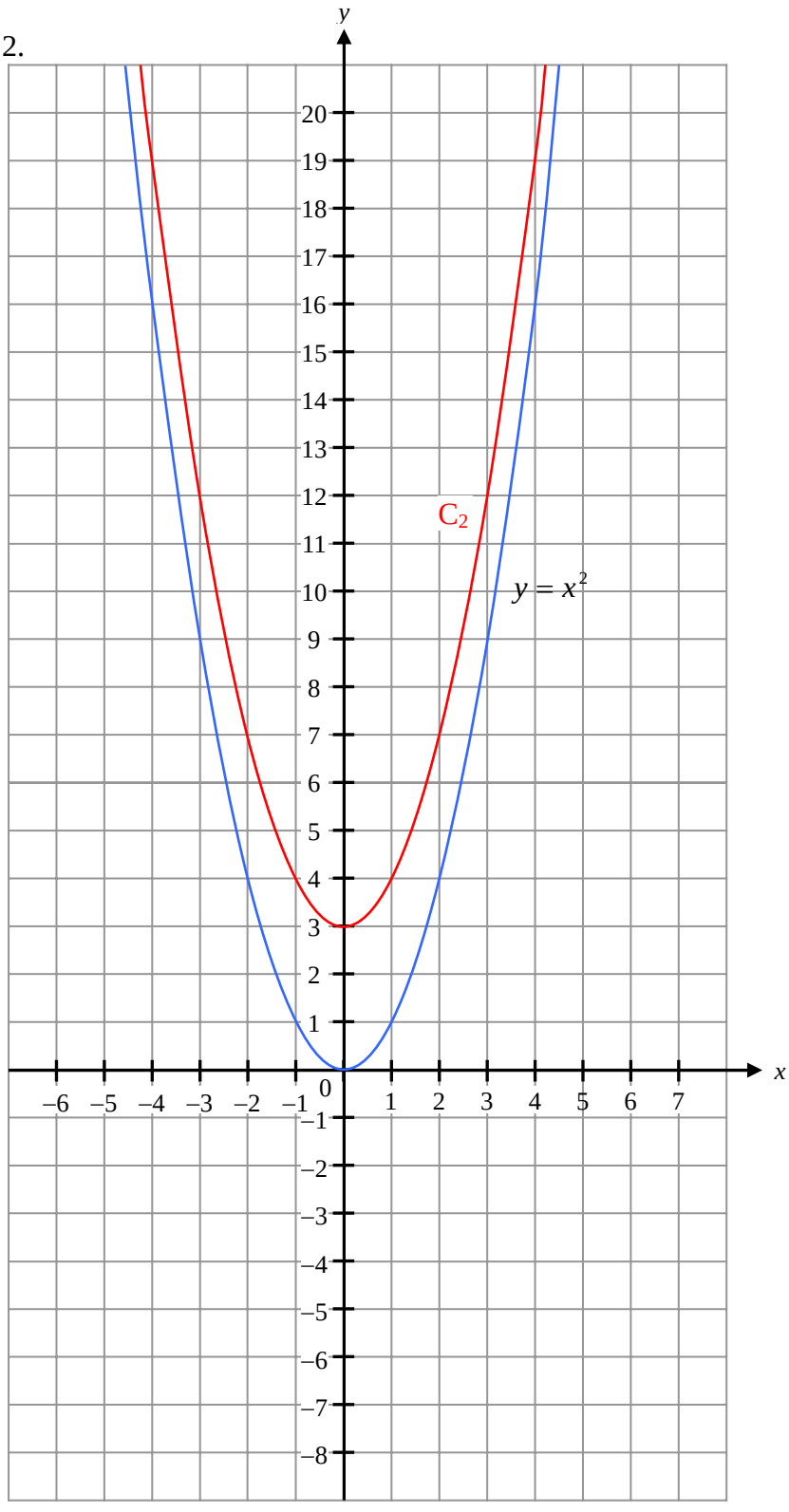
7. The suggested answers for the worksheet are as follows:

Worksheet 1

1. Complete the following table.

x	C_1 $y = x^2$	C_2 $y = x^2+3$
–4	16	19
–3	9	12
–2	4	7
–1	1	4
0	0	3
1	1	4
2	4	7
3	9	12
4	16	19

2.

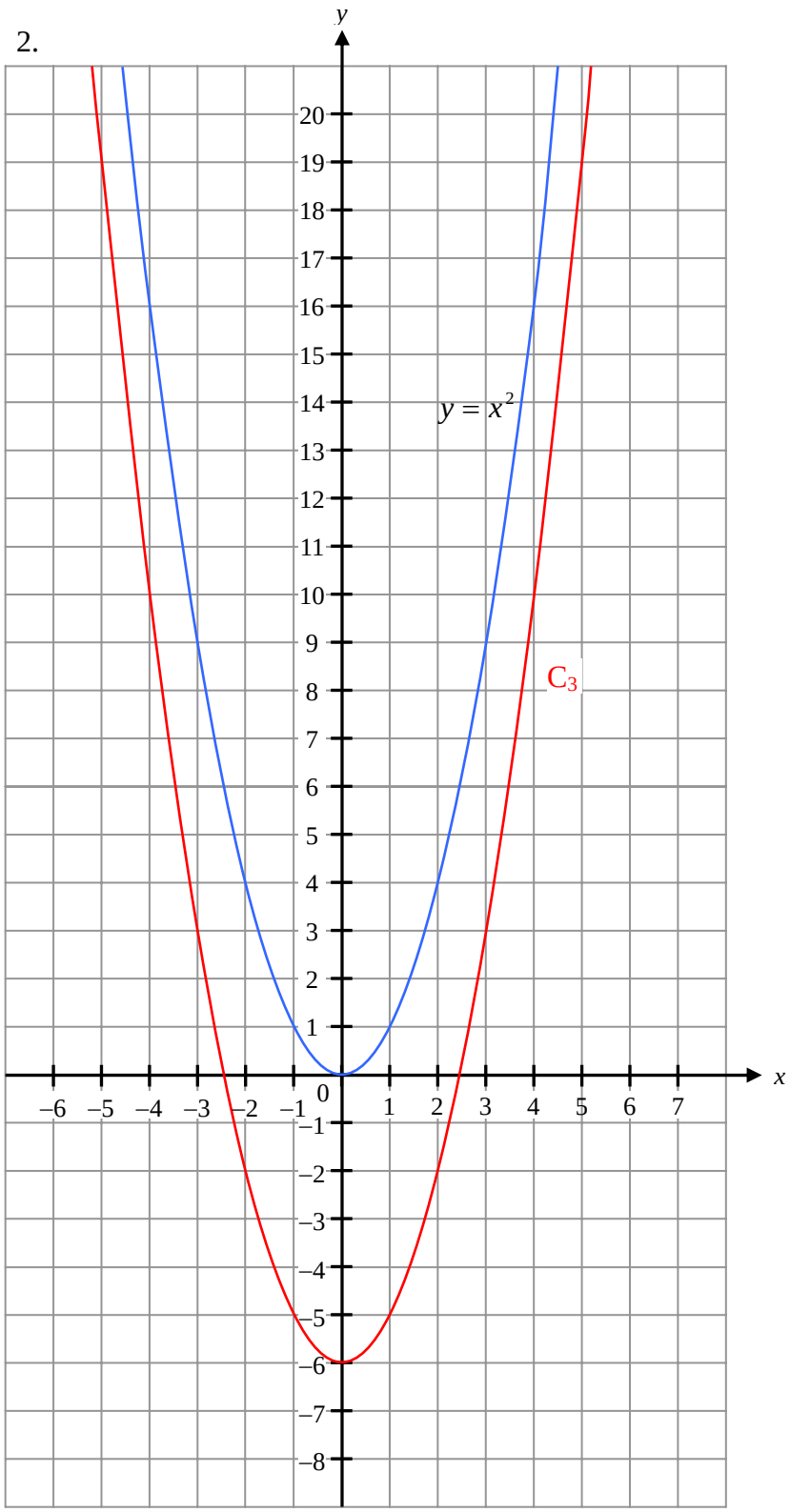


Worksheet 2

1. Complete the following table.

x	C_1 $y = x^2$	C_3 $y = x^2 - 6$
–4	16	10
–3	9	3
–2	4	–2
–1	1	–5
0	0	–6
1	1	–5
2	4	–2
3	9	3
4	16	10

2.



Worksheet 3

1. (a) The graph of function C_2 is translated C_1 downwards by 5 units.

   (b) The equation of C_2 is $y = x^2 - 5$.

2. (a) The graph of function C_3 is translated C_1 downwards by 9 units.

   (b) The equation of C_3 is $y = x^2 - 9$.

3. (a) The graph of function C_4 is translated C_1 upwards by 4 units.

   (b) The equation of C_4 is $y = x^2 + 4$.

4. (a) The graph of function C_5 is translated C_1 upwards by 6 units.

   (b) The equation of C_5 is $y = x^2 + 6$.

5. The table shows the functions after a transformation. Write down the following transformed functions.

	Equation of C_1	Translated upwards/downwards	Equation of C_2 after translation
(a)	$y = x^2 + 1$	Translated *upwards/downwards by 2 units	$y = x^2 + 3$
(b)	$y = x^2 + 5$	Translated *upwards/downwards by 3 units	$y = x^2 + 2$
(c)	$y = x^2 + x$	Translated *upwards/downwards by 1 unit	$y = x^2 + x + 1$
(d)	$y = x^2 + 2x + 1$	Translated *upwards/downwards by 2 units	$y = x^2 + 2x + 3$
(e)	$y = x^3 + 1$	Translated *upwards/downwards by 3 units	$y = x^3 + 4$
(f)	$y = f(x)$	Translated *upwards/downwards by k units	$y = f(x) + k$
(g)	$y = x^2$	Translated *upwards/downwards by 4 units	$y = x^2 - 4$
(h)	$y = x^2 + 2$	Translated *upwards/downwards by 5 units	$y = x^2 - 3$
(i)	$y = x^2 - 3$	Translated *upwards/downwards by 5 units	$y = x^2 - 8$
(j)	$y = x^2 - 5$	Translated *upwards/downwards by 2 units	$y = x^2 - 7$
(k)	$y = x^2 + 3x - 1$	Translated *upwards/downwards by 6 units	$y = x^2 + 3x + 5$
(l)	$y = x^3 - 4$	Translated *upwards/downwards by 1 unit	$y = x^3 - 5$
(m)	$y = f(x)$	Translated *upwards/downwards by k units	$y = f(x) - k$

* Circle whichever is applicable.

6. Write down the algebraic form of the new function after the given transformation.

	Equation of C_1	Translated upwards/downwards	Equation of C_2 after translation
(a)	$y = x^2 + 1$	Translated upwards by 8 units	$y = x^2 + 9$
(b)	$y = x^2 - 1$	Translated upwards by 9 units	$y = x^2 + 8$
(c)	$y = x^2 + 2x + 3$	Translated upwards by 10 units	$y = x^2 + 2x + 13$
(d)	$y = 3x^2 - x + 1$	Translated upwards by 7 units	$y = 3x^2 - x + 8$
(e)	$y = x^3 + 2$	Translated upwards by 2 units	$y = x^3 + 4$
(f)	$y = x^3 - 2$	Translated upwards by 6 units	$y = x^3 + 4$
(g)	$y = f(x)$	Translated upwards by k units	$y = f(x) + k$
(h)	$y = x^2 - 4$	Translated downwards by 12 units	$y = x^2 - 16$
(i)	$y = x^2 + 1$	Translated downwards by 4 units	$y = x^2 - 3$
(j)	$y = x^2 + x$	Translated downwards by 3 units	$y = x^2 + x - 3$
(k)	$y = x^2 + 7x - 3$	Translated downwards by 1 unit	$y = x^2 + 7x - 4$
(l)	$y = x^3 - 1$	Translated downwards by 2 units	$y = x^3 - 3$
(m)	$y = f(x)$	Translated downwards by k units	$y = f(x) - k$

Acknowledgement: Some activities in this exemplar are adapted from the CHC Living Spirit College. Thanks are given to her generosity for granting us to use the materials in this learning package.