Topic Overview	
Topic	BAFS Compulsory Part - Basics of Personal Financial Management C07: Fundamentals of Financial Management – Time Value of Money
Level	S4
Duration	4 lessons (40 minutes per lesson)

Learning Objectives:

1. To understand the fundamental concepts of time value of money;
2. To calculate future value and present value of a single and a series of cash flows;
3. To distinguish between nominal and effective rate of return; and
4. To apply the concepts and calculations of time value of money in personal and corporate financial management.

Overview of Contents:

Lesson 1	Future Value, Simple and Compound Interest
Lesson 2	Present Value and Rule of 72
Lesson 3	Annuity
Lesson 4	Uneven Cash Flows, Nominal and Effective Interest Rate

Resources:

- Topic Overview and Teaching Plan
- Power Point Presentation
- Student Worksheet with Answer

Suggested Activity:

- Problem Solving

Lesson 1	
Theme	Future Value, Simple and Compound Interest
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. Explain the difference between simple and compound interest;
2. Calculate the future value of a single cash flow for one year; and
3. Calculate the future value of a single cash flow for multi-year periods using single and compound interest methods.

Teaching Sequence and Time Allocation:

Activities	**Reference**	**Time Allocation**
Part I: Introduction		
✧ Teacher starts the lesson by taking out a $1,000 bill and asks the class “Who wants this $1,000?” in order to lead to the topic of the time value of money.	PPT#2	5 minutes
Part II: Content		
✧ Teacher introduces the concept of future value and explains how to calculate future value of a single cash flow for one year. ✧ **Activity 1 – Problem Solving (Problem 1)** ■ Students are asked to solve Problem 1 and teacher invites a volunteer to show the calculation to the class. *(Hint: Make sure students are able to perform this basic calculation before introducing the next concept. Teacher can provide more exercises to the students if needed.)*	PPT #3-5 PPT #6-7 Student Worksheet p.1	12 minutes
✧ Teacher extends the calculation of future value to 2 years and explains the concepts of simple and compound interests. ✧ Teacher demonstrates how to calculate the future value of single cash flow for multi-year period using simple and compound interests.	PPT#8-10 PPT#11-16	20 minutes

✧ **Activity 2 – Problem Solving (Problems 2 and 3)** ■ Students are asked to solve Problem 2 and 3 and teacher invites volunteers to show the calculations to the class. *(Hint: Make sure students are able to perform this calculation before moving on. Teacher can provide more exercises to the students if needed.)*	PPT#17-20 Student Worksheet p. 2	
Part III: Conclusion		
✧ Teacher concludes the lesson by recapping different formulas in calculating present value.	PPT#21	3 minutes

Preparation for the next lesson:

Students are asked to review the examples and problems discussed during the class to have a thorough understanding of the concepts and calculations of future value.

Lesson 2	
Theme	Present Value and Rule of 72
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. Explain the concept of discounting;
2. Calculate the present value of a single future cash flow for one year;
3. Calculate the present value of a single future cash flow for multi-year period using annual compounding;
4. Use the Rule of 72 to estimate the number of years required to double the value of an investment; and
5. Use the Rule of 72 to estimate the required rate of return to double the value of an investment in a specific number of year.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher posts a problem related to the calculation of present value to the class.	PPT#22	3 minutes
Part II: Content		
✧ Teacher illustrates the way to calculate present value of a single cash flow for one year. ✧ Teacher further explains the calculation of present value of a single cash flow for multi-year period using annual compounding. ✧ **Activity 3 – Problem Solving (Problem 4)** ■ Students are asked to solve Problem 4 and teacher invites a volunteer to show the calculation to the class. *(Hint: Teacher can provide more exercises to students if needed.)* ✧ Teacher recaps the formulas involving present value calculations.	PPT#23-24 PPT#25-28 PPT#29-30 Student Worksheet p.3 PPT#31	20 minutes

✧ Teacher introduces Rule of 72 to estimate the time in years needed for an investment to double in value. ✧ Teacher demonstrates how to use Rule of 72 and leads students to check the accuracy of the rule. ✧ Teacher explains to students that the rule could be used to estimate the required rate of return for an investment to double in value within a certain number of years. ✧ **Activity 4 – Problem Solving (Problem 5 and 6)** ■ Students are asked to solve Problem 5 and 6. Then teacher asks volunteers to show the calculations to the class. *(Hint: Teacher can provide more exercises to students if needed.)*	PPT#32 PPT#33-35 PPT#36-37 PPT#38-42 Student Worksheet p. 4	15 minutes
Part III: Conclusion		
✧ Teacher concludes the lesson with a review of the key concepts covered.		2 minutes

Preparation for the next lesson:

Students are asked to review the examples and problems discussed during the class in order to have a thorough understanding of the concepts and calculations of present value.

Lesson 3	
Theme	Annuity
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. Explain the features of an annuity;
2. Calculate the present value of an annuity; and
3. Calculate the future value of an annuity.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher posts a problem involving annuity to the class to introduce the concept of annuity.	PPT#43	5 minutes
Part II: Content		
✧ Teacher explains the present value of an annuity can be found by calculating the present values of all cash flows first and then add the present values together to arrive at the answer. ✧ Teacher demonstrates to the class the steps to find the present value of an annuity. ✧ **Activity 5 – Problem Solving (Problem 7)** ■ Students are asked to solve Problem 7 and teacher invites a volunteer to show the calculation to the class. *(Hint: Teacher can provide more exercises to students if needed.)*	PPT#44-47 PPT#48-49 PPT#50-51 Student Worksheet p.5	18 minutes
✧ Teacher explains how to calculate the future value of an annuity and works through the example with students. ✧ **Activity 6 – Problem Solving (Problem 8)** ■ Students are asked to solve Problem 8 and teacher invites a volunteer to show the calculation to the class. *(Hint: Teacher can provide more exercises to students if*	PPT#52-55 PPT#56-57 Student Worksheet p.6	15 minutes

needed.)		
Part III: Conclusion		
✧ Teacher concludes the lesson by reviewing the key concepts covered.		2 minutes

Preparation for the next lesson:

Students are asked to review the examples and problems discussed during the class in order to have a thorough understanding of the concepts and calculations of annuity.

Lesson 4	
Theme	Uneven Cash Flows, Nominal and Effective Interest Rate
Duration	40 minutes

Expected Learning Outcomes:

Upon completion of this lesson, students will be able to:

1. Calculate the present value of a series of uneven cash flows;
2. Calculate the future value of a series of uneven cash flows;
3. Explain the difference between nominal and effective rate of return;
4. Explain the characteristics of flat rate; and
5. Calculate the effective interest rate by the given flat rate.

Teaching Sequence and Time Allocation:

Activities	Reference	Time Allocation
Part I: Introduction		
✧ Teacher extends the discussion of a series of cash flows to uneven cash flows and explains the calculations of future and present value of a series of uneven cash flows.	PPT#58	3 minutes
Part II: Content		
✧ With an example, teacher explains how to calculate the present value of a series of uneven cash flows. ✧ **Activity 7 – Problem Solving (Problem 9)** ■ Students are asked to solve Problem 9 and teacher invites a volunteer to show the calculation to the class.	PPT#59-60 PPT#61-62 Student Worksheet p. 6	5 minutes
✧ With an example, teacher explains how to calculate the future value of a series of uneven cash flows. ✧ **Activity 8 – Problem Solving (Problem 10)** ■ Students are asked to solve Problem 10 and teacher invites a volunteer to show the calculation to the class.	PPT#63-64 PPT#65-66 Student Worksheet p.7	5 minutes

✧ Teacher explains the impact of inflation on the return and the difference between nominal and effective rates of return.	PPT#67-69	10 minutes
✧ Teacher explains what flat rate is and its application. ✧ Teacher demonstrates how to calculate the repayment amount for loans using flat rate and explains the concepts of how the borrower is paying a higher effective interest rate. ✧ Teacher explains how to find the effective interest rate of a loan by the use of the flat rate. ✧ **Activity 9 – Problem Solving (Problem 11)** ■ Students are asked to solve Problem 11 and teacher invites a volunteer to show the calculation to the class.	PPT#70 PPT#71 PPT#72 PPT#73-74 Student Worksheet p. 8	12 minutes
Part III: Conclusion		
✧ Teacher reviews concepts and calculations of time value of money and concludes the lesson.		5 minutes

BAFS Compulsory Part

Basics of Personal Financial Management

Topic C07:

Fundamentals of Financial Management - Time Value of Money

Technology Education Section
Curriculum Development Institute
Education Bureau, HKSARG
April 2009

Introduction

This session aims at providing a fundamental understanding to students on the concepts and calculations of the time value of money. Students will learn the basic skills to solve the problems of time value of money in personal and corporate financial management.

Duration

Four 40-minute lessons

Contents

If you want to have $1,000 from me today ……

- How much would you be willing to pay me back in 1 year's time?
- The value of $1,000 now is different from $1,000 in 1 year's time.

Lesson 1:

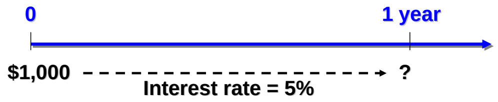
Start the lesson by taking out a $1,000 bill and ask the class "Who wants $1,000?" Then, tell the class that they have to pay back in 1 year's time and ask them how much they would be willing pay back for getting the $1,000 today. Students should know that they have to repay more than $1,000 in 1 year's time.

Points to be highlighted:

- $1,000 now is not the same as $1,000 in the future; and
- The value of $1,000 in the future is higher than the value of $1,000 now. For example, putting the $1,000 now in a deposit and you will get back more than $1,000 in the future as it will earn some interest.

Future Value

- The value of a sum of money in the future
- Example:
  - If you make a 1-year deposit of $1,000 in the bank at 5% interest per year, what is the maturity value (i.e. principal plus interest, or FUTURE VALUE) of the deposit?



Introduce the term "Future Value" which is the value of a sum of money in the future.

Future Value (Cont'd)

earns an interest of $50 (Calculated as $1,000 x 0.05 = $50)

Now $1,000 → 1 year later you'll have $1,050

Explain the calculation of future value with the example.

Explain to the class that interest rates are usually quoted in percent per year and 5% is converted to 0.05 when we calculate the interest amount.

Future Value (Cont'd)

- The interest you will earn is

  = $1,000 × 5%

  = $1,000 × 0.05

  = $50

- Adding the $50 interest to the principal gives you the future value of

  $1,000 + $50 = $1,050

- So, Future value

  = Principal + Principal × interest rate

  = Principal × (1 + interest rate)

  FV = PV × (1 + r)

Summarise the calculation with the formula:

Future value = Principal x (1 + interest rate)

Or FV = PV x (1+r)

(Remarks: For simplicity sake, 1-year period is being used in our discussion here (and annual compounding for the later part). Many banks have calculators on their websites for the calculation of interest or principal plus interest amounts, teacher may show these tools to the class. For example, teacher may go to the following website for demonstration. http://www.bochk.com)

Problem Solving (Problem 1)

- Your mother has bought $4,500 Australian dollars and placed it in the bank on a fixed deposit term of 1 year. If the deposit interest rate is 5.19% per year, how much will she have at the maturity of the deposit?

Activity 1 :

Ask the class to solve Problem 1 in the Student Worksheet p.1 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 1

- At the maturity of the deposit, your mother will have:

$$FV = PV \times (1 + r)$$
$$= A\$4,500 \times (1 + 0.0519)$$
$$= A\$4,733.55$$

Simple Interest

- Suppose you place the \$1,000 deposit for 2 years with an interest rate of 5% p.a.. On SIMPLE INTEREST terms, what will be the future value?

- Simple interest = \$50 per year
- Interest for 2 years = $2 \times \$50$ = \$100
- Future Value = \$1,000 + \$100 = \$1,100

Extend the future value calculation to 2 years. Introduce the concepts of simple and compound interest.

Compound Interest

- But banks normally pay interest once a year and the interest will be added to the principal
- End of 1st year:
  Principal + Interest = $1,000 × (1 + 5%)
  = $1,000 × 1.05
  = $1,050
- End of 2nd year:
  Principal + Interest = $1,050 × (1 + 5%)
  = $1,050 × 1.05
  = $ 1,102.50

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Show the class how to calculate the future value using compound interest. The interest earned in the first year is added to the principal and thus interest is also earned in the second year from the interest of the first year.

Compound Interest (Cont'd)

- Your $1,000 principal earns $50 interest each year
- The $50 interest you earned at the end of the first year will earn $50 × 5% = $2.50 (interest on interest) in the second year
- Total Future Value, thus, equals
  $1,000 + (2 × $50) + $2.5 = $1,102.50

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Show the class the interest earned on the first year's interest (interest on interest) is $2.5.

Finding Future Value

- Suppose you have $10,000 to deposit in a bank and you can earn 6% interest each year over the next 3 years.
- How much would you have at the end of the third year using simple interest?
- How much would you have at the end of the third year using compound interest?
- What is the effect of compounding?

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Reinforce students' understanding on future value calculation by working out another example to find the future value in 3 years time.

Finding Future Value (cont'd)

- SIMPLE INTEREST each year
  = $10,000 × 6%
  = $10,000 × 0.06
  = $600
- 3 years' total interest
  = $600 × 3
  = $1,800
- Total principal + interest using simple interest
  = $10,000 + $1,800
  = $11,800

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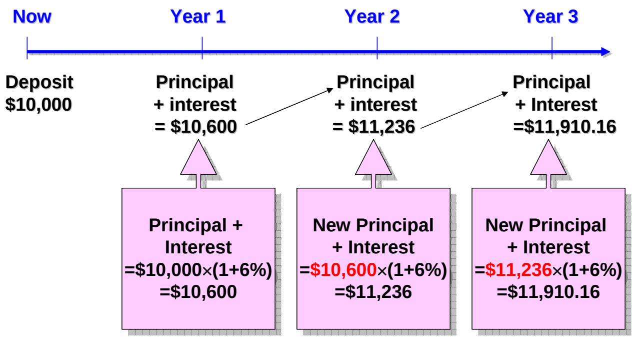
Calculation of the future value using simple interest.

Finding Future Value (cont'd)

Compound Interest



Explain to the class with the time line and show how interest is added to the principal after the 1st year. Second year's interest is calculated by using the increased principal. Similar calculation is done for the third year.

Finding Future Value (cont'd)

- Total principal + interest using COMPOUND INTEREST:

At the end of	Calculation	Principal + Interest
Year 1	$10,000 × (1 + 6%)	$10,600
Year 2	$10,600 × (1 + 6%)	$11,236
Year 3	$11,236 × (1 + 6%)	$11,910.16

Show the calculation of future value using compound interest with a table.

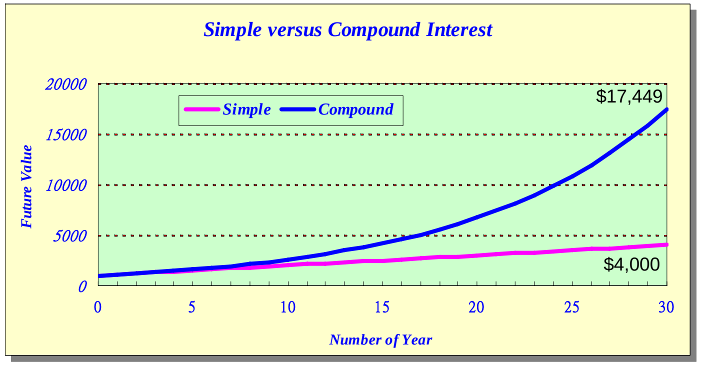
Finding Future Value (cont'd)

- The calculation can also be written as
  $10,000 × 1.06 × 1.06 × 1.06
  = $10,000 × 1.06^3
  = $11,910.16
- Thus, the compounding effect is
  $11,910.16 - $11,800 = $110.16
- So, Future Value
  = Principal × $(1 + \text{interest rate})^{\text{number of years}}$

$$FV = PV \times (1 + r)^n$$

Summarise the calculation of future value using compound interest. Repeat the concept of "interest on interest". Introduce the formula for the calculation of future value.

The Difference between Simple and Compound Interest of 10% on $1,000 for 30 Years is:



Show the chart on the difference in future values using simple and compound interest for a $1,000 principal at an interest rate of 10%. Highlight the striking difference when the time is getting longer. Point out the 'compounding effect' is more significant when the terms of investment is longer.

Problem Solving (Problem 2)

- An investment product promises to pay an 8% return for each year, how much will an initial investment of $100,000 become in 3 years based on compound interest calculation?

Activity 2 :

Ask the class to solve Problem 2 in the Student Worksheet p.2 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 2

- $FV = PV \times (1 + r)^n$

  $= \$100{,}000 \times (1 + 0.08)^3$

  $= \$125{,}971.20$

Problem Solving (Problem 3)

- Your aunt has made a 5-year fixed New Zealand dollar deposit today. The principal amount is NZ$10,000 and interest rate is 6.75% compounded yearly. How much would your aunt get back at the maturity of her deposit?

Activity 2 :

Ask the class to solve Problem 3 in the Student Worksheet p.2 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 3

- $FV = PV \times (1 + r)^n$

  $= NZ\$10{,}000 \times (1 + 0.0675)^5$

  $= NZ\$13{,}862.43$

Future Value Formulas

- A sum of money for a single year

  $FV = PV \times (1 + r)$

- A sum of money for n years using simple interest

  $FV = PV + PV \times r \times n$

- A sum of money for n years using compound interest

  $FV = PV \times (1 + r)^n$

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Conclude Lesson 1 with a recap of the future value formulas.

End of Lesson 1

Would you prefer to get the scholarship now?

- The school has decided to give you a scholarship of $5,000 in 1 year's time, but you may request to be paid now. Would you prefer to get the $5,000 now or a year later? Justify your answer with calculation.

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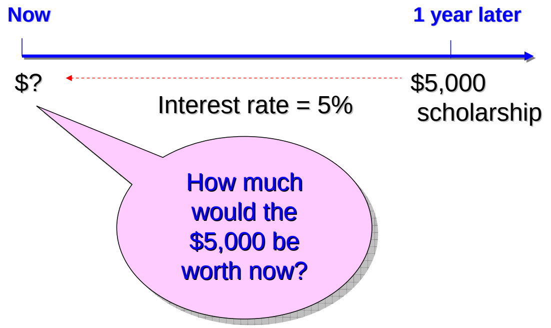
Lesson 2

Let students answer this question.

Introduce the concept of "Present Value" which is the value of a future sum of money now.

Would you prefer to get it now?
(cont'd)



Ask students how much the $5,000 to be received in 1 year would be worth now.

Would you prefer to get it now?
(cont'd)

- Let the amount be PV, if you can deposit it with an interest rate of 5% and get back $5,000 in 1 year time, PV would be

  $PV \times (1 + 5\%) = \$5,000$

  $PV \times 1.05 = \$5,000$

  $PV = \$5,000 / 1.05$

  $= \$4,761.90$
- Thus, $5,000 you receive in 1 year's time is indeed less than $5,000 you receive now.

Explain to students how to deduce the present value of a future amount.

Present Value

- $4,761.90 is PRESENT VALUE of $5,000 in 1 year's time based on the 5% interest rate per annum
- We call this process, "DISCOUNTING"
- The interest rate we used to find the Present Value is called, the "DISCOUNT RATE"

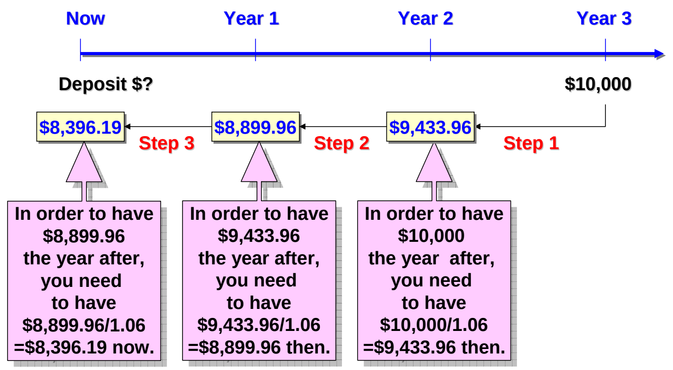
Point out the process of finding out the present value of a future sum of money is called "Discounting" and the interest rate used for discounting is called "Discount Rate".

Finding Present Value

- Today is your 15th birthday and you would want to learn driving when you are 18, and expect that the cost of the driving course would be about $10,000 at that time. If the deposit interest rate is 6% per year, how much would you need to put away now in order to have $10,000 for the driving course three years later?

Extend the calculation of present value for cash flow over 1 year in the future. Point out the compound interest should be used in this case. This example reinforces students' understanding of the calculation of present value.

Finding Present Value (cont'd)



Finding Present Value (cont'd)

Discounting $10,000 from	Present Value
Year 3 → Year 2	$10,000 / 1.06 = $9,433.96
Year 2 → Year 1	$9,433.96 / 1.06 = $8,899.96
Year 1 → Year 0	$8,899.96 / 1.06 = $8,396.19

We can shorten the calculation as:
$\$10,000 / 1.06^3 = \$8,396.19$

Problem Solving (Problem 4)

- How much would you be willing to lend to Thomson with a current interest rate of 3%, if he promises to pay you $1,000 in 1 year time?
- What if Thomson pays you 3 years later?

Activity 3 :

Ask the class to solve Problem 4 in the Student Worksheet p.3 and invite volunteer to show the calculation to the class.

See next slide for the solutions.

Solution to Problem 4

- For 1 year:

  PV= FV / (1 + r)

  = $1,000 / 1.03

  = $970.87
- For 3 years:

  PV= FV / $(1 + r)^n$

  = $1,000 / 1.03^3

  = $915.14

Present Value Formulas

- A sum of money for a single year

  PV = FV / (1 + r)

- A sum of money for n years using annual compounding

  $PV = FV / (1 + r)^n$

Recap the formula for the calculations of present value.

Rule of 72

- A very useful tool in finance and investment
- A quick way to figure out the approximate time (in years) for an investment to double

$$\text{Years to double an investment} \approx \frac{72}{\text{Annual rate of return in percent}}$$

Introduce the Rule of 72 and go through the example to show the mechanism of the rule. Point out that the result from this rule is only an approximation.

Rule of 72 (cont'd)

- A stock is expected to generate a 9% return per year, if you invest $10,000 in the stock, how long would it take for your investment to increase to $20,000?

Ask the class to use the Rule of 72 to solve this problem.

Rule of 72 (cont'd)

- The answer is approximately 8 years, given by:

$$
\begin{aligned}
n &= \frac{72}{r} \\
&= \frac{72}{9} \\
&= 8 \text{ years}
\end{aligned}
$$

where $n =$ years to double an investment

$r =$ Annual rate of return in percent

Check with the class how to use the Rule of 72 to solve this problem.

Rule of 72 (cont'd)

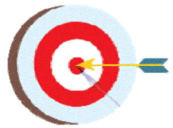
At the end of	Future Value
Year 1	$10,000 × 1.09 = $10,900
Year 2	$10,900 × 1.09 = $11,881
Year 3	$11,881 × 1.09 = $12,950.29
Year 4	$12,950.29 × 1.09 = $14,115.82
Year 5	$14,115.82 × 1.09 = $15,386.24
Year 6	$15,386.24 × 1.09 = $16,771.00
Year 7	$16,771.00 × 1.09 = $18,280.39
Year 8	$18,280.39 × 1.09 = $19,925.63

$$\frac{\$19,925.63}{\$10,000} \approx 2$$

It takes about 8 years to double the investment.

Rule of 72 (cont'd)

- If I want to double my investment in 5 years, what is the annual rate of return to achieve this goal?



Explain to the class that the Rule of 72 can be used to estimate the required return if we want to double an investment within a given period of time.

See next slide for the solution.

Rule of 72 (cont'd)

- By the rule of 72

  $n = 72 / r$

  $\Rightarrow r = 72 / n$

  $= 72 / 5$

  $= 14.4\%$

Problem Solving (Problem 5)

- Today is Jacky's 30th birthday and he is working on a retirement plan. Currently, he has a saving of $100,000 and plan to invest the money in a mutual fund that is expected to have an annual rate of return of 12%. By means of the Rule of 72, estimate:
  - (a) How long will it take for the investment to double?
  - (b) How much would the investment be worth when Jacky is 60 years old?

Activity 4 :

Ask the class to solve Problem 5 in the Student Worksheet p.4 and ask them to tell you the answer.

See next 2 slides for the solutions.

(Remarks: Many students will say the answer for (b) is $500,000 which is incorrect. Their reasoning is that the investment will double every 6 years and in 30 years it will double 5 times, so 5 times $100,000 equals $500,000. But the correct answer should be $3,200,000 calculated as $100,000 times 2^5*.)*

Solution to Problem 5

- (a) By the Rule of 72, it takes about 6 years (i.e. 72/12) for the investment to double.
- (b) The investment period equals 60 – 30 = 30 years, and given the investment will double every 6 years, the investment will double 30 / 6 = 5 times. So $100,000 will grow to $100,000 × 2^5 = $3,200,000.

Solution to Problem 5

At the age of	Investment is worth
30	$100,000
36	$100,000 × 2 = $200,000
42	$200,000 × 2 = $400,000
48	$400,000 × 2 = $800,000
54	$800,000 × 2 = $1,600,000
60	$1,600,000 × 2 = $3,200,000

Problem 6

- Two young people, Marco and Nat have started their new jobs with quite different salaries 12 years ago. Marco started at $5,000 while Nat at $10,000 per month. Macro's salary grew at 18% a year whereas Nat's salary increased at 6% a year.
- They are still working in the same job now, who is making a higher salary?

Activity 4 ：

Ask the class to solve Problem 6 in the Student Worksheet p.4 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 6

- By the Rule of 72, Marco's salary doubled every 4 years (i.e. 72/18=4), and after 12 years his salary has become $\$5,000 \times 2^3 = \$40,000$.
- Nat's salary only doubled once in 12 years (i.e. 72/6=12) to become $20,000.
- Marco is making a higher salary than Nat now.

End of Lesson 2

Which method of payment would you choose?

- A new exciting video game console has recently come onto the market and offers the following two payment methods:

  (a) One time cash price of $9,000; or

  (b) 3 installments of $3,450 at the end of every months

  If the current interest rate is 5% per month, which type of payment method should you choose?

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Time Value of Money

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BAFS Compulsory Part
Learning and Teaching Example

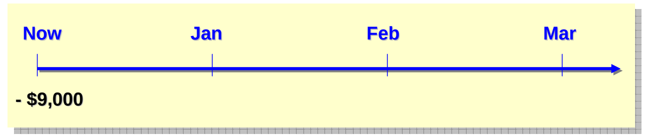
Lesson 3

Ask the class which one they prefer and why.

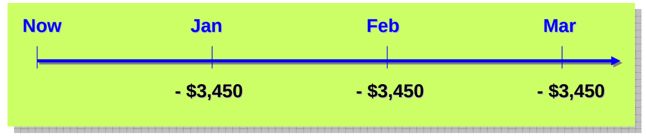
Then walk through the example to show the class how to make the choice.

Which method of payment would you choose?

Payment method (a):



Payment method (b):



Topic C07:
Time Value of Money

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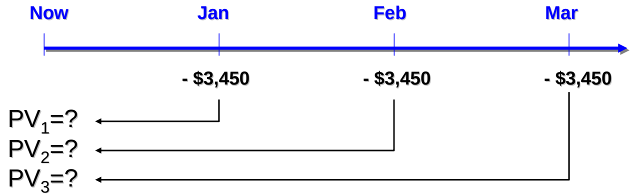
BAFS Compulsory Part
Learning and Teaching Example

Use time line to give students a better idea of the difference between two payment methods.

Which method of payment would you choose?

- To compare the two methods of payment, you can calculate the sum of the present values of the $3,450 payments in the next three months



In order to compare the two methods, we calculate the sum of the present values of the future payments in payment method (b) and use it to compare with the payment in (a).

Which method of payment would you choose?

Calculation of present value:
$PV = FV / (1 + r)^n$

- PV_1= \$3,450/(1+5%)^1
  =\$3,450/1.05
  =\$3,285.71
- PV_2= \$3,450/(1+5%)^2
  =\$3,450/1.05^2
  =\$3,129.25
- PV_3= \$3,450/(1+5%)^3
  =\$3,450/1.05^3
  =\$2,980.24
- $PV_1 + PV_2 + PV_3$ = \$9,395.20
- So you are paying more if choosing payment method (b)

Show the calculation step by step to the class.

Conclude that payment method (a) of paying $9,000 now is more favourable than payment method (b) by installment which is equivalent to paying $9,395.20 now.

Annuity

- The method used in (b) is an ANNUITY payment method
- An annuity represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods
- Payment method (b) shows the calculation of the present value of an annuity

Introduce the concept of "Annuity" which is a finite series of equal cash flows that occur at fixed intervals.

Ask the class to give some examples which have the features of annuity.

Common examples are:

a. Installment payment (or hire purchase);

b. Car loan payment;

c. Mortgage loan payment;

d. Personal installment loan; and

e. Target deposit plan (under this plan, the customers put in an agreed amount by installment over a fixed period. Target savings will be reached in maturity date.)

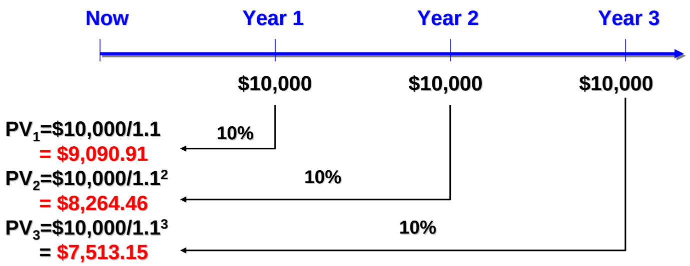
(Remarks: We limit our discussion to 'ordinary annuity', where periodic payments occur at the end of the period. Teacher should highlight to students that there is another type of annuity where periodic payments occur at the beginning of each period, i.e. annuity due.)

Finding Present Value of an Annuity

- A bank is offering an investment product which promises to pay the investor $10,000 at the end of each year for the following 3 years. If an investor expects a return of 10%, how much will she be willing to pay for the investment now?

Walk through the calculation of present value of an annuity in this case to reinforce students' understanding.

Finding Present Value of an Annuity (cont'd)



The investor should be willing to pay = $9,090.91 + $8,264.46 + $7,513.15 = $24,868.52

Topic C07: Time Value of Money — 49 — BAFS Compulsory Part Learning and Teaching Example

The present value of an annuity can be found by first finding the present value of individual payments and then add them up together to get the answer.

Problem Solving (Problem 7)

- Kitty, a rising pop music artist is negotiating with EBI Music Company for the terms of her new 3-year contract. EBI offers two alternatives for her remuneration:

  (a) An upfront payment of $6,000,000; or

  (b) Payment of $2,200,000 at the end of each year for the next three years.

  Suppose the appropriate interest rate is 5%, advise Kitty which package to take.

Topic C07: Time Value of Money — 50 — BAFS Compulsory Part Learning and Teaching Example

Activity 5 :

Ask the class to solve Problem 7 in the Student Worksheet p.5 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 7

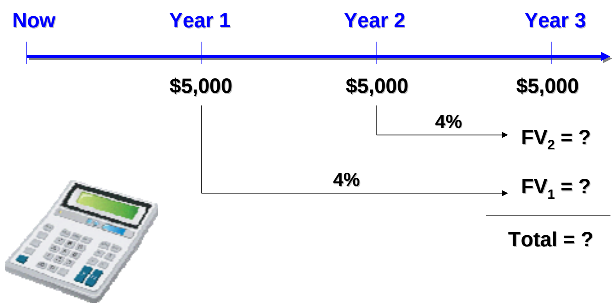
- PV of first \$2,200,000 = \$2,200,000 / 1.05
  = \$2,095,238.10
- PV of second \$2,200,000 = \$2,200,000 / 1.05^2
  = \$1,995,464.85
- PV of third \$2,200,000 = \$2,200,000 / 1.05^3
  = \$1,900,442.72
- PV of annuity = Total PVs
  = \$5,991,145.67 which is less than \$6,000,000.
- Thus, Kitty is better off to be paid \$6,000,000 upfront.

Finding Future Value of an Annuity

- You have decided to save up \$5,000 each year in the coming three years, suppose you can earn 4% interest on your savings, how much would you have at the end of the third year?

Introduce the calculation of the future value of an annuity.

Finding Future Value of an Annuity (cont'd)



The future value of an annuity can be found by first finding the future value of individual payments and then add them up together to get the answer.

Finding Future Value of an Annuity (cont'd)

- The $5,000 you saved up at the end of the first year will become
  $FV_1 = \$5,000 \times (1 + 4\%)^2$
  $= \$5,408$
- The $5,000 you saved up at the end of the second year will become
  $FV_2 = \$5,000 \times (1 + 4\%)$
  $= \$5,200$
- Adding up at the end of the third year, you would have
  $5,408 + $5,200 + $5,000 = $15,608

Show the class step-by-step the calculation of the future value of each cash payment. Point out that the last payment does not earn any interest. Add up the future values to find the answer.

Finding Future Value of an Annuity (cont'd)

Now — Year 1 — Year 2 — Year 3

Year 1: $5,000 | Year 2: $5,000 | Year 3: $5,000

4%: FV_2 = $5,000x1.04 = $5,200

4%: FV_1 = $5,000x 1.04^2 = $5,408

Total = $15,608

Topic C07: Time Value of Money — 55 — BAFS Compulsory Part Learning and Teaching Example

Problem Solving (Problem 8)

- You plan to travel to Japan after your secondary school graduation, and you are expected to save up $2,000 a year for the next 3 years. If you can earn 3.5% interest per year from your savings, how much would you have at the end of the third year?

Topic C07: Time Value of Money — 56 — BAFS Compulsory Part Learning and Teaching Example

Activity 6 :

Ask the class to solve Problem 8 in the Student Worksheet p.6 and invite volunteers to show the calculation to the class.

See next slide for the solution.

Solution to Problem 8

- Total FVs

  $= \$2,000 \times (1+0.035)^2$

  $+ \$2,000 \times (1+0.035) + \$2,000$

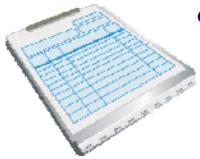
  $= \$2,142.45 + \$2,070 + \$2,000$

  $= \$6,212.45$

Topic C07: Time Value of Money | 57 | BAFS Compulsory Part Learning and Teaching Example

End of Lesson 3

Uneven Cash Flows



- Annuity involves a series of even cash flows, sometimes we may encounter a series of uneven cash flows.
- We can apply the method used in annuity to find the present and future values of a series of uneven cash flows.

Topic C07: Time Value of Money | 58 | BAFS Compulsory Part Learning and Teaching Example

Lesson 4

Start the lesson by introducing a series of uneven cash flows.

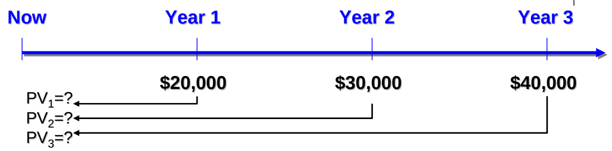
Point out present value and future value of a series of uneven cash flows can be calculated using the same method as in the calculation of those of annuities.

Finding Present Value of Uneven Cash Flows

- There is an investment product which will pay investors $20,000, $30,000 and $40,000 at the end of the first, second and third year respectively. If an investor expects a return of 8%, how much would he be willing to pay for it now?

Demonstrate to the class how to calculate the present value of a series of uneven cash flows.

Finding Present Value of Uneven Cash Flows (cont'd)



Year	Income	Present Value
1	$20,000	PV_1= $20,000 / 1.08 = $18,518.52
2	$30,000	PV2 = $30,000 / 1.082 = $25,720.16
3	$40,000	PV_3 = $40,000 / 1.08^3 = $31,753.29
Total PV = $18,518.52 + $25,720.16 + $31,753.29 = $75,991.97		

Problem Solving (Problem 9)

- Using a discount rate of 7%, what is the total present value of the following series of cash flows?

End of first year	$12,000
End of second year	$22,000
End of third year	$ 5,000

Activity 7 :

Ask the class to solve Problem 9 in the Student Worksheet p.6 and invite volunteers to show the calculation to the class.

See next slide for the solution.

Solution to Problem 9

- PV of $12,000 in the first year
  = $12,000 / 1.07
  = $11,214.95
- PV of $22,000 in the second year
  = $22,000 / 1.07^2
  = $19,215.65
- PV of $5,000 in the third year
  = $5,000 / 1.07^3
  = $4,081.49
- Total PVs = $34,512.09

Finding Future Value of Uneven Cash Flows

- A class of 40 students has decided to set up a fund for a special purpose. Each student will contribute $100, $150 and $200 respectively over the next 3 years. If the interest rate is 4%, how much money will be raised after 3 years?

Explain to the class how to calculate the future value of a series of uneven cash flows. Students should be very familiar with such calculation now.

Finding Future Value of Uneven Cash Flows (cont'd)

Now | Year 1 | Year 2 | Year 3

$4,000 (Year 1) | $6,000 (Year 2) | $8,000 (Year 3)

4%: FV_2 = $6,000 x 1.04
= $6,240

4%: FV_1 = $4,000 x 1.04^2
= $4,326.40

Total = $18,566.40

Problem Solving (Problem 10)

- Based on an interest rate of 7%, what is the future value of the following series of cash flows?

End of first year	$12,000
End of second year	$22,000
End of third year	$ 5,000

Topic C07: Time Value of Money 65 BAFS Compulsory Part Learning and Teaching Example

Activity 8 :

Ask the class to solve Problem 10 in the Student Worksheet p.7 and invite volunteer to show the calculation to the class.

See next slide for the solution.

Solution to Problem 10

- FV of $12,000 in Year 1
  $= \$12,000 \times 1.07^2$
  = $13,738.80
- FV of $22,000 in Year 2
  $= \$22,000 \times 1.07$
  = $23,540
- FV of $5,000 in Year 3
  = $5,000
- Total FVs = $42,278.80

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Flat Rate

- A type of interest rate commonly used
  - Personal installment loan
  - Tax loan
  - Car loan
- The borrower actually pays a higher effective interest rate

Introduce what flat rate is.

Give examples of loans that use flat rates.

Point out that borrowers are paying a rate that is higher than what the flat rate shows.

Flat Rate Example

- A bank is offering a 3-year personal installment loan based on the FLAT RATE of 5% per year.
- For each $1,000 borrowed, total repayment amount is calculated as

  $1,000 + $1,000 × 5% × 3 years = $1,150
- This amount is divided by the number of installments to determine the payment amount ($1,150 / 3 = $383.33)

Explain to the class how flat rate works.

Demonstrate the calculation of repayment amount of an installment loan using flat rate.

Flat Rate Example (cont'd)

- Repayment schedule:

End of Year	Repayment Amount
1	$1,150 / 3 = $383.33
2	$383.33
3	$383.33
Total	$1,150

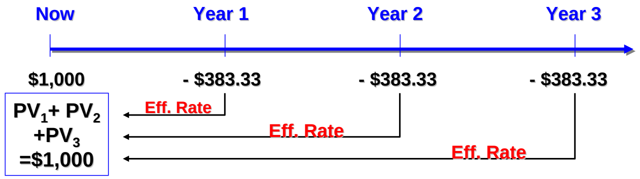
Flat Rate Example (cont'd)

- You can see from the following table, the outstanding principal amount is reduced in Year 2 and 3, the borrower still pays 5% interest on a principal of $1,000.
- Therefore, the effective interest rate is higher than 5%.

End of Year	Principal Repayment	Interest ($1,000 × 5%)	Outstanding Principal
1	$333.33	$50	$666.67
2	333.33	50	$333.34
3	333.34	50	0
Total	$1,000	$150	$1,000

Explain to the class, the borrower is repaying part of the principal starting from the first year but the interest being paid is still $50 which is calculated using a principal of $1,000 and 5% flat rate. The principal is lower but the interest amount remains at $50 and the borrower is paying a higher interest rate.

Flat Rate and Effective Interest Rate



- The effective interest rate is the interest rate that makes the sum of the present values of three $383.33 payments equal to $1,000

Explain how to calculate the effective interest rate on an installment loan using flat rate.

Flat Rate and Effective Interest Rate (Cont'd)

- Through a process of trial and error, we find the effective interest rate to be 7.3269%

$$\frac{\$383.33}{(1+0.073269)^1}+\frac{\$383.33}{(1+0.073269)^2}+\frac{\$383.33}{(1+0.073269)^3} = \$1,000$$

- Which is higher than the 5% flat rate

Through a process of trial and error, the effective rate can be found.

Problem Solving (Problem 11)

- What is the flat rate and effective interest rate for the following installment loan? *(Hint: the rate is between 9 to 10%)*
- For $1,000 loan amount:

End of Year	Repayment Amount
1	$399.29
2	$399.29
3	$399.29
Total	$1,197.87

Activity 9 :

Ask the class to solve Problem 11 in the Student Worksheet p.8 and invite volunteers to show the calculation to the class.

See next slide for the solution.

Solution to Problem 11

- Flat rate = [(1,197.87–1,000)/1,000]/3
  = 0.065957 or 6.5957%
- By trial and error, effective interest rate is found to be 9.6%
- Proof:

  $399.29/1.096 + 399.29/1.096^2 + 399.29/1.096^3$
  = 364.32+332.40+303.29
  = 1,000.01
  (the 1 cent difference is due to rounding)

The End

Topic C07:
Time Value of Money

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BAFS Compulsory Part
Learning and Teaching Example

Review all the key points covered in the lessons.

End of Lesson 4

BAFS Compulsory Part - Basics of Personal Financial Management
Topic C07: Fundamentals of Financial Management - Time Value of Money

Problem Solving

Problem 1: Future Value of a Single Cash Flow

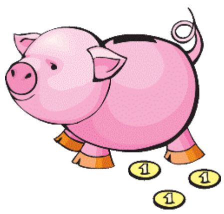
Your mother has bought $4,500 Australian dollars and placed it in the bank on a fixed deposit term of 1 year. If the deposit interest rate is 5.19% per year, how much will she have at the maturity of the deposit?

Problem 2: Future Value of a Single Cash Flow Using Compound Interest

An investment product promises to pay an 8% return for each year, how much will an initial investment of $100,000 become in 3 years based on compound interest calculation?

Problem 3: Future Value of a Single Cash Flow Using Compound Interest

Your aunt has made a 5-year fixed New Zealand dollar deposit today. The principal amount is NZ$10,000 and interest rate is 6.75% compounded yearly. How much would your aunt get back at the maturity of her deposit?



Problem 4: Present Value of a Single Cash Flow

How much would you be willing to lend to Thomson with a current interest rate of 3%, if he promises to pay you $1,000 in 1 year time?

What if Thomson pays you 3 years later?

Problem 5: Rule of 72

Today is Jacky's 30th birthday and he is working on a retirement plan. Currently, he has a saving of $100,000 and plan to invest the money in a mutual fund that is expected to have an annual rate of return of 12%. By means of the Rule of 72, estimate:

(a) How long will it take for the investment to double?

(b) How much would the investment be worth when Jacky is 60 years old?

Problem 6: Rule of 72

Two young people, Marco and Nat have started their new jobs with quite different salaries 12 years ago. Marco started at $5,000 while Nat at $10,000 per month. Macro's salary grew at 18% a year whereas Nat's salary increased at 6% a year.

They are still working in the same job now, who is making a higher salary?

Problem 7: Present Value of an Annuity

Kitty, a rising pop music artist is negotiating with EBI Music Company for the terms of her new 3-year contract. EBI offers two alternatives for her remuneration:

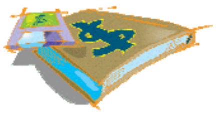
(a) An upfront payment of $6,000,000; or

(b) Payment of $2,200,000 at the end of each year for the next three years.

Suppose the appropriate interest rate is 5%, advise Kitty which package to take.

Problem 8: Future Value of an Annuity

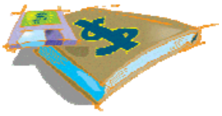
You plan to travel to Japan after your secondary school graduation, and you are expected to save up $2,000 a year for the next 3 years. If you can earn 3.5% interest per year from your savings, how much would you have at the end of the third year?



Problem 9: Present Value of Uneven Cash Flows

Using a discount rate of 7%, what is the total present value of the following series of cash flows?

End of first year	$12,000
End of second year	$22,000
End of third year	$ 5,000



Problem 10: <u>Future Value of Uneven Cash Flows</u>

Based on an interest rate of 7%, what is the future value of the following series of cash flows?

End of first year	$12,000
End of second year	$22,000
End of third year	$ 5,000

Problem 11: Flat Rate and Effective Interest Rate

What is the flat rate and effective interest rate for the following installment loan?
(Hint: the effective interest rate is between 9 to 10%)

For $1,000 loan amount:

End of Year	Repayment Amount
1	$399.29
2	$399.29
3	$399.29
Total	$1,197.87