Mathematics Education
Key Learning Area Curriculum Guide
(Primary 1 – Secondary 6)

Prepared by
the Curriculum Development Council
Preamble

The development of the Hong Kong school curriculum has advanced into a new phase of ongoing renewal and updating. It ushers in a new era for curriculum development to keep abreast of the macro and dynamic changes in various aspects in the local, regional and global landscapes in maintaining the competitiveness of Hong Kong. For the ultimate benefits of our students, schools are encouraged to sustain and deepen the accomplishments achieved since the Learning to Learn curriculum reform started in 2001, and to place new emphases on future needs in curriculum development for achieving the overall aims and learning goals of the school curriculum.

The eight Key Learning Area (KLA) Curriculum Guides (Primary 1 - Secondary 6) have been updated and recommended by the Curriculum Development Council (CDC) to support the ongoing renewal of the school curriculum at the primary and secondary levels.

In updating the KLA Curriculum Guides, the respective KLA committees under the CDC have taken into consideration the concerns, needs and suggestions of various key stakeholders including schools, principals, teachers, students and the public at large. A series of school briefing cum feedback collection sessions coupled with a territory-wide school survey were conducted in 2015 to gauge schools’ views on the major updates of the respective Curriculum Guides.

The eight KLA Curriculum Guides (2017) supersede the 2002 versions. Each KLA Curriculum Guide presents the updated curriculum framework which specifies the KLA’s curriculum aims, learning targets and objectives, delineates the direction of ongoing curriculum development at the KLA level, and provides suggestions on curriculum planning, learning and teaching strategies, assessment, as well as useful learning and teaching resources. In addition, updated examples of effective learning, teaching and assessment practices are provided for schools’ reference. Supplements to some KLA Curriculum Guides and subject curriculum guides are also available to provide further suggestions on their implementation at specific key stages. Schools are encouraged to adopt the recommendations in the KLA Curriculum Guides, taking into account the school contexts, teachers’ readiness and learning needs of their students.

The CDC is an advisory body offering recommendations to the Government on all matters relating to school curriculum development from kindergarten to secondary levels. Its membership includes heads of schools, teachers, parents, employers, academics from tertiary institutions, professionals from related fields or related bodies, representatives from the Hong Kong Examinations and Assessment Authority (HKEAA), and officers from the Education Bureau.
For a better understanding of the interface between various key stages and connections of different learning areas, and how effective learning, teaching and assessment can be achieved, schools should make reference to all related curriculum documents recommended by the CDC and the latest versions of the Curriculum and Assessment Guides jointly prepared by the CDC and the HKEAA for the senior secondary curriculum to ensure coherence in curriculum planning at the school, KLA and subject levels.

As curriculum development is a collaborative and ongoing process, the KLA Curriculum Guides will be under regular review and updating in light of schools’ implementation experiences as well as the changing needs of students and society.

Views and suggestions on the development of the Mathematics Education KLA Curriculum are always welcome. These may be sent to:

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Key Messages

Mathematics Education KLA

Mathematics is essential in the school curriculum as it is a crucial mode of thinking that helps students acquire the ability to explore, conjecture and reason logically, a powerful means of communication, a foundation for the study of other disciplines, and an intellectual endeavor. Mathematics therefore plays an important role in helping students develop necessary skills for lifelong learning.

The Direction of Curriculum Development in Mathematics

In response to the changing needs of society, the rapid development of science and technology, the results of international studies on our education system, as well as views of stakeholders, the Mathematics Education KLA curriculum is developed in a direction to extend the existing strengths, to enhance students’ learning progression and to align with the focal points of ongoing renewal of school curriculum. The focal points that connect with the development of the Mathematics Education KLA include promoting STEM education, information technology in education, and Language across the Curriculum, developing positive values and attitudes, etc.

Aims of Mathematics Curriculum

To develop students’ ability to conceptualise inquire, reason, communicate, formulate and solve problems mathematically; and their capability of appreciating the aesthetic nature and cultural aspects of mathematics.

The Central Curriculum of Mathematics: An Open and Flexible Framework

The central curriculum, in the form of an open and flexible framework, sets out what schools are encouraged to help students develop:

- Subject knowledge and skills as embodied in the learning units under different strands or areas;
- Generic Skills; and
- Positive values and attitudes.

Planning School-based Mathematics Curriculum

- Taking curriculum documents of the Mathematics Education KLA as major references
• Taking into account the school contexts, the overall aims of the Mathematics curriculum and the focal points of the ongoing curriculum renewal (such as STEM education and information technology in education)
• Making use of the flexibility provided by the curriculum framework to cater for learner diversity, to enhance learning progression, and to plan learning and teaching sequences that facilitate cross-KLA learning activities
• Adopting appropriate learning and teaching resources, such as textbooks, e-resources and community resources

Learning and teaching of Mathematics
• Arranging diversified learning activities at different levels, such as hands-on exploratory activities, project work, mathematics reading activities, and activities that based on a topic in Mathematics to integrate relevant learning elements from other KLAs
• Incorporating the use of information technology for interactive learning and self-directed learning
• Adopting different strategies to cater for learner diversity, such as adapting the Mathematics curriculum and using the curriculum space created flexibly for consolidation and enrichment
• Assigning quality homework to consolidate learning, and discouraging mechanical drilling

Assessment
• Arranging assessments to collect ongoing information about the progress of student learning to provide timely and quality feedback for students to improve learning, and for teachers to adjust their teaching strategies
• Providing diversified modes of assessment (such as classroom observation, questioning, open-ended questions, exploratory tasks and projects) for improving learning and teaching
• Making use of suitable assessment tools, such as Learning Progression Framework (LPF) and Student Assessment Repository (STAR) to facilities Assessment for Learning and Assessment as Learning

(For more information on various curriculum matters, please refer to Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (2014) and Secondary Education Curriculum Guide (2017).)
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Chapter 1
Introduction
Chapter 1  Introduction

In response to the changing needs of society, the rapid development of science and technology, the views of stakeholders collected through various surveys and engagement activities as well as the need to align with the direction for the ongoing curriculum renewal of the school curriculum, the recommendations provided in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3) (2002)* have been reviewed. Building on the strengths of Hong Kong students as revealed from international studies and local surveys, the curriculum emphases and content of the Mathematics Education Key Learning Area have been updated to enhance students’ learning progression and to highlight the focal points of the ongoing renewal of the school curriculum, such as Science, Technology, Engineering and Mathematics (STEM) education and IT in education, for further enhancing the quality and effectiveness of learning, hence enabling students to become more effective lifelong learners in the 21st century.

The *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6) (2017)* (this Guide) is prepared by the Curriculum Development Council (CDC) Committee on Mathematics Education. It is an updated version of the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3) (2002)* and has been extended to include the three-year senior secondary Mathematics Education to provide reference for schools in developing a coherent school Mathematics Education curriculum.

The direction for the development of this Guide aligns with the Seven Learning Goals of Primary and Secondary Education (see Appendix 1) and the major recommendations in the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1-6) (2014)* and the *Secondary Education Curriculum Guide (2017)*.

This Guide provides the overall direction for the development of the Mathematics Education curriculum in the five to ten years to come. It reinforces the curriculum emphases provided in the *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 3) (2002)* to enhance learning and teaching and puts forth MRE which take into account the significant development in our society and around the world in various fields, and for the ultimate benefits of student learning. This Guide includes examples relevant to different key stages of learning to illustrate the concepts and ideas introduced and to narrow the gap in curriculum implementation.
Schools should make reference to the following curriculum documents for suggestions on the planning and development of their school Mathematics Education curriculum as well as the strategies for learning, teaching and assessment at different key stages:

- *Mathematics Curriculum Guide (Primary 1 - 6)* (2000);
- *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5)* (1999); and

### 1.1 What is a Key Learning Area?

A Key Learning Area (KLA) is an important part of a curriculum. It is founded on **fundamental and connected concepts within major fields of knowledge** which should be acquired by all students. A KLA provides a context for the development and application of generic skills (e.g. communication, collaboration skills and creativity) and subject-specific skills as well as positive values and attitudes (see Figure 1) through appropriate use of learning and teaching activities and strategies. It serves as a context for the construction of new knowledge and the development of understanding. The studies offered in each KLA may have an academic, social or practical orientation or a combination of these, depending on their purpose(s). They can be organised into subjects, modules, units, tasks or other modes of learning.

![Figure 1](image.png)

*Three Interconnecting Components of the Curriculum Framework*
1.2 Position of the Mathematics Education KLA in the School Curriculum

Students require knowledge and skills that will help them meet the dynamic challenges in the 21st century, which is a knowledge-based information era driven by technology and creativity. Knowledge of Mathematics is a necessity for every individual if they are to contribute towards the development and prosperity of their society. Mathematics and its applications pervade all aspects of life in the modern world. Many of the developments and decisions made in industry and commerce, the provision of social and community services as well as government policy and planning etc., rely on the use of mathematics.

Mathematics is essential in the school curriculum of Hong Kong, as it is:

(a) a powerful means for developing various abilities in a technology-oriented and information-rich society – It helps students acquire the ability to communicate, explore, conjecture, reason logically and solve problems using a variety of methods.

(b) a powerful means of communication – It can be used to present information in many ways like figures, tables, charts, graphs and symbols, which can be processed to generate further information. The presentation skills help students lay a strong foundation for lifelong learning and acquire new knowledge in this rapidly changing world.

(c) a tool for studying other disciplines – It helps students enhance their understanding of the world and provides a basis as well as a foundation to study other disciplines.

(d) an intellectual endeavour and a mode of thinking – It is a creative activity in which students can be fully involved and through which students can demonstrate their imagination, initiative and flexibility of mind.

(e) a discipline, through which students can develop their ability to appreciate the beauty of nature, manage uncertainty and make sound judgements – Mathematical experiences acquired in school enable students to become mathematically literate citizens and contribute towards social prosperity.

Being one of the KLAs that play a more active role in promoting STEM education, the curriculum of the Mathematics Education KLA provides students with a solid knowledge base in mathematics. It also strengthens students’ ability to integrate
and apply the knowledge and skills of STEM-related subjects. As an integral part of general education, mathematics education supports the learning of other subjects. It contributes significantly to the whole-person development of students in primary and secondary schools, prepares them for multiple pathways to post-secondary education and future careers, and hence plays an important role in the Hong Kong school curriculum.

1.3 Rationale and Direction for Development

1.3.1 Rationale for the Development of the Mathematics Education KLA

Before the Learning to Learn Curriculum Reform
In July 1997, an ad hoc committee was set up by the Curriculum Development Council to carry out a review of the Hong Kong Mathematics curriculum. Following two research studies conducted in 1998, the ad hoc committee recommended in its final report (January 2000) that the Mathematics curriculum should be designed according to a set of content-based learning dimensions, the learning of abstract mathematical concepts should be backed up by adequate prior experience of manipulating concrete objects and an abundance of examples, and thinking skills should be developed through mathematical activities. These recommendations had been incorporated in to the Mathematics curriculum as stipulated in the Mathematics Curriculum Guide (P1 – P6) (2000) and Syllabuses for Secondary School – Mathematics (Secondary 1 – 5) (1999).

Learning to Learn Curriculum Reform and Implementation of the New Academic Structure
In 2001, the Learning to Learn curriculum reform was launched to promote a curriculum and pedagogical change at the basic education level to help students become lifelong learners capable of meeting the challenges of a knowledge-based and changing society, globalisation and a competitive economy. In 2005, the report on The New Academic Structure for Senior Secondary Education and Higher Education – Action Plan for Investing in the Future of Hong Kong proposed a 3-year senior secondary and 4-year undergraduate academic system. A more flexible, coherent and diversified senior secondary curriculum was implemented at

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2 The two research studies were: (1) Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries conducted by The University of Hong Kong; and (2) An Analysis of the Views of Various Sectors on the Mathematics Curriculum conducted by The Chinese University of Hong Kong.
Secondary 4 in 2009. The curriculum and assessment reform at the senior secondary level under the New Academic Structure (NAS) is regarded as an extension of the curriculum reform in basic education. The *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6) (2007)* (with updates in November 2015) provides details on the learning, teaching and assessment of the senior secondary Mathematics curriculum (SSMC) under the NAS.

**Review of Senior Secondary Mathematics Curriculum under the New Academic Structure**

There have been reviews on the SSMC in different aspects since its implementation in 2009. The first review was conducted in 2011 and in response to the views collected from the stakeholders, there was fine-tuning of the content and time allocation of the SSMC. The most recent review was conducted in the 2014/15 school year to solicit views of the Mathematics panel heads and teachers on the initial recommendations on the updating of the SSMC at the subject level. Teachers’ views on the curriculum framework of the SSMC were also collected through the questionnaire survey and focus group interviews in the review. It was announced in June 2015 that, as the results of the review, the framework and content of the SSMC was suggested to be kept unchanged for the time being and would continue to be reviewed aiming at the finalised decision to be announced by July 2017.

**Ongoing Renewal of the School Curriculum**

Alongside the implementation of the Learning to Learn curriculum reform, there have been a lot of changes and challenges in society and around the world, including those observed in economic, scientific, technological and social developments. To maintain Hong Kong’s competitive edge and to prepare students well for the local and global changes taking place in various fields, it is necessary to enhance the Learning to Learn curriculum reform, to sustain and deepen the accomplishments achieved and to identify new focuses in the curriculum as we move to a new phase of curriculum renewal and updating.

In late 2015, a school survey was carried out to collect schools’ views on the promotion of STEM education and the updating of the Mathematics Education KLA curriculum (P1 - S6). The results of the survey indicated that it involved schools’ support for the promotion of STEM education as a key emphasis of the Mathematics curriculum, the adoption of e-learning for effective learning and teaching, and the enhancement of the vertical continuity and lateral coherence of the Mathematics curriculum.
Following the results of the NAS review and stepping into the new phase of ongoing curriculum renewal, three Ad Hoc Committees were set up under the CDCC on Mathematics Education in late 2015 to carry out a review of the Mathematics curriculum from P1 to S6 for updating purposes with due regard to the results of the previous school surveys. The direction and strategies for the development of the Mathematics Education KLA curriculum are introduced in the remaining parts of this chapter.

1.3.2 Direction for the Development of the Mathematics Education KLA

In face of the continual local and global changes in various fields, the rapid development of technology, the views of stakeholders, the results of international assessments (e.g. the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS)) on mathematics education in Hong Kong, as well as the direction for ongoing curriculum renewal, the recommendations on planning and implementing the Mathematics curriculum provided in the Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 3) (2002) are revisited. The following focal points of curriculum development are put forth for primary and secondary schools to incorporate into the school Mathematics curriculum to accommodate students’ learning needs arising from the changing contexts and education trends.

(a) Strengthening students’ ability to integrate and apply knowledge and skills through STEM education;

(b) Highlighting the importance of e-learning for enhancing learning and teaching effectiveness, facilitating self-directed learning and nurturing students’ competence in applying information technology for learning;

(c) Highlighting Language across the Curriculum (LaC) in the school Mathematics curriculum, such as promoting reading in Mathematics to develop students’ understanding of the connection of Mathematics with real life and other disciplines; and

(d) Strengthening the development of generic skills and positive values and attitudes in an integrative manner through various Mathematics learning
activities.

The above focal points are also some of the Major Renewed Emphases (MRE) introduced in Booklet 2 of the *Secondary Education Curriculum Guide* (2017). Besides, the development of the Mathematics curriculum is targeted at enhancing students’ learning progressions through the updating of curriculum content. It is also important for the development to be based on the existing strengths.

### 1.4 Strategies for Development

In the ongoing renewal of the Mathematics curriculum, schools could build on their existing strengths, deepen and sustain the accomplishments achieved and identify new areas to focus on, to foster students’ capabilities for whole-person development and lifelong learning. Schools are encouraged to take into account the suggestions and focal points set out in Sections 1.3.2 when planning the school Mathematics curriculum. Schools may select those relevant to their needs, set priorities and integrate them into the school curriculum. The following table summarises schools’ existing strengths and suggested strategies for development to facilitate the ongoing curriculum renewal.

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<td>• Schools agree with the curriculum aims of the Mathematics Education KLA curriculum which cover the development of knowledge, generic skills, and positive values and attitudes.</td>
<td>• Developing the school curriculum continuously by identifying areas for focusing, deepening and sustaining</td>
</tr>
<tr>
<td>• Both students and parents show high regard for Mathematics.</td>
<td>• Participating in research and development activities (such as “Seed” projects) to further develop the school Mathematics curriculum for enhancing students’ whole-person development</td>
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<td></td>
<td>• Enhancing students’ interest and confidence in learning mathematics through various means, such as hands-on activities, STEM activities,</td>
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<tr>
<td>Existing Strengths</td>
<td>Suggested Strategies for Development</td>
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<td>----------------------------------------------------------------------------------</td>
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<tr>
<td>• As revealed in international assessments on Hong Kong mathematics education, (e.g. PISA and TIMSS, Hong Kong has been ranked among the top four of the participating countries/regions in the past decade.</td>
<td></td>
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<tr>
<td>• The proportion of the top performing students is increasing in general, as revealed in TIMSS 1995, 1999, 2003, 2007 and 2011.</td>
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<td>• Most teachers support that STEM education is a new focus of the ongoing renewal of the school curriculum.</td>
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<tr>
<td>• Most teachers support the incorporation of information technology for effective learning, teaching and assessment.</td>
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- Sustaining the existing good practices in learning, teaching and assessment
- Providing students with more opportunities and tools to apply mathematics for problem solving
- Enhancing the design of learning and teaching materials and assessment tasks to cater for students’ diverse abilities, e.g. providing diversified e-learning resources for students to work at their own pace and receive timely feedback.
- Providing STEM learning activities in the best interest of students and within their abilities, e.g. through activities based on topics in the Mathematics curriculum and project work
- Using IT in a well-integrated, pedagogically sound and effective way for the learning and teaching of Mathematics
- Applying IT to facilitate students’ discussion and understanding of abstract concepts
- Supporting students’ self-directed learning by providing suitable
<table>
<thead>
<tr>
<th>Existing Strengths</th>
<th>Suggested Strategies for Development</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e-resources and developing their e-learning strategies</td>
</tr>
<tr>
<td>Mathematics teachers are usually professionally trained.</td>
<td>Arranging teachers’ professional development on focal points of the ongoing curriculum renewal, such as STEM education and IT in education</td>
</tr>
<tr>
<td>Teachers welcome the in-service training provided by the EDB and other professional bodies.</td>
<td>Encouraging teachers to participate in collaborative research projects or study groups for sharing good practices among schools</td>
</tr>
<tr>
<td></td>
<td>Extending teachers’ understanding of the Mathematics curriculum across different key stages for enhancing vertical continuity</td>
</tr>
<tr>
<td>Teachers show high regard for assessing students’ ability through formative and summative assessment.</td>
<td>Adopting diversified modes of assessment, such as hands-on tasks, open-ended questions and problem-based tasks to assess students’ diverse abilities</td>
</tr>
<tr>
<td>Schools are making use of the internal and external assessment results to review and adjust their school curriculum and the learning and teaching strategies adopted.</td>
<td>Deepening the use of more informative feedback to promote Assessment as Learning, in addition to Assessment for Learning, to help students set goals, and monitor, reflect on and evaluate their own learning</td>
</tr>
<tr>
<td>Most schools have taken school-based measures to embrace</td>
<td>Offering enhancement/supporting measures to both ends of the student</td>
</tr>
</tbody>
</table>
### Existing Strengths

- learner diversity, e.g. organising remedial classes for students weak in mathematics.
- There is flexibility in the curriculum design. Enrichment elements (e.g. Enrichment Topics in the primary and junior secondary Mathematics curricula) and Further Learning Units are available in the Mathematics curriculum.

### Suggested Strategies for Development

- population
  - Planning a school curriculum which is adjusted and adapted to meet the needs of both less able students and more able students
  - Making use of the flexibility provided by the Mathematics curriculum to cater for learner diversity and adopt diversified learning and teaching strategies

Although the focal points of the ongoing curriculum renewal suggested in the previous section are not unfamiliar to schools, they are highlighted and supported through the following measures:

- Professional development programmes on the focal points under different categories including curriculum planning, learning and teaching and knowledge enrichment organised for curriculum leaders and teachers;
- Collaborative research and development projects and study groups organised for initialising, promoting, sustaining and improving the incorporation of the focal points in the learning, teaching and assessment of Mathematics (e.g. STEM education and IT in education); and
- Others, such as provision of resource packages, newsletters and reading materials.

Suggestions on curriculum planning, learning and teaching, assessment and learning and teaching resources, and more elaboration on the focal points of ongoing curriculum renewal mentioned in this chapter are discussed in Chapters 3 to 6.
Chapter 2
Curriculum Framework
Learning to Learn 2+ - The Hong Kong School Curriculum

A broad and balanced curriculum with diversification and specialisations (choices) for academic, professional and vocational development according to students' needs.

Nurturing lifelong & self-directed learning capabilities

Multiple pathways

SEVEN LEARNING GOALS

Fostering whole-person development

Five Essential Learning Experiences:
- Moral and Civic Education
- Intellectual Development
- Community Service
- Physical and Aesthetic Development
- Career-related Experiences

Core Subjects:
- Chinese Language
- English Language
- Mathematics
- Liberal Studies

Electives:
- 20 Elective Subjects
- Applied Learning
- Other Languages

Other Learning Experiences:
- Moral and Civic Education
- Aesthetic Development
- Physical Development

Four Key Tasks: Towards major renewed emphases (MRE) at the JS level and beyond
- STEM education & ITE
- Values education (incl. MCE & Basic Law education)
- Language across the Curriculum (incl. reading), etc.

Values & attitudes:
- Seven priority values:
  - Perseverance
  - Respect for Others
  - Responsibility
  - National Identity
  - Commitment
  - Integrity
  - Care for Others

Generic skills:
- Basic skills:
  - Communication Skills
  - Mathematical Skills
  - IT Skills

- Thinking Skills:
  - Critical Thinking Skills
  - Creativity
  - Problem Solving Skills

- Personal & Social Skills:
  - Self-management Skills
  - Self-learning Skills
  - Collaboration Skills

Values, attitudes, skills and knowledge:
- Language
- Nature & Living
- Self & Society
- Arts & Creativity
- Physical Fitness & Health
Chapter 2  Curriculum Framework

2.1 Aims of the Mathematics Education KLA Curriculum

Students need mathematics to meet the dynamic challenges of their future studies, careers or daily life in an information-rich society with rapid development in technology. The overall aims of the Mathematics Education KLA curriculum are to develop in students:

(a) the ability to think critically and creatively, to conceptualise, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines;

(b) the ability to communicate with others, express their views clearly and logically in mathematical language;

(c) the ability to manipulate numbers, symbols and other mathematical objects;

(d) number sense, symbol sense, spatial sense, measurement sense and the capacity to appreciate structures and patterns; and

(e) a positive attitude towards mathematics learning and an appreciation of the aesthetic nature and cultural aspect of mathematics.

The focal points of curriculum renewal suggested in the previous chapter, including the promotion of STEM education, Information Technology in Education and Language across the Curriculum, echo the aims of developing students’ abilities to formulate and solve problems in daily life and other disciplines, ability to communicate with others clearly and logically. The development of Generic Skills and positive values and attitudes, being a continuous emphasis, is also a part of the curriculum aims.

2.2 Components of the Curriculum Framework

The curriculum framework for Mathematics Education KLA is the overall structure for organising the learning, teaching and assessment of Mathematics together with curriculum management, leadership and planning to achieve the overall aims and
learning targets of the Mathematics Education KLA.

Figure 2 shows a diagrammatic representation of the Mathematics Education curriculum framework. The central part of the framework comprises a set of interlocking components including subject knowledge organised under strands, generic skills, and values and attitudes, which sets out what students should learn and develop in the Mathematics Education KLA.

Curriculum management, leadership and planning, as well as effective learning, teaching and assessment of Mathematics involve not only the central part, but also the learning needs of students in the contemporary contexts, including the development of students’ abilities in using language and information technology for learning. As the learning of Mathematics is also connected with other KLAs/subjects, one of the main concerns is integrating and applying knowledge and skills in different subjects, especially in promoting STEM education. Further, effective use of resources and partnership between schools, the EDB and other organisations also lead to successful implementation of the Mathematics Education curriculum.

Further details on the knowledge, generic skills and values and attitudes of the Mathematics curriculum are given below:

2.2.1 Strands\(^3\), Learning Targets and Objectives

Strands are categories of mathematical knowledge and concepts for organising the curriculum. Their main function is to organise mathematical contents for the purpose of developing knowledge, generic skills, and values and attitudes as a holistic process. There are basically three strands in the Mathematics Education curriculum, namely “Number and Algebra”, “Measures, Shape and Space” and “Data Handling”. At the primary level, these three strands are subdivided into five strands (see Figure 2).

\(^3\) The term “Strands” has been referred to as “Dimensions” in earlier curriculum documents such as the Mathematics Curriculum Guide (P1 – P6) (2000) and Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5) (1999).
Figure 2  Diagrammatic Representation of the Mathematics Education KLA Curriculum Framework

* Flexibility in curriculum content is provided by the setting of foundation topics, non-foundation topics and enrichment topics (see Section 2.3 and 2.4 for details). A Further Learning Unit is also included in each key stage.

# Module 1 of the Extended Part consists of “Foundation knowledge”, “Calculus” and “Statistics” and Module 2 consists of “Foundation knowledge”, “Algebra” and “Calculus”.

---

Overall Aims and Learning Targets of the Mathematics Education KLA Curriculum

Curriculum Management, Leadership & Planning

Effective Learning, Teaching and Assessment

in response to the needs of students and the contemporary context

Generic Skills

Knowledge*

Strands of KS 1 and KS2
- Number
- Algebra
- Measures
- Shape & Space
- Data
- Handling

Strands of KS3
- Number & Algebra
- Measures
- Shape & Space
- Data
- Handling

Strands of KS4 Compulsory Part
- Number & Algebra
- Measures
- Shape & Space
- Data
- Handling

Areas of KS4 Extended Part
- Foundation knowledge
- Algebra
- Calculus
- Statistics

Integration and Application (STEM Education)

Values and Attitudes

Resources & Partnership

Information Technology in Education

Language across the Curriculum

---

* Module 1 of the Extended Part consists of “Foundation knowledge”, “Calculus” and “Statistics” and Module 2 consists of “Foundation knowledge”, “Algebra” and “Calculus”.

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To ensure meaningful and effective learning, there must be a coherent plan for students’ learning at the primary and secondary levels. The set of learning targets and objectives, which are geared toward the overall aims of the Mathematics, Education curriculum are organised progressively and systematically across Key Stage 1 (Primary 1 - 3), Key Stage 2 (Primary 4 - 6), Key Stage 3 (Secondary 1 - 3) and Key Stage 4 (Secondary 4 - 6).

As the content of the Extended Part of the senior secondary Mathematics curriculum are interwoven, they are not organised under strands but grouped under the areas of “Foundation knowledge”, “Algebra”, “Calculus” and “Statistics”. Module 1 of the Extended Part consists of “Foundation knowledge”, “Calculus” and “Statistics” while Module 2 consists of “Foundation knowledge”, “Algebra” and “Calculus”. At each key stage, in addition to the curriculum content in the strands and areas, a Further Learning Unit is designed to enhance students’ ability to inquire, reason and conceptualise mathematical concepts and to allow students to integrate and apply knowledge and skills learned in different strands and areas.

The learning content of each strand or area are subdivided into learning units which could, on one hand, reflect the relationship between learning content of similar nature, and on the other hand, enable teachers and students to relate the content in different units.

As mentioned in Section 1.3.1 in Chapter 1, there are updates in the learning units of the Mathematics curriculum at different levels in order to align with the direction of the development of mathematics education and the ongoing renewal of school curriculum. The main purposes of the updates are:

- to enhance the interface of the curriculum across key stages;
- to enhance the support to other subjects;
- to enhance the organisation of curriculum content for betterment of learning and teaching; and
- to provide more specific descriptions on the depth and breadth of curriculum content.

The updated learning units of the curriculum for KS1 to KS4 are shown in the tables on the next pages. Before the implementation of the updated curriculum, schools are advised to make reference to the following documents for Learning
Targets, Learning Units and Learning Objectives of the primary, junior secondary and senior secondary Mathematics curriculum:

- *Syllabuses for Secondary Schools – Mathematics (Secondary 1–5) (1999)*; and
## Overview of Learning Units

### The Learning Units for Key Stage 1 (P1 – P3)

<table>
<thead>
<tr>
<th>Number</th>
<th>Measures</th>
<th>Shape and Space</th>
<th>Data Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Numbers to 20</td>
<td>Length and distance (I)</td>
<td>29. 3-D shapes (I)</td>
</tr>
<tr>
<td>2.</td>
<td>Basic addition and subtraction</td>
<td>Hong Kong money (I)</td>
<td>30. 2-D shapes</td>
</tr>
<tr>
<td>3.</td>
<td>Numbers to 100</td>
<td>Length and distance (II)</td>
<td>31. Directions and positions (I)</td>
</tr>
<tr>
<td>4.</td>
<td>Addition and subtraction (I)</td>
<td>Time (I)</td>
<td>32. Angles</td>
</tr>
<tr>
<td>5.</td>
<td>3-digit numbers</td>
<td>Length and distance (III)</td>
<td>33. Directions and positions (II)</td>
</tr>
<tr>
<td>6.</td>
<td>Addition and subtraction (II)</td>
<td>Time (II)</td>
<td>34. Quadrilaterals (I)</td>
</tr>
<tr>
<td>7.</td>
<td>Basic multiplication</td>
<td>Hong Kong money (II)</td>
<td>35. 3-D shapes (II)</td>
</tr>
<tr>
<td>8.</td>
<td>4-digit numbers</td>
<td>Weight (I)</td>
<td>36. Quadrilaterals (II)</td>
</tr>
<tr>
<td>9.</td>
<td>Addition and subtraction (III)</td>
<td>Length and distance (IV)</td>
<td>37. Triangles</td>
</tr>
<tr>
<td>10.</td>
<td>Basic division</td>
<td>Time (III)</td>
<td>38. Pictograms (I)</td>
</tr>
<tr>
<td>11.</td>
<td>5-digit numbers</td>
<td>Capacity</td>
<td>39. Bar charts (I)</td>
</tr>
<tr>
<td>12.</td>
<td>Multiplication (I)</td>
<td>Time (IV)</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>Division (I)</td>
<td>Weight (II)</td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>Four arithmetic operations (I)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>Fractions (I)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Further Learning Unit

40. Inquiry and investigation

Note: Learning units in the overview are not arranged in the order of teaching.
<table>
<thead>
<tr>
<th>Number</th>
<th>Algebra</th>
<th>Measures</th>
<th>Shape and Space</th>
<th>Data Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Fractions (II)</td>
<td>27. Speed</td>
<td>28. Area (III)</td>
<td>34. Symmetry</td>
</tr>
<tr>
<td>7.</td>
<td>Decimals (I)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Large numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Fractions (III)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Decimals (II)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Decimals (III)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>Fractions (IV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>Decimals (IV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>Decimals (V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>Percentages (I)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>Percentages (II)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Further Learning Unit**

41. Inquiry and investigation

Note: Learning units in the overview are not arranged in the order of teaching.
<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>Measures, Shape and Space</th>
<th>Data Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Basic computation</td>
<td>15. Errors in measurement</td>
<td>28. Organisation of data</td>
</tr>
<tr>
<td>2. Directed numbers</td>
<td>16. Arc lengths and areas of sectors</td>
<td>29. Presentation of data</td>
</tr>
<tr>
<td>3. Approximate values and numerical</td>
<td>17. 3-D figures</td>
<td>30. Measures of central tendency</td>
</tr>
<tr>
<td>4. Rational and irrational numbers</td>
<td>19. Angles and parallel lines</td>
<td></td>
</tr>
<tr>
<td>5. Using percentages</td>
<td>20. Polygons</td>
<td></td>
</tr>
<tr>
<td>7. Algebraic expressions</td>
<td>22. Similar triangles</td>
<td></td>
</tr>
<tr>
<td>8. Linear equations in one unknown</td>
<td>23. Quadrilaterals</td>
<td></td>
</tr>
<tr>
<td>9. Linear equations in two unknowns</td>
<td>24. Centres of triangles</td>
<td></td>
</tr>
<tr>
<td>11. Polynomials</td>
<td>26. Rectangular coordinate system</td>
<td></td>
</tr>
<tr>
<td>12. Identities</td>
<td>27. Trigonometry</td>
<td></td>
</tr>
<tr>
<td>13. Formulae</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. Linear inequalities in one unknown</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Further Learning Unit**

32. Inquiry and investigation

Note: Learning units in the overview are not arranged in the order of teaching.
<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>Measures, Shape and Space</th>
<th>Data Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quadratic equations in one unknown</td>
<td>10. Equations of straight lines</td>
<td>15. Permutations and combinations</td>
</tr>
<tr>
<td>5. More about equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Variations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Arithmetic and geometric sequences and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>their summations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Inequalities and linear programming</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. More about graphs of functions</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Further Learning Unit**

19. Further applications
20. Inquiry and investigation

Note: Learning units in the overview are not arranged in the order of teaching.
### The Learning Units for Key Stage 4 (S4 – S6)

#### Extended Part: Module 1

<table>
<thead>
<tr>
<th>Foundation Knowledge</th>
<th>Calculus</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Exponential and logarithmic functions</td>
<td>4. Differentiation of a function</td>
<td>11. Discrete random variables</td>
</tr>
<tr>
<td></td>
<td>5. Second derivatives</td>
<td>12. Probability distribution, expectation and variance</td>
</tr>
<tr>
<td></td>
<td>6. Applications of differentiation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7. Indefinite integration and its applications</td>
<td>13. The binomial distribution</td>
</tr>
<tr>
<td>9. Approximation of definite integrals using the trapezoidal rule</td>
<td>9. Applications of the binomial and the Poisson distributions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16. Basic definition and properties of the normal distribution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>17. Standardisation of a normal variable and use of the standard normal table</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18. Applications of the normal distribution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>19. Sampling distribution and point estimates</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20. Confidence interval for a population mean</td>
<td></td>
</tr>
</tbody>
</table>

**Further Learning Unit**

21. Inquiry and investigation

*Note: Learning units in the overview are not arranged in the order of teaching.*
### The Learning Units for Key Stage 4 (S4 – S6)

#### Extended Part: Module 2

<table>
<thead>
<tr>
<th>Foundation Knowledge</th>
<th>Calculus</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Odd and even functions</td>
<td>6. Limits</td>
<td>12. Determinants</td>
</tr>
<tr>
<td>5. Introduction to the number $e$</td>
<td>10. Definite integration</td>
<td>16. Scalar product and vector product</td>
</tr>
<tr>
<td>11. Applications of definite integration</td>
<td></td>
<td>17. Applications of vectors</td>
</tr>
</tbody>
</table>

#### Further Learning Unit

- 18. Inquiry and investigation

Note: Learning units in the overview are not arranged in the order of teaching.
2.2.2 Generic Skills

Generic skills can be seen as both process skills and learning outcomes in the Mathematics Education KLA. They are **developed through the learning and teaching of Mathematics** and serve as a **means to help students acquire and master the mathematical knowledge and concepts**. It should be noted that generic skills are **not to be added to but embedded in the learning and teaching of Mathematics**.

Nine generic skills have been identified as essential for student learning for the 21st century in the school curriculum since 2001.

- Collaboration Skills
- Communication Skills
- Creativity
- Critical thinking Skills
- Information technology Skills
- Mathematical Skills
- Problem solving Skills
- Self-learning Skills
- Self-management Skills

“Mathematical skills” and “self-learning skills” have been referred to as “numeracy skills” and “study skills” respectively in earlier curriculum documents. “Numeracy Skills” is renamed to “Mathematical Skills” to represent a comprehensive set of skills in various areas such as measurement, logical thinking, describing patterns and data handling for applications in different disciplines. “Study Skills” is replaced by “Self-learning Skills” for nurturing self-directed lifelong learners.

Based on past experience of implementing the curriculum reform and in response to the dynamic changes in society and recent research, the nine generic skills are grouped in three clusters of related skills, namely Basic Skills, Thinking Skills and Personal and Social Skills, for better integrative understanding and application (see the table below).
<table>
<thead>
<tr>
<th>Basic Skills</th>
<th>Thinking Skills</th>
<th>Personal and Social Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Skills</td>
<td>Critical Thinking Skills</td>
<td>Self-management Skills</td>
</tr>
<tr>
<td>Mathematical Skills</td>
<td>Creativity</td>
<td>Self-learning Skills</td>
</tr>
<tr>
<td>IT Skills</td>
<td>Problem Solving Skills</td>
<td>Collaboration Skills</td>
</tr>
</tbody>
</table>

Daily life applications and higher order thinking skills are emphasised in mathematics education at both the primary and secondary levels. The Mathematics Education curriculum provides numerous opportunities for the development of generic skills such as critical thinking skills, creativity and problem solving skills through problem solving activities in real life and in mathematical contexts.

Details of the nine generic skills and how the Mathematics Education KLA contributes to the development of these skills are available in Appendix 2 of this Guide.

When completing a more complicated learning task, the generic skills are often applied in an integrative manner rather than in isolation. Some generic skills are more likely to be used together. For example, collaboration skills are often used with communication skills and problem solving skills; critical thinking skills tend to be used with problem solving skills and creativity. The former cluster of generic skills can be referred to as “collaborative problem solving skills” and the latter as “holistic thinking skills”. Schools are encouraged to plan learning and teaching activities in a holistic manner whereby the clusters of skills can be effectively developed and applied through classroom learning experiences.

Further elaborations on “collaborative problem solving skills” and “holistic thinking skills” and examples on how the Mathematics Education KLA contributes to the development of these integrative uses of generic skills are provided in Appendix 3.

**2.2.3 Values and Attitudes**

Besides knowledge and skills, the development of positive values and attitudes is also important in mathematics education. According to the revised Moral and Civic
Education Curriculum Framework (2008), seven priority values and attitudes are identified to reflect the uniqueness of Hong Kong as an international city in which both Chinese and Western cultures and values co-exist and interact. They are perseverance, respect for others, responsibility, national identity, commitment, integrity, and care for others. They are of vital importance for students’ whole-person development to meet their own needs as well as those of society.

In the Mathematics Education KLA, values education can be carried out through relevant topics and appropriate learning and teaching activities that help students apply and reflect on the priority values and attitudes, or other relevant ones, which permeate the curriculum in different key stages. The following objectives illustrate how the Mathematics curriculum is related to the development of positive values and attitudes and aim at facilitating the planning of relevant learning experiences in the Mathematics Education KLA. These objectives, however, are neither exhaustive nor do they imply that the related values and attitudes should progress in the sequence presented. In fact, they can be realised at all key stages to different extents.

- Display perseverance in solving challenging mathematical problems.
- Show respect for and acceptance to others in seeking different solutions to a mathematical problem, or in comparing strategies for completing a mathematical project/task.
- Understand and take up one’s responsibilities in group work and develop a sense of commitment by taking up different roles for completing group tasks.
- Foster a sense of integrity in discussing the misuse of statistics in different social contexts.
- Think independently in solving mathematical problems.
- Share ideas and experience, and work co-operatively with others in accomplishing mathematical tasks and solving mathematical problems.
- Be open-minded, willing to listen to others in the discussion of mathematical problems, respect others’ opinions, and value and appreciate others’ contributions.
- Develop interest in learning mathematics.
- Show keenness to participate in mathematical activities.
- Show confidence in applying mathematical knowledge in daily life, clarifying one's argument and challenging others' statements.
- Appreciate the preciseness, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

2.2.4 Developing Generic Skills, and Values and Attitudes

Generic skills, values and attitudes are not developed in a vacuum. They are expected to be fostered through the learning of mathematical knowledge in the content areas. It is desirable for teachers to help students cultivate them through planned learning activities. Figure 3 illustrates how they intertwine to form a reference grid.

![Figure 3](image)

The Mathematics Education KLA provides meaningful contexts for the development of generic skills through appropriate learning and teaching activities and specific topics alongside the specific skills of Mathematics. Schools should plan learning and teaching activities for the Mathematics Education KLA in a holistic manner whereby individual or cluster of generic skills could be suitably and effectively applied and developed. Teachers are encouraged to

- embed elements of generic skills in the setting of the learning goals, as well as the design of the curriculum, learning activities and assessment plans for their schools;
- blend naturally the learning elements of generic skills with subject knowledge, values and attitudes;
• organise learning activities (such as projects, performance tasks and mini surveys) that require students to apply generic skills and reflect on their use of generic skills; and

• create authentic learning environments that bridge students' learning of these generic skills with real world issues.

Examples included in this Guide can illustrate how to link the learning targets and generic skills in learning and teaching. The following table summarises the main learning targets and generic skills from four examples (one from each key stage).

<table>
<thead>
<tr>
<th>Examples</th>
<th>Objective(s)</th>
<th>Main generic skills applied / developed</th>
</tr>
</thead>
</table>
| Example 1 Hand in Hand (Key Stage 1)          | • To understand even and odd numbers                                         | • Communication
|                                               |                                                                              | • Critical thinking
|                                               |                                                                              | • Problem solving                                           |
| Example 8 Knowing Your Community (Key Stage 2)| • To find the eight compass points with a compass                           | • Collaboration                                             |
|                                               | • To become familiar with the district in the vicinity of the school         | • Creativity                                                |
|                                               | • To sketch the plan of the district in the vicinity of the school; to be    | • Critical thinking                                         |
|                                               |   aware of the facilities and services offered in the community             | • Problem solving                                           |
|                                               | • To present the collected data with statistical charts                     |                                                             |
| Example 15 Slopes of Perpendicular Lines     | • To enhance the understanding of relation between slopes of perpendicular   | • Information technology                                    |
| (Key Stage 3)                                 |   lines with the help of IT tools                                           | • Problem solving                                           |
|                                               |                                                                              | • Self-learning                                              |
| Example 20 Modelling the Spread of a Disease  | • To let students recognise the mathematics behind everyday life and apply   | • Critical thinking                                         |
| (Key Stage 4)                                 |   information technology to solve problems                                  | • Problem solving                                           |
**Examples** | **Objective(s)** | **Main generic skills applied / developed**
--- | --- | ---
|  | • To let students recognise mathematics as a powerful tool for planning |  

Besides linking the learning targets and generic skills, positive values and attitudes can also be developed in the context of different learning activities. Such integration comes naturally in the learning and teaching activities for the Mathematics Education KLA. Annotations on Example 8 are provided below for illustration.

In Example 8, the generic skills of collaboration skills, mathematical skills, critical thinking skills, problem solving skills and creativity are involved in the learning activity. While collecting data and constructing graphs in groups, students have to apply collaboration skills when they work out the roles of individual members and share the responsibilities. They also need to learn to respect others during discussion, be responsible and put effort into their part of the project and work cooperatively. Mathematical skills and their prudence are utilised when students formulate and solve the problems. Critical thinking and problem solving skills are involved when students are required to choose relevant information to solve the problems. Through the problem–solving journey, students’ perseverance is enhanced. Their creativity is also developed and strengthened through the process of formulating solutions, presenting ideas in their own ways and suggesting ways of improving their solutions. Moreover, the activity enhances students’ interest in learning mathematics and sensitivity towards the importance of mathematics in daily life.

### 2.3 Curriculum Organisation

Flexibility and diversification are provided in the Mathematics curriculum to help schools and teachers build on their strengths and adopt different modes of curriculum planning to meet different purposes of learning and teaching. At both primary and secondary levels, a “strand approach” is adopted with generic skills, values and attitudes incorporated into the content (see Figure 3 of Section 2.2.4) in the learning and teaching process.
The learning units for each strand are developed by key stages in both the primary and secondary curricula. Teachers are free to select the learning units for planning their school curriculum for each year level among those suggested in the key stage concerned, provided that the units selected are organised logically and coherently. The same idea applies to the learning units for the areas in Module 1 and Module 2 of the senior secondary Mathematics curriculum.

Enrichment Topics are provided in the primary Mathematics curriculum and the junior secondary Mathematics curriculum, while Foundation Topics, Non-foundation Topics\(^4\) are provided in the junior secondary Mathematics curriculum and the Compulsory Part of the senior secondary Mathematics curriculum to suit the different abilities of students. (see Section 2.4 for details)

Schools should make reference to the Basic Education Curriculum Guide (P1 - 6) (2014) and the Secondary Education Curriculum Guide (S1 - 6) (2017) for the suggested time allocation for the Mathematics Education KLA for different key stages, and exercise school-based lesson time planning. Sufficient lesson time should be allocated for Mathematics at all key stages to provide teachers with curriculum space to rearrange or to adapt the content and depth of the teaching materials, to incorporate e-learning strategies into the learning and teaching process, and to organise exploratory or cross-curricular activities based on topics in the Mathematics curriculum. There is lesson time reserved for the Further Learning Units in the curriculum for teachers to incorporate different learning activities in the lessons.

Teachers have the flexibility to design their school Mathematics curriculum to cater for the needs of their students. For example, teachers of Key Stage 3 are suggested to teach the topic Linear Equations in One Unknown at a lower level. After that, they are free to choose the teaching sequence for the following combinations of topics:

(a) Linear Inequalities in One Unknown

(b) Formulas followed by Linear Equations in Two Unknowns

(c) Identities followed by Factorisation of Simple Polynomials

\(^4\) The terms “Foundation Part” and “Non-foundation Part” have been used in the Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5) (1999) instead.
The teaching sequence can be (b), (a), (c) or (b), (c), (a) or some other sequences that suits the needs and abilities of students (see Figure 4). In addition, teachers are giving a free hand to teach the combinations of topics anytime at the key stage. All the topics mentioned become the prerequisites for learning the related topics at Key Stage 4.

2.4 Core and Extension

Apart from meeting different purposes of learning and teaching, the issue of learner diversity has also been considered in the development of the Mathematics curriculum for primary and secondary schools.

The Foundation Topics of the junior secondary Mathematics curriculum and the Compulsory Part of the senior secondary Mathematics curriculum represent the
topics that all students should strive to learn. They constitute a coherent set of essential concepts and knowledge, and are selected in accordance with the following principles in mind:

(a) to include basic concepts and knowledge necessary for the learning content in the Junior Secondary Mathematics Curriculum and the Compulsory Part, and for simple applications in real-life situations; and

(b) to cover topics from different areas to enable students to develop a coherent body of knowledge and to experience mathematics from an all-around perspective.

There are also contents beyond those in the Foundation Topics in terms of depth and breadth. They are identified as Non-foundation Topics and cover a wider range of contents. Teachers can judge for themselves the suitability and relevance of the Non-foundation Topics for their students at KS3 and KS4. On the other hand, no division between Foundation and Non-foundation Topics is made in the primary Mathematics curriculum as most of the concepts at the primary level are basic and fundamental.

To cater for students who have different needs, interests and orientations, the senior secondary Mathematics curriculum includes an Extended Part in addition to the Compulsory Part. The Extended Part is designed for students who need more mathematical knowledge and skills for their future studies and careers. Two modules are offered for students to choose in the Extended Part. Module 1 emphasises applications rather than mathematical rigour and provides students with intuitive concepts of calculus and statistics. Module 2 emphasises understanding of mathematics for further progress in mathematically inclined disciplines and provides students with a concrete foundation in algebra and calculus. The contents of Module 1 and Module 2 are built upon the study of the Foundation and Non-foundation Topics in the Compulsory Part. It is advisable for students to study both the Foundation Topics and Non-foundation Topics in the Compulsory Part if they study the Extended Part.

For more able students at the primary and junior secondary levels, teachers can incorporate some of the suggested enrichment topics such as “curve stitching” at the primary level and “exploring the formula for external point of division” at the junior secondary level) into the curriculum at their discretion to extend students’ horizon and exposure in mathematics. Teachers may also adopt suitable enrichment
topics other than those suggested in the curriculum to suit the interest and abilities of their students. At the senior secondary level, the learning unit “Further applications” is incorporated into the Compulsory Part to enable students to recognise and appreciate the interconnection between the different parts of mathematics they have learnt at both the junior and senior secondary levels. It is also designed for students to integrate and apply knowledge and skills learned in the strands to solve problems in real life as well as in mathematical contexts. It allows teachers to plan different learning activities (such as cross-curricula activities based on mathematics topics) for students. In order to enhance students’ ability to inquire, communicate, reason and conceptualise mathematical concepts, the further learning unit “Inquiry and investigation” is incorporated into the Mathematics curriculum across key stages. Teachers can make good use of the time reserved to engage students in learning activities from different learning units.
Chapter 3
Curriculum Planning
Chapter 3  Curriculum Planning

3.1 A Balanced Curriculum

The structure of the Mathematics curriculum adopts a strand approach. The curriculum at the primary level has 5 strands while at the secondary level 3 strands (see Section 2.2.1 for more details). Designing the Mathematics curriculum in a framework of strands enables the learning objectives and students’ progress to be structured so that a balanced repertoire of mathematical knowledge, generic skills, as well as values and attitudes can be provided. Since students have different learning styles and abilities, teachers have to design their school-based curriculum to cater for the reality of their school. To ensure the internal consistency and the balance of the curriculum, opportunities should be provided to students to master all fundamental knowledge in each strand at each key stage, especially Key Stage 3 and Key Stage 4, at which more flexibility is provided by the curriculum on the planning of topics of different strands for each year level. Care should be taken to avoid over emphasising or putting insufficient lesson time and resources on a particular strand.

In designing school-based Mathematics curriculum, the following matters should be considered: (a) curriculum aims of the central curriculum; (b) focal points of the ongoing curriculum renewal, e.g. STEM education and Information Technology in Education (ITE); (c) cognitive development of students; (d) learner diversity; (e) pedagogical strategies for deep learning in Mathematics; (f) interface between levels; (g) lateral coherence with other subjects; (h) diversified modes of assessment; (i) provision of resources and support; and (j) students’ multiple pathways. The following section gives some basic principles for developing a school-based Mathematics curriculum.

3.2 Central Curriculum and School-based Curriculum Development

The central Mathematics curriculum in this Guide is presented in the form of an open and flexible framework of learning content, generic skills, and values and attitudes which all students are entitled to (please refer to Section 2.1 and 2.2). Schools may adapt the central Mathematics curriculum to develop their own school-based curriculum to suit the needs and abilities of their students, and
contexts of the schools through varying the organisation of

- content, contexts and examples;
- learning and teaching strategies;
- pace of learning and teaching; and
- modes of assessment.

Below are some essential aspects for planning and developing the school-based Mathematics curriculum:

(a) Curriculum documents of the Mathematics Education KLA should be taken as major references on curriculum aims, framework and content as well as lesson time.

(b) Schools are encouraged to set and work on clear and manageable aims and targets taking into account the school contexts, the overall aims of the Mathematics curriculum and the focal points of the ongoing curriculum renewal (such as STEM education and Information Technology in Education, see Section 3.3 and 3.4 below for details).

(c) Schools could make use of the flexibility provided by the curriculum framework to provide students (may be streamed into different groups) with curriculum content fitting their needs and abilities, for example,

i) adapting the depth of treatment of the learning objectives and the logical coordination of the learning content;

ii) planning the learning and teaching of Non-foundation Topics at the secondary level;

iii) selecting some of the enrichment topics in the primary and junior secondary curriculum (or other suitable enrichment topics) on the basis of students’ abilities and interests and the time available to extend students’ horizon and exposure in mathematics;

iv) allowing suitable senior secondary students to take modules in the Extended Part and arranging suitable and flexible lesson time for different groups of students taking the Compulsory Part only and the Compulsory Part plus a module across year levels.

(d) Teachers could adopt different sequencing of learning units for each year level. For example, teachers can

i) arrange the learning sequence so that students learn all strands spirally in
each year;

ii) reserve more periods in S1 to revise and consolidate students’ learning in primary schools and focus only on the Foundation Topics of the curriculum over S2 and S3 to cater for students learning at a slower pace (A more detailed exposition of catering for learner diversity at the school level can be found in Section 4.4.2); or

iii) make use of most of the allocated lesson time for Mathematics in S4 to teach the Compulsory Part first until students are more mathematically mature and are equipped with enough Mathematical concepts and knowledge for the learning of the Extended Part.

(e) Before a new school year, teachers could set and work on learning activities to be carried out in the school year. Teachers are encouraged to provide primary and junior secondary students with more opportunities to engage in hands-on exploratory activities that help develop their abilities to inquire, communicate, reason and apply mathematical concepts. The inclusion of tasks or projects on real-life problem would provide more opportunities for students to engage in a deeper learning.

(f) Teachers could make use of the Learning Progression Framework (LPF) as a reference for planning the assessment of their school-based curriculum. LPF describes the progression of students’ learning in Mathematics across primary and secondary levels. It provides descriptors of students’ learning outcomes as a common language for teachers to describe students’ performance and progress in Mathematics learning. (Please refer to Section 5.2.3 for more elaborations) LPF of Mathematics is available at http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/index.html, placed under the links “Primary One to Six”, “Secondary One to Three” and “Secondary Four to Six”.

(g) Teachers could choose and adapt appropriate textbooks and other learning resources. Good management of resources of different kinds, including real models and e-resources, facilitates the effective discharge of related learning and teaching strategies; and

(h) Schools are encouraged to formulate an assessment and homework policy that promotes Assessment for Learning and Assessment as Learning. Setting and working on the methods and modes of assessment for students of different key stages are essential for achieving the curriculum aims, including the nurturing of self-directed learners.
Curriculum development is an on-going process. Schools should develop their own school-based curriculum whenever appropriate and feasible. They should also encourage the professional development of teachers and collaboration with other stakeholders to achieve the curriculum aims, learning targets and objectives in the Mathematics curriculum framework.

School may refer to Chapter 2 of the *Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 2 of the *Secondary Education Curriculum Guide* (2017) for more ideas on school-based curriculum development.

### 3.3 Strengthening STEM Education

STEM is an acronym that refers collectively to the academic disciplines of Science, Technology, Engineering and Mathematics. The promotion of STEM education aligns with the worldwide education trend of equipping students to meet the changes and challenges in our society and around the world with rapid economic, scientific and technological developments.

In the curriculum context of Hong Kong, STEM education is promoted through Science, Technology and Mathematics Education KLAs. The promotion of STEM education aims to strengthen the Science, Technology and Mathematics Education to nurture diversified talents in the science and technology fields for enhancing the international competitiveness of Hong Kong. The specific objectives include:

- To develop among students a solid knowledge base and to enhance their interests in Science, Technology and Mathematics for further studies and careers in meeting the changes and challenges in the contemporary world
- To strengthen students’ ability to integrate and apply knowledge and skills, and to nurture students’ creativity, collaboration and problem solving skills, as well as to foster their innovation and entrepreneurial spirit as required in the 21st century
- To strengthen the professional capacity of and collaboration among teachers in schools and the partnerships with community stakeholders
To nurture talents and develop experts in STEM areas so as to contribute to the development of Hong Kong and our nation.

In STEM education, Mathematics serves as a discipline that equip students with computational skills, geometric knowledge, data handling skills and logical reasoning skills that facilitate students to integrate and apply their knowledge and skills across different disciplines in solving real-life problems with practical solutions and innovative designs. STEM is not a new strand in the Mathematics curriculum, but it places more emphasis on the applications of mathematics in different context, with the integration of science and technology elements at different levels. In other words, through creating more opportunities for students to apply the mathematical knowledge and skills in identifying and modelling real-life problems that may or may not have a definite solution, and in formulating solutions and eventually solving the problems, STEM education could be strengthened in Mathematics.

On the other hand, real-life problems and the elements of science and technology can be useful examples and context for the learning of abstract mathematical concepts, and thus facilitate the learning of subject knowledge of Mathematics.

When designing school-based curriculum, it is desirable to provide all students with the opportunities to participate in STEM learning activities and let them experience the process of integrating and applying their mathematical knowledge and skills together with those of other STEM subjects. The followings are two suggested approaches that may serve the purpose:

- **Learning activities based on a topic in Mathematics for students to integrate relevant learning elements from other KLAs.** For example, in the topic “Laws of Integral Indices”, students apply the index laws for handling numbers represented in the scientific notation. Related learning elements (such as representing and comparing sizes of cells using scientific notation) from other KLAs could be incorporated in classroom learning activities. After the activities, students could also be assessed for their understanding on the related concepts and skills through problems with cross-KLA elements. There are a number of topics in the curriculum which have closer relations with other STEM KLAs, such as “Ratios” in Number and Algebra, “Mensuration” in Measures, Shape and Space, and “Statistical Graphs” under Data Handling, to list just a few. (Please refer to Section 4.3 for other examples.) In employing
this approach in curriculum planning, teachers may consult their colleagues in other STEM KLAs on students’ prerequisite knowledge in designing the learning and teaching activities. A more detailed illustration on the learning and teaching strategies can be found in Chapter 4.

- **Projects for students to integrate relevant learning elements from different KLAs.** Projects on authentic problems provide opportunities for students to integrate learning elements of different subjects. As the project progresses, students explore the issues in real-life contexts, and the relevant learning elements from Science Education, Technology Education and Mathematics Education KLAs are brought in naturally by the students themselves. Teachers from different KLAs can jointly decide some themes of study for students to start with in the curriculum planning stage. Students may try to tackle some open problems on these themes. In mathematical perspectives, students have more opportunities to go through a complete cycle of mathematical modelling, namely identifying, formulating and solving the problem. Teachers of different KLAs will be the facilitators to support students in different genres of knowledge and skill sets. As a positive consequence, this approach also provides an opportunity for teachers of these KLAs to collaborate and further enhance the effectiveness of learning and teaching. Some examples of implementing this approach in learning and teaching will be discussed in Chapter 4.

Depending on the needs, interests and workloads of students, schools can adopt one or both of the approaches, or other approaches whatever appropriate. In any cases, emphasis should be shifted from doing routine calculations and drilling exercises to problem-based exploratory or mathematical modelling activities in learning and teaching. This encourages students to reach a deeper learning.

EDB will continually support schools in strengthening STEM education through (1) renewing related curricula, (2) strengthening the provision of quality learning experiences to students, (3) offering KLA-based and cross-KLA resource materials to schools, (4) organising professional development programmes for curriculum leaders and teachers, (5) synchronising the contributions from different community key players, and (6) reviewing the development of STEM education and consolidating good practices for dissemination.
3.4 Strengthening Information Technology in Education

With the rapid development of information technology (IT), the learning and teaching of Mathematics can very much be enhanced through proper strategies and careful planning in employing suitable e-resources. In planning the use of IT in the learning and teaching of Mathematics, there are few major concerns: (i) the readiness of hardware infrastructure; (ii) the competencies of teachers and students in basic operations of the commonly used software packages in Mathematics education; (iii) the necessity and advantages in using information technology in the learning and teaching of topics in the Mathematics curriculum; and (iv) the degree of integration of technology with related pedagogy for enhancing the effectiveness of learning and teaching.

With the launching of the Fourth Strategy on Information Technology in Education (ITE4), it is anticipated that the IT infrastructure in schools will be updated and classrooms of school premises will be covered with a stable and high capacity Wi-Fi network. Lessons will thus be enabled with connections with the Internet. Students and teachers can make use of appropriate resources such as cloud storage, cloud computing, and e-platform to enrich the interaction in Mathematics lessons. Some established platforms with interactive e-learning resources for learning and teaching, compatible with widely-used dynamic geometry software and computer algebra systems, can be found on the Internet. These learning and teaching resources and platforms can be used in both desktop computers and mobile devices. Teachers may also beware of the system requirements of certain software packages for Mathematics, especially those for 3-dimensional simulations, in order to ensure a smooth lesson flow.

To carry out the lessons with suitable learning and teaching materials, teachers may consider preparing the materials through self-developing or customising existing resources. In both cases, teachers need to have some understanding of the operations of the software packages they are preparing to use. Teachers can choose the appropriate software packages according to the nature of the learning activities, such as demonstration, simulation, mathematical experiment, and exploration. The developed or customised e-resources could be shared amongst the panel within the school, or even as an inter-school collaboration, with the aid of IT. A carefully planned collection of e-learning materials may form a resource bank for students’ self-directed learning in Mathematics, but it should be noted that learning and teaching instructions should be integrated with the learning and teaching resource
packages.

Last but not least, teachers are encouraged to take into consideration the different advantages of employing e-learning materials in the learning and teaching of certain topics in Mathematics during the curriculum planning stage. For example, in topics involving Mathematical modelling such as Probability or Permutation and Combination, well designed e-learning materials can allow students to conduct mathematical experiments which involve a large amount of repeated computations and may not be done in traditional computing equipment. Computer algebra system and dynamic geometry software can also shorten the computation time in exploration activities and provide good virtual models of complicated geometric figures, when physical models are hard to find in the market. Such appropriate use of IT could enhance learning effectiveness and provoke students’ deep learning. In order to maximise the efficiency in using IT in Mathematics, teachers should consider the following key points on the design of the e-learning materials:

- Clear focus towards a specific learning objective should be identified. Modulised design in the same e-learning package should be employed if it serves more than one focus in the learning objectives.
- Instructions on how the e-learning materials are used should be provided so that students can have a clear target in doing exploratory or problem solving tasks.
- The limitations of the e-learning materials in aspects such as concept building, accumulation of computation experiences, as well as mathematical proofs should be figured out. Other learning and teaching activities should not be neglected, but could be used as the complement measures to ensure a comprehensive introduction of mathematical concepts.

3.5 Strengthening Values Education

Some may regard Mathematics as a neutral and value-free subject and Mathematics in school should likewise be taught in a value neutral way. In fact, most values appear to be learnt and taught implicitly rather than explicitly in Mathematics classrooms. For example, in cases like students need to check the reasonableness of results when they make estimations, or mathematics teachers caution their students to be careful when reading tables and graphs during the lessons, the values and attitudes like reasonableness and carefulness are implicitly taught. Therefore,
through mathematics education, values and attitudes such as carefulness, being systematic and analytic are nurtured in students’ mind. When applying mathematics to solve daily life problems, students have to analyse the situation or problems clearly. They choose and decide which strategies or methods are more appropriate to use. They then apply these methods systematically. In doing so, they can appreciate the beauty and importance of mathematics.

To increase students’ awareness of values, the following strategies can be implemented in mathematics learning and teaching:

- **Routine problems:** The learning of Mathematics should be both challenging and enjoyable experience for all students. The problems should be reasonably difficult and thus requiring efforts from students in attempting to solve them. Students will develop values such as willingness to try, persistence and perseverance when they take up challenges. Likewise, the mathematical problems set must be accessible to the students so that they are able to solve them without too much difficulty and with confidence. They should thus derive a sense of achievement and feel motivated. Nevertheless, the questions set must not be too easy. Students will become bored and the process of learning could become unenjoyable and meaningless.

- **Non-routine problems (including open-ended questions):** Non-routine problems or open-ended questions can be introduced to reinforce or extend students’ understanding of concepts, to help students develop problem solving strategies, and to encourage students to be logical. To solve these problems, students need to discriminate what knowledge is required for certain situations. Sometimes teachers may give problems which contain too much information or insufficient information, so students need to determine what is relevant. Openness, perseverance and creativity are therefore nurtured in students as they cannot just use the bookish knowledge which they have been taught. They need to apply general knowledge and common sense, make their own hypothesis, or accept situations with undetermined solutions.

- **Designing problems:** Students can make up their own problems to enhance their understanding about Mathematical knowledge. This can encourage them to be flexible, and to realise that there can be more than one way of looking at a problem. Further, the teacher can set a theme for the problems that the students make up, such as environment protection, which can help them focus on the underlying values as well as the mathematics.
• **Develop an understanding of the history of mathematics:** Teachers can organise a one-off activity, or a collection of activities around a topic to engage student interest. For example, teachers may assign students to carry out a short research project about the historical development of solving linear, quadratic and cubic equations in one variable. Students are also encouraged to investigate some aspects of Chinese mathematics, for example, the use of abacus for arithmetic calculation; the use of counting board to solve systems of simultaneous linear equations; and the Chinese approach of proving the Pythagoras’ Theorem. In ancient times, Chinese mathematics was very advanced and many important mathematical concepts had been developed, such as the study of circles and π. The ancient Chinese mathematicians also made great contributions on astronomy, perfecting the calendar, algorithm development and algebra. The hardship and persistence of Chinese mathematicians and their contributions may be introduced in the mathematics classrooms.

• **Inquiry and Investigation:** Inquiry and investigation approach encourages critical and logical thinking, being creative and open-minded. The process develops the attitudes of collaboration and respect to one another if group work is involved. Students are also expected to apply mathematical knowledge in a confident and self-motivated way. The use of this approach makes it difficult for students to just carry out routine tasks without thinking about what they are doing. Teachers can provide some guiding questions, for example: Does it always work? Why does that happen? How many cases are there? Is there any connection between this and that? At the last part of the activities, students could be invited to compare their work with classmates, especially if they get different solutions, to discuss who is right. This approach encourages students to be flexible, and to develop the ability to respond to unexpected situations or situations that do not have an immediate solution. It also helps students to develop perseverance in the face of failure. Students can construct their own ideas about mathematics and to be responsible for their own learning.

In conclusion, the above strategies can strengthen values education, involve important mathematics skills and are essential to students’ whole-person development.
3.6 Smooth Transition between Different Key Stages and Multiple Pathways

Students adapt to changes during the transition from Kindergarten to P1 and also between key stages. Their new learning needs arising from the promotion to new schools or a new key stage should be addressed carefully. In planning the school curriculum, students’ development of knowledge, generic skills, values and attitudes through the key stages are taken into account to ensure a smooth transition.

3.6.1 Smooth Transition between Kindergarten and Primary School

Concerning the transition between kindergarten and primary school, there are some points to note:

- Children at the pre-primary level are expected to acquire preliminary mathematics concepts (e.g. counting and comparing) through hands-on activities, games, story telling and daily life experiences. Through diversified activities, children’s communication skills, thinking ability and problem solving skills are nurtured. Number sense and spatial sense can also be fostered with the manipulation of real objects. At this stage, emphasis is being put on developing children’s interest in learning mathematics. Learning activities on different learning areas are usually integrated through thematic approach and playing.

- At the junior primary level, the approach integrating mathematics with children’s experience and focusing on the application of mathematics in daily life are suggested to be continued. Diversified learning activities such as manipulation of real objects, making measurements and construction of lines and shapes, are suggested in the curriculum to arouse students’ interest and develop their basic mathematical knowledge and skills.

- Teachers teaching Primary 1 are advised to consolidate the mathematical concepts in the Number strand and Shape & Space strand as the concepts are foundation knowledge for further studies. The teaching should correspond to children’s development and thinking ability. For example, when geometric shapes are taught, students should be allowed to manipulate and observe geometric shapes and to describe the shapes in mathematical language, such as the proper names of shapes in mathematics and terms like “straight lines”.

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At primary level, homework could be used for consolidation of classroom learning and facilitation of self-learning but the quantity of assignments should be reasonable. Schools are encouraged to regularly review their homework policy in consultation with their key stakeholders, including parents. Schools should also ensure that students have sufficient time for rest and leisure to develop an active and healthy lifestyle.

At the upper primary level, mathematical concepts are further strengthened through diversified learning activities with more emphasis on students’ thinking process and acquisition of concepts. As compared with junior primary, more comprehensive learning activities and exploratory activities are designed for students to enhance their higher order thinking skills. At this stage, students should be encouraged to use appropriate mathematical language along with objects, pictures, diagrams, words and symbols to represent their work. The ability to use abstract symbols should be encouraged and gradually developed.

3.6.2 Smooth Transition between Primary School and Secondary School

The curriculum in junior secondary is built on the learning outcomes of the primary level. It continues the development of the learning of primary mathematics and equips students with prerequisite knowledge and skills for their further learning in senior secondary.

Concerning the transition between primary and junior secondary, there are some points to note:

- Getting familiar with the curriculum as well as the learning and teaching approaches of primary level is essential for secondary teachers to achieve a connected progression in students’ learning. As students from different primary schools may have different mathematics backgrounds, revisiting the foundation knowledge that students have acquired at the primary level can effectively prepare students for the learning content in KS3. Teachers can make use of assessment and consolidation activities to ensure that students have firm understanding on the prerequisite knowledge before proceeding to the new content.

- At primary level, students have learnt several areas of basic mathematical knowledge and the applications of the knowledge in simple situations. In
junior secondary, teachers can progressively introduce more complicated application problems, including some demands a more integrative use of mathematical knowledge in different learning units.

- As the junior secondary level plays the role of transition from the more concrete mathematics content in primary to the more abstract mathematical content in senior secondary along with students’ cognitive development, learning and teaching approaches achieving such a transition are encouraged. For example, for learning units in the Measure, Shape and Space strand, teachers can provide hands-on activities, such as sketching the 2-D representation of simple solids, paper folding and geometric construction, to facilitate students’ development of spatial sense and act as stepping stones for the learning of deductive geometry.

3.6.3 Smooth Transition between Junior Secondary and Senior Secondary Levels

As far as content is concerned, the senior secondary curriculum is built upon the solid foundation laid down in the previous key stages. Just like the layout in the KS3, the contents of the Compulsory Part are categorised into Foundation Topics and Non-foundation Topics. Students learning only the Foundation Topics at KS3 have sufficient background knowledge to study the Foundation Topics in the Compulsory Part of the KS4 curriculum.

In terms of learning and teaching, there are some points to note:

- In junior secondary, teachers explain the concepts in a concrete manner with simple language to facilitate students’ learning; however, in senior secondary, teachers have to present concepts more abstractly and rigorously by using more advanced mathematical language such as set, logic, function etc. Students need to differentiate and integrate the concepts consists of symbols and words in a more abstract way.

- To enhance student’s interest in learning mathematics in KS3, teachers use materials which are more daily life related, concrete, easy to follow, understand and resolve. But in senior secondary, teachers use more advanced mathematical languages, more abstract and rigorous learning and teaching materials, create a more mathematically rich learning environment and delve into deeper understanding of mathematical concepts and knowledge, all these lead to a higher demand on knowledge and skills of students. Students may
find it difficult to adapt to the new learning environment at the beginning, which may in turn diminish their interest in studying mathematics. Teachers are therefore advised to closely observe students’ learning behavior and provide timely assistance.

- Most topics in senior secondary level requires students to integrate the knowledge and skills grasped in KS3, for example, solving 3-D problems in trigonometry is an integration of both trigonometry and 3-D figures in KS3. Integrated learning arrangements should be designed to encourage students to reason about mathematical relations. However, it is crucial for teachers to consolidate and reify students’ previous knowledge so as to pave a way for them to build up new knowledge and connect the knowledge with those gained in KS3. In particular, teachers have to help students to clear out those misconceptions obtained in KS3.

- To develop students’ ability to explore different problem solving strategies and to integrate various parts of mathematics, the learning unit “Further Applications” is added in the senior secondary. Teachers can make good use of the learning unit to guide students to integrate various parts of mathematics which they have learnt, and thus recognise the inter-relationships between the concrete objects in junior forms and abstract notions in senior forms, and strengthen their abilities in applying mathematics in problem solving.

3.6.4 Supporting Learners in Educational and Vocational Pathways

While the primary Mathematics curriculum focusses on developing students’ understanding of basic mathematical concepts and acquisition of basic computation skills, the junior and senior secondary Mathematics curriculum aims to prepare students for a range of post-secondary pathways including tertiary education, vocational training, employment, etc. The senior secondary Mathematics curriculum, as introduced in Section 2.3 and 2.4, consists a Compulsory Part to provide knowledge and skills necessary for students’ different career pathways; and an optional Extended Part for students who need more mathematical knowledge and skills for their future studies and careers.

The Mathematics curriculum equips students with mathematical knowledge and skills for vocational and professional education and training (VPET), and allowing them to explore and develop their potential talents and career interests. For
example, students develop through the Mathematics curriculum essential skills in computation, measurement and data handling for applied learning courses (e.g. courses in the area of studies “Engineering and Production”).

3.7 Cross KLA Links

Mathematics is the foundation of many other disciplines and provides supporting knowledge for them. It is linked to the other KLAs by providing a basis for making investigations as well as a tool for analyzing data, representing findings and models with symbols, graphs and charts, and for theorizing. And in reverse, other KLAs provide opportunities for students to apply mathematics in real-life situations.

It is important to relate students’ daily life experiences to Mathematics. For some topics (like “Using percentages” and “Uses and abuses of statistics”), integration with other KLAs (like Science Education or Personal, Social and Humanities Education) is one of the ways of organizing students’ learning experiences mathematically. Integrated learning removes the boundaries of subjects and reflects the interdependent nature of reality and the complexities of life. It provides students with a holistic context for learning and enables students to make connections between what they are learning in class and problems in the real world. An example showing how this is done can be found in Example 8 at the end of this Guide. Some of the links between the Mathematics Education KLA and the other KLAs are exemplified in the following paragraphs.

Cross KLA Links
In Chinese Language Education and English Language Education, Mathematical concepts are essential if students are to understand essays with mathematical and statistical content. When reading non-continuous texts, students have to extract relevant information from tables, charts and graphs, etc. to explain facts, infer results and draw logical conclusions.

In Arts Education, lines and shapes are important elements to create pictures, models and buildings, and patterns and symmetry are often explored in creative dance. Students can also apply tessellation in designing different patterns of tiles.

In Physical Education, Mathematics can help analyse sports data and design an appropriate strategy for improving sporting performance.
In the Personal, Social and Humanities Education KLA, a variety of Mathematical tools and procedures are used for making rational and responsible social decisions, such as identifying patterns and trends in statistical data and assessing validity in personal and social issues. Mathematical models are also used for theorizing in Social Sciences, and in particular in Economics.

In the Science Education KLA, laws and formulae are represented in Mathematical language. Mathematical methods are employed to solve problems, organise data and generalize experimental findings, and mathematical models are used to represent physical phenomena. In the Technology Education KLA, mathematical knowledge supports the writing of programmes. Mathematical models are used in computer simulations to explore the feasibility of realising some design ideas, and tables and charts are important tools in the representation of technical information. In the General Studies for Primary Schools, mathematical knowledge supports students to participate in design-and-make activities and project work. The role of Mathematics in STEM education is discussed in Section 3.4.

Linkage with Liberal Studies
Independent Enquiry Study (IES) is an integral component in the curriculum of Liberal Studies. It contributes to the curriculum aims by helping students develop higher-order thinking skills and communication skills through investigative exploration of issues. Students have to collect data, evidence and information through means appropriate to their enquiry. They also have to analyse and evaluation information in order to construct knowledge, propose solutions to real issues, and make decisions.

In conducting an issue of IES, students should know how to use a number of methods of data collection and analysis to suit different topics and contexts. The Mathematics curriculum can provide a solid background to the methods used in quantitative researches (questionnaire survey, statistics, etc.). Students should be aware of the limitations in data collection in terms of questionnaire design, sampling methods and sample sizes. As students have learnt a variety of statistical graphs or charts, such as pictogram, bar chart, pie chart, stem-and-leaf diagram, histogram, and broken line graph in Mathematics, they can choose an appropriate statistical graph to present a set of data in their issues of IES. They should be able to understand that different choices of statistical graphs may lead to different interpretations of the same set of data and result in conclusions in different perspectives.
3.8 Time Allocation

The suggested time allocations for the primary Mathematics curriculum and secondary Mathematics curriculum can be found in Chapter 2 of the *Basic Education Curriculum Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014), and Booklet 2 of the *Secondary Education Curriculum Guide* (2017) respectively. They are also summarised in the table below:

<table>
<thead>
<tr>
<th>Level</th>
<th>Suggested time allocation</th>
<th>Lesson time (over 3 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KS1 (P1-P3)</td>
<td>12 – 15%</td>
<td>285-356 hours</td>
</tr>
<tr>
<td>KS2 (P4-P6)</td>
<td>12 – 15%</td>
<td>285-356 hours</td>
</tr>
<tr>
<td>KS3 (S1-S3)</td>
<td>12 – 15%</td>
<td>331-413 hours</td>
</tr>
<tr>
<td>KS4 (S4-S6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compulsory Part</td>
<td>10 – 12.5%</td>
<td>250-313 hours</td>
</tr>
<tr>
<td>Compulsory Part with a module</td>
<td>15%</td>
<td>375 hours</td>
</tr>
</tbody>
</table>

The suggested time allocations are provided to aid schools and teachers to plan their lessons. Schools could adjust the time allocation required for the curriculum concerned to suit the abilities of their students.

Schools are also encouraged to use the designated “flexible time” to conduct cross-curricular learning activities, e.g. values education, Reading across the Curriculum and STEM-related activities, to ensure the whole-school curriculum is broad and balanced, in terms of promoting whole-person development. The flexible time for the primary and junior secondary levels is 19% and 8% respectively in each key stage. As for the senior secondary level, the 10-15% time allocation is set aside for Other Learning Experiences (OLE). Schools should give due consideration to the overall planning and co-ordination among different KLAs and subjects with flexibility to organise OLE inside and outside the school timetable.
Chapter 4
Learning and Teaching
Chapter 4  Learning and Teaching

4.1  Guiding Principles

In selecting or devising approaches and strategies for effective learning and teaching of Mathematics, the following principles should be noted:

- The development of lifelong and self-directed learners is the main concern, with emphases on nurturing students’ skills to face the challenges of the contemporary world.
- All students are able to learn, but they may learn at different pace and styles.
- The learner-centered approach should continue be emphasized, with the harnessing of information technology for effective learning and teaching.
- There should be a balance between what students are expected to learn in the central Mathematics curriculum and aspects of school-based curriculum development based on the needs of the students in school.

The primary, junior secondary and senior secondary Mathematics curricula have been developed in line with these principles. Their main focus is not only on what mathematical topics should be learnt but also on how mathematics is effectively learnt. The acquisition of generic skills and the fostering of positive values and attitudes should be embedded within the learning of mathematical content. The teachers’ role is to help students to learn how to learn mathematics and develop their self-directed learning skills.

In respond to the economic, scientific and technological developments in the contemporary world, the learning and teaching of Mathematics should include elements on the development of students’ ability to integrate and apply the knowledge and skills of Science, Technology and Mathematics subjects. The use of IT for effective and interactive learning should also be promoted.

As students possess different learning abilities and styles, teachers have to master their learning characteristics to raise the effectiveness of learning and teaching. There is flexibility in the Mathematics curriculum. For example, the arrangement of foundation and non-foundation topics in the junior secondary Mathematics
curriculum provide teachers with a viable way to adapt the mathematics curriculum to suit the needs of students of different abilities. Strategies on catering for learner diversity are discussed in Section 4.3.

A learner-centered approach is recommended. Due consideration should be taken of students’ abilities, experience, interests and learning styles when designing a learning and teaching approach. Diversified learning and teaching activities are encouraged to suit the different abilities and interests of students. To align with the worldwide education trend of equipping students to meet the changes and challenges in our society and around the world with rapid economic, scientific and technological developments, greater emphasis is placed on getting students to apply mathematical concepts to solve problems as this provides students with the motivation for learning mathematics. STEM education is promoted through solving daily life problems with practical solutions and innovative designs. The integration of IT in the learning and teaching of Mathematics for facilitating understanding of concepts, timely assessments and students’ self-directed learning is also encouraged.

Apart from the formal Mathematics curriculum, mathematics-related extra-curricular activities also play an important role in mathematics learning. It is generally agreed that well chosen and organised mathematics-related activities help to promote students’ interest in learning the subject and nurture students’ ability to apply mathematics. Examples include mathematical games, puzzles, competitions, trails, projects, etc.

### 4.2 Approaches to Learning and Teaching

To address the needs of our students to face the challenges of the 21st Century and to help them develop the capability of learning to learn, appropriate learning and teaching approaches and strategies are essential for facilitating students’ construction of mathematical knowledge, and their development of generic skills and positive values and attitude. Approaches such as direct instruction, enquiry learning and co-construction could be adopted flexibly to facilitate students’ acquisition, connection and construction of mathematical concepts and knowledge.

**Direct instruction**: When adopting the direct instruction approach, teachers provide information and explanation on the learning content and present factual
materials in a logical manner. Teachers’ lecturing breaks complex ideas into simpler pieces/steps. This approach is most relevant to contexts which involve explanation, demonstration or modelling to enable students to gain knowledge and understanding of particular concepts. Although direct instruction is more teacher-centered, it can be interactive, and be enhanced by the aid of IT for visualizing abstract concepts.

**Enquiry learning**: This approach places emphasis on the process and action undertaken by the learner. It focuses on students’ thinking and inquiring process, which promote critical thinking and problem solving skills. Students are invited to make connections between facts, make predictions, and debate alternatives. The approach essentially relies on peer interaction in groups or the whole-class. The enquiry activities involve open-ended questions, group discussion, investigations, experiments, hands-on exercises, and application software for exploration.

**Co-construction**: This approach requires the whole-class to form a community of learners, and to learn through student-student and student-teacher interactions. Discussion is provoked by creating a problem solving environment allowing students to exchange their ideas freely. During the knowledge construction process, teachers provide demonstrations, prompts, feedback, and opportunities for students to explain their arguments, while students develop social skills, organise thinking and develop rational arguments.

Teachers’ flexible and integrative use of approaches allows different learning objectives of the school-based curriculum be achieved effectively. No matter which approach do they deploy, teachers are more than a transmitter of “truth” or “knowledge”. They have the roles of resource person, collaborator and facilitator of learning. Teachers have the responsibility of delivering clear explanation, designing and conducting exploratory activities in lessons, providing suitable hints and feedback, creating an open atmosphere for discussion and showing concern for students’ progress through suitable assessments.

Among various learning and teaching strategies, the Four Key Tasks and Life-wide Learning (LWL) are proposed to help students develop independent learning capabilities and to enable students to gain experiences that are more difficult to acquire in ordinary classroom setting, within and across Key Learning Areas.

It is important to develop students’ competencies in connecting learning to real-life
contexts in a proactive manner, which would results in deep learning. Teachers may adopt approaches and strategies mentioned above to design or identify challenging tasks to enhance students’ intrinsic motivation in learning and to help them apply their learning in real-life contexts. The use of the Four Key Tasks and LWL, together with the role of e-learning and homework in the learning and teaching of Mathematics, are discussed below.

4.2.1 The Four Key Tasks

The Four Key Tasks, namely moral and civic education, reading to learn, project learning and information technology for self-directed learning can help students to develop independent learning capabilities and hence help to realize the spirit of “learning to learn” in schools. Over the past decade, schools have been familiar with incorporating the Four Key Tasks into their school-based curriculum. Schools are recommended to build on their existing strengths to flexibly adopt them with deepened understanding to focus on the focal points for ongoing renewal of the school curriculum.

Moral and Civic Education

- Mathematics education plays a role as part of the school curriculum in providing holistic and balanced learning experiences, creating conducive learning environment and enhancing students’ understanding of positive values from different perspectives for the implementation of moral and civic education. In Mathematics, the moral and civic elements could be introduced in the following ways:

  - Inclusion of open-ended questions in the learning and teaching of Mathematics to develop students’ attitude of respecting others. In tackling open-ended questions, students not only need to evaluate their own solutions or strategies, but also evaluate alternative solutions suggested by others. Through discussion or debating on the solutions or strategies, students are led to accept different points of view and respect for others and the value of moral and civic education is then built among students intrinsically.

  - The inclusion of daily life examples in the mathematics classroom will enhance students’ awareness of its relation to real life. For example, for the learning and teaching of statistics, teachers could make use of the
environmental statistics on air, waste, noise and water provided by the Environmental Protection for classroom activities. Teachers may use statistical diagrams to illustrate the wastes dumped in landfill to let students aware of the importance of environmental protection.

- By project work, students are provided with opportunities to take up responsibilities in group work and develop a sense of commitment by taking up different roles for completing the group tasks. Leadership, social skills and a sense of collaboration are also fostered.

**Reading to Learn: Towards Reading across the Curriculum**

- In a knowledge-based society, schools play an essential role to equip students with reading to learn skill. In general, schools have established a reading culture for students of different levels. With the implementation of reading scheme of Mathematics, or systematic provision of mathematics reading materials, students’ interest and knowledge in mathematics are enhanced, and at the same time, students are provided with opportunities to apply and consolidate reading skills and strategies developed in language lessons.

- Schools are encouraged to build on the groundwork of the present achievements in promoting “Reading to Learn” and further motivate learners to read mathematics reading materials to broaden their knowledge base and enhance students’ literacy and ability in handle multimodal reading materials made up of text, diagrams and mathematical symbols. To strive for improvement and enhancement of the quality of “Reading to Learn” in Mathematics and to promote “Reading across the Curriculum”, schools could review their own strategies in promoting “Reading to Learn”. Suggestions on strategies include:

  - The effective use of library resources facilitates the promotion of “Reading to Learn”. Schools can borrow from the public library, or purchase for their school library, mathematics reading materials that are appropriate to students’ levels, language competence and interests.

  - Appropriate e-reading materials could be provided for students to read online for the development of independent learning. This includes articles of mathematical journals or magazines. Students can read those materials by devices such as mobile phones, tablets, or desk-top computers. If the reading materials chosen are relating to the Mathematics curriculum, they
can act as students’ pre-lesson preparation or as their further studies materials.

- Teachers could categorise reading materials to fit students of diversified interest and abilities. Reading materials of diversified themes, e.g. applications of mathematics in daily life, history of mathematics, and stories of famous mathematicians, could help students connect their learning experiences within the content of Mathematics to the mathematics in daily life, cultural aspects of mathematics and other disciplines.

- Besides reading records, teachers may design suitable post-reading activities for students to share their reading experience. This can also reinforce students’ development of conceptions of the application of mathematics in different disciplines or the cultural aspects of mathematics. To cater for students’ different characteristics, teacher may allow students to make their presentation in varied ways, such as verbal reports, drawings or role-play.

- Mathematics teachers could collaborate with English and Chinese teachers to help students transfer reading skills and strategies to the reading of mathematics books or passages for enhancing effectiveness.

- Teacher could become role models of the students and share some mathematics recreational book reviews with them. It encourages students to develop their reading habits and eventually establishes a good reading culture in schools.

(Please see Example 22 of this Guide for suggestions on organising and implementing reading schemes for Mathematics.)

Project Learning

- Apart from normal classroom learning and teaching of Mathematics, project learning is an effective mean to enrich students’ learning and arouse their interests in mathematics. It provides alternative learning experiences and is a powerful learning and teaching strategy to promote self-directed, active, and self-reflective learning. It enables students to connect knowledge, generic skills, values and attitudes when constructing their mathematical knowledge.

- The theme of project learning in mathematics can be a specific mathematical
topic, such as prime numbers, or a study on using mathematics to solve a real-life problem that connects mathematics to the real world and other disciplines. Project learning is therefore an effective approach for promoting cross-KLA learning and STEM education (See Section 3.4 and Section 4.3). When themes on STEM education are involved, collaboration among KLAs is strongly recommended to help students connect and integrate different learning experiences.

- Project Learning provides substantial opportunities for students’ development of the nine generic skills. It enhances students’ mathematical skills as data analysis and data presentation by charts and graphs are often involved. Other elements of mathematical skills, such as performing calculations and describing patterns might also be applied, depending on the themes of projects. Besides Mathematical skills, other generic skills are also nurtured in project learning. Collaboration skills, self-learning skills and self-management skills are applied when students are required to plan and manage their individual and group work systematically. While creativity, critical thinking and problem solving skills are nurtured when students study the project questions, investigate them from different perspectives and explore different approaches to tackle them.

- Some issues only arise in group work and while handling them, students learnt to listen and respond to alternative ideas and perspectives, respect others, delegate responsibilities, and resolve conflicts. Positive values and attitudes were provoked through these processes.

- In project learning, the role of teachers changes from knowledge transmitter to tutor, adviser and facilitator. Teachers need to provide continuous supports, advices, monitoring and regular feedbacks to students to develop their research skills in project learning. Once the students are becoming more independent in the learning process, teachers can merely act as facilitators. Well established guideline and clear learning objective can help students conduct their projects effectively. Teacher may use samples of projects to exemplify the key elements of good projects and lead students to improve the quality of their projects.

- Examples 3, 9, 12 and 13 of this Guide illustrate how projects can be adopted for the learning and teaching of mathematics. Indeed, there are many other topics and types of projects, such as surveys, investigations on applications of
mathematics, studies on the history of mathematics or famous mathematics problems, and biographies of mathematicians. The areas of study in project learning should be carefully chosen to suit students’ interests and abilities. The depth of treatment of the projects can also vary according to students’ abilities. Similarly, different levels of guidance can be provided to cater for students’ abilities, the focus and the difficulty and nature of the project. Projects can be done individually or in groups depending on their nature. Collaboration and communication skills, however, will be more effectively developed by projects done in groups. Project work can also be considered as an effective tool of Assessment for Learning. It could be assessed holistically by some pre-determined criteria (see Section 5.2).

**Information Technology for Self-directed Learning**

- The rapid advancement in information technology in the past decades has lead to its wide applications in education. The use of mobile devices and application software in lessons has become increasingly popular, and has brought a considerable impact on the learning and teaching of Mathematics. As discussed in Section 3.4, the use of IT in Mathematics education is to harness its advantage to enhance student learning. Instead of merely applied, it should be used with aligned effective pedagogies and meaningful learning tasks for promoting student learning. The pedagogies and technology chosen should fit the characteristics of the topic to be delivered. Teachers could refer to the TPACK\(^5\) framework, which describes the integration of technological knowledge, pedagogical knowledge and content knowledge, for more ideas on the integration of information technology with suitable pedagogy for the effective learning of a certain topic.

- Nowadays, most classrooms in Hong Kong are internet-connected to make e-resources accessible. With the help of Wi-Fi network and mobile devices, mathematics lessons are provided with multimedia resources, authentic data, application software packages, communication/sharing platforms and other e-resources for interactive learning and exploratory activities. With the help of software packages, abstract concepts can be presented in multiple representations to enhance students’ understanding. Moreover, with the

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\(^5\) TPACK, stands for Technological Pedagogical and Content Knowledge, is a framework that helps us understand and describe the kinds of knowledge needed by a teacher for developing effective pedagogical practice in a technology enhanced learning environment. More information about TPACK is available at [http://www.matt-koehler.com/tpack/tpack-explained/](http://www.matt-koehler.com/tpack/tpack-explained/)
introduction of IT to classrooms, there is a paradigm shift of mathematics teaching from teacher-centered approach to a more interactive learner-centered approach. During the lesson, students could make use of the Wi-Fi network to share their ideas with teachers and classmates.

- Nurturing students to apply IT tools and resources in learning Mathematics is important. This also equips students with tools for self-directed learning. When learning mathematical concepts, students can use application software for simulations, analysis and modelling. For example, the dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum. With the help of IT tools, students can easily plot graphs for visualizing mathematical relations, propose conjectures and make connection between different areas of mathematics. Appropriate use of IT tools can save students’ effort in routine computations or presentations and create time and space for deeper understanding and thinking. Examples 15 and 19 at the end of this Guide show how IT can be utilized in the learning and teaching of Mathematics.

In many cases, more than one of the key tasks are connected to achieve a number of learning objectives. For example, the project of investigating the work of ancient mathematicians requires students to use IT skills to search for information and evaluate information. Students also need to read books on history of mathematics. Past endeavors of ancient mathematicians and collaboration among students during the project are conducive to the development of perseverance, self-management and other qualities promoted through Moral and Civic Education. The Four Key Tasks should be flexibly used in the learning and teaching of Mathematics to enhance students’ capabilities for constructing knowledge, to promote the development and applications of generic skills in an integrative manner and to nurture positive values and attitudes.

(School may refer to Chapter 3 of the Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6) (2014) and Booklet 6 of the Secondary Education Curriculum Guide (2017) for more ideas on the Four Key Tasks.)
4.2.2 Life-Wide Learning

Learning is not confined to classroom. The provision of life-wide learning (LWL) opportunities enables students to learn mathematics from experience in real contexts. This sort of learning experience cannot always be provided in the classroom. The choice of life-wide learning activities should be both school-based and learner-centered.

At senior secondary, LWL is effectively carried out through various experiential activities organised by the school for the curriculum component of OLE. These learning experiences with quality reflection are essential for students’ whole-person development and fostering their lifelong learning.

There are many opportunities for students to learn mathematics through experience outside classrooms. Typical examples include:

- Conducting real statistical survey for the election of student union in the school;
- Inviting guest speakers to deliver talks on some interesting mathematics topics or real-life applications of mathematics, such as cryptography, history of mathematics, paper folding of polyhedron, mathematics games, etc.;
- Setting up mathematics corner or mathematics room for students to play mathematics games, investigate mathematics puzzles and construct 3-D figures;
- Encouraging students to participate in different mathematics competitions, such as:

<table>
<thead>
<tr>
<th>Competitions</th>
<th>Organising bodies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Project Competition for Secondary Schools</td>
<td>Mathematics Education Section, EDB</td>
</tr>
<tr>
<td>Mathematics Book Report Competition for Secondary Schools</td>
<td>Mathematics Education Section, EDB</td>
</tr>
<tr>
<td>Statistical Project Competition for Secondary School Students</td>
<td>Hong Kong Statistical Society and the EDB</td>
</tr>
<tr>
<td>Statistics Creative-Writing Competition for Secondary School Students</td>
<td>Hong Kong Statistical Society and the EDB</td>
</tr>
</tbody>
</table>
### Competitions

<table>
<thead>
<tr>
<th>Competitions</th>
<th>Organising bodies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong Mathematics Olympiad</td>
<td>Mathematics Education Section, EDB and Department of Mathematics and Information Technology of EdUHK</td>
</tr>
<tr>
<td>Hong Kong Mathematics Creative Problem Solving Competition for Primary Schools</td>
<td>Gifted Education Section, EDB and Hong Kong Federation of Education Workers in collaboration with Hong Kong Aided Primary School Heads Association</td>
</tr>
<tr>
<td>Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools</td>
<td>Gifted Education Section, EDB and Hong Kong Federation of Education Workers in collaboration with Hong Kong Aided Primary School Heads Association</td>
</tr>
</tbody>
</table>

School may also refer to Chapter 6 of the *Basic Education Guide — To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 7 of the *Secondary Education Curriculum Guide* (2017) for more ideas on life-wide learning.

#### 4.2.3 e-Learning

e-Learning refers to an open and flexible learning mode involving the use of electronic media, including use of digital resources and communication tools to achieve the target learning objectives. In the context of Mathematics Education, e-learning can be integrated for enhancing learning and teaching through,

- **Teachers’ use of IT for presentation of abstract concepts**
  Teachers could apply different software package, e.g. graphing tools, virtual 3-D manipulatives and dynamic geometry software for multiple representations of abstract concepts so as to enhance students’ understanding.

- **Students’ application of software for exploration**
  Students could apply their IT skills for inquiry and investigation using suitable application software. With the help of graphing software, spreadsheet or computer algebra system, students could concentrate more on analyzing the problem or formulating solutions. (See Example 15 of this Guide)

- **Interaction and collaboration on e-platform**
  The school e-platform enhances students’ interaction and collaboration both inside and outside the classroom. Students can extend their learning by participating in mathematical activities, discussions or collaborative projects on e-platforms. Teachers may also use the platform to upload notes, past
papers and videos for students to revise.

- *e-Platform for learning outside classroom*
  With the e-learning environment, students are equipped with the tools for independent learning outside the classroom. For example, students could be assigned to study a new mathematical concept before lessons via passages or short videos on the school e-platform and use the lesson time for further learning through discussion, collaboration, problem solving or exploration. In such a way, students are expected to have a greater engagement in self-directed learning.

Along with the increasing use of IT, students have more opportunities to apply their information technology skills. In the learning and teaching of Mathematics, students have to search information efficiently and evaluate it critically. They also need to organise data, analyse the results and make presentation systematically.

It should be stressed that IT is a useful mean to facilitate learning and teaching but the importance of teachers’ professionalism to provoke the learning of mathematics through IT should be highlighted. To provide support to mathematics teachers, the Education Bureau has developed the One-stop Portal to help teachers select and develop appropriate teaching materials to suit their students’ needs. Please refer to Section 6.1 for more details.

### 4.2.4 Meaningful Homework

Homework assignment is an extension and consolidation of learning. Quality homework encourages independence in learning after lessons. Its purposes are:

1. To develop and consolidate what students have discovered or learned
2. To reinforce and maintain the concepts and skills developed during lessons
3. To enable students’ self-learning
4. To encourage students to think independently, analyse, formulate and solve problems, and to assess situations and make decisions
5. To lead students to appreciate that mathematics is meaningful, enjoyable and useful, and develop their interests in learning Mathematics.
6. To help teachers evaluate students’ learning performance and progress
7. To help teachers identify students’ learning problems and adjust their teaching plans and strategies.

Apart from written homework, student assignments may include discussions through e-platforms, group work, book reading, project work and other activities. When setting assignments, the following should be borne in mind:

1. The quantity of each assignment should be appropriate in order to motivate students to learn. It is undesirable to assign too many at one time. The frequency should be regulated so that students will not be unduly overburdened. Coordination among subjects is desirable.

2. Each assignment should have a clear learning objective. Questions selected should be geared to the objective. Too hard or too easy questions should be avoided.

3. The exercises of a good textbook could be the main part of the assignment. Those designed by teachers themselves or derived from other sources could be complementary or supplementary, e.g. fundamental questions for less able students and more demanding questions for abler students.

4. Variety of questions in each assignment is essential because it maintains students’ interest and also provides different ways of learning.

5. Selection of questions should cater for students’ learner diversity. Core questions should be identified and assigned to students of general ability. Less should be laid on the two extremes.

6. For topics on application of mathematics, exercises should be designed with emphasis on practicality and closely related to daily life situations.

(School may refer to Chapter 8 of the Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6) (2014) and Booklet 4 of the Secondary Education Curriculum Guide (2017) for more ideas on meaningful homework.)

4.3 Learning and Teaching for STEM Education

As discussed in Section 3.4, promoting STEM education in Mathematics is a focal point in the ongoing renewal of school curriculum to enable students to become
more effective lifelong learners in the era of rapid advancement of technology. When arranging STEM learning activities in Mathematics for students, schools can adopt the following two approaches:

**Approach One: Learning activities based on topics of Mathematics Education KLA for students to integrate relevant learning elements from the Science Education and Technology Education KLAs**

![Figure 5](image)

In this approach, the design and development of the learning activity is based on a particular topic of Mathematics curriculum (see Figure 5). For example, in the learning unit “Speed” at P6, students could be requested to design and build rubber band powered toy cars (see Example 6). Students recognise and apply the related concepts through the attempts to improve the speed of the toy cars. Relevant learning elements are drawn from other KLAs such as recognizing energy conversion. Students also need to integrate and apply the knowledge and skills they have learned in Mathematics and General Studies.

At the secondary level, knowledge and skills of subjects of Science or Technology Education can also be integrated with different topics in the Mathematics curriculum. For example, in the learning and teaching of volume of spheres, teachers could use designing a measuring spoon as a real-life application of the mathematical knowledge (see Example 18). The technological knowledge in the design of the product can also be embedded, such as how the thickness of the material affects the volume and design.
Below is a table showing some examples of STEM activities and the related topics in the junior secondary Mathematics curriculum.

<table>
<thead>
<tr>
<th>Topics</th>
<th>Examples of STEM-related activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Algebra Strand</td>
<td>Scientific notation: The applications of scientific notation in science, technology, or engineering can be discussed in lessons to illustrate the advantages of using this notation to present and compare values. For example, students could be asked to compare the sizes of different cells in our body, or the computational speeds of different computer processing units.</td>
</tr>
<tr>
<td></td>
<td>Ratios: Students could be asked to estimate the nutrition value of packed food by applying knowledge on ratios and by referring to the nutrition information on food labels.</td>
</tr>
<tr>
<td></td>
<td>Binary numbers: The application of binary numbers in computer systems could be used as an example of applications of mathematics. In a computer, the binary system is adopted for the expression of numbers and characters, and for performing logical and mathematical operations. Interested students could be asked to investigate the amount of data that an 8-digit binary number holds.</td>
</tr>
<tr>
<td>Measures, Shape and Space Strand</td>
<td>Error in measurement: Students could be asked to design a method to investigate the error of measurement by some common GPS tracking apps. Students may compare the measurement results of the apps with those of other measuring tools, and discuss the reliability of the apps and ways to reduce errors in measurement.</td>
</tr>
<tr>
<td></td>
<td>Volumes of 3-D figures: Students could be asked to design a funnel or a box with specified volume and shape that would use the ...</td>
</tr>
<tr>
<td>Topics</td>
<td>Examples of STEM-related activities</td>
</tr>
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<td>------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>least amount of material to manufacture. Students could use spreadsheet software and apply the formulas for the volume and surface area of a circular cone or a cuboid for exploration. Students could also use 3D printers to realise their design for verification of their calculations.</td>
<td></td>
</tr>
<tr>
<td>Trigonometric ratios</td>
<td>Students could apply knowledge of trigonometric ratios to authentic problems. For example, students could find the distance between the school campus and a nearby building by measuring the angles of elevation of the building from two different levels using angle-measuring apps in a tablet computer or a mobile phone.</td>
</tr>
<tr>
<td><strong>Data Handling Strand</strong></td>
<td></td>
</tr>
<tr>
<td>Organising data and constructing graphs</td>
<td>Students can conduct a cross-disciplinary project on healthy diet menu for the school lunch box supplier. Students could study and estimate the nutritional values of different food and conduct a survey to collect food preference of their fellow students. They can then design a diet menu meeting students’ nutritional needs and their preference.</td>
</tr>
<tr>
<td>Uses and misuses of graphs</td>
<td>When doing project work for Science, Technology, Mathematics or cross-KLA topics, students can collect data from secondary sources (e.g. the average annual rainfall in Hong Kong and the energy consumption in Hong Kong) to assist their exploration. Students need to interpret the data and present the results appropriately. Sometimes if the data come from graphs in printed and electronic media, students need to evaluate whether the graphs is misleading and to avoid misinterpretation.</td>
</tr>
</tbody>
</table>
**Approach Two:** Projects for students to integrate relevant learning elements from different KLAs

In this approach, a project learning activity is adopted and it can be used to integrate all the related learning elements from Science, Technology and Mathematics Education KLAs (see Figure 6). Students start with an authentic object of studies, for example, designing a healthy diet menu for a school lunch box supplier. If the project is carried out by upper primary students, they could apply their mathematical knowledge on fractions and decimals, their computation skills, together with the knowledge on food pyramid and nutrition in General Studies to design the menu. Primary teachers could guide students to apply the concepts of percentages on studying nutrients in different food and to construct statistical charts for presentation and analysis. For junior secondary students, their investigation could go further by applying their knowledge in Science about contents of food, nutrient science and balanced and healthy diet to formulate a theoretical framework in designing the lunch menu. They could apply suitable statistical methods learnt in Mathematics to collect and present data, such as students’ food preference, and use an appropriate choice of measures of central tendency. They could apply the concepts of ratio and errors in calculating food ingredients. For senior secondary students who have equipped with knowledge of linear programming, they could be guided to extent the use of these mathematical concepts to formulate a mathematical model for this real-life scenario, and with the aids of information technology that the mathematical model could be solved for a menu with optimized nutrition. They might also make use of knowledge on food preparation in Technology Education curriculum to actualise a practical menu.
Teachers of different subjects can be the facilitators in their own subject perspectives from time to time to support students, and students would endeavour to integrate and apply their knowledge and skills as the cross-disciplinary project progresses.

It should be reminded that projects in STEM-related topics are deep learning tasks that encourage students’ collaborative explorations and integration of knowledge for solving real-life problems.

Teachers may refer to Appendix 4 for some suggested e-resources for arranging learning and teaching activities on STEM education.

4.4 Embracing Learner Diversity

Every student is unique. They are different in level of maturity, gender, personality, ability, aspiration, interest, learning motivation, culture, language and socioeconomic background. Their intelligence, thinking mode and learning styles influence their learning characteristics. To address the diversity, it requires the use of appropriate learning and teaching strategies, the provision of different learning experiences and pathways for students to realise their potential. Catering for learner diversity is not intended to narrow the gap between individuals or even out their abilities and performances, but to enable all students to learn and perform to the best of their abilities. The ultimate goal is to help all students, whether they are gifted, having special educational needs (SEN), more able or less able, to achieve maximum growth as learners. Three aspects in planning strategies to cater for learner diversity, namely the central curriculum aspect, the school aspect and the classroom aspect, are suggested below.

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6 The current gifted education policy of the HKSAR Government encourages mainstream schools to nourish gifted students through school-based gifted programmes. Support services are provided by Gifted Education Section, Curriculum Development Institute, Education Bureau and The Hong Kong Academy for Gifted Education (HKAGE) together with tertiary institutes and educational organisations/bodies. Teachers are encouraged to nominate their gifted students with outstanding performance/great potential in Mathematics to join the HKAGE programmes.

7 On the principle of “one curriculum framework for all”, students with special educational needs (SEN) follow the mainstream school curriculum and are offered essential life-long learning experiences. Materials and teaching packages on curriculum for students with SEN have been uploaded to the website: http://www.edb.gov.hk/en/curriculum-development/major-level-of-edu/special-educational-needs/index.html for teachers’ reference.
4.4.1 Central Curriculum Aspect

In designing the central curriculum, the needs of students at both ends of the ability scale are equally important. Opportunities to learn should be maximised for all students. Attention should not be placed only on low academic achievers. The needs of the more able students should also be catered for. Flexibility in the curriculum organisation is provided in the Mathematics curriculum of Hong Kong. Further details can be found in Section 2.4.

4.4.2 School Aspect

Nowadays, both primary and secondary schools in Hong Kong have been developing their school-based Mathematics curriculum with reference to the central curriculum. It is a general practice that panel heads, in collaboration with other panelists, make diagnoses of students’ general strengths and weaknesses in mathematics, as well as their needs, to provide references for the planning of school-based mathematics curriculum. Strategies in catering for student diversities at the school level include:

(a) Adopting school level arrangements such as providing additional lessons to students in need and adopting ability grouping strategies like streaming, split class, withdrawal and cross-level subject setting.

For more able or gifted students, teachers can group them in a class for mathematics pull-out programme within or after regular school hours to allow systematic training.

(b) Adjusting the inclusion of Non-Foundation Topics of the secondary Mathematics curriculum or the depth of treatment of topics in the primary Mathematics curriculum while selecting topics as the common core learning contents for all students.

(c) Arranging the learning contents in a logical sequence for each year level taking into consideration the cognitive development, the mathematical abilities, learning styles and interests of students; the learning objectives of each learning unit; the inter-relation of learning units; and the inter-relation of mathematical learning at different year levels.

For less able students in schools, it is necessary to review the essential topics regularly. The spiral approach assists students to refresh their knowledge.
However, it should be noted that the teaching of too many topics in a single year and fragmentation of learning (i.e. breaking learning down into unconnected bits of knowledge or skills) should be avoided. In addition, measures like organizing bridging programs can be introduced to ensure that students of different abilities can follow.

For more able students or gifted students, teachers can employ curriculum enrichment, compaction and acceleration to cater for their learning needs. Curriculum compaction refers to the skipping of certain basic concepts or basic skills of certain topics that students have mastered. Teachers could then focus more on the harder part of the topics or introduce enrichment topics related to the topics. Teachers can encourage students’ self-directed learning by selecting suitable basic contents for pre-lesson study. Nevertheless, teachers should secure students’ mastering of basic skills through appropriate assessments. Curriculum acceleration refers to learning and teaching at a pace faster than the suggested one in the central curriculum.

(d) Choosing an appropriate textbook, selecting proper learning and teaching resource and adapting or producing instructional materials. Schools may use different textbooks for different ability groups of students in the same year level, or use the same textbook but centrally produce different instructional materials to support students in different classes. Teachers can provide learning and consolidation tasks fitting students’ abilities by providing different scaffolding for students of different abilities.

(e) Designing a wide variety of learning activities such as, weekly questions posted in the mathematics bulletin boards, mathematics books reading scheme, poster design, mathematics club, etc. Students with different inclinations and abilities may participate in different activities that suit their needs or interest. Students could be encouraged to participate in outside school activities like Mathematics Creative Problem Solving Competition for Primary Schools, Hong Kong Mathematics Olympiad (HKMO), Mathematics Project Competition for Secondary Schools, Mathematics Book Report Competition for Secondary Schools, Hong Kong Statistical Project Competition for Secondary School Students and Hong Kong Statistics Creative-Writing Competition for Secondary School Students, etc.

(f) Formulating the assessment policies and the method of recording and reporting to provide feedback for learning and teaching. Schools may empower teachers to use diversified modes of assessment to suit the needs of
individual classes and allow certain percentages, say 5% to 10%, of students’ mathematics scores to teachers’ own discretion. Teachers may design their own test papers, project works, daily marks, etc., which account for these 5% to 10% marks.

Schools may also consider using different question papers for classes of different mathematical abilities. Such practice can provide flexibility for each class to assess what they have learnt and at their particular ability levels. Information provided would reflect the performance of students suitable to the ability criteria and would serve the purpose of Assessment for Learning.

(g) Caring for non-Chinese speaking (NCS) students. In general, these students have a lower Chinese language competence and English is adopted as the medium of instruction. Some learning and teaching materials produced by the EDB have been translated in English and are assessable in the Internet. Besides language, there might be differences in cultural background and learning experience between students with a Chinese cultural background and NCS students. For example, NCS students might have a different counting system in their own culture. The learning content of Mathematics might need to be tailored or simplified to suit the needs of NCS students. Using more graphical illustrations in instructional materials may help NCS students to grasp the content.

(h) Providing gifted students with information on off-site support programmes. Off-site support is an individualised educational arrangement for the gifted students who require resource support outside the regular school. The Gifted Education Section of Education Bureau and The Hong Kong Academy for Gifted Education cooperate with or/and commission tertiary institutes (or educational organisations/bodies) to provide challenging off-site enrichment and extension learning opportunities for exceptionally gifted students nominated by schools.\(^8\)

(i) The school-based curriculum should be adapted to the needs of students with SEN with consideration given to the differences in their pace and style of learning. Curriculum adaptation is not equivalent to trimming but about tailoring the learning objectives, content, materials, teaching strategies and learning environment according to the learning needs of students with SEN.

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Schools are encouraged to help students with SEN identify learning objectives, achievement targets and expected level of performance according to their prior knowledge, abilities and learning needs.

4.4.3 Classroom Aspect

General Strategies
No matter how the curriculum is designed or how students are organised in schools, it is important that the class teacher should be flexible enough to adjust his/her teaching plan to suit the needs of students. Below are some of the strategies for teachers in designing their classroom activities:

- **Diagnosis of Students’ Needs and Differences**
  Teachers need to gather background information of students, including their interests, their strong and weak areas. Teachers’ observation of students’ performance in class and in assignments and tests is also a reliable basis for diagnosis.

- **Variation on Level of Difficulties and Contents Covered**
  Based on the above findings, teachers can plan relevant learning activities for each lesson. For example, primary teachers could include hands-on activities, or other activity-based learning in their lessons that fit the needs of students. Teachers have to select, adapt or design materials to suit the range of abilities of their students. Too easy or too difficult tasks will not stimulate and sustain student’s internal drive to learn. For less able students, tasks should be relatively simple and fundamental in nature as these activities can give students greater sense of satisfaction and hence greater confidence. On the other hand, for more able students, tasks assigned should be challenging enough to cultivate as well as to sustain their interest in mathematics learning. Teachers may prepare different sets of activity materials for students of different abilities in a class, or prepare one set of materials with a core part for all students, a part that reinforce the prerequisite knowledge for the less able students and a part that challenges the more able students. Students could then choose the parts that suits their ability and needs.

For the planning of learning content for classes with different abilities, below is one example in the Key Stage 3 concerning the learning unit on the
rectangular coordinate system in the Measures, Shape and Space strand. For less able students, teachers can consider not to include the non-foundation topic about the formula of internal points of division. For more able students, teachers can cover all learning objectives and select the enrichment topic, which is about the formula for external point of division.

- **Variation in Questioning Techniques**
  On the other hand, appropriate questioning can aid students to achieve the learning goals effectively and improve themselves. Through providing students with different clues when asking questions, teachers can enable students with different abilities to learn the same topic at the same year levels. In general, teachers can ask simple and straightforward questions to less able students and comparatively more challenging questions to more able ones. It should be noted that feedback from teachers is essential in facilitating students’ learning. Even for the less able students, teachers can request them to modify their answers, to explain their strategies of solving the problems instead of giving the solutions right after they give wrong answers.

Teachers may decide what questions to ask, in terms of level of thinking, to cater for different students. There are questions on memory/information recall, on interpretation, comparison and explanation, and also on new ideas or alternatives. Questions of thinking levels fitting students’ progress in development of concepts can greatly assist students learning.

- **Variation in Clues provided in Tasks**
  Teachers can also provide students with the same task or exercise but with additional supports such as diagrams to aid comprehension and structuring long question for less able students. For more able students, teachers may provide fewer hints in the process of solving problems. Further, open-ended problems (such as Examples 2, 5 and 14 at the end of this Guide) and graded exercises can also be used to motivate students to solve the problems with strategies suited their abilities and concerns.

- **Variation in Approaches in Introducing Concepts**
  Teachers can introduce mathematical concepts with different approaches. Concrete examples may be used to illustrate the concepts for less able or young students but can use symbolic language for more able or more mature students. For example, diagram of dots arranged in a triangular pattern can be
used to illustrate the sequence of triangular numbers for less able students, whereas the idea of sum of arithmetic sequence can be introduced to more able students.

Teachers could adopt multiple means of presentation to cater for different learning styles. For example, in the learning and teaching of topics relate to 3-D figures, diagrams, real models of 3-D figures for hands-on manipulation, virtual 3-D figures by software packages can be used to address the needs of visual, auditory and kinesthetic learners. Example 17 at the end of this Guide shows a learning activity designed to cater for learners of different learning styles.

- **Variation in Peer Learning**
  Besides whole-class teaching, teachers can also consider different grouping strategies to cater for the needs of different students. However, it should be noted that the way the groups are formed, the suitability of the tasks designed for the groups, the durability of the grouping and the ongoing assessment of the group dynamic are ingredients for successful collaborative learning. Further, it is very important to build up the collaborative instead of competitive atmosphere that is found undesirable in effective learning.

  Teachers may consider grouping students with similar learning abilities, different learning abilities, or in different group size for collaborative learning. However, care must be taken to avoid labeling effect on grouping students of same abilities especially for a long duration. Heterogeneous groups may lead to both positive academic and remedial outcomes. When students of different abilities are grouped together, the high-ability students benefit from group interaction as much as the average or the less able students. For maximum communication among members, group size should not be too large. Groups of 3 to 4 students work quite well.

- **Importance in Arousing Learning Motivation**
  Motivation is probably one of the most important factors in affecting learning performance because a well-motivated learner is more determinative to achieve and to overcome a lot of learning difficulties. Motivation is not constant over time but may change according to the circumstance and disposition of the learner. Teachers must be aware of the possibilities of such changes and be flexible enough to adjust their strategies when necessary. It is
crucial for teachers to plan learning activities with particular attention paid to initiate their students’ motivation.

- **Variation in Using e-Learning Tools**
  
  e-Learning packages usually provide different levels of exercises or activities. Appropriate use of IT provides teachers with a way to cater for learner diversity as it allows students with different abilities to learn at different paces. The e-features to record students’ performance also provide information for teachers to diagnose students’ misconceptions or general weaknesses so that they can re-adjust the teaching pace and the teaching strategies. e-Learning tools and resources can also engage different types of learners, and to enrich their learning experiences.

**Strategies for Students with SEN**

- **Diagnosis of Students’ Needs and Differences**
  
  Teachers need to recognise the needs of students. For some SEN students, they may have problems in building up number concepts in their long-term memory, and retrieving procedures or strategies when solving problems. Even though SEN students give a correct answer, they may not show any confidence towards their work.

- **Variation on Level of Difficulties and Contents Covered**
  
  Students with SEN may show anxiety to their mathematics learning even though they give a correct answer on their work. Teachers should recognise their learning abilities and adopt appropriate strategies in learning and teaching. For example, for SEN students having a weaker working memory, teachers should simplify the instruction of learning activities in the lessons into small steps or use teaching aids such as counting blocks and pictorial cards, for helping them to follow the activities and understand the concepts. Solving one type of problem with many methods and requesting for a prompt reply in mental calculation also make some SEN students feel overburdened and anxious about mathematics. Teachers could concentrate on some strategies and encourage SEN students to use jottings to reduce their burden. Providing a successful learning experience is most important for SEN students with difficulties in learning mathematics.
Below is an example in the P3 unit “Multiplication (I)” in the Number strand of the primary Mathematics curriculum.

\[
\begin{array}{ccc}
1 & 2 \\
\times & 3 \\
\hline
3 & 0 \\
\hline
6 \\
\hline
3 & 6
\end{array}
\]

Students need to know that the values of “1” in 12 is 10 and the result of 10 × 3 is 30 before performing the multiplication. Connecting the new technique with their previous one by writing “30” instead of “3” in the calculation can effectively reduce the loading of their working memory.

When students are requested to perform multiplication with multiplier 1 digit and multiplicand 2 digits, teachers usually started the discussion for less able students by adopting calculations without carrying, and together with counting blocks, to help students understand the calculation in column form. Same strategies can also be adopted for SEN students with modifications to reduce their burden on memory.

Strategies for Gifted Students

- **Diagnosis of Students’ Needs and Differences**
  To get assistance on identifying more able students or gifted students, teachers may refer to the information provided by the Gifted Education Section of Education Bureau.

- **Variation on Level of Difficulties and Contents Covered**
  In the learning and teaching of conditions for congruent triangles, teachers can extend the concept of fixing a triangle to fixing a quadrilateral to challenge gifted students. Teachers may also ask students to guess what condition(s) are sufficient for identifying congruent triangles.

  In the learning and teaching of the methods to construct angle bisectors, perpendicular bisectors and special angles by compasses and straight edges, teachers can prompt students to connect the geometric construction with the angle bisector theorem and the perpendicular bisector theorem.

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• **Variation in Clues provided in Tasks**

  Teachers can adopt tiered assignments to cater for learner diversity. Tiered assignments are differentiated learning tasks developed by teachers which based on students’ abilities to meet their individual needs. It provides a better matching between students and their learning needs, and involves different levels of difficulty, complexity, abstractness, depth and creativity. According to Heacox (2002), there are six ways to structure graded assignments: (1) by challenge level; (2) by complexity; (3) by resources; (4) by process; (5) by outcome; and (6) by product. To prepare for a tiered assignment, teachers may consider first the instructional level of average students. The assignment can be modified to become more challenging by increasing the level of difficulty and complexity for the mathematically gifted students. More details can be found in the resources<sup>10</sup> produced by Gifted Education Section of Education Bureau and The Hong Kong Academy for Gifted Education.

• **Variation in Approaches in Introducing Concepts**

  For average students, teachers are advised to let students build and manipulate the concrete models by themselves before learning the 2-D representations of a 3-D solid. For more able students or gifted students, teachers may skip the use of concrete models by using appropriate computer program to demonstrate the effect of rotation of a 3-D solid on the 2-D representations.

• **Importance in Arousing Learning Motivation**

  Teachers can use some mathematics paradoxes to provoke gifted students’ curiosity. For example, teachers may demonstrate the proof of 0.999… = 1 to challenge students’ intuitive view on rejecting the equality of 0.999… and 1.

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Chapter 5
Assessment
Chapter 5 Assessment

5.1 Guiding Principles

Assessment is the practice of collecting evidence about student learning, interpreting information and making judgement about students’ performance with a view to providing feedback to students, teachers, schools, parents, other stakeholders and to the education system, which are fundamental for facilitating and improving students learning. It is not a separate add-on activity but an integral part of the learning and teaching process. It also plays a critical role in helping students become self-directed learners. Suitable assessment enables

- students to understand their abilities and hence improve their ways of learning;
- teachers to understand the performance of their students and the effectiveness of the learning and teaching strategies adopted so that suitable and timely measures can be provided; and
- parents to understand the performance of their children so that they can, in collaboration with teachers, provide suitable support to help the learning of their children.

The assessment framework shown in Figure 7 shows that learning and assessment should be interrelated. As reflected in the Mathematics curriculum, both learning process and content of mathematics are important in mathematics learning. Therefore, to obtain a complete picture of students’ performance, assessments should be aligned with the aims and objectives of the curriculum. In general, assessment in mathematics should not only focus on the answers provided by students in solving problems, but it should also reflect students’ learning process, the skills used in solving mathematical problems and the development of thinking abilities as well as values and attitudes. A balanced and diversified assessment is definitely helpful to obtain a comprehensive view of students’ achievement and performance, but teachers should note the following points (see the table on p.79) when collecting information about performance of students through different assessment activities.
A FRAMEWORK OF SCHOOL ASSESSMENT PRACTICES

FORMATIVE ASSESSMENT
(informs learning and teaching)

SUMMATIVE ASSESSMENT
(measures attainment)

Leads to more successful results

Learning and Teaching Process
- Sharing learning objectives with students
- Effective questioning (e.g. want / pause time, a variety of question types – open / closed questions, content-centred to student-centred)
- Observation (e.g. body language, facial expression)
- Peer learning (e.g. listening and reflecting on other students’ answers in whole class setting)
- Effective feedback (e.g. clear advice for improvement / reinforcement)
- Active involvement of students in their own learning
- Raising of students’ self-esteem

Feedback Loop

Internal Assessments
- Diversity
  - Different modes of assessment
    (e.g. pen and paper tests, projects, portfolio, performance tasks, self-reflection, etc.) to match learning objectives and processes
  - Different parties (e.g. self / peer / teachers / parents)
  - Different strategies to assess the quality of learning (e.g. setting assessments that are both challenging and suitable for students’ competence other than reward and punishment)
- Tests which are used diagnostically to inform learning and teaching
- Opportunities for students learn, reflect and correct rather than compare marks with others

External Tests
- e.g. Hong Kong Diploma of Secondary Education Examination, Territory-wide System Assessment
- Tests / examinations which are used to assign grades or levels (e.g. end of school term / year)
- Recording
  - For tracking students’ learning progress
- Reporting
  - Qualitative feedback, reducing reliance on grades and marks

Recording
  - For tracking students’ learning progress

Reporting
  - Qualitative feedback, reducing reliance on grades and marks

Curriculum Development Institute, 2017
(Adapted from Shirley Clarke)
Suitable Assessment | Unsuitable Assessment
---|---
• Helps students develop confidence and interest | • Causes students’ anxiety and undue pressure, and in extreme cases, students may lose confidence and interest in learning
• Enables teachers to provide immediate feedback to enhance learning and teaching | • Reduces learning and teaching time in class, increases teachers’ workload unnecessarily and hence poses pressure on both students and teachers
• Addresses different learning objectives | • Overemphasizes drilling

Schools’ assessment policy should aims at achieving a better balance across assessment of different modes and purposes. Aspects to be considered while formulating the assessment policy include:

- What are the expected learning outcomes at the level concerned?
- What assessment activities will be adopted in assessing students’ performance in the various aspects of mathematics and cross-KLA areas (e.g. STEM education)?
- Will the assessment activities include life-wide learning activities and activities other than the paper-and-pencil type, e.g. performance tasks?
- What assessment criteria will be used in activities like projects, oral presentation, etc.?
- Have learner diversity be considered in the design of assessment activities and criteria?
- Will e-assessment be considered to facilitate Assessment for Learning of some suitable topics?
- What is the number of tests/examinations in the school year?
- What assessment data will teachers receive and how teachers use it to inform learning and teaching and school-based curriculum planning?
- What kind of feedback will be provided to students and is it comprehensive enough for students’ self-directed learning?
Assessment in appropriate frequency is useful to inform student learning. Over-assessment disrupts learning and teaching and causes overburden to students and teachers. In planning assessment policy, enough space has to be reserved for students’ learning and teachers’ teaching. Over-assessment should be avoided. (School may refer to Chapter 5 of the Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6) (2014) and Booklet 4 of the Secondary Education Curriculum Guide (2017) for more ideas on school policy on assessment.)

5.2 Formative and Summative Assessment

By considering purposes, one can distinguish between formative and summative assessment. The former one focuses more on providing a comprehensive and summary description of student performance and progress in learning while the latter one is mainly diagnostic aiming to identify students’ strengths and weaknesses and to provide feedback for the review of teaching strategies.

5.2.1 Purposes of Assessment

In general, assessment can be divided into the following three categories according to the purposes of assessment:

- **Assessment of Learning**
  - Providing evidence to make judgements on student achievement against learning targets, objectives or standards at a certain point of time
  - Being summative in nature and results in a grade or certification

- **Assessment for Learning**
  - Collecting ongoing information about the progress of student learning to provide timely and quality feedback for students to improve learning, and for teachers to adjust their teaching strategies.
  - Being formative and diagnostic
Assessment as Learning

- Using learning tasks and feedback collected by students to enhance own learning
- Being formative and provoking students’ understanding of their learning, evaluation of learning effectiveness, adjustment on learning strategies, planning for follow-up actions and setting of future learning goals

When formulating appropriate modes of assessment for Mathematics, the formative “Assessment for Learning” and “Assessment as Learning” can be used to collect evidence of the process of student learning, provide feedback to enhance learning and establish students’ roles and responsibilities in their learning. “Assessment of Learning”, being summative, is usually conducted at the end of a teaching module, a school term or a school year, to summarise students’ learning performance. No matter what kind of assessment is undertaken, students should be encouraged to make use of assessment feedback for self-reflection and teachers could carefully analyse the results of formative and summative assessment to revise class instructions.

5.2.2 Modes of Assessment

Different modes of assessment serve for different purposes. Teachers may use them deliberately to focus on the learning process, learning progress and product of learning. As both the learning process and product are important in the mathematics curriculum, different modes of assessment in the form of various assessment activities are necessary. Various assessment activities are needed to provide teachers with opportunities to collect, judge and interpret information about students’ performance in the areas of mathematical knowledge, attitude and generic skills. Designing appropriate and diversified assessment activities according to learning targets and students’ learning needs is essential in the Assessment as Learning process. Some common assessment activities in mathematics include:

- **Class discussion or oral presentation**
  In the learning and teaching of mathematics, discussion, questioning and answering between teacher and students (or among students) are often involved. Discussion in class not only enables teachers to discover what students understand about a particular topic, but also provides opportunities
for students to present their views. It helps foster their communication skills. Teacher could also understand students’ attitudes and abilities in applying thinking skills through discussions. Criteria that can be used include: Can they explain how they got to the solutions they have put forward and what strategies they have employed? Do they know how to raise questions? Do they raise questions actively? Constructive feedback from teachers allows students to understand the correctness of their response and achieve an in-depth analysis of the topic. Topics suitable for discussion include: Why \( \frac{1}{5} + \frac{1}{3} \neq \frac{2}{5} \)? Why \((x + y)^2 \neq x^2 + y^2\)? How can the height of the school building be estimated? Are the areas of figures with the same perimeter equal? How can a right-angled triangle be constructed by using a straight rule and a pair of compasses? How statistics be applied in real life?

- **Classwork and homework**
  It is a common practice for teachers to assign classwork and homework to students. These can help students consolidate concepts in mathematics (see Section 4.4) and help teachers assess the performance of their students. It is important to give an appropriate amount of assignments and to ensure that they are at a suitable level of difficulty. Teacher may consider assigning different classwork to students of different abilities. Moreover, each assignment should be appropriately related to specific objectives. It is inappropriate, for example, to give students an assignment on the drilling of addition and subtraction while the aim is to assess the application of addition and subtraction. Finally, classwork and homework should not be confined to routine mathematical problems. They may include open-ended questions, reading assignments, hands-on assignments and preparatory work for discussion in class. For example, newspaper cutting on the uses of statistics in daily life can be used for discussion on the misuse of statistics; containers marked with volume in litre collected at home can be used for measuring activities in school.

- **Project work**
  Project is a useful activity to assess students’ performance. In addition, project learning is an effective learning and teaching strategy to promote self-directed learning and enable students to connect knowledge, generic skills, values and attitudes. It can also foster students’ critical thinking skills, creativity and problem solving skills. Teachers should note that collaboration and communication skills will be more effectively developed by projects done in
groups rather than individually. Teachers may assess student performance using the following criteria:

1. comprehension of the project;
2. use of strategy and approach;
3. coverage, depth, accuracy of content;
4. presentation and communication; and
5. attitude.

It should be noted that students’ reflections and peer assessment can be an integral part of assessment and parents can also contribute their views. Using assessment rubrics is a good practice for these two kinds of assessments for achieving self-directed learning. Information technology may be employed to facilitate assessment of project learning. There can be sharing among students and submission of students’ work through the school online platform.

Typical examples of project work include investigating students’ favourite extra-curricular activities, comparing the heights of boys and girls in the school, an investigation into the relationship between students’ eyesight and time spent watching TV or using display screen equipment, story of π, design of containers that with the optimal use of materials, sports and mathematics, statistical survey on students’ time management, etc. Cross-KLA topics or topics on real-life problems can also be assigned for promoting STEM education. Detailed examples can be found at the end of this Guide (Examples 10 and 12).

- **Exploratory Task and Performance Task**

  Exploratory task and performance tasks are class activities that require students to apply various skills to perform mathematical investigations or solve problems in a hands-on way. Through getting students to complete the tasks in groups, teachers can look at students’ problem solving skills and collaboration skills. Criteria of assessment can also include comprehension of the problem, the use of strategy and approach, the degree of participation and attitude, etc. Typical examples of exploratory task and performance task include constructing parallel lines, measuring volumes of irregular objects, geometric constructions by compasses and straightedge (or by dynamic geometry software), constructing special sequences by spreadsheet, making
3-D models of polyhedrons, constructing loci by dynamic geometry software, etc.

- Tests and examinations
  Generally, there are both tests and examinations in schools. Nevertheless, teachers should pay attention to the following points when setting tests/examination papers.
  - The coverage in the paper should be complete and the item format should be diversified
  - Each item should have clear assessment objective(s)
  - The levels of difficulty of items should reflect students’ abilities and cater for their learner diversity
  - The number of items in each paper should be reasonable
  - The language used in the paper should be simple and clear

Before setting a test or an examination paper, teachers should prepare a table of specifications with allocation of marks on different learning units/objectives being clearly shown. Appropriate amount of marks should be allocated so as to reflect the aims and focus of the paper and to ensure the proper coverage of topics. The paper should embrace various types of item, like explanatory-type items, fill-in-blank items, multiple-choice items, etc. to assess students’ knowledge in various aspects of mathematics. Open-ended questions should also be included to assess students’ thinking abilities like critical thinking skills, creativity and communication skills. Examples of open-ended questions can be found at the end of the Guide (Examples 2, 5 and 14).

The incorporation of peer assessment and self-assessment in various assessment activities is also beneficial as they allow students to learn from peers and understand their own learning. The assessments also direct students to consider the main learning objectives, skills involved and perform self-reflection on the learning process and attitude. Learner diversity should also be considered in designing assessment activities. For example, a balance among the number of elementary, intermediate and advanced problems should be achieved in tests and examinations.

The incorporation of e-assessment may facilitate learning and teaching by
providing instant feedback to both students and teachers. Students can perform self-reflections using the feedback when as they are still working on their tasks while teachers can understand students’ performance when instruction is still in progress. E-Assessment can also help adjust the levels of difficulty of assessment items to cater for learner diversities. Besides, students’ work and assessment data can be stored for future analyses.

On the whole, the most appropriate method or procedure for gathering information is best decided by considering the purpose for which the information will be used and the kind of performance that will provide the information required. No matter what kind of assessment activities are adopted in collecting information of students’ learning, it should be borne in mind that the main purpose of assessment is to enhance the learning and teaching process.

5.2.3 Promoting Formative Assessment

Assessment of Learning has long been a usual practice of schools. Many schools had also incorporated the idea of Assessment for Learning and Assessment as learning into their assessment policies with improving effectiveness. Besides, schools have put effort on using diversified modes of assessment to get a full picture of student learning and at the same time cater for learner diversity and enhance self-directed learning. In recent years, Learning Progression Framework (LPF) and Student Assessment Repository (STAR) have been developed as tools for schools to adopt Assessment for Learning and Assessment as Learning. Below introduces the details of LPF and STAR.

Learning Progression Framework

In support of the curriculum reform and the recommendations detailed in the Report Learning to Learn - The Way Forward in Curriculum Development (CDC, 2001), schools are encouraged to put emphasis on Assessment for Learning as an integral part of the learning-teaching-assessment cycle. In addition, the Curriculum Development Council has endorsed the development of a Learning Progression Framework (LPF) to articulate learners’ performance in Chinese Language, English Language and Mathematics from primary one to secondary six. It also help teachers plan strategically how to enhance Mathematics learning and teaching in the learning-teaching-assessment cycle.
The LPF represents the growth of learners on a developmental continuum as they work towards the Learning Objectives of the Mathematics curriculum. LPF is made up of Learning Outcomes (LOs) organised under three strands and divided into eight levels of attainment. They are expressed in the form of outcome statements which give a general description of learner performance. There are pointers as specific examples of what learners are able to do in demonstrating the LOs, which support the outcome statements. It should be noted that the pointers are meant to elucidate the LOs. They are neither prescriptive nor exhaustive, and must not be regarded as a checklist for mathematical development. The LPF is available at the website http://www.edb.gov.hk/en/curriculum-development/kla/ma/res/index.html, placed under the links “Primary One to Six”, “Secondary One to Three” and “Secondary Four to Six”.

Student Assessment Repository
Student Assessment Repository (STAR) is an online assessment bank developing for the three subjects of Chinese Language, English Language and Mathematics with a view to enhancing teachers’ assessment literacy and improving students’ learning through promoting Assessment for Learning and Assessment as Learning by means of technology. The STAR platform will be made accessible to both students and teachers.

STAR platform including diversified assessment items on mathematical knowledge and skills, which are designed by making reference to the learning outcomes and pointers of LPF and to suit the online environment. Assessment tasks in the STAR platform can be assigned by teachers or initiated by students and reports for diagnosing students’ performance will be generated after completion. By using STAR, teachers can understand students’ attainment for planning remediation and progression in teaching or guiding students to use STAR for self-directed learning.

5.3 Internal and External Assessment

5.3.1 Internal Assessment

Internal assessment refers to the assessments of different modes and scales that schools employ as an integral part of learning and teaching. Teachers are encouraged to incorporate diversified modes of formative and summative assessment tasks suggested in Section 5.2.2 to provide timely feedback for
enhancing learning and teaching.

As discussed in Section 5.1 and illustrated in Figure 7, feedback is a crucial component of the learning and teaching cycle. Without proper feedback, assessment becomes meaningless. However, it should be noted that feedback does not mean “praise” or “blame” and is not the same as “guidance”. Effective feedback must be able to help students know what they can and cannot do and where their strengths and weaknesses lie, and it must also tell them what to do to improve. Teachers are encouraged to note the followings:

- Feedback from assessment can be in verbal or in written form and should be related to the particular qualities of their work with advice on what they should do to improve. Comparisons among students should be avoided.
- Timely feedback on the basis of formative assessment should be given wherever possible, as it is effective. For example, homework should be marked and promptly returned to students with appropriate comments whenever necessary.
- STAR platform or school-based e-assessment platform can help teachers get a quick analysis of students’ learning and provide timely feedback to students. By selecting suitable assessment items for students, teachers can provide more meaningful feedbacks to students according to the data provided by the e-assessment platform. Students can then acquire a clearer picture of their own performance and learn to adjust their learning.
- Feedback from summative assessment activities can provide information for students to plan their subsequent study, and for teachers to plan the next teaching sequence, and to adjust the breadth and depth of the school-based mathematics curriculum for the subsequent term or year.
- Communication between teachers and parents is essential for integrating learning across school and home. Based on the evidence collected from assessment activities, more information on how to improve children’s learning could be provided to parents through different channels. Through the feedback from teachers, parents can better understand the learning targets and collaborate with teachers to support children’s learning.

In sum, schools need to formulate their policy on internal assessment according to their context, teachers’ experience and students’ needs and interests. The
assessment policy should be balanced in terms of assessment activities and reporting of the information collected. Learning objectives, learning activities and assessment tasks should be aligned to ensure that what is intended will be properly taught and successfully learned.

5.3.2 External Assessment

External assessments, including public assessment, are assessments conducted by external organisations (such as the Hong Kong Examinations and Assessment Authority) for evaluating the learning progressions or outcomes of students. For Mathematics, students have to take various external assessments after their completion of their learning at several key stages for different purposes.

- Primary and junior secondary: Territory-wide System Assessment (TSA) for Mathematics at P3, P6\textsuperscript{11} and S3 (Assesses students’ performance on Basic Competencies, which are the essential knowledge and skills acquired by students in relation to the learning targets and objectives set out in the curriculum for each key stage)

- Senior secondary: Hong Kong Diploma of Secondary Education (HKDSE) Examinations for Mathematics (Compulsory Part, Extended Part - Module 1 and Extended Part - Module 2)

Feedback from external assessment also plays an important role in informing learning and teaching. The results of HKDSE Examination provide feedback to students on their performance and to teachers and schools on the quality of the teaching provided. Regarding TSA, each participating school will receive a school report containing information on students’ performance in different strands of the curriculum. With the report, schools can identify students’ strengths and weaknesses and adjust the teaching time, sequence and strategies accordingly. For example, if the performance of P3 students in identifying 2-D shapes is not as good as that of the average of Hong Kong students, the school may adjust the learning and teaching strategies of this topic in P3 and related topics in the upper primary. However, when designing related classwork and homework, schools are advised to help students consolidate and extend their learning based on their established foundation and to avoid mechanical drilling.

\textsuperscript{11} TSA at P6 is implemented in alternate year starting from 2011.
Chapter 6
Learning and Teaching Resources
Chapter 6  Learning and Teaching Resources

6.1  Quality Textbooks and Other Learning and Teaching Resources

Learning and teaching resources provide students with a basis for enriching their learning experiences. The effective use of learning and teaching resources helps students to extend and construct knowledge and consolidate what they have learned; and is therefore an important factor affecting the successful implementation of the school-based Mathematics curriculum. Schools need to select, adapt and, where appropriate, develop the relevant resources to support student learning.

Among the many learning and teaching resources, textbooks are usually regarded as the most common ones. However, textbooks are only tools to bring about learning and not the curriculum itself. Teachers are encouraged to refer to the curriculum documents of primary, junior secondary and senior secondary Mathematics to acquire understanding on the aims and objectives of the curriculum before teaching:


(The documents can be found on the homepage of Mathematics Education Section at http://www.edb.gov.hk/cd/maths/)

Other than textbooks, teachers should select relevant learning and teaching resources to achieve the learning objectives.

6.1.1  Textbooks

Nowadays students learn in a wide learning environment and from a variety of learning resources, resulting in broadened learning experiences. However, even in such an environment, textbooks still play an important role in mathematics instruction. They influence the learning and teaching that takes place in most classrooms. Textbooks are also students’ self-directed learning materials for
preparation and revision purposes. Therefore, well-written mathematics textbooks should align with the aims of the mathematics curriculum and facilitate students to learn.

A set of guiding principles for quality textbooks has been formulated for schools’ reference in choosing textbooks appropriate to their students’ interests and abilities. It is accessible at http://www.edb.gov.hk/en/curriculum-development/resource-support/textbook-info/GuidingPrinciples/index.html.

Schools can make reference to the Recommended Textbook List (RTL) and the Recommended e-Textbook List (eRTL) respectively at www.edb.gov.hk/rtl and www.edb.gov.hk/ertl when selecting textbooks. Printed or electronic textbooks placed on the RTL or eRTL are written in line with the curriculum documents issued by the CDC and deemed acceptable in terms of content, learning and teaching, structure and organisation, and language used.

The following shows some major considerations in the selection of mathematics textbooks:

- Whether the sequencing of topics is compactable with the school-based curriculum and meets students’ abilities for effective development of mathematical concept, generic skills, values and attitudes promoted in the curriculum.
- Whether different learning activities are included for students to investigate, discuss, make conjectures, and test hypotheses.
- Whether the examples and illustrations of mathematical concepts and skills meet the needs and abilities of students.
- Whether the learning activities provided by the textbooks allow teachers to engage students in applying mathematics to solve problems, including real-life problems.
- Whether peer and self-assessments are incorporated in learning activities to help students assess their own learning and reflect on what and how they have learned for further improvements.

In choosing mathematics textbooks for use by students, schools should set up mathematics textbook selection committees to evaluate the relevance and
suitability of the textbook in meeting the educational needs and abilities of their students. The extent to which teachers can exercise professionalism in adapting the contents to meet the learning and teaching objectives should also be taken into consideration.

6.1.2 Learning and Teaching Resources Other than Textbooks

Textbook is only one of the many learning and teaching resources. Other resources for the learning and teaching of mathematics may include:

1. physical models/teaching aids (such as counters, blocks and geometric models),
2. audio-visual materials (such as ETV),
3. printed resources (such as library books, magazines, journals, and reference materials developed by the EDB),
4. e-resources (such as commonly used application software for desktop computers or mobile devices and resources available on the school e-platform) including Internet resources (such as web-based graphing tools and short videos on mathematics topics), and
5. community resources (such as statistical reports and graphs in the newspaper and resources provided by professional bodies).

Using the textbook as the only resource is not sufficient to achieve the aims and objectives of the Mathematics curriculum. Adaptation of textbooks and other learning and teaching resources may be necessary to meet the needs and abilities of different students. When new concepts are to be constructed, manipulative materials can be provided and suitable apps can be used to help students to visualise abstract concepts. When skills are to be sharpened or facts to be memorized, consolidation activities are needed. When independent work is prescribed, activity cards, projects, and application software can be used. Providing quality learning and teaching resources to support mathematics curriculum is therefore basic to effective teaching. At the same time, teachers could develop their learning and teaching resources to suit their students.

A list of suggested websites and application software for learning and teaching of mathematics is provided in Appendix 4 for teachers’ reference.
6.1.3 Resources in Support of Curriculum Development

*Education Bureau Resources*

To support the implementation of Mathematics curriculum and enhance the effectiveness of learning and teaching of Mathematics, EDB developed a number of resource materials for teachers of both primary and secondary levels. These resources cover learning, teaching and assessment of Mathematics. Below are some examples:

- The One-stop Portal for Learning and Teaching Resources
  *(http://minisite.proj.hkedcity.net/edbsp-eng/eng/home.html)*
  It provides learning and teaching materials for different learning units of different levels.

- Learning and teaching resource packages, such as Addenda Series for Primary Mathematics

- Printed materials, such as Mathematics Cabinet Series and School Mathematics Newsletter

- Website of Mathematics Education Section, EDB

- The Assessment Tasks Reference
  *(www.hkedcity.net/edbatr)*
  It provides assessment tasks on various topics

- Web-based Learning and Teaching Support (WLTS)
  *(http://wlts.edb.hkedcity.net/)*

- Educational Television (ETV) programmes
  *(https://www.hkedcity.net/etv/en/?currId=71)*

For more details of the EDB resources, please refer to Appendix 6 or visit the homepage of Mathematics Education Section at *http://www.edb.gov.hk/cd/maths/*

*Collaborative Projects*

To facilitate the implementation of the primary and secondary Mathematics curriculum and further support teachers, a series of collaborative research and development (“Seed”) projects in schools were done since September 2001. These projects are geared towards promoting the learning capabilities of students,
generating good practices, suggesting useful teaching methodologies and
developing a critical mass in order to achieve the aims of the school-based
curriculum.

Since 2001, more than 30 seed projects had been conducted by the Mathematics
Education Section. More than a hundred primary schools had participated in the
seed projects, and the same for secondary schools. The key concerns of these
projects include:

1. curriculum planning, learning and teaching strategies in Mathematics;
2. development of critical thinking and creativity through learning activities;
3. Assessment for Learning; and
4. key emphases of curriculum development, e.g. self-directed learning and
   STEM education.

Outcomes of these projects are used to produce learning and teaching packages,
and disseminated in the professional development programmes. For the list of
projects conducted in the past years, please refer to Appendix 7 or visit

Community Resources
Materials around such as advertisement leaflets, statistical reports presented on the
Internet, graphs printed in the newspaper could supply up-to-date information that
cannot be found in other sources and they could easily arouse students’ interest in
learning. Associations/organizations in the community also provide resources such
as seminars and forums to familiarize teachers with current development in
mathematics education. Some of these are listed in Appendix 6 for teachers’
reference.

6.2 Effective use of Learning and Teaching Resource

Appropriate use of learning and teaching resources helps create meaningful
learning experiences for students, not only it can facilitate the interaction between
teachers and students but also broaden students’ learning experiences and cater for
their learner diversity. Apart from textbooks and reference books, resources such as
real models of 3-D figures, online learning materials, computer software packages,
mobile application software, e-learning platforms, and community resources, do play a key role in student’s learning. Therefore an effective way of using these learning and teaching resources is important. For instance, it is worthwhile to consider the following points when preparing the lessons:

1. The learning objectives as stated in the curriculum documents.

2. Whether the learning and teaching resources
   - provide a means for students to acquire the mathematical concepts or master the skills;
   - promote students’ interest and active involvement in learning mathematics;
   - provide students with the opportunity to explore, discuss and co-construct knowledge to nurture their mathematical thinking and logical reasoning;
   - provide different levels of difficulty and learning experiences to cater for students’ diversified needs and abilities;
   - can complement or extend the content of the mathematics textbook, enable students to make connections and achieve a wider understanding of concepts and skills; and
   - help students develop their positive values and attitudes as well as various generic skills and self-directed learning strategies towards the learning of mathematics knowledge and skills.

3. Adapting the curriculum contents, learning and teaching resources, and textbooks to meet the needs and abilities of different students.

4. Choosing on-line resources from reliable sources, such as getting statistical data from websites of government departments or official websites of organisations.

Nowadays, the application of e-resources has become a usual practice. Teachers make use of the interactive and real-time functions of e-resources in their lessons to cater for the diversified needs of students and to build up an effective learning community in class so as to enhance students’ learning effectiveness. Besides, teachers are empowered to play a better role of facilitators and students are provided with more supports for their self-directed learning. However, students should be reminded to use non-electronic resources whenever appropriate so that they can understand the mathematical concepts and skills more comprehensively. Teachers are encouraged to bring real-life scenarios to the classroom to allow
students to apply what they have learnt in real-life situations.

6.3 Resource Management in Schools

Proper use of resource materials in teaching can enhance students’ learning effectiveness and make learning and teaching more interesting. Below are some suggestions on resources management:

- Schools should make good and flexible use of grants to build up a suitable stock of both electronic and non-electronic resource materials for the learning and teaching of mathematics.

- An up-to-date list of resources, with resources grouped under proper categories, is suggested to be uploaded to the school intranet for teachers’ easy reference. Teachers are encouraged to make flexible use of different kinds of learning and teaching materials.

- If space is available, new resources and learning packages should be displayed for teachers’ information and perusal.

- Small workshops or experience-sharing sessions can be conducted to provide teachers with a better understanding of the resources available.

- The stock of library books on academic and recreational aspects of mathematics for teachers and students could be increased. It is also advisable to compile or update the list of available mathematics journals and reference books in the school library for teachers’ perusal.

- Regarding e-resources, teachers are encouraged to
  - check that webpages or on-line tools are still readily available on the Internet;
  - compare software of similar functions and choose an appropriate one for their students;
  - choose an appropriate e-platform to facilitate sharing and communication among students and teachers, such as sharing learning and teaching resources with appropriate measures to protect intellectual property rights; and
  - form communities of practice to investigate, reflect on and share experiences in the use of different types of e-resources.
(School may refer to Chapter 7 of the *Basic Education Guide – To Sustain, Deepen and Focus on Learning to Learn (Primary 1 – 6)* (2014) and Booklet 10 of the *Secondary Education Curriculum Guide* (2017) for more ideas on learning and teaching resources.)
Examples
This Curriculum Guide includes a number of examples, which illustrate how the spirit of the Mathematics curriculum can be realised through activities, how the focal points of the ongoing curriculum renewal can be promoted, how generic skills can be fostered and how learner diversity can be catered for. The table below shows the main concerns of the examples.

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*Note: “Mathematical Skills” is not included in the list as all examples involve development and applications of Mathematics Skills in different aspects.
Example 1
Hand in Hand

Key Stage: 1

Strand: Number

Learning Unit: Numbers to 20

Objective: Understand odd and even numbers

Prerequisite Knowledge: Understand numbers 1-10

Resources Required: (i) Beads (ii) Worksheets

Description of the Activity:
Activity 1
1. The teacher chooses a group of students (not more than 10 students), for example, 7 students. Within the group, every student tries to find one partner and stand together hand in hand. Students record the results on the Worksheet “Hand in Hand”.

<table>
<thead>
<tr>
<th>No. of Students</th>
<th>Draw lines joining hands to indicate how the classmates stand hand in hand</th>
<th>Does everyone get a partner?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7</strong></td>
<td>![Diagram of students hand in hand]</td>
<td>X</td>
</tr>
</tbody>
</table>

2. Repeat Step 1 with different numbers of students in a group and record the results on the worksheet.

Questions for discussion:
1. In a group of students, every two students hold hands together. Will everyone always has a partner?
2. If there are 7 students in a group, how many students could be added so that each student has a partner? At least how many students should be added?
3. In what situation can each student has a partner? In what situation that a student is left without a partner?
4. The teacher chooses a group of students (not more than 10) to enter the “three-legged race” competition. The rules of the competition require teams of 2 students. How many students can be chosen? (More than one answer can be given.)

**Activity 2**
1. The teacher gives each student 10 beads. The teacher says a number, e.g. 5, and each student takes out the correct number of beads and arranges them in pairs.
2. Students record the results in Part A of the Worksheet “Odd and Even Numbers”.
3. Students observe and compare the results, and divide the numbers into two types. Then the teacher introduces the concept of “odd numbers” and “even numbers”.

Questions for discussion:
1. Observe the arrangements of beads, what are the similarities and differences?
2. Based on the above observations, how many types of numbers can 1 to 10 be divided into?
3. Which numbers can be grouped into the same type? What is the common characteristic of these numbers?

**Activity 3**
1. Students generalise a conclusion for the discussion in Activity 2, and complete Part B of the Worksheet “Odd and Even Numbers”.
2. Students observe the results in Part B of the Worksheet and discuss, and find out the relationship between odd numbers and even numbers.

Questions for discussion:
1. Which numbers are odd numbers? Which numbers are even numbers?
2. What is the relationship between the order of appearing of odd numbers and even numbers?
3. You have odd number of beads and if you want to make it even, how many more beads should be added? What is the least number of beads that should be added? How many beads should be taken away? What is the least number of beads that should be taken away?
4. You have even number of beads and if you want to make it odd, how many more beads should be added? What is the least number of beads that should be added? How many beads should be taken away? What is the least number of beads that should be taken away?
This example mainly involves the following generic skills:

1. Communication Skills
   - Comprehend and act appropriately on spoken instructions, for example, by following the teacher’s instructions to draw and to record the findings on the worksheet
   - Present results of tasks with drawings and symbols
   - Discuss with others in accomplishing tasks, for example, by finding out the differences between odd and even numbers

2. Critical Thinking Skills
   - Reason inductively when exploring the characteristics of numbers
   - Draw logical conclusions based on adequate data and evidence

3. Problem solving Skills
   - Adopt various ways of solving problems, for example, by finding out the ways to make an odd number even or make an even number odd
Hand in Hand

Can you record how the classmates stand hand in hand?

<table>
<thead>
<tr>
<th>No. of students</th>
<th>Draw lines joining hands to indicate how the classmates stand hand in hand</th>
<th>Everyone get a partner? (✓ or ×)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Odd and Even Numbers

(A) Draw the arrangements of beads in the table below.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</tr>
</tbody>
</table>

(B) Colour the boxes with **odd numbers** in red, **even numbers** in blue.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</tr>
</tbody>
</table>
Example 2
Sharing Game

Key Stage: 1

Strand: Number

Learning Unit: Division (I)

Objective: (i) Understand the meaning of equal sharing (ii) Perform equal sharing of objects

Prerequisite Knowledge: Basic multiplication

Resources Required: (i) Pencils, beads (ii) Worksheet

Description of the Activity:

Activity 1
1. The teacher shows students 6 pictures of apple. One student tries to share the apples evenly between two classmates. The student repeats the sharing using 7 apples.
2. The teacher asks students to pay attention to the sharing process and helps students to recognise the concept of equal sharing.
3. Students repeat the sharing of apples among three and four classmates. By comparing the sharing processes and the respective results, students further recognise the concept of sharing.

Questions for discussion:
1. If we do not know the total number of objects, how could we share the objects evenly between 2 people?
2. What are the similarities and differences in the processes and results between the cases of sharing 6 apples and 7 apples between two people?
3. When 6 apples are shared between two people, will each one get the same number of apples? Can all the apples be shared evenly? How about sharing among three or four people?
4. What are the similarities and differences in the processes and results when 6 apples are shared among two, three or four people?
5. If all the apples have been shared, but the amount of apples each person get is not the
same, can this still be considered as equal sharing?

**Activity 2**
1. The teacher shows students 12 pencils. Students guess how many persons can share the pencils evenly and the number of pencils each person can get.
2. Students perform the real sharing and see if their guess is correct or not.
3. The teacher reminds students to pay attention to the sharing processes and results. The teacher guides students to find out the relationship between the amounts of objects being divided and the numbers of people involved in the sharing.

Questions for discussion:
1. Can you guess how many people can share the pencils evenly? How many pencils will each person get?
2. Are your answers correct? How do you guess the answers?
3. What other method(s) can be used to speed up the sharing process apart from sharing the pencils one by one?
4. How is the change in the number of pencils each person gets if more people share the pencils?
5. What is the relationship between the number of pencils, the number of people sharing the pencils and the number of pencils each person gets under the situation of equal sharing?

**Activity 3**
1. Students take the Worksheet “Sharing Game” and 18 beads.
2. Students complete Questions 1 to 3.
3. The teacher guides students to find out the multiplication relation among the quantities.

Questions for discussion:
1. Will each person get the same number of pencils when 15 pencils are all shared among four people?
2. How many beads will each person get when 18 beads are shared evenly between two people?
3. If each person gets 9 beads, then how many beads in total do two people get? How is it calculated?
4. What is the relationship between the total number of beads, the number of people and the number of beads each person gets under equal sharing? Does this relationship still hold when the beads cannot be shared evenly?
Activity 4
1. After students have returned the beads to the teacher, they complete Question 4 of the Worksheet. The teacher encourages students to write down their ideas.
2. Students share their answers of Question 4e.
3. The teacher guides students to observe the answers in the worksheet and try to find out the characteristic or pattern of the sharing process. Students share their findings with others.

Questions for discussion:
1. In each activity of sharing objects, can the objects be shared evenly every time?
2. How to tell the objects are shared evenly?
3. When the objects are shared evenly, how can we find out the number of objects that each person gets?
4. If a certain number of objects can be shared evenly into two portions, what is the characteristic of the amount?

This example mainly involves the following generic skills:

1. Communication Skills
   - Explain how students know that their answers are correct
   - Explain how the problems are solved
   - Explain the differences between the cases

2. Problem solving Skills
   - Transfer knowledge of basic multiplication and solve the problems
   - Use real objects and diagrams to assist the thinking process
   - Record results systematically
Sharing Game

1. 

a) Can the apples be shared evenly between two people? (Yes / No)  
   If yes, each one can get _____ apples.

b) Can the apples be shared evenly among four people? (Yes / No)  
   If yes, each one can get _____ apples.

2. 

a) Can the pencils be shared evenly between two people? (Yes / No)  
   If yes, each one can get _____ pencils.

b) Can the pencils be shared evenly among three people? (Yes / No)  
   If yes, each one can get _____ pencils.

c) Can the pencils be shared evenly among five people? (Yes / No)  
   If yes, each one can get _____ pencils.

d) Can the pencils be shared evenly among four people? (Yes / No)  
   If yes, each one can get _____ pencils.
3.  

a) Can the beads be shared evenly between two people? (Yes / No)  
   If yes, each one can get _____ beads.

b) Can the beads be shared evenly among three people? (Yes / No) 
   If yes, each one can get _____ beads.

c) Share among six people, each one can get _____ beads.

d) Share among nine people, each one can get _____ beads.

e) Can the beans be shared evenly among four people? (Yes / No) 
   If yes, each one can get _____ beads.

f) 18 beads also cannot be shared evenly among ______ people.

4.  

a) Evenly share between two people, each one can get _____ beads.

b) Evenly share among four people, each one can get _____ beads.

c) Evenly share among five people, each one can get _____ beads.

d) Evenly share among ten people, each one can get _____ beads.

e) How do you get the answers? Try to explain.
Example 3
Making Your Own Measuring Cup

Key Stage: 1

Strand: Measures

Learning Unit: Capacity

Objectives: To consolidate students’ concept on measurement and capacity through making a measuring cup

Prerequisite Knowledge: (i) Compare the capacity of containers directly
(ii) Recognise “litre” and “millilitre” (or “L” and “mL”) as one of the standard units of capacity

Resources Required: Transparent containers with capacity more than 1L, measuring cups (500mL), some daily life containers of known capacities, papers strips, and adhesive tape

Description of the Activity:

Activity 1:
1. The teacher shows some transparent containers with capacity more than 1L, which are commonly found in our daily live.
2. Students form groups of four or five. Each group is given a 500mL measuring cup and a transparent container.
3. The teacher asks the students to fill the measuring cup with 500mL of water and then pour the water into the transparent container.
4. Students stick a paper strip vertically on the transparent container, and mark “500mL” on the paper strip at the height of water level.
5. Students pour another 500 mL water into the transparent container and mark “1L” on the paper strip at the height of water level (as shown in the figure on the next page).
Questions for Discussion:
1. Can the container be used as a measuring cup to measure the capacity of some other containers?
2. What are the drawbacks of this self-made measuring cup? (can measure 500mL and 1L only)
3. How do you improve it?
Notes for teachers:
1. The teacher should make sure that students have a clear direction to improve their self-made measuring cup.
2. The teacher may use containers with a uniform cross-section or other containers for the activities.
3. If the cross sections of the container are not uniform, the marking of “500mL” may not lie exactly at the middle between the marking of “1L” and the lower end of the paper strip.

Activity 2:
1. Students are given a period of time (e.g. two weeks) to improve the self-made measuring cup.
2. Then each group presents how they improve the measuring cup and the problems they encountered.
3. The teacher provides some containers (with capacity between 500mL and 1L preferred) for each group to test their measuring cup.

Notes for Teachers:
1. The teacher may allow students to conduct the first part of Activity 2 at home.
2. The teacher can decide whether to give hints to students in accordance with students’ abilities.
3. The teacher can hint students to repeat the method used in Activity 1 (to mark “500mL” and “1L”) to locate markings like “200mL”, “400mL”, etc. using some small measuring cups at home.
4. Students should be given adequate time to do the project.
5. Suggested criteria for assessing students’ performance:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description of the Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand the theme of the project</td>
<td>• Do students mark “500mL” and “1L” on the appropriate positions?</td>
</tr>
<tr>
<td></td>
<td>• Do students provide appropriate new markings?</td>
</tr>
<tr>
<td>Using appropriate strategies and methods</td>
<td>• Do students use appropriate methods to locate the marking of “1L” instead of doubling the length of the water level of 500mL on the strips?</td>
</tr>
<tr>
<td></td>
<td>• Do students develop a strategy to add some more markings on the paper strip?</td>
</tr>
<tr>
<td>Criteria</td>
<td>Description of the Criteria</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Accuracy</td>
<td>• Do student give markings accurately with respect to the water level?</td>
</tr>
<tr>
<td></td>
<td>• Do students demonstrate a strategy (e.g. repeat the measure for several times) to locate the markings in a more reliable manner?</td>
</tr>
<tr>
<td>Presentation and</td>
<td>• Can students elaborate their ideas on the design clearly?</td>
</tr>
<tr>
<td>communication</td>
<td>• Can students present their process on measurement clearly?</td>
</tr>
<tr>
<td>Attitude</td>
<td>• Do students work collaboratively?</td>
</tr>
<tr>
<td></td>
<td>• Do students participate actively in the task?</td>
</tr>
</tbody>
</table>

The criteria suggested are by no means exhaustive and each criterion above is not of the same importance.

This example mainly involves the following generic skills:

1. Collaboration Skills
   • Participate actively in discussion in selecting a suitable strategy for locating the markings for different capacities
   • Listen to others with patience, and explain own ideas in response to group members’ views in selecting a suitable design for the measuring cups

2. Critical Thinking Skills
   • Find some reasons to account for the non-uniform spreading of markings.
   • Identify advantages and limitation of their measuring cup and suggest some ways to improve

3. Problem solving skills
   • Develop a strategy to put new markings at appropriate positions
Example 4

Discovering Symmetry

Key Stage: 2

Strand: Shape & Space

Learning Unit: Symmetry

Objectives:
(i) To recognise symmetrical shapes
(ii) To identify symmetrical shapes
(iii) To make symmetrical shapes

Prerequisite Knowledge: Recognise regular and irregular 2-D shapes

Resources Required: Handicraft paper, scissors and worksheets

Description of the Activity:

Activity 1
1. Each student is given a pair of scissors and a piece of handicraft paper. They cut out some simple shapes such as a heart or a butterfly in a simple way.
2. The teacher selects some of the students’ shapes that are more or less symmetrical and posts them on the blackboard for discussion.

Questions for discussion:
1. What is the fastest way to cut the shapes on the blackboard?
2. Are there any differences between the shapes cut by folding the paper and those did not cut by folding the paper?
3. For the shapes cut by folding the paper, what are the features of the two halves of the folding line?

Activity 2
1. The teacher introduces the concepts of symmetrical shapes and lines of symmetry.
2. Each student is given some handicraft paper. They are asked to create symmetrical shapes by folding the handicraft paper.
3. The teacher selects some of students’ shapes and posts them on the blackboard.
Questions for discussion:
1. Which of the shapes are symmetrical? What are their common properties?
2. How do we identify symmetrical shapes?
3. Are there any other methods to identify symmetrical shapes?
4. What are the differences between the shapes made by folding the paper once and twice?

Notes for Teachers:
1. Teacher can prompt students to fold the paper more than once when they have finished their first shape.
2. Besides folding, mirror can also be used for identifying symmetrical shapes. Students can also be guided to observe that the line segment joining “overlapping points in folding” is perpendicular to the line of symmetry and also be cut into two equal line segments by the line of symmetry.

Activity 3
1. The teacher groups students into groups of three or four.
2. Each group is given the “Symmetrical Shapes” worksheet. (The teacher could also provide students with enlarged copies of the 2-D shapes in the worksheet. They are mainly used to assist students in identifying which shapes are symmetrical.)
3. The teacher asks students to identify the symmetrical shapes and add lines of symmetry on those symmetrical shapes.

Questions for discussion:
1. Which shapes in the worksheet are symmetrical shapes?
2. Where are the lines of symmetry for each symmetrical shape? (The teacher can post the shapes on the blackboard and ask students to draw the lines of symmetry.)
3. Which of the symmetrical shapes have more than one line of symmetry?
4. In our daily life, which shapes are symmetrical? (After the discussion, the teacher can ask students to collect some pictures of symmetrical shapes at home.)

Notes for Teachers:
1. According to students’ abilities, the teacher could decide if it is necessary to provide students with the enlarged symmetrical shapes in the Worksheet, so as to let students verify the symmetry of the shapes by folding, measuring or other methods.
This example mainly involves the following generic skills:

1. Communication Skills
   - Describe the features of symmetric shapes
   - Discuss the advantages and disadvantages on the methods of identifying symmetrical shapes

2. Creativity
   - Create symmetrical shapes by folding the handicraft paper

3. Critical Thinking Skills
   - Justify if the shapes are symmetrical according to the concept and properties of symmetrical shapes
Symmetrical Shapes

For the shapes below, identify the symmetrical shapes and add lines of symmetry on those symmetrical shapes.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10.
Example 5
Finding Area

Key Stage: 2

Strand: Measures

Learning Unit: Area (I)

Objective: To consolidate the concept of area

Prerequisite Knowledge: Finding the area of figures using the standard unit: square centimeter (cm²)

Resources Required: Worksheet (Grid paper and dotted paper)

Description of the Activity:
1. The teacher distributes the grid paper to students and guide them to answer the following questions:
   (a) Try to draw three figures that have the same area as the shaded region but have different shapes.
   (b) How do you know that each figure drawn has the same area as the shaded region?

2. After students complete the drawing of figures, the teacher could invite students to share their drawing strategies.

3. (Extension) The teacher distributes the dotted paper to students and guide them to answer the following questions:
   (a) Is the area of the shaped region on the dotted paper the same as that on the grid paper?
   (b) Try to create some figures on the dotted paper that are different (in terms of both shape and area) from the shaped region.
   (c) What are the areas of the figures created? (in cm²)

Notes for Teachers:
1. Sufficient time should be allowed for students to draw the figures.
2. Figures drawn could be some common ones. They could also be irregular polygons.
3. Students’ understanding on the concept of area and creativity in drawing the figures are the foci of this activity.

4. Teachers can use the following assessment criteria to assess students’ performance:
   (a) Can students understand the concept of area? (Are the figures drawn closed figures?)
   (b) Can students measure areas using square centimetres (cm²) correctly?
   (c) Can students reach the requirements on the number and area of figures to be drawn?
   (d) Can students conclude their strategies applied in drawing the figures?

This example mainly involves the following generic skills:

1. Communication Skills
   • Explain, with the aid of diagrams, the reasons why the figures have the same area

2. Creativity
   • Draw figures other than the common ones
   • Use different methods of finding the areas of figures

3. Problem solving Skills
   • Use simple methods to find the areas of figures
   • Draw figures using different strategies
   • Apply previous knowledge learnt from the past to draw figures
Grid paper:

Dotted paper:
Example 6

Rubber Band Powered Car

Key Stage: 2

Strand:
Mathematics: Measures (Learning Unit: Speed)
General Studies: Science and Technology in Everyday Life
(Core Elements: efficient transfer of energy and the interaction between energy and materials)

Objectives:
(i) To consolidate the concept of speed
(ii) To use “metres per second” (m/s) and “kilometres per hour” (km/h) as the unit of speed
(iii) To recognise efficient transfer of energy and the interaction between energy and materials
(iv) To design and build models by using different materials and to test selected functional characteristic of the model built with the chosen materials

Prerequisite Knowledge: Understanding the relationship among speed, time and distance

Resources Required: Worksheets, stopwatches, measuring tapes and calculators

Related Links: Scientific Investigation Series - Potential Energy Car
www.hkedcity.net/edb/teachingresources/project/?p=science&path=/car
Scientific Toy DIY - Rubber Band Powered Car
sites.google.com/site/sciencemagician/ke-xuediy/xiang-pi-jin-dong-li-che-guang-die-che

Description of the Activity:
Activity 1
To consolidate students’ concept of speed by teacher’s questions raised.
Questions for discussion:
1. If A and B run 100 metres in 20 seconds and 30 seconds respectively. Who runs faster on average?
2. If C and D run 50 metres and 60 metres in 10 seconds respectively. Who runs faster on average?
3. What is the relationship between time and speed if they run the same distance?
   What is the relationship between distance and speed if they run at the same time?
4. What information must be obtained to determine their running speed if they do not start in the same place?

Notes for Teachers:
Students are divided into groups and provided the related links. They could prepare and make the rubber band powered car prior to the lesson.

Activity 2
1. The teacher read out Part 1 of Worksheet as an introduction. Students are grouped in pairs. Students discuss the ways to compare 2 persons’ speeds when the distances they run and the time they spend are different. The teacher guides students to find the formula which is used to calculate the average speed.

2. Each group is given the Worksheet. Students complete Part 1 of Worksheet and present the results.

3. Students collect the data and complete Part 2 of Worksheet by using their rubber band powered cars.

Questions for discussion:
1. If we obtain the distance an object moves and the time it spends to move, how can we calculate the average distance it moves in 1 second?
2. How can we calculate the average time it spends to move 1 metre?

3. Which method do you like to use to compare their speeds? Why?

4. The distances the cars of two groups run and the time they spend are different. How can you compare their speeds?

Notes for Teachers:
1. Activities aim at measuring the speed of their rubber band powered cars, not the speed racing. There is no need to arrange each car starting at the same place and same time.
2. The teacher can remind students to refer to the design and materials used by each other groups in order to improve their own rubber band powered car.

Activity 3
To discuss with students the energy conversion and the ways to improve the structure of the car.

Questions for discussion:
1. When a student is scrolling the rubber band, what kind of energy is stored?

2. When students put their cars on the ground and release the wheels, what kind of energy will be converted from the energy stored in the rubber band? What is the relationship between this kind of energy and the speed of their cars?

3. How to increase the speed and running time of the rubber band powered car?

Notes for Teachers:
1. To save time, the teacher may allow students to complete Part 2 of Worksheet using a calculator.

2. The teacher can remind students how to use the stopwatch and measuring tape for measurement.

Integration and Application:
Science Education: Conversion of energy

Technology Education: Model design and choosing appropriate materials

Mathematics Education: Concept of speed and measurement of speed
This example mainly involves the following generic skills:

1. Collaboration Skills
   - Collaborate in groups
   - Share responsibilities and understand the roles of individual members in the construction of the rubber band powered car

2. Creativity
   - Improve and refine the designs of the rubber band powered car

3. Problem solving Skills
   - Propose different solutions to increase the speed and running time of the rubber band powered car
   - Try out and improve the solutions through different experiments
Worksheet

1. The running records of Student A and Student B are as follows:

<table>
<thead>
<tr>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100 m</td>
</tr>
<tr>
<td>B</td>
<td>150 m</td>
</tr>
</tbody>
</table>

How to compare the speed of A and B?

<table>
<thead>
<tr>
<th>method 1</th>
<th>method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A has run _____ m in 1 second on average.</td>
<td>A has spent _____ s to run 1 m on average.</td>
</tr>
<tr>
<td>B has run _____ m in 1 second on average.</td>
<td>B has spent _____ s to run 1 m on average.</td>
</tr>
<tr>
<td>Student _____ runs faster, because on average the distance he/she runs is _____ in 1 second.</td>
<td>Student _____ runs faster, because on average he spends _________ time to run 1 m.</td>
</tr>
</tbody>
</table>

Which method do you like to use to compare their speeds? Why?

2. Complete the following table and calculate the speed of rubber band powered car.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Time (s)</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Remark: The speed calculated can be rounded off to 1 decimal place.
Example 7

Cross Sections of 3-D Shapes

Key Stage: 2

Strand: Shape and Space

Learning Units: 3-D Shapes (III)

Objectives:
(i) Recognise the concepts of the cross sections of a prism and a cylinder
(ii) Recognise that the sizes and shapes of cross sections of a prism and a cylinder, which are parallel to the bases, are the same as that of the bases

Prerequisite Knowledge:
(i) Identify different 3-D shapes
(ii) Identify different 2-D shapes

Resources Required:
(i) objects or plasticine shaped in 3-D figures
(ii) physical models of 3-D figures (include cuboids, triangular prisms, cubes, triangular pyramids, rectangular pyramids, cylinders and cones)
(iii) tablet computer
(iv) e-resources on cross sections of 3-D solids (such as https://sites.google.com/site/hkgeogebra/ks2/ss/3d_sections)

Description of the Activity:

Activity 1
1. The teacher shows some plasticine or objects shaped in prisms or circular cylinders (such as fruits, cakes, etc.) to students, and cuts the objects or plasticine by a plastic knife or ruler. The teacher then introduces the cutting planes to students and calls the planes “Cross Sections”.

2. Students are requested to guess the shapes of cross sections of circular cylinders and circular cones, which are parallel to the base of the two 3-D shapes, by observing the physical models given by the teacher.

3. Students then use the tablet computers to access the e-resources to verify their guesses.
4. The teacher could request students to discuss the similarities and differences between the cross sections of circular cylinders which are parallel to their bases and those of circular cones, with the help of tablet computers.

Questions for Discussion:
1. If a cylinder is cut in a direction parallel to the bases, are the shapes of cross sections obtained equal? How about cones?
2. If a cylinder is cut in a direction parallel to the bases, are the sizes of cross sections obtained equal? How about cones?
3. (Further discussion) If you want to get cross sections with same shapes and sizes, how do you cut a cylinder/cone?
4. (Further discussion) Can you get a rectangular section when cutting a cylinder/cone? How do you cut it?

Notes for Teachers:
1. If needed, the teacher may let students cut real objects to develop the concept of cross sections before using the software for exploration. The teacher should notice that cross sections which are parallel to the bases should be the major topic for discussion.
2. Students are requested to discuss the cross sections of cylinders first as their cross sections are simple.
3. Students are reminded to capture the screen of the software, in order to facilitate explanation and discussion.

Activity 2
1. Students are requested to explore the cross sections of triangular prisms which are parallel to their bases and those of triangular pyramids, with the help of tablet computers. The teacher should provide assistance to students whenever necessary.

Questions for Discussion:
1. What are the differences between the cross sections of a triangular prism which are parallel to the bases and those of a circular cylinder?
2. What are the differences between the cross sections of a triangular pyramid which are parallel to the base and those of a circular cone?
3. What are the similarities and differences between the cross sections of triangular prisms which are parallel to the bases and those of triangular pyramids?
4. (Further discussion) Which types of polygonal cross sections can be obtained by cutting a triangular prism/pyramid?
Notes for Teachers:
1. When students have acquired enough experience on operating the software, teachers can let them explore the cross sections of cuboids and rectangular pyramids for their self-learning.

This example mainly involves the following generic skills:

1. Critical thinking
   - Compare the similarities and differences between cross sections of circular cylinders which are parallel to their bases and those of circular cones through observations, discussions, making guesses and verifications

2. Information Technology
   - Use software packages for learning the concepts of cross sections

3. Self-learning
   - Take initiative in identifying and organising main points from sources, for example, record the figures of cross sections in different 3-D shapes for comparison
Example 8
Knowing Your Community

Key Stage: 2

Strand: Shape & Space, Data Handling

Learning Units: Directions and Positions (III) and Uses and Abuses of Statistics

Objectives:
(i) To find the eight compass points with a compass
(ii) To become familiar with the district in the vicinity of the school
(iii) To sketch a plan of the district in the vicinity of the school
(iv) To recognise the facilities and services offered in the community
(v) To present the collected data with statistical charts

Prerequisite Knowledge:
(i) Finding the four directions with a compass
(ii) Constructing a simple bar chart

Resources Required: Compasses

Description of the Activity:

Activity 1:
1. Students use compasses to find the four directions and describe the community facilities within the school vicinity in the four directions.
2. Students use compasses to find the eight compass points and record the community facilities within the school vicinity in the eight directions.

Questions for discussion:
1. If you are in a fixed position of the school campus, will the directions of the community facilities determined by the compass change when you change the direction faced?
2. Will the directions of the community facilities determined by the compass change when you are in different positions of the school campus?

Activity 2:
Students are divided into groups to perform the following tasks:
1. Sketch the plan of the area in the vicinity of the school, with the help of compasses.
For example,

(Since distances between facilities are not the main concern of this activity, students can estimate them intuitively.)

2. Collect information on the community facilities and services available within the school vicinity, for example, the number of markets, parks, and libraries. Choose one or two community facilities for investigation in detail, e.g. the scale of the facilities, the frequency with which the facilities are used by the residents of the community. (More able students may collect information by conducting a small-scale survey.)

3. Organise the information collected and present the data with appropriate statistical charts, such as bar charts. The teacher can encourage students to make use of information technology in presenting their findings. For example, students can record data and draw statistical charts by spreadsheet and take pictures of the facilities with mobile digital devices.

4. Prepare a report and present the findings orally.

5. (Extension) Suggest ways to improve the community facilities.

Questions for discussion:
1. What kinds of facilities are provided in the vicinity of the school?
2. Which types of facilities do you think are in excess? Why?
3. Which types of facilities do you think are insufficient? Why?
4. If you were a town planner, how would you redesign this district?
5. How can you protect and improve the facilities and environment of the community?

**Notes for Teachers:**
1. Students should be given adequate time to do the project.
2. Teachers should advise students not to use a maps (included printed map, online map or apps) when they are sketching the plan of the vicinity of the school, and advise them to apply the knowledge on compasses points and get to know the community through sketching the plan. However, student could use a map if the location and surrounding environment of the school makes the sketching process difficult.
3. Teachers should give explicit instructions and suggest methods for the collection of data.

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Share responsibilities and understand the roles of individual members in collecting data and presenting projects results
   - Discuss and exchange ideas with others in sketching the plan, and decide the information to be collected and the strategies to be used through discussion
   - Participate actively in the project and listen to others patiently in discussions
   - Understand that there might be different views in ways of improving the community facilities from members with different cultural backgrounds

2. Creativity
   - Propose creative solutions to problems, present own views on the provision of community facilities and suggest ways of improving

3. Critical Thinking Skills
   - Extract, classify and organise information collected from the area in the vicinity of the school
   - Understand the concepts of relevance and irrelevance, for example, in identifying the relevant information useful for supporting the suggested ways to improve the community facilities

4. Problem solving Skills
   - Choose the relevant information to solve problems, for example, by identifying the statistical data necessary for giving suggestions for improving the community facilities
• Make use of various tools in solving problems, for example, using a tally in counting the frequency with which the facilities are used, and spreadsheet software to record data and construct statistical charts.
Example 9
Extra-Curricular Activities

Key Stage: 2

Strand: Data Handling

Learning Units: Bar Charts (II)

Objectives:
(i) To use appropriate methods to collect and organise data
(ii) To use relevant statistical charts to represent collected data
(iii) To read and discuss charts constructed
(iv) To make recommendations based on the data collected

Prerequisite Knowledge: Experience of collecting data and drawing block graphs and bar charts

Resources Required: Grid paper

Description of the Activity:
1. Students form groups of 4 students.
2. The teacher guides each group to fix a project topic about extra-curricular activities with students, for example, investigating how much time students spent on extra-curricular activities, which extra-curricular activities are more popular, the usage rates of venues for extra-curricular activities.
3. Students, working in groups, discuss the method to collect information about the topic on extra-curricular activities that decided.
4. Students collect data and organise the data collected.
5. Students select suitable and relevant statistical charts to present the data.
6. Students read and discuss the charts with groupmates and make recommendations about extra-curricular activities based on the set of data collected, for example, which activities should be organised or which kind of activity room should have the number increased.
7. Each group report to the class orally and prepare a report.

Notes for Teachers:
1. The teacher prepares information about the extra-curricular activities organised at school before the project.
2. Assessment of students’ performance in the project should not only focus on the product (the written report). Their attitude towards the project and their participation, etc. should also be considered (see point 5 below for the assessment criteria).
3. Part of the project can be done at home and students should be given adequate time for discussion and preparation of the project.
4. The teacher should give students suggestions as to the time schedule for the project and tell them by when they should have completed their work.
5. Suggested criteria for assessing students’ performances:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description of the Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand the theme of the project</td>
<td>• Do students demonstrate a thorough understanding of the theme of the project?</td>
</tr>
<tr>
<td></td>
<td>• Do students know how to carry out the project and what information they are expected to collect?</td>
</tr>
<tr>
<td>Using appropriate strategies and methods</td>
<td>• Are proper methods of data collection designed and relevant information collected?</td>
</tr>
<tr>
<td></td>
<td>• Do students use appropriate statistical charts to represent the data collected?</td>
</tr>
<tr>
<td></td>
<td>• Do students analyze and compare data?</td>
</tr>
<tr>
<td>Accuracy</td>
<td>• Do students construct statistical charts accurately?</td>
</tr>
<tr>
<td></td>
<td>• Do students analyze the statistical chart accurately?</td>
</tr>
<tr>
<td></td>
<td>• Do students make feasible recommendations based on their analysis?</td>
</tr>
<tr>
<td>Presentation and Communication</td>
<td>• Can students present their work in a systematic and logical way?</td>
</tr>
<tr>
<td></td>
<td>• Do students report their work accurately?</td>
</tr>
<tr>
<td>Attitude</td>
<td>• Do students exercise a good team spirit?</td>
</tr>
<tr>
<td></td>
<td>• Are students actively involved in the activity?</td>
</tr>
<tr>
<td></td>
<td>• Have students reviewed their work in a timely manner?</td>
</tr>
</tbody>
</table>

The criteria suggested are by no way exhaustive and each criterion above is not of the same importance.

This example mainly involves the following generic skills:
1. Collaboration Skills
   • Collaborate in groups and share responsibilities, understand the roles of individual members in collecting data, preparing statistical charts and presenting the project results
   • Share ideas freely with others in conducting the project
   • Discuss, negotiate and compromise with others in designing methods for collecting data and presenting the results of the project
   • Share ideas in making recommendations on extra-curricular activities at school

2. Communication Skills
   • Use suitable communicative skills to collect information
   • Discuss with group members during the process
   • Report to the class and discuss with classmates and teacher during reporting

3. Critical Thinking Skills
   • Read and discuss charts, find out the important or critical parts
   • Draw logical conclusions and make feasible and logical recommendations based on the information collected
Example 10
Four Seasons

Key Stage: 2

Strand:
Mathematics: Data Handling (Learning Unit: Broken line graphs)
General Studies: People and Environment (Learning Objective: to identify and describe climate and seasonal changes and their effects)

Objectives: (i) To read and discuss broken line graphs
(ii) To construct broken line graphs
(iii) To recognise the temperature change in the four seasons
(may collaborate with General Studies)

Prerequisite Knowledge: (i) Finding the average of a group of data
(ii) Reading and constructing bar charts of large frequency counts

Resources Required: World maps, globes and computers

Related Links: Hong Kong Observatory: www.hko.gov.hk
Bureau of Meteorology, Australia: www.bom.gov.au

Description of the Activity:
Activity 1
The teacher asks students to record the daily temperature of 7 days in a week. It is not necessary to guide the students to record the data in any means or in any forms. Students may collect the data from various sources, e.g. thermometer, weather report. One week later, the teacher discuss with students the data collected.

Questions for discussion:
1. Why are the data collected by everyone on each day different?
2. If the temperature of one day is selected to represent that of the week, which day should be selected?
Notes for Teachers:

1. The teacher may guide students to account for the difference. For example, the temperature varies within a day and the readings are not the same when they are measured at different time of a day. Measuring at different places or by different instruments may also lead to inconsistent results. Students are guided to conclude that some conditions or criteria are necessary in the setting of data collection.

2. By using the result in the discussion, the teacher may introduce that different regions use maximum temperature, minimum temperature or average temperature for describing the temperature of a particular period of time. The following links can be referred to:
   - Climatological Information Services (Hong Kong Observatory):
     www.weather.gov.hk/cis/climat_e.htm
   - Climate statistics for Australian location (Bureau of Meteorology, Australia):
     www.bom.gov.au (follow the path Bureau Home > Climate > Climate Data Online)

3. Through discussion, the teacher can introduce the use of the average of a group of data to represent the group in daily life situations.

Activity 2
According to the result of Activity 1, the teacher guides the students to agree with a scheme for measuring temperature. Students will measure the temperature of 7 days in another week in accordance with the scheme.

Questions for discussion:
1. Are the temperatures measured at one or some particular times on each day? If so, which moments are the most appropriate?
2. If the temperatures are measured at different time of a day, is the maximum temperature or minimum temperature selected to be the target data for recording? How can you collect the data for the maximum temperature or minimum temperature?

Notes for Teachers:
To arouse students’ interest, the teacher can introduce some instruments that are commonly used by the Hong Kong Observatory for measuring temperature. The following links can be referred to:
- Conventional Instruments installed at an Automatic Weather Station in Hong Kong (Hong Kong Observatory)
  http://www.hko.gov.hk/education/article_e.htm?title=ele_00454
Activity 3

1. The teacher introduces and discusses with students the temperature information available in the Internet, e.g. the monthly mean of maximum daily temperature of Hong Kong available in the webpage of the Hong Kong Observatory.
2. The students collect temperature information from the webpage of Hong Kong Observatory and Australia’s Bureau of Meteorology.
3. The students construct broken line graphs by Excel under the guidance from the teacher.
4. There can be discussion on the points to note in the construction of broken line graphs by Excel.
5. The students read and discuss the characteristics of the broken line graphs.
6. The teacher shows the broken line graphs constructed by using the temperature information of the two places and guides the students to compare and find their differences. Students can explore the reason to account for the phenomenon.

Questions for discussion:

1. When is summer in Hong Kong? When is winter in Hong Kong?
2. When is summer in Australia? When is winter in Australia?
3. Ying Ying’s grandma lives in Australia. If Ying Ying is going to visit her during Christmas holiday, should Ying Ying wear winter clothes or summer clothes?
4. Describe the changes of temperature of the two places. Why is there such a difference?

Notes for Teachers:

1. When the students are guided to construct broken line graphs by Excel, the teacher should guide the students to use an appropriate graph.
2. When the students have finished their construction, they should be reminded to check if the graph has provided adequate information, such as title and labels for the two axes, etc.
3. For the ease of teacher’s preparation, two sets of temperature data of Hong Kong and Australia are included in the Information Sheet.
4. The teacher may use the monthly mean of daily maximum/minimum temperature of two places for comparison.
5. If necessary, data from places other than Hong Kong and Australia can be used to construct broken line graphs. They can be used as reference and to support students’ conclusion.
Integration and Application:

Science Education: Revolution of the Earth around the Sun
Technology Education: Construction of graphs by IT
Mathematics Education: Data collection and handling

This example mainly involves the following generic skills:

1. Creativity
   - Design own scheme in recording the daily temperature.

2. Critical Thinking Skills
   - Explore reasons to account for the differences in daily temperatures recorded by classmates and the difference in seasonal temperature variations between Hong Kong and Australia, and justify the reasons.

3. Information Technology Skills
   - Use a spreadsheet program to construct statistical charts.
### Monthly Means of daily maximum, mean and minimum temperature recorded at the Hong Kong Observatory and Bureau of Meteorology, Australia between 1981-2010

<table>
<thead>
<tr>
<th>Month</th>
<th>Hong Kong Mean Daily Maximum (°C)</th>
<th>Hong Kong Mean Daily Minimum (°C)</th>
<th>Perth, Australia Mean Daily Maximum (°C)</th>
<th>Perth, Australia Mean Daily Minimum (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>18.6</td>
<td>14.5</td>
<td>31.9</td>
<td>17.3</td>
</tr>
<tr>
<td>February</td>
<td>18.9</td>
<td>15.0</td>
<td>32.1</td>
<td>17.6</td>
</tr>
<tr>
<td>March</td>
<td>21.4</td>
<td>17.2</td>
<td>29.9</td>
<td>16.2</td>
</tr>
<tr>
<td>April</td>
<td>25.0</td>
<td>20.8</td>
<td>26.1</td>
<td>13.4</td>
</tr>
<tr>
<td>May</td>
<td>28.4</td>
<td>24.1</td>
<td>22.2</td>
<td>10.9</td>
</tr>
<tr>
<td>June</td>
<td>30.2</td>
<td>26.2</td>
<td>19.2</td>
<td>9.0</td>
</tr>
<tr>
<td>July</td>
<td>31.4</td>
<td>26.8</td>
<td>18.1</td>
<td>8.1</td>
</tr>
<tr>
<td>August</td>
<td>31.1</td>
<td>26.6</td>
<td>18.7</td>
<td>8.2</td>
</tr>
<tr>
<td>September</td>
<td>30.1</td>
<td>25.8</td>
<td>20.4</td>
<td>9.3</td>
</tr>
<tr>
<td>October</td>
<td>27.8</td>
<td>23.7</td>
<td>22.9</td>
<td>10.5</td>
</tr>
<tr>
<td>November</td>
<td>24.1</td>
<td>19.8</td>
<td>26.5</td>
<td>13.3</td>
</tr>
<tr>
<td>December</td>
<td>20.2</td>
<td>15.9</td>
<td>29.2</td>
<td>15.2</td>
</tr>
</tbody>
</table>

**Note:**

The sources of two sets of data are the Hong Kong Observatory ([www.weather.gov.hk](http://www.weather.gov.hk)) and the Bureau of Meteorology, Australia ([www.bom.gov.au](http://www.bom.gov.au))

* To make the data comparable and more reliable, data of thirty years (1981 – 2010) of two places are extracted.

# Perth is chosen from Australia as it lies in the same time zone as Hong Kong.
Example 11
Investigating Errors of Measurements by GPS Tracking Apps

Key Stage: 3

Mathematics Education
Strand: Measures, Shape and Space
(Learning Unit: Error in Measurement)

Technology Education
Knowledge contexts: Information and Communication Technology
(Module: Computer Systems)

Objectives:
(i) To consolidate the concept of error in measurement
(ii) To understand and apply ICT as a prime tool for learning and in our daily life

Prerequisite Knowledge:
(i) Calculate different types of errors
(ii) Use rate, ratio and proportion to solve real-life problems

Resources Required: Scaled floor plans of the school playground, trundle wheels, and tablet computers with GPS tracking apps installed
Description of the Activity:
1. The teacher introduces the activity to students and revises the concept of ratio and proportion.

2. The teacher gives each student a scaled floor plan of the school playground.

3. Students, working in groups, measure the dimensions of the playground on the floor plan and calculate the actual dimensions of the playground by consideration the scale ratio of the floor plan.

4. Students discuss how to verify the answer found in (3).

5. Students use trundle wheels and GPS tracking apps at the same time to measure the dimensions of the playground.

6. Students compare the results obtained in (3) and (5) and by taking the measurements by trundle wheels as reference, compare the errors of the results obtained from measuring the floor plan of the school playground and by GPS tracking apps.

7. Students discuss which method (scaled floor plan or GPS) is more reliable and how to reduce the errors in measurement.

Notes for Teachers:
1. It is desirable for students to work in small groups.

2. The teacher should allow ample opportunities for students to discuss and draw conclusion by themselves instead of giving them straightforward hints.

3. The teacher should prepare prior information about the dimensions of the school playground.

4. The teacher should install the GPS tracking apps in the tablets before the lessons.

5. The teacher could also conduct the activity in a standard athletic ground and asks students to measure the lengths of tracks for studying the errors of measurements by GPS tacking apps.

6. GPS function is widely used nowadays. Teachers could encourage students to find out the principle and usage of GPS from the Internet.
This example mainly involves the following generic skills:

1. Communication Skills
   • Understand, analyse and respond to teacher’s spoken instructions and instructions on worksheets
   • Use appropriate language and mathematical expressions to present the methods and results of calculations
   • Discuss and work with others to accomplish tasks, for example, determining the most appropriate way to take the measurement through discussion

2. Critical Thinking Skills
   • Understand the restrictions of real measurement
   • Evaluate the ways of finding the actual dimensions of the school playground
   • Draw logical conclusions based on adequate data and evidence, for example, by comparing different methods and related errors in measuring to conclude the most appropriate method for taking measurements

3. Information Technology Skills
   • Use the GPS tracking apps in the tablet to carry out the exploratory activities

4. Problem solving Skills
   • Compare the results of different measurement methods and justify the method selected
Example 12
Design a Healthy Diet Menu for a School Lunch Box Supplier

Key Stage: 3

Key Learning Areas: Science, Technology and Mathematics Education KLA

Learning elements:

<table>
<thead>
<tr>
<th>KLA</th>
<th>Learning Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Education</td>
<td>• Common food substance</td>
</tr>
<tr>
<td></td>
<td>• Function of food substance</td>
</tr>
<tr>
<td></td>
<td>• Food pyramids</td>
</tr>
<tr>
<td></td>
<td>• Balanced diet</td>
</tr>
<tr>
<td></td>
<td>• Healthy lifestyles</td>
</tr>
<tr>
<td>Technology Education</td>
<td>• Food groups, dietary goals and eating habits</td>
</tr>
<tr>
<td></td>
<td>• Meal planning</td>
</tr>
<tr>
<td></td>
<td>• Principles and skills, hygienic and safe practices in food preparation</td>
</tr>
<tr>
<td></td>
<td>• Food product development – using a design cycle to create and develop food products to meet the design specifications of a task e.g. address the health concerns of teenagers and sensory requirements of the products</td>
</tr>
<tr>
<td></td>
<td>• Health lifestyle / sedentary lifestyle / unhealthy lifestyle</td>
</tr>
<tr>
<td>Mathematics Education</td>
<td>• Approximation and estimation</td>
</tr>
<tr>
<td></td>
<td>• Collect and organise data</td>
</tr>
<tr>
<td></td>
<td>• Construction and interpretation of statistical graphs</td>
</tr>
<tr>
<td></td>
<td>• Measures of central tendency</td>
</tr>
</tbody>
</table>

Objectives: To integrate and apply knowledge and skills in Science, Technology and Mathematics Education KLAs to solve real-life problems

Description of the Activity:
In this activity, teacher adopts a cross-disciplinary approach to integrate the learning of Science, Technology and Mathematics Education KLA. The project itself is arranged as an independent activity. Learning elements from different KLAs would be drawn in by the
students themselves or by the teachers during the course of the project learning activity.

- In the beginning, the teacher chooses an authentic problem which most students would be concerned. Quite often, there are students complaining about the taste, quality and quantity of the lunch boxes provided by the school canteen. On the other hand, the nutritional values of the lunch boxes are also an issue related to the health of teenagers. Teachers can ask students to conduct a project work to design a healthy diet menu for the lunch box supplier, to meet with the needs of healthy and quality food in school.

- Starting from the essential question, plenty of learning opportunities can be provided for the students to construct, integrate and apply knowledge and skills from different KLA. Students can use IT skills to search information about food and diet, including the functions of food substances, nutritional values, recommended daily intake, etc. Students can apply mathematical skills to calculate and analyse the nutritional values of different food. Students could collect data on food preference of their fellow students, and prepare food for tasting. After proper analysis, student can submit the healthy diet menu to the school lunch box provider for reference.

- During the progress of the project, teachers can provide proper guidance and provide feedback, resources and assistance to their students when needed.

This example mainly involves the following generic skills:

1. Collaboration Skills
   - Share responsibilities and understand the role of each member in the project
   - Agree on suitable strategies for carrying out the project through discussion

2. Creativity
   - Under the constraints in the aspects of nutrition and proportion of ingredients, students designed their menu by applying their creativity.

3. Problem solving Skills
   - Identify the problems associated with the project
   - Make use of data collected to design the menu
Example 13
Mathematics Magic

Key Stage: 3

Strand: Number and Algebra

Learning Unit: Algebraic Expressions
Laws of Integral Indices

Objectives: To apply algebraic language and concepts on place value to explore the principles behind some number games

Prerequisite Knowledge: (i) Representing word phrases by algebraic expressions
(ii) Representing a number in the sum of products of the digit and place value of each place

Description of the Activity:
1. The teacher helps students to form groups for the project. Since the complexity of the project is not great, the group size could be less than four.

2. The teacher shows an example of “mathematics magic”, such as an age guessing game, and explains to students the topic for the project.

An example of a simple game:
(a) You choose a two digit number,
(b) add 6 to it, multiple the sum by 2,
(c) do something “reverse”, that is, subtract 6 from the product in (b) and divide the result by 2,
(d) subtract the original two digit number from the result of (c).
(e) I know what you get. It is 3.

An example of a more complicated one:
(a) You are asked by the “mathematics magician” to use the format yyyymm to form two numbers, one from the present year and month and one from your year and month of birth, and then subtract the second number from the first.
(b) Then you find the difference between the result of (a) and the number formed
by reversing the digits of it, e.g. if the result of (a) is 12345, then calculate 54321 – 12345, the larger one minus the smaller one.
(c) You choose a digit from the result of (b) to delete and tell the magician the sum of the remaining digits.
(d) Although the magician does not know your year and month of birth, he can tell the digit that you choose to delete.
(Reference: http://www.guokr.com/article/92761/)

3. Students are required to
(i) search for some “mathematics magic” or number guessing games, from books or the Internet, that the principles behind is not provided;
(ii) investigate the principles of the games or magic;
(iii) make comparisons between different games or magic; and
(iv) design their own mathematics magic.

4. The teacher should keep track of the progress of each group and provide guidance/assistance whenever necessary.

5. Students are required to write a brief report and present their findings to their classmates.

**Notes for Teachers:**
1. According to students’ abilities, the teacher could choose to give further explanations to the examples provided to students. For example, for the first game mentioned above, teachers could demonstrate the use of algebraic language to represent the process:

   \[ [(x + 6) \times 2 - 6] \div 2 - x = (2x + 6) \div 2 - x = 3 \]

2. There are different types of number games for the project. Here are some examples:

   **Guessing numbers or dates**
   - http://mall.cnki.net/magazine/article/ZXSX201013026.htm

   **Using binary numbers**
   - http://www.guokr.com/article/437284/
   - https://scratch.mit.edu/projects/91474307/
3. Below are some suggested criteria for assessing students’ performance in the project:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description of the Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the theme of the project</td>
<td>• Do students demonstrate a thorough understanding of the theme of the project?</td>
</tr>
<tr>
<td></td>
<td>• Do students know how to carry out the project and what information they are expected to collect?</td>
</tr>
<tr>
<td>Using appropriate strategies and methods</td>
<td>• Do students use algebraic expressions to investigate the principles of the games/magic?</td>
</tr>
<tr>
<td></td>
<td>• Do students adopt feasible strategies to investigate the principles of the games/magic? Are variables be set systematically?</td>
</tr>
<tr>
<td>Accuracy</td>
<td>• Are the algebraic expressions formulated correctly?</td>
</tr>
<tr>
<td></td>
<td>• Are the algebraic manipulations carried out correctly?</td>
</tr>
<tr>
<td></td>
<td>• Do the principles suggested completely explain the games/magic?</td>
</tr>
<tr>
<td>Creativity</td>
<td>• Can the group design their own game/magic?</td>
</tr>
<tr>
<td>Presentation and communication</td>
<td>• Can students present their exploration process in a systematic and logical way?</td>
</tr>
<tr>
<td></td>
<td>• Is the report clear, concise and matching the theme?</td>
</tr>
<tr>
<td>Collaboration</td>
<td>• Is the division of duties among students in the group appropriate?</td>
</tr>
<tr>
<td></td>
<td>• Is there a good collaboration among members?</td>
</tr>
</tbody>
</table>

The criteria suggested are by no way exhaustive and each criterion above is not necessarily of the same importance.

4. Each group, after completing the project, will be invited to present their findings to their classmates. The teacher may invite those groups that have designed their games to play the game with their classmates during their presentation. There should also be time for each group to respond to questions raised by classmates and the teacher.

5. It should be emphasized that the process of the project is considered to be more important than the corresponding results/findings in this activity. Encouragement should be rendered to the members of groups who participate in the project but end up with unsatisfactory/incorrect results.
6. Students should be discouraged from spending too much time on the art design of the report. For instance, sophisticated computer art-work in the report is not necessary. In addition, the teacher should give students suggestions on the time schedule for the project and indicate by when they should have completed their work.

7. Students should be given sufficient time to complete the project.

This example mainly involves the following generic skills:

1. Collaboration Skills
   • Share responsibilities and understand the role of each member in the project
   • Select suitable strategies for investigating the principles of the games through discussion and by considering supporting reasons

2. Communication Skills
   • Discuss and exchange ideas openly with other members
   • Put forward ideas in a systematic and logical way during discussion
   • Give a concise and relevant report after the completion of the project
   • Respond to questions raised by the teacher and classmates during oral presentation

3. Critical Thinking and Problem solving Skills
   • Identify the main task of the project
   • Make use of appropriate knowledge and skills to find out the principles of the games
Example 14
Open-ended Geometric Problem

Key Stage: 3

Strand: Measures, Shape & Space

Learning Units: Congruent triangles
Centres of Triangles

Objective: To explore relations on angles, line segments and areas related to triangles

Prerequisite Knowledge: (i) Conditions for congruent triangle
(ii) Properties of isosceles triangles
(iii) Centres of triangles

Description of the Activity:
Students are asked to answer the following question.

Problem
The figure below shows an isosceles triangle $ABC$ with $BA = BC$. $G$ is the centroid of $\triangle ABC$. $Q$ and $R$ are points on $AC$ and $AB$ respectively such that $BQ$ and $CR$ are line segments passing through $G$. Write down as many equalities related to the figure as possible. Justify your answers. Your equalities may involve angles, line segments and areas of geometric figures.
Notes for Teachers:
This is an open-ended question. Students’ performance can be assessed on their ability to list all equalities involving angles, line segments and areas with justifications. Teachers are free to adjust points, marks or grades assigned to students if they only provide some of the equalities.

This example mainly involves the following generic skills:

1. Communication Skills
   • Describe findings and explain conjectures using mathematical language
   • Formulate and write simple geometric proofs involving angles, lines and triangles with appropriate symbols and reasons

2. Critical Thinking Skills
   • Categorize information using various basic geometric facts, for example, locating equal angles related to congruent triangles
   • Reason deductively in the process of obtaining triangles equal in area

3. Problem solving Skills
   • Choose relevant information and geometric facts to solve problems
   • Understand the problem by noting the important information given in the question
   • Apply knowledge learnt to solve new problems
Example 15

Slopes of Perpendicular Lines

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: The Rectangular Coordinate System

Objective: To enhance the understanding of relation between slopes of perpendicular lines

Prerequisite Knowledge: (i) The changes to the coordinates of a point after a rotation about the origin through multiples of 90° (ii) Finding the slope of a straight line (iii) The relation between the slopes of parallel lines

Resources Required: (i) Dynamic Geometry apps for tablets (e.g. GeoGebra) (ii) A video clip on the topic “Coordinates of Points rotate about the Origin” (iii) tablets computers

Description of the Activity:

Pre-lesson Preparatory Activity

The teacher asks students to watch the video clip “Coordinates of Points rotate about the Origin” at home and answer the question “What are the coordinates of a point \( P(s, t) \) after a rotation of 90° about the origin?” before the lesson.

Notes for Teachers:

Activities 1 and 2 below are designed to guide students to discover and understand the relation between slopes of perpendicular lines in the lessons. Before the activities, the teacher may first give a brief review on concepts introduced in the pre-lesson activity to see whether students have any questions about them.
Activity 1

1. Students are asked to use the freeware GeoGebra to
   
   (a) construct a straight line $L$, which passes through the origin $O$, and the point $P(6, 5)$,
   
   (b) rotate the straight line $L$ through $90^\circ$ about the origin to form a new straight line $L_1$,
   
   (c) consider the point $Q$, which is the image of $P$ after a rotation of $90^\circ$ about the origin, mark the point $Q$ on $L_1$ by rotating $P$.

2. Students are then asked to
   
   (a) find the slopes of $L$ and $L_1$ by considering the coordinates of $P$ and $Q$ respectively,
   
   (b) observe the relationship between the slopes of $L$ and $L_1$ and draw a conclusion.

3. repeat steps (1) and (2) above a few times with different coordinates of $P$ and verify the conclusion drawn in step 2(b).

4. During the construction process, the teacher may prompt students with the following questions:
   
   (a) [in step 1(b)] What is the relationship between $L$ and $L_1$?
   
   (b) [in step 1(c)] What have you got for the coordinates of $Q$?
   
   (c) [in step 2(a)] How do the results relate to the coordinates of $P$?
   
   (d) [in steps 2(b) and 3] What conclusion can you draw? Can you think of another way to present your conclusion?

5. Let students have sufficient time for discussion and exploration before drawing any conclusion.

Activity 2

1. Teacher may then repeat Activity 1, but this time the line $L$ passes through the points $P(5, 0)$ and $Q(1, 6)$ and ask students to rotate the line about point $R(3, 3)$ and consider
the slopes of the two lines.

2. Students are required to perform group discussion on whether the conclusion in Activity 1 step 3 still holds for lines not intersecting at the origin. Students are required to provide a logical explanation on their conclusion.

Notes for Teachers:

1. It is desirable for students to work in small groups.

2. The teacher should allow ample opportunities for students to discuss and draw conclusions by themselves instead of giving them straightforward hints.

This example mainly involves the following generic skills:

1. Information Technology Skills
   - Use dynamic geometry apps and tablet computers to facilitate learning

2. Problem solving Skills
   - Identify the main focus of the problem by building connection between the prerequisite knowledge of point rotation with the perpendicularity of straight lines.

3. Self-learning Skills
   - Check the mastery of prerequisite knowledge for the learning of the new topic through the pre-lesson preparatory activity
Example 16
Volume of Frustums

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: Mensuration

Objective: To let students appreciate the mathematical achievements of ancient China through understanding the methods in finding the volume of frustums in *The 9 Chapters on the Mathematical Art* (九章算術); and to enhance students’ understanding of the cultural aspect of mathematics.

Prerequisite Knowledge: (i) Properties of similar triangles
(ii) Method of dissection in finding volumes
(iii) Finding the volume of rectangular pyramids

Resources Required: Cube blocks

Extended Reading Materials: Chapter 5 “Shanggong” (商功) of *The 9 Chapters on the Mathematical Art*

Description of the Activity:
1. Students are asked to find the volume of a frustum of square base, with lengths of upper square and lower square 40 cm and 50 cm respectively, and with height 50 cm. In this activity, students would try to derive the formula of the volume of a frustum of square bases, namely $V = \frac{1}{3}(a^2 + b^2 + ab)h$, where $a$, $b$, $h$ and $V$ being the length of upper square, length of lower square, height and volume of the frustum respectively.

2. The teacher introduces the method of dissection (棊驗術) used in the problem of “Square Pavilion” (方亭) in Chapter 5, *The 9 Chapters on the Mathematical Art*:
   a. As the basic units in the discussion of the volume of a solid, the solids “li-fang” (立方), “qian-du” (壍堵), “yang-ma” (陽馬) and “bie-nao” (鱉臑) are introduced to students, which may be done with the aid of computer software on
cutting a *li-fang* gradually, with an example of illustration as Figure 1a and 1b below:

![Figure 1a](image1.png) ![Figure 1b](image2.png)

b. Students are guided to explore the ratio of the volumes of *li-fang*, *qian-du* and *yang-ma* (or cuboid, right-angled triangular prism and rectangular pyramid with the vertex vertical above one of the vertices of the rectangular base) of the same dimension (i.e. *qian-du* and *yang-ma* being cut from the *li-fang* being considered);

c. Students are guided to explore how a square-based frustum could be vertically dissected into *li-fang*, *qian-du* and *yang-ma* and fill in a table as below, which corresponds to the top view of dissection.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>yang-ma</em></td>
<td><em>qian-du</em></td>
<td><em>yang-ma</em></td>
</tr>
<tr>
<td><em>qian-du</em></td>
<td><em>li-fang</em></td>
<td><em>qian-du</em></td>
</tr>
<tr>
<td><em>yang-ma</em></td>
<td><em>qian-du</em></td>
<td><em>yang-ma</em></td>
</tr>
</tbody>
</table>

Figure 2 below shows an example of dissection. Students should note that the dimensions of *li-fang*, *qian-du* and *yang-ma* resulted from the dissection are not necessarily the same (i.e. not being cut from the same cuboid).

![Figure 2](image3.png)

d. Students are guided to consider multiplying the number of solids in the table with the same number so as to represent the total volume in each cell of the table.
as integral multiples of some *li-fangs*, i.e. after multiplying the table by 6;

<table>
<thead>
<tr>
<th>2 li-fang</th>
<th>3 li-fang</th>
<th>2 li-fang</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 li-fang</td>
<td>6 li-fang</td>
<td>3 li-fang</td>
</tr>
<tr>
<td>2 li-fang</td>
<td>3 li-fang</td>
<td>2 li-fang</td>
</tr>
</tbody>
</table>

(*li-fangs* in the above table may have different dimensions)

e. Students are given some cube blocks to visualise the table as a model (as in Figure 3 below), and hence guided to explore how the formula of the volume of a frustum in Step 1 can be derived through the model.

![Figure 3](image)

Remark: Students should be reminded that the dimensions of the solids represented by each cell in the tables of (c) and (d) above may **NOT** be the same, and that the term “li-fang” does not necessarily mean a cube as the modern terminology does. The cube blocks should only be understood as a tool to simulate the abstract ideas of the method introduced.

3. **(Enrichment)** Students discuss the generalisation of the formula of volume to general prismatoids (i.e. a polyhedron with all vertices lying on two parallel planes) of square or rectangular bases (such as wedges and truncated wedges) by using a similar method as in Step 2 on frustums of rectangular bases. Students are expected to appreciate that the method stated in *The 9 Chapters on the Mathematical Art* is indeed a more generalised and powerful method to find the volume of prismatoids as compared with applying the properties of similar triangles.
Notes for teachers:

1. The key feature of this method of dissection builds on the use of the basic units, known as *qi* (棊) in the original text. In this example, the basic units *li-fang*, *qian-du* and *yang-ma* are used. It should be emphasised that the ratio of the volumes of these basic units is fixed if they are of the same dimensions.

2. It should be noted that when comparing the basic units of two different cells in the table in Activity 2c or 2d, they may not be of the same dimensions, which means that there is no assumptions that the frustum is made from a right pyramid. Hence, instead of finding the volumes of each solid after dissection, the multiplication done in Activity 2d is essential to form the parts of solids of which the volumes can be found using the given dimensions in the question.

3. After a suitable multiplication to represent each cell of the table in *li-fang*, there is no need to know all the dimensions of each basic unit. Only the lengths and widths of the upper and lower bases together with the height of the prismatoids are sufficient for finding the volume. In other words, students could be guided to understand that the information about relative positions of the upper and lower bases is not necessary to find the volume of this of kind prismatoids. To students with higher ability and more interest in mathematics, this can serve as an entry point to the important concept of Cavallieri’s Principle.

4. Teacher could use of the 3-D printing technology to make a model for illustrating the dissection of the rectangular based frustum (as in Figure 3 below). This allows students to understand the concept of the method of dissection through hands-on manipulation of the model.

![Figure 3a](image1.png)  ![Figure 3b](image2.png)

5. Though the original Chinese terminologies used in *The Nine Chapters on the Mathematical Art* may arouse the interests of some of the students, the use of these terminologies is not essential if it would hinder students’ understanding on the mathematical concepts. However, more capable students in understanding classical Chinese texts should be encouraged to read the original remarks by LIU Hui (劉徽) to taste the mathematical quality of ancient Chinese mathematicians. Samples of
images of the book can be found in some free electronic library (Figure 4).

This example may serve the purpose of promoting students’ understanding, appreciation and interest in mathematics through the cultural and historical aspects.

This example mainly involves the following generic skills:

1. Creativity
   - Appreciate and elaborate an alternative methods in finding the volume of a frustum

2. Communication skills
   - Understand and analyse the classical Chinese text and translate them into modern mathematical language and illustration

3. Problem-solving skills
   - Deriving the formula of volumes of frustums by referring to solids with known formulae of volume, through planned dissections and multiplications.
Example 17
Surface Areas of Prisms

Key Stage: 3

Strand: Measures, Shape & Space

Learning Unit: Mensuration

Objectives: (i) To recognise the concept of surface areas of prisms
    (ii) To investigate the relation between the dimensions of a square-based prism and its surface area

Prerequisite Knowledge: Finding areas of simple polygons and volumes of prisms

Resources Required: Empty boxes (tissue box, cereal box or chocolate box, etc.), colour pens, scissors, a set of unit cubes, worksheets and computers

Description of the Activity:
Activity 1
This activity allows students to recognize the concept of surface areas of prisms.
1. Students are divided into groups of four. The teacher distributes to each group some empty boxes (tissue box, cereal box or chocolate box, etc.) and Worksheet 1.
2. After showing different empty boxes to students, teachers can guide students to consider the following questions:
   (a) Regarding the surface of each box, how many faces are there?
   (b) What kinds of two-dimensional figures (i.e. rectangle, triangle, etc.) are the faces of the boxes?
   (c) Which edges of the boxes can determine the surface areas of the boxes?
   (d) Are there any congruent faces?
3. Each group is firstly requested to cut the boxes along its edges to form plane nets.
4. Each group measures the lengths of suitable edges of the boxes for calculating the surface areas. Then students are required to complete Worksheet 1.

Notes for Teachers:
1. Teacher should guide students to recognize that the surface area of a prism is the sum
of all the faces.

2. The main focus of the activity is to determine which edges or lengths are needed to be measured in order to calculate the surface area.

**Activity 2**
This activity allows students to explore the setting of dimensions of a square-based prism with a fixed volume, for the least surface area.

1. Students are divided into groups of four. Each group is given 64 unit cubes and Worksheet 2. Students have to use the cubes to make square-based prisms of different base lengths \(a\) unit and heights \(h\) unit, and calculate the surface area \(A\) square unit) of the prism for each case.

2. Students are required to record the data and complete the table in Worksheet 2.

3. Students try to find the values of \(a\) and \(h\) such that the square-based prism has the least surface area.

**Notes for Teachers:**
In this activity, students should discover that the surface area of square-based prism will not always increase when the height of the prism decreases from 64 units to 1 unit.

**Activity 3**
This activity allows students to investigate the prism as in Activity 2 using the graphical method. The dimensions of \(a\) and \(h\) will not be limited to only some values as in Activity 2. In this activity, \(a\) can take any positive value.

1. Students write down the formula for volume \(V\) cubic unit) and surface area \(A\) square unit) of the square-based prism in terms of \(a\) and \(h\).

2. From the two expressions in 1, students write \(A\) in terms of \(a\) and \(V\).

3. Consider \(V = 64\) and \(a\) takes the values 1, 2, 3, …, 12, students use a spreadsheet software to construct a table showing the corresponding values of \(h\) and \(A\).
4. Students plot the graph of $A$ against $a$ with $V = 64$ using a spreadsheet software or any graphing software.

5. Students find out from the graph the values of $a$ such that $A$ is minimum. Students repeat using other values of $V$ (such as 125, 216, and 1 000) and try to draw conclusion on the dimensions of a square-based prism of fixed volume that has the least surface area.

**Notes for teachers:**

1. For Activities 1 and 2, there is more usage of concrete models. Such teaching method favors those students who learn best through classroom activities and hands-on exploratory tasks.

2. In all the activities, teachers are suggested to adopt strategies like questioning, group discussion and group reporting to enhance interaction. Such teaching method favors those students who learn best by oral lectures, work in a group and discussions.

3. Activity 2 could be skipped if students are comfortable with abstract algebraic manipulations as the table constructed in Activity 3 may serve a similar purpose.

4. The graphical method in Activity 3 is not a proof for the minimum surface area. Students could be encourage to learn calculus at senior secondary if they want to prove that $A$ is minimum when $a = h$. 


This example mainly involves the following generic skills:

1. Communication Skills
   - Describe the correspondence between the dimensions of square-based prism to their surface area verbally with simple and appropriate mathematical terms
   - Present the results logically with appropriate drawings and graphs and mathematical expressions

2. Critical Thinking Skills
   - Determine the dimensions of a square-based prism that lead to the least surface area from the results of calculations

3. Problem solving Skills
   - Answer the problem by considering all the cases in Activity 2 and the data computed by the spreadsheet software
   - Compare different approaches in tackling the same problem
**Worksheet 1**  
*Surface areas of Prisms*

<table>
<thead>
<tr>
<th>Sketch of the prism</th>
<th>Sketch of the net (mark the dimension required to calculate the surface area in the net)</th>
<th>Surface area (correct to the nearest cm²)</th>
</tr>
</thead>
<tbody>
<tr>
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</table>
**Worksheet 2**

**Surface Area of Square-based prism**

Use the 64 cubes provided to make square-based prisms of different base length \((a\ \text{unit})\) and height \((h\ \text{unit})\) and to find out the dimensions which result in the least surface area.

![Diagram of a square-based prism](Image)

Record the base lengths and heights of the square prisms that you have constructed in the table below.

<table>
<thead>
<tr>
<th>Sketch of the prism</th>
<th>Base length (a\ \text{unit})</th>
<th>Height (h\ \text{unit})</th>
<th>Surface area (A\ \text{sq. unit})</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
Example 18
Flipping Measure Spoons

Key Stage: 3

Strand: Measures, Shape and Space

Learning Unit: Mensuration

Objective: Understand the applications of calculating volumes and lengths in industrial designs, with the consideration of appropriately chosen materials.

Prerequisite Knowledge: (i) Basic concepts on calculating circumferences and the volume of a sphere
(ii) Understand and use the relationships between sides and volumes of similar figures
(iii) Imagine the 3-D objects from given 2-D representations

Relationship with other KLA(s) in STEM Education:
“Particle model for the three states of matter”, “Corrosive nature of acids”, and “Materials of the modern world” in Science (S1 – 3) Curriculum Framework 2016 (Provisional Final Draft) of Science Education KLA.

Resources Required: 1. Internet resources on the information of flipping measuring spoons and the physical and chemical properties of silicone.
2. Tablet computers with Internet connection.

Scenario:
A measuring spoon is an important utensil in cooking for a fairly accurate measurement of the amounts of ingredients during the preparation. Traditionally, one measuring spoon is used to measure one fixed amount (conventionally 1 cup, 1/2 cup, 1 tablespoon, 1/2 tablespoon, 1 teaspoon, 1/2 teaspoon, etc.). However, with the aid of extensible and flexible materials such as silicone rubber, a flippable measuring spoon can be designed so that when flipping, two fixed amount can be measured using the same measuring spoon (See the reference link below: https://www.youtube.com/watch?v=yKTgaZnkaqc)
This task is to discuss the industrial design of such flipping measuring spoons by applying students’ mathematical knowledge in finding various measurements and scientific knowledge in explaining why a specific material is or is not suitable for making a piece of kitchen utensil.

Description of the Activity:

Preparatory Activity (Optional)
Students familiarise themselves in finding the radius of a hemisphere from given volume. For example, if the measure of a tablespoon is 15 cm$^3$, and assuming the shape of a measuring spoon is a hemisphere, students are requested to find the radius of the measuring spoons that can be used to measure one tablespoon and 1/2 tablespoon.

Teachers may suggest students to use the knowledge of similar figures in finding the ratio of the radii between the two measuring spoons by considering the ratio between their volumes.

Activity 1:
Teachers may play the video clip (link provided above) of the flipping measuring spoons to introduce the ideas behind the industrial design of the flipping measuring spoons which can measure two fixed volumes.

After the video, teachers may use the cross-section model of the flipping measuring spoons in Figure 1a and 1b to further facilitate students’ understanding on the design:

Remarks: Teachers can also use 3-D graphics software packages to illustrate the design of the measuring spoon if necessary (see Figure 2a and 2b):
Questions for discussion:
1. If the flipping measuring spoon is designed to measure one tablespoon and 1/2 tablespoon of ingredients respectively, and assuming that in both cases the interior is a hemisphere, what should the radii of the two hemispheres be?
2. Let \( t \) be the thickness of the material for making the flipping measuring spoon and \( r \) be the radius of the circular mount, suggest one formula which links up the relationship between \( t \) and \( r \) such that the capacities of the two states of the measuring spoon are respectively one tablespoon and 1/2 tablespoon.
3. Teachers can discuss with students on the errors the assumption on hemispheres created, and how they could be reduced by varying \( t \) and \( r \).

**Activity 2:**
Students are asked to search on the Internet on some physical and chemical properties of silicone, such as its melting point, durability, flexibility, conductivity, toxicity, chemical reactivity, acid resistance, and so on to critically analyse whether this material is good for making kitchen utensils. Teachers are encouraged to collaborating with Science teachers, to form a stronger linkage of the lesson to Science Education KLA. Related science topics in junior secondary includes “Particle model for the three states of matter”, “Corrosive nature of acids”, “Materials of the modern world”, etc.

Questions for discussion:
1. Why is considering the melting point of a matter important in choosing the material to make kitchen utensils? Besides the melting point, what are the other factors one should consider concerning temperature?
2. In order to make the design of the flipping measuring spoons possible, what important physical properties of silicone are featured? Are there any other matters which possess the same physical properties?
3. Compare silicone and plastic. Which materials should be preferred as the material for making kitchen utensils in considering its health impact and environmental impact?

This example mainly involves the following generic skills:

1. Critical thinking skills
   - Critically discuss the advantages and limitations of the design, and the materials used in the flipping measure spoon
   - Employ the ideas of mathematical modelling as a powerful tool to quantitatively simulate and solve the problems of practicability in designing the flipping measure spoon

2. Self-learning skills
   - Autonomously gather useful information on the materials used in the flipping measure spoon through various sources
   - Initiate own enquiry to modify and refine the designs for better purposes of the flipping measure spoon
**Example 19**

**Translations of Functions**

**Key Stage:** 4

**Strand:** Number and Algebra

**Learning Unit:** More about graphs of functions

**Objective:** Understand the vertical and horizontal translation of a function $f(x)$ from graphical and symbolic perspectives.

**Prerequisite Knowledge:**
(i) Basic concepts of translation in rectangular coordinates system
(ii) Using algebraic and graphical methods to represent functions
(iii) Basic operation skills of dynamic geometry software

**Resources Required:** Dynamic geometry software such as *GeoGebra*

**Preparation:**
1. In order to utilise the teaching resources in conducting learning activities, teachers may consider developing or customising electronic worksheets according to students’ needs, with aids of suitable software packages.

2. The following questions should be addressed in developing or customising the learning and teaching materials and electronic worksheets:
   - What are the key elements students should observe?
   - How can functions of the software be used to highlight these key elements?
   - What are the key elements that may not be effectively demonstrated by functions of the software?

3. The following steps are the suggestions on creating the electronic worksheet for exploring the translations of graphs of functions using *GeoGebra*:
   3.1 Create two sliders, labelled with $a$ and $b$.
   3.2 Create a function $f(x)$ by keying in “$f(x) = x$” in the input field.
   3.3 Create an input box which is linked to the function $f(x)$ so that students can
freely key in any functions in \( x \) to define \( f(x) \).

3.4 Create two functions \( g(x) \) and \( h(x) \) such that
\[
g(x) = f(x) + a \quad \text{and} \quad h(x) = f(x + b).
\]

3.5 Add a point \( P \) on the graph of \( y = f(x) \).

3.6 Define another two points \( P' = P + (0, a) \) and \( P'' = P + (-b, 0) \).

3.7 Define another two points \( Q' \) and \( Q'' \) on the graphs of \( y = g(x) \) and \( y = h(x) \) respectively such that \( PQ' \) and \( PQ'' \) are respectively a horizontal and vertical line.

Remarks:

a. It can be done by adding a horizontal and vertical line at \( P \) and locate \( Q' \) and \( Q'' \) by finding the correct intersections.

b. \( Q' \) and \( Q'' \) may not exist depending on the type of \( f(x) \). Also, \( Q'' \) may not be unique. However, these drawbacks can be used to create the contrast of whether one can view a horizontal translation as a vertical translation, or vice versa.

3.8 Add the line segments \( PP', PP'', PQ' \) and \( PQ'' \), and measure the lengths.

3.9 Add a check box, labelled with “show distance”, and link it with the objects \( PP', PP'', PQ' \) and \( PQ'' \). Right-click the check box and rename it as \( s \).

3.10 Add two check boxes, labelled with “vertical translation” and “horizontal translation”. Rename the check boxes as \( t \) and \( u \). For the check box of “vertical translation”, link it with the objects \( g(x) \), slider \( a \), \( P', Q', PP' \) and \( PQ' \). For the check box of “horizontal translation”, link it with the objects \( h(x) \), slider \( b \), \( P'' \), \( Q'' \), \( PP'' \) and \( PQ'' \).

3.11 Right click \( PP' \) and \( PQ' \). In the “Advanced” page in “Object Properties”, type “\( s \& t \)” in “Condition to Show Object”, as illustrated Figure 1:

![Figure 1](image)

Similarly, set the condition to show \( PP'' \) and \( PQ'' \) as “\( s\&u \)”.

3.12 Set the condition to show the check box “show distance” as “\( t|u \)”, i.e. \( t \) or \( u \).
Description of the Activity:

**Activity 1a**

1. The teacher revises some basic concepts of translation in rectangular coordinate system, including the algebraic expression \((x, y) \rightarrow (x + a, y + b)\) to describe a translation.

2. In the *GeoGebra* electronic worksheet, the teacher could ask students to enter any single-variable quadratic functions in \(x\) and check the “vertical translation” module (See Figure 2) using their tablet or notebook computers.

3. By dragging the slider, students are asked to observe and verbally describe the translation of the graphs. Students are guided to discuss the relationship between the sign of the slider and the direction of translation.

4. The teacher then asks students to check the “show distance” box and move the point on the graph of the original function (See Figure 3). Students are guided to observe the changes of horizontal and vertical distances between the graphs at different positions.
5. Students are guided to observe and discuss the relationship between the values of the slider, the horizontal and vertical distances between the graphs, the algebraic expression of the functions shown, and the displacement of the translation. Students are then asked to make hypothesis on describing the vertical translation of a function algebraically.

6. Students can then be asked to type in any other functions of different types to verify their hypothesis.

**Activity 1b**
1. The teacher provides a definition of vertical translation of a function.
2. Students are guided to consider the graph of a function \( f \) as a point set with points \((x, f(x))\) and hence discuss how to describe the graph of the translated function \( f' \) algebraically in terms of \( f \).
3. The teacher then asks students to verify and hence prove that the vertical distances between the graphs of the functions \( y = f(x) \) and \( y = f(x) + a \) are always the same at different values of \( x \).
4. The teacher guides students to draw the conclusion on the effect of transforming \( f(x) \) through \( f(x) + k \).

**Activity 2a**
1. In the GeoGebra electronic worksheet, the teacher asks students to uncheck the “show distance” box and “vertical translation” module, enter any single-variable quadratic functions in \( x \) and check the “horizontal translation” module (See Fig. 4) using their tablet or notebook computers.

![Figure 4](image-url)
2. By dragging the slider, students are asked to observe and verbally describe the translation of the graphs. Students are guided to discuss the relationship between the sign of the slider and the direction of translation.

3. The teacher then asks students to check the “show distance” box and move the point on the graph of the original function (See Figure 5). Students are guided to observe the changes of horizontal and vertical distances between the graphs at different positions.

4. Students are guided to observe and discuss the relationship between the values of the slider, the horizontal and vertical distances between the graphs, the algebraic expression of the functions provided by GeoGebra, and the displacement of the translation. Students are then asked to make hypothesis on describing the horizontal translation of a function algebraically.

5. The teacher asks students to type in any other functions of different types to verify their hypothesis.

**Activity 2b**
1. The teacher provides a definition of horizontal translation of a function.

2. Students are guided to consider the graph of a function $f$ as a point set with points $(x, f(x))$ and hence discuss how to describe the graph of the translated function $f'$ algebraically in terms of $f$.

3. The teacher then asks students to verify and hence prove that the horizontal distances between the graphs of the functions $y = f(x)$ and $y = f(x - b)$ are always the
same at different values of $y$.

**Activity 2c** (Enrichment learning activity for more able students)

1. The teacher asks students to observe the vertical translation and horizontal translation of the linear function $y = x$ on whether there is any difference between the two translation.

2. The teacher asks students to enter any single-variable linear function in $x$ and check the “vertical translation” module using their tablet or notebook computers.

3. By checking the “show distance” box, the teacher asks students to drag the slider and move the point on the graph of the original function to observe the changes of horizontal and vertical distances between the graphs at different positions.

4. Students are guided to discuss whether the translation $(x, f(x)) \rightarrow (x, f(x) + a)$ is a vertical or horizontal translation when $f$ is a linear function.

5. Similarly, students are asked to explore and discuss whether the translation $(x, f(x)) \rightarrow (x - b, f(x))$ is a vertical or horizontal translation when $f$ is a linear function.

6. Students are guided to show that for all linear functions $f(x)$, $f(x-p)$ can always be written in form of $f(x) + q$ for some $q$.

7. **(As a further enrichment)** Students are guided to discuss the relationship between the values $p$, $q$ and the slope of $f(x)$, and hence draw the conclusion that only linear functions have such properties that a vertical translation is at the same time a horizontal translation.

**Notes for Teachers:**

1. Teachers are encouraged to discuss the horizontal translation after the vertical translation. With appropriate design, it is also good for students to compare and contrast the different effects of a vertical translation and a horizontal translation to the same function.

2. While the use of information technology can significantly provide the graphs of all functions for the discussion, teachers should be aware that it may not directly help students conceptualising the transformation algebraically. Hence, traditional consolidation activities are essential.

3. Teachers can use Activity 2c as an enrichment activity to stretch the potential and
interest of students who are more able in Mathematics.

4. To cater for the diverse learning style, teachers can rewrite the form of transformation of \( y = f(x) + k \) to \( y - k = f(x) \) and \( y = f(x + k) \) to \( y = f(x - k) \), and help students to recognise the effects of the translation by observing the position and sign of the constant \( k \). (Remark: Such written forms, together with a similar treatment of other transformations such as \( y = f(kx) \), and \( y = kf(x) \), can be explained to students with high mathematical ability and strong interests in mathematics using the concepts of transforming axis of the coordinate system, which exceed the curriculum requirement, as an enrichment activity.)

This example mainly involves the following generic skills:

1. **Creativity**
   - Making hypothesis based on one’s own observations

2. **Information Technology Skills**
   - Use of GeoGebra to explore properties of transformations of functions
Example 20

Modelling the spread of a disease

Key Stage: 4 (Compulsory Part and Module 1)

Learning Unit: Further Application
Inquiry and investigation

Objectives:
(i) To help students relate STEM education with the real life
(ii) To let students recognise the mathematics behind everyday life and apply information technology to solve problems
(iv) To let students recognise mathematics as a powerful tool for planning

Prerequisite Knowledge:
(i) “Exponential and logarithmic functions” in Compulsory Part
(ii) “Simple idea of probability” in junior secondary
(iii) Topics on calculus in Extended Part Module 1

Modelling the spread of a disease
Background information:
Bird flu, SARS and Ebola were examples of fatal epidemics that emerged in a large scale in past two decades. They badly threatened human lives in the world. Below are two mathematical models on the spread of diseases.

Basic Assumptions for the following two models:
1. The population is in a closed area.
2. The total number of the population is fixed.
3. The disease is transmitted by direct individual contact.
4. The recovered people are not considered.

Model 1: A simple epidemic model
The spread of the disease starts from an infected person. The person directly spreads the disease to two susceptible persons. The two new infected persons then each transmit the disease to two more persons (as in the following figure), and so on. The process is continued until everyone is infected.
The number of persons infected at each stage is a power of 2. We can model the situation by the exponential function: $y = 2^x$. We can take the chance to review the characteristics of exponential function.

Teachers may raise the questions about the graph of the function:
- Does the graph of the exponential function get steeper or less steep as $x$ increases?
- Does the graph have a $y$-intercept? If so, what is it? If not, explain why not.
- Does the graph have an $x$-intercept? If so, what is it? If not, explain why not.

Discussion questions on this model:
- How many steps are needed to infect all the people in the classroom? How about the whole school?
- If we start the spread from another infected person, will we get the same result?
- What are the differences if 3 persons were infected at each stage?
- How many steps do we need to infect all the people in HK in this model?
- What are the limitations of this model?

**Note for teachers:**
It is the simplest way to model the spread of disease but the model has many restrictions and seems unrealistic. Real epidemics do not really follow this model. The model does not take into account people who recover and are then immune. The model also does not consider isolated communities.
**Model 2:** Counter Plague model:

We use a dice to simulate the variability of the number of infected people in the model. The number of newly infected people caused by each infected person at each step is determined by the value of tossing a fair dice, so the infection rate is not fixed. We would investigate two scenarios for the model:

(i) infection rate greater than 1

(ii) infection rate less than 1

---

**Case 1:**

<table>
<thead>
<tr>
<th>Number on dice</th>
<th>Number of newly infected people</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

The expected rate of infection = \( \frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 = \frac{4}{3} \) which is greater than 1. **Epidemics take off.**

---

**Case 2:**

<table>
<thead>
<tr>
<th>Number on dice</th>
<th>Number of newly infected people</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

The expected rate of infection = \( \frac{1}{6} \times 2 + \frac{1}{6} \times 1 + \frac{1}{6} \times 1 + \frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 0 = \frac{2}{3} \) which is less than 1. **Epidemics die out.**

---

**Classroom Activity:**

Teachers could ask students to form small groups and provide each group with one dice and about 30 counters. For each group, students follow the following steps:

1. Chose a case from Case 1 and Case 2.
2. Place one counter on the table top to represent one infected person, i.e. the originally
infected person.
3. Throw the dice once. Suppose the dice shows 4. In Case 1, this means that two new persons are infected. Two more counters have to be placed on the desk to indicate the number of infected person. In case 2, no one is infected and the disease dies out.
4. Repeat for EACH newly infected persons and place the appropriate number of counter on the desk.
5. Repeat until either the epidemic dies out or you run out of counters.
6. Record the progress of the epidemic on a graph.
7. Run the simulations several times for the two cases and compare the graphs.

Discussion questions on this model:
• Does the epidemic take off or die out in each case?
• On average, how many steps does the epidemic run in each case?
• What aspects of the model help us to understand how epidemics progress?
• Are there important factors which the model does not include? Can you think of ways to improve it?

Challenging Problem:
Suppose a population is threatened by an infectious disease. The population can be divided into two groups, the healthy and the infected. Let $p$ be the probability that a healthy person gets the disease, $r$ be the probability that an infected person recovers in each month. Initially there are 8 healthy people and 2 infected people.

Setting up the model
Students may try to model what happens to the 10 people over a period of 10 months by tossing a fair 6-sided dice. Assume no one dies in 10 months and we roll the fair dice for each person in each month. If we get a 1, 2 or 3 for a healthy person, then he becomes infected that month ($p = \frac{1}{2}$). If we get a 4 or 5 for an infected person, then he recovers that month ($r = \frac{1}{3}$). For other outcomes, their situations remain unchanged.

Theoretical suggested solution
Let $x$ be the proportion of population that are infected, we can present the rate of change of $x$ by the following expression:
\[
\frac{dx}{dt} = p(1-x) - rx
\]
\[
\frac{dx}{dt} = \frac{1}{2}(1-x) - \frac{1}{3}x
\]
\[
\frac{dx}{dt} = \frac{3-5x}{6}
\]

Notes for Teachers:
(i) From the given condition, students are able to set up the expression for rate of change of \(x\). It is not required to solve differential equation in our curriculum. Teachers may demonstrate the technique of solving the problem for the more able students.

\[
\frac{dx}{dt} = \frac{1}{2} - \frac{5}{6}x
\]

\[
\int \frac{6}{3-5x} \, dx = \int dt
\]

\[-\frac{6}{5} \ln(3-5x) = t + k \quad \text{where } k \text{ is a constant}
\]

When \(t = 0, x = 0.2\), so \(k = -\frac{6}{5} \ln 2\), thus

\[-\frac{6}{5} \ln(3-5x) = t - \frac{6}{5} \ln 2
\]

\[-t = \frac{6}{5} \ln \left(\frac{3-5x}{2}\right)
\]

\[e^{-\frac{5}{6}t} = \frac{3-5x}{2}
\]

So \(x = \frac{3}{5} - \frac{2}{5} e^{-\frac{5}{6}t}\) and when \(t\) tends to infinity, \(x\) tends to \(\frac{3}{5}\). The graph is:

What happens to the solution if the value of \(p\) and \(r\) is changed?
(ii) Another way of dealing with this problem is to give the suggested solution \( x = \frac{3}{5} - \frac{2}{5} e^{-\frac{z}{6}} \) to students and ask them to verify that this solution satisfies the differential equation.

Reference:
(i) A simple activity on Model 1
    https://motivate.maths.org/content/DiseaseDynamics/Activities/StandingDisease

(ii) We can simulate the Counter Plague model by the following simulation program:
    http://motivate.maths.org/content/sites/motivate.maths.org/files/e-CounterPlague-1.s.wf

(iii) Demonstration video about the Counter Plague model can be found in the following link:
    https://motivate.maths.org/content/MathsHealth/ModellingEpidemics/video/CounterPlague

This example mainly involves the following generic skills:

1. Critical Thinking Skills
   • Compare mathematical model and real situation, analyse the shortcomings of the model

2. Problem Solving Skills
   • Formulate a mathematical solution when tackling a real-life problem
   • Use real objects to simulate abstract mathematical context
Example 21

Return and Risk

Key Stage: 4

Strand: Data Handling

Learning Unit: More about probability
Measures of dispersion

Objective: Understand the use of expected value and standard deviation to describe return and risk

Pre-requisite Knowledge: (i) understand the meaning of probability
(ii) recognise the meaning of expectation
(iii) understand the concept of standard deviation for both grouped and ungrouped data sets

Description of the Activity:
Two games will be played to introduce the basic idea of return and risk, which are described by expectation and standard deviation of the given data. Players need to consider both return and risk in making their decisions.

Game 1
1. Students are required to form groups of about 2 to 4 students.
2. Before the game, the teacher gives each group 10 chocolates as the initial number. Each group will also be distributed a pack of 52 playing cards.
3. Each group will need to shuffle the playing cards and randomly pick a card each time to determine the change to the number of chocolates. Students need to record the changes (the teacher needs not collect or give chocolates at this stage).

- Scenario 1
  If the group picks a club, the reward is 10 chocolates; otherwise, they lose 5 chocolates.
- Scenario 2
  If the group picks a red card, the reward is 7 chocolates; otherwise, they lose 3 chocolates.
• **Scenario 3**
  If the group picks a red card less than 4, the reward is 40 chocolates; otherwise, they lose 10 chocolates.

If the number of chocolates drops to zero or negative, or increases to 40 or above, a game is over.

4. All the groups need to play the game for at least three times within 10 minutes. They can stop after three times or continue playing the game until the time is up. If a group at a particular step of a game has a total of 40 chocolates or more, then that group can keep the 10 chocolates at hand.

5. Before each time, each group needs to toss a fair six-sided die to decide which scenario to play. If they get “1” or “6”, they will play the game in Scenario 1. If they get “3” or “4”, they will play the game in Scenario 2. If they get “2” or “5”, they will play the game in Scenario 3.

6. Each group needs to calculate the expected value of the game under different scenarios according to the theoretical situation and decide under which scenarios to play if they are free to choose.

Questions for discussion:
1. Under the rule in step 5 above, does each group have equal chance of choosing the three scenarios?
2. In the long run, which scenario should each group choose to play the game to get a greater reward? Please explain.

**Notes for Teachers:**

1. Based on the theoretical probability, the expected gain of chocolates in each scenario is as follows:
   - **Scenario 1**
     The expected gain of chocolates = \( \frac{13}{52} \times 10 + \left( 1 - \frac{13}{52} \right) \times (-5) = -1.25 \)
   - **Scenario 2**
     The expected gain of chocolates = \( \frac{26}{52} \times 7 + \left( 1 - \frac{26}{52} \right) \times (-3) = 2 \)
• **Scenario 3**

The expected gain of chocolates = \( \frac{4}{52} \times 40 + \left(1 - \frac{4}{52}\right) \times (-10) = -6.15 \)

From the result, it seems that we should choose Scenario 2 in the long run to get a greater reward and the case in Scenario 3 will bring us a great loss. The expected value reflects the risk behind the large reward in Scenario 3.

2. In practice, as students only play the game at limited number of times, they are expected to recognise that the actual gain of chocolates in a particular scenario may be very different from what reflected by the expected gain; and the meaning of each expected value in relation to the risk of losing chocolates under the corresponding scenario.

**Game 2**

Players need to concern and analyse the variability of return in making their decision. Suppose we have two plans, the expected returns of them are the same. It is quite reasonable to choose the plan with lower fluctuation in return. Variance and standard deviation of the return could be used to measure the level of risk in relation to the fluctuation in return.

1. Students are required to form groups of about 2 to 4 students.

2. The teacher distributes 10 chocolates and two dice to each group. Then the teacher asks each group to play the game according to the following rules:

   **Scenario 1**

   (i) Throw two dice and subtract the smaller number from the larger number. We call the results “Dice Differences”.

   (ii) At each throw, if the “Dice Difference” is 0 or 1, each group can get 2 chocolates. Otherwise, each group needs to return the teacher 1 chocolate.

   (iii) The game of a group will stop when all the chocolates have been returned to the teacher after a throw.

   (iv) If a group still owns chocolates after a throw, it needs to continue throwing dice until it has performed 20 throws.
(v) Each group needs to record their results at each throw and calculate their final number of chocolates for comparison.

Scenario 2
(i) Throw two dice and add the two numbers of the outcome. We call the results “Dice Sums”.
(ii) At each throw, if the “Dice Sum” is 5 or below, each group can get 9 chocolates. Otherwise, each group needs to return 3 chocolates to the teacher.
(iii) The game of a group will stop when all the chocolates have been returned to the teacher after a throw.
(iv) If a group still owns chocolates after a throw, it needs to continue throwing dice until it has performed 20 throws.
(v) Each group needs to record their results at each throw and calculate their final number of chocolates for comparison.

Each group gets chocolates of the amount recorded at the end of the game. The group with the largest number of chocolates wins the game.

Questions for discussion:
1. According to the rules, what are the expected gains of chocolates in Scenario 1 and Scenario 2? Are the games fair?
2. Suppose you need to choose between Scenario 1 and Scenario 2, which scenario will you choose in order to win the game? Please give justification for your suggestion. You may consider the standard deviation of the gain in number of chocolates for each scenario.
3. Are there any drawbacks if we use standard deviation to describe the risk of playing a game?

Notes for Teachers:
The possible outcomes for throwing two dice are listed as follows:
Example 21

### 1. Based on the theoretical probability, the expected gain of chocolates in each scenario under the given rules is as follows:

**Scenario 1**

The expected gain of chocolates = \( \frac{16}{36} \times (+2) + \left( 1 - \frac{16}{36} \right) \times (-1) = \frac{1}{3} \)

**Scenario 2**

The expected gain of chocolates = \( \frac{10}{36} \times (+9) + \left( 1 - \frac{10}{36} \right) \times (-3) = \frac{1}{3} \)

It seems that in the long run, each group can gain chocolates. The expected return is the same. The game is favourable to the players.

### 2. Each group needs to record their results like the following tables. We may use online simulator (such as [http://www.virtualdiceroll.com/](http://www.virtualdiceroll.com/)) to get the outcomes of virtual dice throws. The following tables list simulated results under the two scenarios:

#### Scenario 1

<table>
<thead>
<tr>
<th>1st set</th>
<th>Dice diff.</th>
<th>Gain</th>
<th>Number of chocolates</th>
<th>2nd set</th>
<th>Dice diff.</th>
<th>Gain</th>
<th>Number of chocolates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(6,2)</td>
<td>4</td>
<td>-1</td>
<td>1.</td>
<td>(3,5)</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>2.</td>
<td>(6,3)</td>
<td>3</td>
<td>-1</td>
<td>2.</td>
<td>(1,3)</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3.</td>
<td>(1,1)</td>
<td>0</td>
<td>2</td>
<td>3.</td>
<td>(3,6)</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4.</td>
<td>(1,5)</td>
<td>4</td>
<td>-1</td>
<td>4.</td>
<td>(2,4)</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>5.</td>
<td>(3,5)</td>
<td>2</td>
<td>-1</td>
<td>5.</td>
<td>(4,1)</td>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>
### Example 21

<table>
<thead>
<tr>
<th>1st set</th>
<th>Dice diff.</th>
<th>Gain</th>
<th>Number of chocolates</th>
<th>2nd set</th>
<th>Dice diff.</th>
<th>Gain</th>
<th>Number of chocolates</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>(3,1)</td>
<td>2</td>
<td>−1</td>
<td>7</td>
<td>6.</td>
<td>(2, 2)</td>
<td>0</td>
</tr>
<tr>
<td>7.</td>
<td>(2,2)</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>7.</td>
<td>(6, 4)</td>
<td>2</td>
</tr>
<tr>
<td>8.</td>
<td>(5,5)</td>
<td>0</td>
<td>2</td>
<td>11</td>
<td>8.</td>
<td>(5, 6)</td>
<td>1</td>
</tr>
<tr>
<td>9.</td>
<td>(6,2)</td>
<td>4</td>
<td>−1</td>
<td>10</td>
<td>9.</td>
<td>(3, 5)</td>
<td>2</td>
</tr>
<tr>
<td>10.</td>
<td>(1,4)</td>
<td>3</td>
<td>−1</td>
<td>9</td>
<td>10.</td>
<td>(2, 5)</td>
<td>3</td>
</tr>
<tr>
<td>11.</td>
<td>(1,5)</td>
<td>4</td>
<td>−1</td>
<td>8</td>
<td>11.</td>
<td>(4, 5)</td>
<td>1</td>
</tr>
<tr>
<td>12.</td>
<td>(3,6)</td>
<td>3</td>
<td>−1</td>
<td>7</td>
<td>12.</td>
<td>(2, 5)</td>
<td>3</td>
</tr>
<tr>
<td>13.</td>
<td>(4,4)</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>13.</td>
<td>(4, 5)</td>
<td>1</td>
</tr>
<tr>
<td>14.</td>
<td>(6,5)</td>
<td>1</td>
<td>2</td>
<td>11</td>
<td>14.</td>
<td>(5, 5)</td>
<td>0</td>
</tr>
<tr>
<td>15.</td>
<td>(6,3)</td>
<td>3</td>
<td>−1</td>
<td>10</td>
<td>15.</td>
<td>(6, 6)</td>
<td>0</td>
</tr>
<tr>
<td>16.</td>
<td>(2,3)</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>16.</td>
<td>(5, 4)</td>
<td>1</td>
</tr>
<tr>
<td>17.</td>
<td>(3,6)</td>
<td>3</td>
<td>−1</td>
<td>11</td>
<td>17.</td>
<td>(1, 5)</td>
<td>4</td>
</tr>
<tr>
<td>18.</td>
<td>(3,6)</td>
<td>3</td>
<td>−1</td>
<td>10</td>
<td>18.</td>
<td>(6, 6)</td>
<td>0</td>
</tr>
<tr>
<td>19.</td>
<td>(1,4)</td>
<td>3</td>
<td>−1</td>
<td>9</td>
<td>19.</td>
<td>(1, 1)</td>
<td>0</td>
</tr>
<tr>
<td>20.</td>
<td>(6,2)</td>
<td>4</td>
<td>−1</td>
<td>8</td>
<td>20.</td>
<td>(5, 6)</td>
<td>1</td>
</tr>
</tbody>
</table>

Standard deviation of gain = 1.4  
Standard deviation of gain = 1.5

### Scenario 2

<table>
<thead>
<tr>
<th>1st set</th>
<th>Dice sum</th>
<th>Gain</th>
<th>Number of chocolates</th>
<th>2nd set</th>
<th>Dice sum</th>
<th>Gain</th>
<th>Number of chocolates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(6,2)</td>
<td>8</td>
<td>−3</td>
<td>7</td>
<td>1.</td>
<td>(3, 5)</td>
<td>8</td>
</tr>
<tr>
<td>2.</td>
<td>(6,3)</td>
<td>9</td>
<td>−3</td>
<td>4</td>
<td>2.</td>
<td>(1, 3)</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>(1,1)</td>
<td>2</td>
<td>9</td>
<td>13</td>
<td>3.</td>
<td>(3, 6)</td>
<td>9</td>
</tr>
<tr>
<td>4.</td>
<td>(1,5)</td>
<td>6</td>
<td>−3</td>
<td>10</td>
<td>4.</td>
<td>(2, 4)</td>
<td>6</td>
</tr>
<tr>
<td>5.</td>
<td>(3,5)</td>
<td>8</td>
<td>−3</td>
<td>7</td>
<td>5.</td>
<td>(4, 1)</td>
<td>5</td>
</tr>
<tr>
<td>6.</td>
<td>(3,1)</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>6.</td>
<td>(2, 2)</td>
<td>4</td>
</tr>
<tr>
<td>7.</td>
<td>(2,2)</td>
<td>4</td>
<td>9</td>
<td>25</td>
<td>7.</td>
<td>(6, 4)</td>
<td>10</td>
</tr>
<tr>
<td>8.</td>
<td>(5,5)</td>
<td>10</td>
<td>−3</td>
<td>22</td>
<td>8.</td>
<td>(5, 6)</td>
<td>11</td>
</tr>
<tr>
<td>9.</td>
<td>(6,2)</td>
<td>8</td>
<td>−3</td>
<td>19</td>
<td>9.</td>
<td>(3, 5)</td>
<td>8</td>
</tr>
<tr>
<td>10.</td>
<td>(1,4)</td>
<td>5</td>
<td>9</td>
<td>28</td>
<td>10.</td>
<td>(2, 5)</td>
<td>7</td>
</tr>
<tr>
<td>11.</td>
<td>(1,5)</td>
<td>6</td>
<td>−3</td>
<td>25</td>
<td>11.</td>
<td>(4, 5)</td>
<td>9</td>
</tr>
<tr>
<td>12.</td>
<td>(3,6)</td>
<td>9</td>
<td>−3</td>
<td>22</td>
<td>12.</td>
<td>(2, 5)</td>
<td>7</td>
</tr>
<tr>
<td>13.</td>
<td>(4,4)</td>
<td>8</td>
<td>−3</td>
<td>19</td>
<td>13.</td>
<td>(4, 5)</td>
<td>9</td>
</tr>
<tr>
<td>14.</td>
<td>(6,5)</td>
<td>11</td>
<td>−3</td>
<td>16</td>
<td>14.</td>
<td>(5, 5)</td>
<td>10</td>
</tr>
<tr>
<td>15.</td>
<td>(6,3)</td>
<td>9</td>
<td>−3</td>
<td>13</td>
<td>15.</td>
<td>(6, 6)</td>
<td>12</td>
</tr>
<tr>
<td>16.</td>
<td>(2,3)</td>
<td>5</td>
<td>9</td>
<td>22</td>
<td>16.</td>
<td>(5, 4)</td>
<td>9</td>
</tr>
<tr>
<td>17.</td>
<td>(3,6)</td>
<td>9</td>
<td>−3</td>
<td>19</td>
<td>17.</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>
According to the simulations, it seems that players can gain more chocolates by choosing Scenario 2 (a maximum of 28 in both trials), but the risk arising from the fluctuation of returns is great. The group may lose all their chocolates.

3. For interested students, the teacher could provide further reading materials, for example: the booklet *Applications of Mathematics: Investment Portfolios and Market Efficiency* (downloadable from http://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/res/portfolio.pdf)

This example mainly involves the following generic skills:

1. **Critical Thinking Skills**
   - Students need to make logical judgements based on adequate data and evidence, for example, each group may stop playing the game 1 after three rounds.

2. **Problem Solving Skills**
   - Students need to identify the problems associated with the rules of the game.
   - Students need to choose relevant information and strategies to solve problems.
Example 22
Mathematics Reading Scheme

Key Stage: 1-4

Dimension/Strand: All

Reading to learn is one of the Four Key Tasks in the curriculum reform aiming to enhance students’ skills in learning to learn and self-directed learning. Through adopting Reading across the Curriculum, reading activities can promote cross-discipline learning. This example serves to suggest reading programme with activities that can be used:

(i) to develop students’ interest and habit in reading in Mathematics and enhance their literacy;
(ii) to develop students’ generic skills (such as communication skills and self-learning skills);
(iii) to provide opportunities for students’ to link mathematical knowledge with content of other subjects (Learning across the Curriculum); and
(iv) to broaden students’ understanding on the applications of mathematics in real life and the cultural aspect of mathematics.

Planning of Reading Programme/Activities
Below are some suggestions to help schools to set up a Mathematics reading programme:

- Schools could set up long term and short term objectives of the reading programme and implement the reading programme by stages
- Schools could formulate their reading programme for promoting reading in Mathematics with reference to
  - schools’ major concerns,
  - students’ reading competency,
  - students’ interest and ability in Mathematics,
  - teachers’ experiences in promoting reading, and
  - reading resources available for students.
- Schools could build up the collaboration between the Mathematics panel, other subject panels and the school library for organising reading programme for students
- Teachers could review and categorise Mathematics reading materials which suit the interest and ability of the targeted students and plan in advance to increase the quantities of good Mathematics reading materials in classrooms or in the school library. Teachers are encouraged to make good use of resources of public libraries.
- Teachers could design suitable post-reading activities to reinforce students’ learning
and help students discover the mathematics concepts, application of mathematics in different disciplines or the cultural aspects of mathematics in the reading materials.

- Schools could choose appropriate methods of evaluation with suitable success criteria, and evaluate the effectiveness of the implementation plan regularly.

Schools examples:

School A (Primary)

Background: Reading to learn is a major concern of the school in the current school development cycle. Students of the school are keen on reading but they have very little experience in reading mathematics books. Parents are supportive to school’s policies and the reading scheme. The school has provided more mathematics books for lower primary students in the school library.

<table>
<thead>
<tr>
<th>Implementation plan of the Reading Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target: Primary 2 Students</td>
</tr>
<tr>
<td>Objectives:</td>
</tr>
<tr>
<td>1. To create a positive atmosphere of learning Mathematics through reading</td>
</tr>
<tr>
<td>2. To arouse students’ interest and develop reading habits in reading Mathematics books or passages</td>
</tr>
<tr>
<td>3. To enhance students’ communication skills through sharing activities</td>
</tr>
<tr>
<td>Activities and Strategies:</td>
</tr>
<tr>
<td>- Introduction of the Reading Scheme</td>
</tr>
<tr>
<td>To brief parents and targeted students the objectives, activities and details of the scheme in a morning assembly in September.</td>
</tr>
<tr>
<td>- Reading with parents</td>
</tr>
<tr>
<td>Teachers deliver one mathematics book or a reading passage each time in a reading bag for students to read at home with their parents and fill-in the record book. Four times in each term.</td>
</tr>
<tr>
<td>- School-wide Reading Programme</td>
</tr>
<tr>
<td>Cooperate with the School-wide Reading Programme of the school library to encourage students to borrow mathematics books in the library. Students have</td>
</tr>
</tbody>
</table>
to record their reading in a record book. They receive awards after the completion of the Programme and can earn an extra gift for every three mathematics books read.

- **Reading with buddies**
  Arrange students of P.5 and P.6 to be Mathematics Ambassadors to read books with P.2 students in the morning reading period once a week.

- **Good books or passages election**
  Students share one of the books or passages read and elect the ten most popular books or passages after the sharing session in each term.

---

**School B (Secondary)**

Background: The school has carried out a reading pilot scheme in S1 last year to arouse students’ interest and to help students develop reading habits in reading mathematics books. Teachers participated in the pilot scheme share students achievement and the effectiveness of the pilot scheme within schools’ Mathematics Department. Built on experience and good practices, they are keen to extend the reading scheme to all junior forms.

---

**Implementation plan of the Reading Scheme**

**Target:** Junior secondary students

**Objectives:**
1. To enhance students’ thinking skills, problem solving skills and creativity through reading in mathematics
2. To broaden students’ knowledge and exposure in mathematics
3. To nurture students’ understanding of the cultural aspects of mathematics
4. To help students to link the learning of Mathematics with other subjects, e.g. language subjects and History.

**Activities and Strategies:**
- **Mathematics Reading Group**
  Students work in groups of four to study on a theme set by the teachers. They
are required to search and read relevant books or information from the Internet. The themes of S1, S2 and S3 are “Stories of Mathematicians”, “Mathematics games and puzzles” and “Mathematics in daily life” respectively.

- **Books Recommendation**
  Teachers regularly share and recommend relevant books or reading materials from the Internet for students to read in order to help them carry out their study.

- **Reading Worksheet**
  The teachers design different worksheets on each theme for students to complete. For example, S1 students are required to select a mathematician, read his life and contributions and set a question on what they have found out about the mathematician. For S2 students, they have to read books about mathematical games and puzzles, and then post challenging problems for their classmates.

- **Sharing Sessions**
  After the group work, students present and share their reading experiences and achievements in the class. They share the questions posted in the worksheet for class discussion.

- **Publication of Students’ Work**
  Teachers compile a collection of students’ work into a book to showcase students’ achievement in reading.
Example 22

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Appendices
Appendices

1. Seven Learning Goals of the Primary and Secondary Education 197
2. Development of Generic Skills in the Mathematics Curriculum 199
3. Integrative Application of Generic Skills 222
4. Suggested Websites and Application Software (Apps) for Learning and Teaching of Mathematics 229
6. Learning and Teaching Resources List for Mathematics 237
7. List of Collaborative Research and Development (“Seed”) Projects for Mathematics 244
Appendix 1 : Seven Learning Goals of the Primary and Secondary Education

The two sets of learning goals for the primary and secondary education were updated in 2014 and 2016 respectively to respond to the latest changes from a natural continuum of progressive milestones that students are expected to achieve upon completion of the two levels of education based on the seven areas covering knowledge, generic skills, and values and attitudes for whole-person development and lifelong learning. The learning goals at the two levels are defined using slightly different diction and sequences due to consideration of the students’ developmental and growth needs and learning experiences.

<table>
<thead>
<tr>
<th>Seven Learning Goals of Primary Education</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Basic Education Curriculum Guide</em> (Primary 1 – 6) (2014)</td>
</tr>
</tbody>
</table>

- Know how to distinguish right from wrong, fulfil their duties as members in the family, society and the nation, and show acceptance and tolerance towards pluralistic values;
- Understand their national identity and be concerned about society, the nation and the world, and to fulfil their role as a responsible citizen;
- Develop an interest in reading extensively and cultivate a habit of reading;
- Actively communicate with others in English and Chinese (including Putonghua);
- Develop independent learning skills, especially self-management skills and collaboration skills;
- Master the basics of the eight Key Learning Areas to prepare for studying in secondary schools; and
- Lead a healthy lifestyle and develop an interest in aesthetic and physical activities and an ability to appreciate these activities.
Updated Seven Learning Goals of Secondary Education

To enable students to

- become an informed and responsible citizen with a sense of national and global identity, appreciation of positive values and attitudes as well as Chinese culture, and respect for pluralism in society

- acquire and construct a broad and solid knowledge base, and to understand contemporary issues that may impact on students’ daily lives at personal, community, national and global levels

- become proficient in biliterate and trilingual communication for better study and life

- develop and apply generic skills in an integrative manner, and to become an independent and self-directed student for future study and work

- use information and information technology ethically, flexibly and effectively

- understand one’s own interests, aptitudes and abilities, and to develop and reflect upon personal goals with aspirations for further studies and future career

- lead a healthy lifestyle with active participation in physical and aesthetic activities, and to appreciate sports and the arts
Appendix 2 : Development of Generic Skills in Mathematics Curriculum

Nine generic skills have been identified as essential for student learning for the 21st century in the school curriculum since 2001. They are grouped in three clusters of related skills, namely “basic skills”, “thinking skills” and “personal and social skills”, for better integrative understanding and application:

<table>
<thead>
<tr>
<th>Basic Skills</th>
<th>Thinking Skills</th>
<th>Personal and Social Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Skills</td>
<td>Critical Thinking Skills</td>
<td>Self-management Skills</td>
</tr>
<tr>
<td>Mathematical Skills*</td>
<td>Creativity</td>
<td>Self-learning Skills*</td>
</tr>
<tr>
<td>Information Technology Skills</td>
<td>Problem Solving Skills</td>
<td>Collaboration Skills</td>
</tr>
</tbody>
</table>

The 9 generic skills and how mathematics education contributes to the development of them are illustrated in this appendix.

* “Mathematical skills” and “self-learning skills” have been referred to as “numeracy skills” and “study skills” respectively in Learning to Learn: Life-long Learning and Whole-person Development (2001)
Appendix 2

Collaboration Skills

Problem solving, planning and making decisions in a small group require Collaboration Skills, namely, the skills of communication, appreciation, negotiation, making compromises and asserting leadership. Learners with these skills will be able to effectively engage and contribute in teamwork tasks.

The expected achievements of the learners in this generic skill cannot be suitably classified according to key stages

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understanding the nature of group work</strong>&lt;br&gt;Learners will learn to&lt;br&gt;• recognize the need of team work and that the team has a shared responsibility&lt;br&gt;• recognize that individuals as well as the team have to take the consequences for their own actions</td>
<td>Learners&lt;br&gt;1. participate in doing mathematical group work like collecting data, measuring objects and presenting project results, and understand the benefit of working as a team&lt;br&gt;2. accepts and follows the group decision on individual role in doing mathematical group work with the understand that member’s actions affect the product of group work (e.g. whether a correct solution could be found)</td>
</tr>
<tr>
<td><strong>Desirable dispositions for group work</strong>&lt;br&gt;Learners will learn to&lt;br&gt;• be open and responsive to others’ ideas; appreciate, encourage and support the ideas and efforts of others&lt;br&gt;• be active in discussing and posing questions to others, as well as in exchanging, asserting, defending and rethinking ideas&lt;br&gt;• recognise and avoid stereotyping; withhold premature judgment until the facts are known&lt;br&gt;• be willing to adjust their own behaviour to fit the dynamics of various groups and situations</td>
<td>Learners&lt;br&gt;1. discuss and exchange ideas and findings openly with others in completing tasks and solving mathematical problems&lt;br&gt;2. exercise patience and listen to others in the discussion of mathematical problems (e.g. when investigating number patterns or formulating proofs of geometric problems)&lt;br&gt;3. value the contributions of others and put effort to achieve synergy in accomplishing mathematical tasks or solving mathematical problems together&lt;br&gt;4. appreciate different solutions to mathematical problems presented by others (e.g. using different approaches to prove mathematical theorems)&lt;br&gt;5. participate actively in discussion, and pose questions in clarifying arguments in the solution of mathematical problems and offer improvements and corrections with justifications (e.g. discussing the strategies to be adopted in investigating practical statistical problems)</td>
</tr>
<tr>
<td>Expected Achievements</td>
<td>Examples of Implementation in Mathematics Education</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td><strong>Skills for group work</strong></td>
<td>Learners will learn to</td>
</tr>
<tr>
<td><strong>Goals setting</strong></td>
<td>select a strategy and plan cooperatively to complete a task in a team</td>
</tr>
<tr>
<td><strong>Role taking</strong></td>
<td>understand the strengths and weaknesses of members and maximize the potential of the team</td>
</tr>
<tr>
<td></td>
<td>clarify and accept various roles and responsibilities of individual members in a team and be willing to follow team rules</td>
</tr>
<tr>
<td><strong>Synergising</strong></td>
<td>liaise with members for views and resources</td>
</tr>
<tr>
<td></td>
<td>negotiate and compromise with others</td>
</tr>
<tr>
<td><strong>Reflection</strong></td>
<td>reflect on and evaluate the strategy used by the group and make necessary adjustments</td>
</tr>
</tbody>
</table>

1. share with groupmates their experience in solving mathematical problems and select with agreed justifications a suitable strategy |
2. take up group tasks according to their strengths and weaknesses and be respectfully to the role assignment (e.g. the members with good presentation skills, IT skills, modelling skills, and others, take up the related roles in accomplishing the mathematical tasks) |
3. clarify arguments objectively and rationally in solving mathematical problems and look for a logical structure in addressing mathematical challenges (e.g. when examining the appropriateness of a particular problem-solving strategy) |
4. liaise, negotiate and compromise with others in selecting suitable strategy for solving a mathematical problem (e.g. choose between a deductive or analytic approach in solving a geometrical problem, and negotiate with group members to set the theme and research method of a statistical study) |
5. evaluate the strategy adopted and suggest improvements (e.g. evaluating the effectiveness of data collection by small teams, or presenting the results by a single member and suggest a more effective method)
Communication Skills

Communication Skills refer to the abilities to achieve the desired outcomes or goals in a process where two or more people interact (be it in a face-to-face or virtual context) through expressing or receiving messages using verbal and non-verbal means. To communicate effectively, learners should learn to listen, speak, read and write competently. Not only should they express themselves in an accurate, organised and proper manner, but they should also understand and respect others’ views and expectations, and use appropriate information and means to convey a message in accordance with the purpose, context and audience. They should also evaluate the effectiveness of their communication and identify areas of improvement to achieve the best results.

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Stage One (Lower Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. retrieve information from pictograms or block graphs according to the spoken instructions from teachers</td>
</tr>
<tr>
<td>• comprehend and act appropriately on spoken instructions</td>
<td>2. interpret drawings, figures (e.g. rectangles and cubes) and symbols (e.g. +, −, ×, ÷) and describe them in plain language (e.g. 2 + 3 as 2 plus 3) and appropriate mathematical terms (e.g. a cube has six faces)</td>
</tr>
<tr>
<td>• comprehend the explicit messages of information from different media</td>
<td>3. present findings with drawings and symbols</td>
</tr>
<tr>
<td>• use clear and appropriate means of communication, both verbal and non-verbal, to express meaning and feelings</td>
<td>4. present data with tables and graphs (e.g. block graph)</td>
</tr>
<tr>
<td>• work and discuss with others to accomplish simple tasks</td>
<td>5. compare fractions and give verbal explanation to the result of comparison</td>
</tr>
<tr>
<td></td>
<td>6. express simple daily life problems in mathematical language (e.g. use symbols like $4 \times 3$)</td>
</tr>
<tr>
<td></td>
<td>7. work in small groups to discuss the method to measure the length and width of the classroom</td>
</tr>
<tr>
<td><strong>Key Stage Two (Upper Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. interpret drawings, symbols (e.g. %), tables and graphs (e.g. read and discuss broken line graphs)</td>
</tr>
<tr>
<td>• comprehend and respond to different types of text</td>
<td>2. describe and explain findings/results of mathematical tasks in both oral and written forms (e.g. the average score of a student’s performance in tests, the favorite fruit)</td>
</tr>
<tr>
<td>• comprehend and infer the messages of information from different media</td>
<td></td>
</tr>
</tbody>
</table>
### Expected Achievements

<table>
<thead>
<tr>
<th>Key Stage Three (Junior Secondary)</th>
<th>Key Stage Four (Senior Secondary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learners will learn to</td>
<td>Learners will learn to</td>
</tr>
<tr>
<td>• understand, analyse, evaluate and respond to a range of different types of text</td>
<td>• listen and read critically, evaluate the messages of information from different media and express fluently in accordance with the audience and reader</td>
</tr>
<tr>
<td>• synthesise the messages of information from different media</td>
<td>• use appropriate means of communication to inform, entertain, persuade and argue and achieve</td>
</tr>
<tr>
<td>• use appropriate language and/or other forms of communication to present information and different points of view, and to express feelings</td>
<td>• work and negotiate with others to solve problems and accomplish tasks</td>
</tr>
<tr>
<td>• work and negotiate with others to develop ideas and accomplish tasks</td>
<td>• reflect and improve on the effectiveness of their own communication</td>
</tr>
</tbody>
</table>

### Examples of Implementation in Mathematics Education

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-verbal means of expression to convey information and opinions, and to explain ideas</td>
<td>3. present results of tasks with appropriate drawings and symbols and present data with tables, charts and graphs (e.g. bar charts, broken line graphs)</td>
</tr>
<tr>
<td>• work and negotiate with others to develop ideas and accomplish tasks</td>
<td>4. present solutions of problems logically (e.g. use of “=” properly)</td>
</tr>
<tr>
<td></td>
<td>5. express simple problems in mathematical language</td>
</tr>
<tr>
<td></td>
<td>6. discuss with others in accomplishing tasks such as projects on finding volumes of irregular objects</td>
</tr>
<tr>
<td>Key Stage Three (Junior Secondary)</td>
<td>Learners</td>
</tr>
<tr>
<td>Learners</td>
<td>1. interpret a daily life problem from the media and identify mathematical elements within (e.g. identify the spread rate of an epidemic)</td>
</tr>
<tr>
<td></td>
<td>2. describe findings or explain conjectures in both oral and written forms using mathematical language (e.g. the two triangles are congruent)</td>
</tr>
<tr>
<td></td>
<td>3. choose appropriate statistical charts/graphs to present data and use appropriate mathematical terminology or symbols in explaining ideas</td>
</tr>
<tr>
<td></td>
<td>4. formulate and write simple geometric proofs involving 2-D rectilinear figures with appropriate symbols, terminology and reasons</td>
</tr>
<tr>
<td></td>
<td>5. respond appropriately to others’ mathematical arguments using precise mathematical vocabulary in both oral and written forms</td>
</tr>
<tr>
<td></td>
<td>6. recognise the differences between the language used in a mathematical context and that in daily life (e.g. the word “similar”) and describe mathematical ideas and procedures in ordinary language</td>
</tr>
<tr>
<td></td>
<td>7. present possible solutions to a problem (e.g. explain calculation strategies) to others using mathematical language and ordinary language appropriately</td>
</tr>
<tr>
<td>Key Stage Four (Senior Secondary)</td>
<td>Learners</td>
</tr>
<tr>
<td>Learners</td>
<td>1. demonstrate a clear understanding of mathematical content or application of mathematics through logical presentation (e.g. write and present more complex geometric proofs with appropriate symbols and terminology)</td>
</tr>
</tbody>
</table>
| | 2. intertwine ordinary language, mathematical language, diagrams and graphs to describe and explain mathematical concepts or solutions of real-life problems (e.g. to answer which group of
<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected outcomes</td>
<td>athletes has a more stable performance by statistical measures)</td>
</tr>
<tr>
<td>• resolve conflicts and solve problems with others to accomplish tasks</td>
<td>3. explain how a mathematical concept is applied to an authentic situation in a way that a layman can understand (e.g. the Richter Scale) and further elaborate using appropriate graphs or charts after receiving feedback from others</td>
</tr>
<tr>
<td>• evaluate the effectiveness of their communication with others from different perspectives for further improvement</td>
<td>4. produce concise and structured report on statistical survey for group discussion and evaluate the effectiveness of the graphs chosen for presenting the findings</td>
</tr>
</tbody>
</table>
Creativity

Creativity is manifested in new ideas, acts, or products. It emerges spontaneously or through deliberate processes of divergent and convergent thinking. It involves the integration of general or domain-specific knowledge for a meaningful purpose. Creativity brings in changes or transformations.

The expected achievements of the learners in this generic skill cannot be suitably classified according to key stages.

Development of creativity involves the following abilities, dispositions and favourable factors.

1. Abilities

<table>
<thead>
<tr>
<th>Abilities</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity</td>
<td>To discern details from observation and quickly respond to stimulus</td>
</tr>
<tr>
<td>Fluency</td>
<td>To generate numerous ideas promptly</td>
</tr>
<tr>
<td>Flexibility</td>
<td>To adapt varied ideas and to initiate new thoughts for action</td>
</tr>
<tr>
<td>Originality</td>
<td>To produce unusual, novel and unique ideas</td>
</tr>
<tr>
<td>Elaboration</td>
<td>To expand, refine and embellish ideas</td>
</tr>
</tbody>
</table>

2. Dispositions

<table>
<thead>
<tr>
<th>Dispositions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curiosity</td>
<td>Showing interest and desire to find out more</td>
</tr>
<tr>
<td>Risk-taking</td>
<td>Showing courage and determination to deal with uncertainties or ambiguities</td>
</tr>
<tr>
<td>Imagination</td>
<td>Enjoying fantasising and generating new ideas</td>
</tr>
<tr>
<td>Complexity</td>
<td>Being attracted to intricacies and novelty; embracing challenges</td>
</tr>
</tbody>
</table>

3. Favourable Factors for Nurturing Creativity

<table>
<thead>
<tr>
<th>Factors</th>
<th>Corresponding actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place</td>
<td>• To foster a supportive environment (open, inviting and accepting atmosphere; resourceful, safe and stimulating environment)</td>
</tr>
<tr>
<td>Person</td>
<td>• To recognise and accommodate the wide range of attributes and dispositions of learners (strengths, weaknesses, learning styles, learning needs, motivation and readiness)</td>
</tr>
</tbody>
</table>

Appendix 2

<table>
<thead>
<tr>
<th>Factors</th>
<th>Corresponding actions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Process</strong></td>
<td>To identify and develop learners' potential for creative acts</td>
</tr>
<tr>
<td></td>
<td>• To open up alternatives for learners to explore personal interest</td>
</tr>
<tr>
<td></td>
<td>• To provide interesting and stimulating themes conducive to arousing creative acts and satisfying a craving</td>
</tr>
<tr>
<td></td>
<td>• To expose learners to various stages of creating (preparation, incubation, illumination and verification)</td>
</tr>
<tr>
<td></td>
<td>• To value attempts to present new ideas and encourage further refinements</td>
</tr>
<tr>
<td><strong>Product</strong></td>
<td>• To encourage creative actions and output (ideas, plans, methods, solutions, products, theories)</td>
</tr>
<tr>
<td></td>
<td>• To value the experience of creation and celebrate learners’ creative output</td>
</tr>
<tr>
<td></td>
<td>• To encourage learners to persuade others (especially experts in the field) to accept their creative output</td>
</tr>
</tbody>
</table>

Examples of Implementation in Mathematics Education:

<table>
<thead>
<tr>
<th>Key Stage 1</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>propose applications of division in daily life</td>
</tr>
<tr>
<td>2.</td>
<td>design a method to compare the capacities of two irregular containers</td>
</tr>
<tr>
<td>3.</td>
<td>design methods to draw parallel lines and perpendicular lines using objects in simple 2-D or 3-D shapes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Stage 2</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>create polygons with the same perimeter or area but in different shapes</td>
</tr>
<tr>
<td>2.</td>
<td>design symmetrical figures or patterns</td>
</tr>
<tr>
<td>3.</td>
<td>create different methods to find the perimeter of a circular object</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Stage 3</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>design patterns based on tessellations of a plane using triangles or quadrilaterals</td>
</tr>
<tr>
<td>2.</td>
<td>design a card robot formed by different regular polyhedra and prisms</td>
</tr>
<tr>
<td>3.</td>
<td>participate in group project to design a ramp for wheelchairs users for a building in their communities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Stage 4</th>
<th>Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>design encryption and decryption methods to transmit a message to their classmates</td>
</tr>
<tr>
<td>2.</td>
<td>design a container of tennis balls that fulfills some preset requirements on shape, capacity and volume of material used</td>
</tr>
<tr>
<td>3.</td>
<td>propose alternative solutions to problems</td>
</tr>
</tbody>
</table>
Critical Thinking Skills

Critical thinking is drawing out meaning from available data or statements, examining and questioning their accuracy and credibility, in order to establish one’s views and evaluate the arguments put forward by oneself and others.

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Stage One (Lower Primary)</strong></td>
<td>Learners</td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. sort objects using various criteria such as shapes and sizes</td>
</tr>
<tr>
<td>• extract, classify and organise information</td>
<td>2. choose the right tools to measure objects such as using measuring tapes to measure the circumference of a round table</td>
</tr>
<tr>
<td>• identify and express main ideas, problems or core issues</td>
<td>3. reason inductively (e.g. when exploring the commutative property of addition)</td>
</tr>
<tr>
<td>• understand straightforward cause-and-effect relationships</td>
<td>4. choose relevant information in solving a problem (e.g. tell the number of toys from the information that there are 3 trains, 5 apples and 6 dolls in a box)</td>
</tr>
<tr>
<td>• distinguish between obvious fact and opinion</td>
<td>5. check the reasonableness of the answer to a problem (e.g. is the amount of water drunk by a student per day too large to be realistic?)</td>
</tr>
<tr>
<td>• notice obvious contradictions, seek clarifications and make simple predictions</td>
<td></td>
</tr>
<tr>
<td>• draw simple but logical conclusions not contradictory to given data and evidence</td>
<td></td>
</tr>
<tr>
<td><strong>Key Stage Two (Upper Primary)</strong></td>
<td>Learners</td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. categorise information using various criteria such as considering properties of length and parallel sides when discussing properties of quadrilaterals</td>
</tr>
<tr>
<td>• make inductions/ inferences from sources</td>
<td>2. choose appropriate methods and units to measure objects, such as using the method of displacement to measure the volumes of irregular objects</td>
</tr>
<tr>
<td>• cross-reference other sources to determine the reliability of a source</td>
<td>3. reason inductively (e.g. when exploring the formula for the area of a rectangle)</td>
</tr>
<tr>
<td>• understand the concepts of relevance and irrelevance</td>
<td>4. check the reasonableness of the solution to a problem (e.g. the steps for solving a problem might be unusually complicated)</td>
</tr>
<tr>
<td>• distinguish fact and opinion as well as source and evidence</td>
<td>5. ask questions about mathematics during exploration (e.g. ask questions about the relation between circumferences and diameters of circles after related measurements of circular objects)</td>
</tr>
<tr>
<td>• recognise obvious inconsistencies, omissions, assumptions, stereotypes and biases</td>
<td></td>
</tr>
</tbody>
</table>
### Expected Achievements

**Key Stage Three (Junior Secondary)**

Learners will learn to
- identify the issue at stake
- clarify and define key words to guide thinking
- compare information from different sources, note contrasts and similarities, and determine its reliability
- differentiate between fact, opinion and reasoned judgment
- recognise that information providers’ value orientations and ideologies would affect the perspectives or judgments of sources
- recognise and challenge stereotypes, emotional factors, propaganda and fallacies
- draw and test conclusions as well as hypotheses, identify reasonable alternatives and predict probable consequences
- admit their own limitations, shortcomings or errors of the thinking process

**Examples of Implementation in Mathematics Education**

Learners
1. use deductive reasoning to study the properties of geometric figures, such as “the sum of exterior angles of a convex polygon is 4 right angles”
2. generalise observations in symbolic forms from concrete experiences (e.g. when generalizing the index laws from observing several examples in numbers)
3. judge whether the information given about a problem is relevant or not (e.g. identifying extraneous information given in a geometric problem)
4. examine the reasonableness of the solution to a problem and evaluate the strategy adopted (e.g. evaluate the effectiveness of using the graphical method to solve simple linear equations)
5. compare different ways of approaching traditional mathematical problems (e.g. different proofs of Pythagoras’ Theorem or different ways of tiling a floor using triangular tiles of a single size and shape)

**Key Stage Four (Senior Secondary)**

Learners will learn to
- differentiate between real and stated issues, false and accurate representations, and relevant and irrelevant evidence
- differentiate between sophisticated fact, opinion and reasoned judgment
- recognise and challenge subtle or fundamental assumptions, permeating value orientations and ideologies
- recognise that the selection and deployment of information / facts are affected by personal perspectives
- draw warranted conclusions, predict and assess probable consequences and make reasoned judgment in reading, writing, and speech
- apply appropriate thinking skills to evaluate and reflect on their thinking process and suggest ways for improvement

**Examples of Implementation in Mathematics Education**

Learners
1. investigate and judge the validity of arguments derived from data sets
2. assess statistical investigations presented in different sources such as news media, research reports, etc. and be aware that different motive, perspectives and values may affect the conclusion (e.g. a company might abuse statistical measures to boost its sales)
3. compare and evaluate the effectiveness of graphical method and algebraic method in solving quadratic inequalities
4. construct, check and amend deductive proofs of geometric properties involving circles
5. reflect on problem solving method to see if there are any mistakes or imperfections (e.g. reflect on whether the value of common ratios is considered before applying the formulae for finding infinite sums of geometric sequences)
Information Technology Skills

**Information Technology Skills** are the ability to use IT critically to search, select, analyse, manage and share information. Mastery of IT Skills facilitates collaborative learning, problem solving and self-directed learning.

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Stage One (Lower Primary)</td>
<td>Learners</td>
</tr>
</tbody>
</table>
| Learners will learn to | 1. use suitable application software to investigate number patterns and properties of numbers (e.g. odd, even, ascending, descending)  
2. use suitable application software to create and explore geometric patterns (e.g. apps on tangram)  
3. search from the Internet the lengths of the cross-harbour tunnels |
| • operate computers or mobile devices |  |
| • input Chinese characters |  |
| • use e-resources to support learning with the help of teachers |  |
| • recognise some methods to locate and access information with given searching criteria |  |
| • generate, present, and safely share ideas with IT tools in learning activities |  |
| Key Stage Two (Upper Primary) | Learners |
| Learners will learn to | 1. use suitable application software to investigate the properties of shapes, draw and create geometric patterns (e.g. draw squares, rectangles, triangles, circles and create geometric patterns with these figures)  
2. use a spreadsheet to record data and create graphs for doing statistical projects (e.g. input data in a spreadsheet and present the data with line graphs or bar charts)  
3. use the information obtained through the Internet for self-directed learning and when doing projects (e.g. symmetric objects in daily life) and share the results through school e-platforms |
| • use a variety of software for word-processing, calculation, image-processing and other learning activities |  |
| • produce multimedia presentation with simple design |  |
| • search, select and prudently share information via computer networks and other media |  |
| • process information and produce user-generated content using IT tools |  |


User-generated content refers to content that is produced and shared by end-users of digital media.
### Appendix 2

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Stage Three (Junior Secondary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. use graphing software for various computational and exploratory activities (e.g. draw straight lines and explore their condition for perpendicularity)</td>
</tr>
<tr>
<td>• use appropriate IT tools to facilitate learning</td>
<td>2. use suitable application software to explore the relations of numbers (e.g. number patterns), algebraic formula (e.g. formulae of areas and volumes) and graphical representations (e.g. finding solutions of simultaneous equations by the graphical method)</td>
</tr>
<tr>
<td>• use IT tools and strategies for processing and presenting information</td>
<td>3. use suitable application software to construct appropriate statistical diagrams (e.g. frequency polygon, histogram) to represent given data and to find simple statistical measures (e.g. mean, mode)</td>
</tr>
<tr>
<td>• produce multimedia presentation with appropriate design for different purposes</td>
<td>4. use dynamic geometry software to explore properties of 2-D rectilinear figures (e.g. the relationship among the angles or sides of a parallelogram) and to visualise geometric properties of 2-D and 3-D figures (e.g. uniform cross-section of prisms)</td>
</tr>
<tr>
<td>• communicate and collaborate with others via computer networks and other media</td>
<td>5. use the information obtained through the Internet, for doing projects (e.g. study the climate change in Hong Kong) and in self-directed learning, and discuss the reliability of information through school e-platforms</td>
</tr>
<tr>
<td>• verify and evaluate the accuracy and reliability of information</td>
<td></td>
</tr>
</tbody>
</table>
Mathematical Skills include the ability to perform computations and estimations of numbers in various forms, to describe spatial relationships between objects, to perform measurements, to manage data, to employ logical reasoning for drawing valid conclusions, and to apply mathematical concepts in different contexts.

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Stage One (Lower Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. describe and compare the number of objects with natural numbers</td>
</tr>
<tr>
<td>• perform comparison and basic computations of whole numbers</td>
<td>2. perform properly basic computations involving whole numbers</td>
</tr>
<tr>
<td>• describe shapes, sizes and positions</td>
<td>3. sort and name objects (e.g. biscuits boxes, ice-cream cones) of different geometric shapes</td>
</tr>
<tr>
<td>• apply the knowledge of measurement, and use appropriate units and tools for measurement</td>
<td>4. recognise basic directions: east, south, west and north</td>
</tr>
<tr>
<td>• present data by means of, and retrieve information from simple charts and graphs</td>
<td>5. measure the length and width of a classroom or a playground with suitable instruments, e.g. a metre ruler and a trundle wheel, to prepare suitable amount of decoration materials</td>
</tr>
<tr>
<td>• perform simple deductions with the use of basic logical concepts, such as “and”, “or”, “all”, “some”, “because”, “if … then” and “contradiction”</td>
<td>6. read simple statistical charts (e.g. block graphs and simple pictograms)</td>
</tr>
<tr>
<td>• apply simple mathematical knowledge in daily life</td>
<td>7. form numbers that satisfy two conditions, e.g. use four numerals to form a 4-digit number which is odd and smaller than a fixed value</td>
</tr>
<tr>
<td><strong>Key Stage Two (Upper Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. choose the correct forms of numbers in presenting information (e.g. using percentages to describe the size of the portion of students with glasses in the class)</td>
</tr>
<tr>
<td>• perform computations and simple estimations involving whole numbers, fractions, decimals, and percentages, such as estimating expenses</td>
<td>2. estimate the total expenditure of purchases made in a week</td>
</tr>
<tr>
<td>• use simple geometric properties, such as symmetry, parallel and perpendicular, to describe shapes, sizes and positions more accurately</td>
<td>3. use the 8 compass points, e.g. north-east, south-west, for locating the relative positions of buildings</td>
</tr>
<tr>
<td>• apply strategies and formulae in measurement</td>
<td>4. apply formulae to find the area of a desk top and the capacity of a locker in the classroom</td>
</tr>
<tr>
<td>• collect and process data, present data</td>
<td>5. collect classmates’ preferences on food and drinks offered in the school Christmas party and present the</td>
</tr>
</tbody>
</table>
### Expected Achievements

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>by means of suitable statistical charts and graphs and retrieve information from charts and graphs</td>
<td>data using suitable statistical graph for making decisions</td>
</tr>
<tr>
<td>• perform deductions, such as syllogism and provide counter examples</td>
<td>6. provide counter examples to disprove the statement “a 2-D figure with four equal sides must be a square”</td>
</tr>
<tr>
<td>• apply mathematical concepts in daily life</td>
<td>7. use equations to solve simple problems</td>
</tr>
</tbody>
</table>

#### Key Stage Three (Junior Secondary)

Learners will learn to:
- handle very large or very small numbers and negative numbers with a sense of scale
- perform numerical manipulations, such as percentage changes, and perform estimations with appropriate strategies
- describe the rules of arrangement of objects or occurrence of events, such as the pattern formed by a set of shapes or the trend of population growth
- describe spatial relationships between objects using distance, angle, scale, bearings and gradient
- choose appropriate tools and strategies to find measurements according to the degree of accuracy required by the specific purpose
- use different methods for handling (i.e. collecting, organising, analysing, and presenting) quantitative information and make reasonable interpretation of the results
- estimate risks and chances through the use of elementary probability
- perform deductions and verifications, and check their validity
- apply various mathematical concepts in authentic situations

#### Key Stage Four (Senior Secondary)

Learners will learn to:
- evaluate the appropriateness of tools and strategies for handling quantitative
<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>information</td>
<td>Mathematics assessments</td>
</tr>
<tr>
<td>• use quantitative information for making informed decisions in different contexts</td>
<td>2. perform project learning by studying statistical information from the Hong Kong Yearbook or the website of the Hong Kong Observatory</td>
</tr>
<tr>
<td>• evaluate processes of deductions to avoid committing logical fallacies</td>
<td>3. check whether all conditions are fulfilled before applying a theorem</td>
</tr>
<tr>
<td>• apply various mathematical concepts in different contexts with appropriate strategies and be aware of the need to make adaptations in new situations</td>
<td>4. study on the wealth disparity between the more developed countries and the less developed ones (through organizing and analysing numerical information in statistical charts to support the reasoning on social issues)</td>
</tr>
</tbody>
</table>
### Problem Solving Skills

Problem solving involves using various skills to resolve a difficulty. The process includes investigating the problem, synthesizing information and generating ideas to determine the best course of action. Learners need to adjust and evaluate strategies, as well as consolidate experience for knowledge construction.

<table>
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</thead>
<tbody>
<tr>
<td><strong>Key Stage One (Lower Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. identify key information from word problems (e.g. operations required or quantities to be computed)</td>
</tr>
<tr>
<td>• develop ideas about the problem and identify related sources of information</td>
<td>2. solve problems by simple computations (e.g. using addition to find the sum of money used in buying goods)</td>
</tr>
<tr>
<td>• identify, under guidance, one or more ways of tackling the problem</td>
<td>3. adopt various ways of solving problems (e.g. using drawing and manipulatives to do addition)</td>
</tr>
<tr>
<td>• choose and implement a solution plan, using support and advice given</td>
<td>4. solve problems by choosing the correct data (e.g. choosing the correct combination of coins for one exact bus fare)</td>
</tr>
<tr>
<td>• follow the given step-by-step methods to check and describe the outcomes</td>
<td></td>
</tr>
<tr>
<td><strong>Key Stage Two (Upper Primary)</strong></td>
<td><strong>Learners</strong></td>
</tr>
<tr>
<td>Learners will learn to</td>
<td>1. use different approaches to solve problems (e.g. finding the area of a polygon by means of counting the number of squares, dissecting the polygon into parts or using formulae)</td>
</tr>
<tr>
<td>• identify the problem and describe its main features</td>
<td>2. make use of various tools to solve problems (e.g. measuring tapes and metre rules)</td>
</tr>
<tr>
<td>• propose alternative courses of action for solving it</td>
<td>3. choose the correct data and relevant information to solve problems (e.g. identify the correct height and base in finding the area of a triangle)</td>
</tr>
<tr>
<td>• plan and try out the selected option, obtain support and make changes when needed</td>
<td>4. compare new problems with similar ones solved before and try to apply past experience to solve the problems (e.g. comparing the problem of finding the area of a trapezium to that of a triangle)</td>
</tr>
<tr>
<td>• develop an appropriate method to measure the effectiveness of the solution plan adopted</td>
<td>5. perform computations in different sequences and compare the computation processes</td>
</tr>
<tr>
<td>• gain insights from the problem solving process</td>
<td></td>
</tr>
<tr>
<td>Expected Achievements</td>
<td>Examples of Implementation in Mathematics Education</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td><strong>Key Stage Three (Junior Secondary)</strong>&lt;br&gt;Learners will learn to&lt;br&gt;• explore the problem and identify its main focus&lt;br&gt;• suggest and compare the possible outcomes of each alternative course of action and justify the option selected&lt;br&gt;• execute the planned strategy, monitor progress and make adjustment when necessary&lt;br&gt;• evaluate against established criteria the quality of outcomes, and review the effectiveness of the problem solving process&lt;br&gt;• formulate personal views, and paraphrase or construct analogies to explain how the problem is solved</td>
<td>Learners&lt;br&gt;1. understand the given conditions of a geometric problem, identify the proposition to be proved, and devise a plan for proving it using appropriate theorems&lt;br&gt;2. apply analytic and deductive approaches for solving geometric problems&lt;br&gt;3. judge and evaluate arguments of their own or others in presenting a solution to a mathematical problem (e.g. evaluating the reasons supporting that “mode” is the best measure of the most popular size of shoes in a market)&lt;br&gt;4. formulate a mathematical solution when tackling a problem in an authentic situation (e.g. finding the floor area of a flat from its layout plan)</td>
</tr>
<tr>
<td><strong>Key Stage Four (Senior Secondary)</strong>&lt;br&gt;Learners will learn to&lt;br&gt;• recognize the complexity of the problem and search for appropriate information required to solve it&lt;br&gt;• formulate feasible strategies to achieve optimal results, considering both long and short term objectives&lt;br&gt;• modify objectives or strategies and suggest remedial or enhancing measures to cope with circumstantial changes or difficulties&lt;br&gt;• evaluate the overall strategy and outcomes, and anticipate future problems that may be incurred&lt;br&gt;• consolidate experience on problem solving for knowledge construction</td>
<td>Learners&lt;br&gt;1. search for appropriate information when doing a project on comparing different mortgage plans of different banks&lt;br&gt;2. evaluate the anticipated amount of computations need for solving a problem (e.g. solving simultaneous equations involving a quadratic one) and choose appropriate tools for computation or use another strategies to reduce computation time&lt;br&gt;3. review and modify the questionnaire design of a statistical survey on weekly time used on mobile devices (e.g. number of options, ranges of time durations) before data collection&lt;br&gt;4. evaluate a sampling method and find out its limitations</td>
</tr>
</tbody>
</table>
## Self-learning Skills

Self-learning Skills refer to the ability to initiate, plan, carry out, evaluate and adjust learning activities autonomously. Learners with advanced Self-learning Skills can select or design effective strategies for in-depth learning. These skills help learners enhance academic performance and self-efficacy. Self-learning Skills form the core part of lifelong learning and help learners acquire new knowledge to adapt to the fast changing world.

<table>
<thead>
<tr>
<th>Expected Achievements</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
</table>
| **Key Stage One (Lower Primary)** Learners will learn to  
• consciously listen and read to learn; actively present their learning  
• concentrate and pay attention to instructions  
• identify and retain main ideas  
• collect information from given sources and organize it into pre-determined categories  
• try out different means to present ideas and demonstrate learning  
• develop simple learning plans to meet short term targets  
• show interest in enquiring further | Learners  
1. listen to instruction carefully, participate actively and show interest in learning activities in the class (e.g. exploring different types of triangles by using straws of equal or different lengths to form triangles)  
2. understand concepts and do not learn only by rote memorization (e.g. students have to understand the concept of multiplication instead of just memorizing the multiplication tables)  
3. present simple problems in mathematical languages including symbols and graphs (e.g. pictograms) |
| **Key Stage Two (Upper Primary)** Learners will learn to  
• take initiative in the enquiry learning area selected by themselves  
• actively locate required information from different media  
• take initiative in identifying and organising main points from sources, e.g. note-taking, mind mapping  
• decide on the most suitable means to present ideas and demonstrate learning  
• seek help appropriately when necessary  
• manage time to complete tasks according to plan | Learners  
1. try to understand concepts and not to learn only by rote memorization (e.g. recognize the connection between the formulae of areas of different figures instead of memorize the formulae by rote)  
2. search for reference information from various sources including libraries, reference books, Internet, etc.  
3. use different aids to build up mathematical concepts (e.g. use a table of integers from 1 to 100 to study the patterns of multiples)  
4. use diagrams, pictures and charts to help understand mathematical concepts (e.g. mark the locations of moving objects at different time to understand the idea of speed)  
5. learn from mistakes made in homework exercises, tests,
### Expected Achievements

- make use of feedback to reflect on the effectiveness of different learning tactics
- etc. and learn from feedback given by teachers and classmates in solving mathematical problems

### Examples of Implementation in Mathematics Education

**Key Stage Three (Junior Secondary)**

Learners will learn to

- initiate learning activities and apply relevant personal strengths to overcome challenges
- set learning plans with stage-wise goals
- identify lines of reasoning and possible hidden ideas in sources
- function effectively in a group to achieve the learning goals
- decide on the most suitable means to manage and present knowledge
- adjust the learning strategies to improve learning effectiveness

**Learners**

1. initiate and plan exploratory activities for knowledge construction (e.g. participate in a mathematics project competition)
2. learn from mistakes made in solving mathematical problems and identify their own incorrect concepts
3. identify key similarities and differences from working among various types of mathematical problems
4. use diagrams, pictures and charts to present and help understand mathematical concepts (e.g. use a diagram to present the classification of quadrilaterals)
5. contribute constructive ideas in group discussion for explaining mathematical results or suggesting strategies for solving mathematical problems

**Key Stage Four (Senior Secondary)**

Learners will learn to

- initiate challenging learning activities and develop relevant personal strengths to overcome challenges
- plan and set goals for self-initiated enquiries
- autonomously select or design more effective learning strategies for in-depth learning
- evaluate key ideas, opinions and arguments identified from sources independently and synthesize them to construct and develop their own interpretation
- evaluate and suggest ways to improve the effectiveness of learning strategies
- learn beyond the prescribed curriculum and apply knowledge in a variety of contexts

**Learners**

1. generalise observations to a higher level of abstraction for transferring strategies to a wider range of problems (e.g. recognise some typical counting problems for solve counting problems of similar structures)
2. initiate and plan exploratory activities for learning beyond the prescribed curriculum (e.g. do project work on solving real-life problems, such as redesigning the layout of the school carpark)
3. select more effective learning strategies, such as exploration examples with increasing complexity for learning the properties of arithmetic sequences and geometric sequence instead of rote memorization of related formulae
4. acquire further understanding of mathematical concepts by exchanging and discussing ideas with others, testing the hypothesis, searching for information from the Internet, etc., and evaluate effectiveness of different strategies
5. solve real-life and mathematical problems by studying the information for clues, presenting the problem in mathematical language, or integrating knowledge and skills of various strands of the curriculum
## Self-management Skills

Self-management Skills comprise essential life skills and desirable personal qualities such as maintaining emotional stability, making decisions and exercising self-discipline. Self-management Skills enable learners to embrace challenges encountered on a personal or team basis.

The expected achievements of the learners in this generic skill are classified according to level of mastery.

<table>
<thead>
<tr>
<th>Elements of Self-management Skills</th>
<th>Beginning</th>
<th>Developing</th>
<th>Mastering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learners will learn to</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-worth</td>
<td>express positive statements about themselves</td>
<td>identify and apply personal skills, attitudes and values to overcome challenges</td>
<td>uphold, synthesize and renew their own beliefs and values</td>
</tr>
<tr>
<td>Goal setting and tracking</td>
<td>set goals to assist their learning and personal development</td>
<td>set and keep track of realistic goals</td>
<td>set, keep track of, and be reflective on and accountable for goals which work towards excellence in their life</td>
</tr>
<tr>
<td>Decision making</td>
<td>make decisions in daily life situations with supporting reasons</td>
<td>list out and evaluate pros and cons of a suggestion, and estimate the consequences of a decision</td>
<td>consider all factors, such as technical, ethical, resource and community considerations before making a decision</td>
</tr>
<tr>
<td>Confidence, resilience and adaptability</td>
<td>develop confidence and resilience in simple tasks and appreciate the progress made</td>
<td>demonstrate motivation, confidence, commitment and adaptability when faced with new or difficult situations, and derive satisfaction from accomplishments and efforts</td>
<td>demonstrate confidence and adaptability in adversities, tolerate ambiguities and appreciate lessons learnt from mistakes</td>
</tr>
<tr>
<td>Appropriate expression of emotions</td>
<td>understand, accept and appropriately express emotions</td>
<td>describe their genuine feelings, such as joy and disappointment and identify factors contributing to these feelings</td>
<td>use appropriate means to contain or release their emotions</td>
</tr>
<tr>
<td>Managing resources</td>
<td>demonstrate care for personal properties and shared resources</td>
<td>treasure and make good use of time, money and other resources</td>
<td>suggest ways for effective, equitable and ethical use of resources</td>
</tr>
</tbody>
</table>
## Appendix 2

### Elements of Self-management Skills

<table>
<thead>
<tr>
<th>Elements of Self-management Skills</th>
<th>Beginning</th>
<th>Developing</th>
<th>Mastering</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learners will learn to</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keeping promises to others</td>
<td>keep promises and fulfill obligations</td>
<td>assess feasibility before making promises</td>
<td>make determined efforts to keep promises; take responsibility and make up for broken promises obliged by circumstances</td>
</tr>
<tr>
<td>Self-discipline</td>
<td>exercise self-control against distractions, and focus on and complete given tasks at hand within a given time</td>
<td>extend self-control in scope and duration over personal impulses through developing positive thinking and self-affirmation</td>
<td>exercise self-control naturally as a habit of mind</td>
</tr>
<tr>
<td>Reflective Practice</td>
<td>review their learning readily to know more about themselves and how they work</td>
<td>form habits of reviewing their learning and identify factors that contribute to or hinder their learning effectiveness</td>
<td>sustain self-improvement by paying attention to and making judicious use of feedback</td>
</tr>
</tbody>
</table>

### Examples of Implementation in Mathematics Education

<table>
<thead>
<tr>
<th>Elements of Self-management Skills</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
</table>
| **Self-worth**                     | Learners:  
1. appreciate their positive learning attitudes or effort put in learning Mathematics  
2. identify their strength among the strands of Number and Algebra, Measures, Shape and Space and Date Handling  
3. reflect on their learning attitude to different mathematics topics as well as their mathematics ability attained and plan for improvement |
| Goal setting and tracking          | Learners:  
1. set learning goal in learning Mathematics (e.g. understanding a difficult concept/theorem)  
2. review their learning goals by considering their assessment results in Mathematics  
3. consult teachers for advice on their learning goals in mathematics learning and monitor the progress for meeting the goals |
| Decision making                    | Learners:  
1. explain their solutions logically  
2. choose appropriate data collection methods for statistical surveys  
3. choose strategies for solving mathematical problems by considering various factors such as tools available, time constraints, presentation of solutions and whether a general solution is available |
<table>
<thead>
<tr>
<th>Elements of Self-management Skills</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
</table>
| Confidence, resilience and adaptability | Learners:  
1. verify their computation results to ensure their accuracy  
2. apply mathematics to solve problems independently, being self-reliant and confident  
3. Examine and correct wrong calculations, algebraic manipulations or deductive proofs, discover incorrect concepts behind and correct their understanding on the related topics |
| Appropriate expression of emotions | Learners:  
1. communicate their own feeling to others when facing failure in solving problems  
2. express the sense of accomplishment after solving a difficult question  
3. release one's negative feeling of failure by taking a break when they get stuck by a difficult problem |
| Managing resources | Learners:  
1. bring along with set squares, rulers, calculators or other learning tools to the lessons when required  
2. use calculators, or application software, to facilitate their enquiry activities  
3. apply suitable application software in project work, for sharing data with groupmates, conducting simulations, plotting graph, etc. |
| Keeping promises to others | Learners:  
1. solve difficult problems with persistence  
2. discuss with groupmates of project work to arrange jobs and complete their own parts on schedule  
3. inform groupmates of project work when faced with unexpected circumstances, such as break down of computer or loss of data |
| Self-discipline | Learners:  
1. work neatly and tidily in accomplishing tasks (e.g. statistical projects) and doing mathematical problems (e.g. drawing geometrical figures with rulers and pencils)  
2. participate and focus on learning tasks even not being closely overseen by teachers  
3. persist in following learning plans, overcome difficulties and distractions, enjoy the outcomes of studies and appreciate their own effort when goals are achieved |
### Elements of Self-management Skills

<table>
<thead>
<tr>
<th>Reflective Practice</th>
<th>Examples of Implementation in Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Learners:</td>
</tr>
<tr>
<td></td>
<td>1. make use of assessment results to understand their learning progress</td>
</tr>
<tr>
<td></td>
<td>2. self-evaluate their own strengths and weaknesses in mathematics learning</td>
</tr>
<tr>
<td></td>
<td>3. evaluate and make use of others’ opinions in accomplishing tasks (e.g. different ways of collecting data, different strategies for solving geometrical problems) in order to improve the problem solving methods applied</td>
</tr>
</tbody>
</table>
Appendix 3 : Integrative Application of Generic Skills

When completing a more complicated learning task, the generic skills are often applied in an integrative manner rather than in isolation. Below shows the descriptors of two such integrative applications of generic skills, namely “collaborative problem solving skills” and “holistic thinking skills”. Examples on the applications of the skills in Mathematics are also provided after each set of the descriptors.

**Collaborative Problem Solving Skills**
Collaborative Problem Solving Skills refers to students’ ability to solve problems with synergised efforts through effective division of labour, as well as incorporation of information from multiple sources of knowledge, perspectives and experiences. Compared to individual problem solving, collaborative problem solving has distinct advantages because it enhances the creativity and quality of solutions through stimulation brought by the ideas of other group members\(^\text{16}\). In the 21st Century, it is particularly important for people with different perspectives and talents to solve problem as a team with the effective use of communication technology.

The expected achievements of the learners in “Collaborative Problem Solving Skills” are classified according to level of mastery.

Learners will learn to

<table>
<thead>
<tr>
<th>Beginning</th>
<th>Developing</th>
<th>Mastering</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Collaboration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• be ready to act responsively and reach the goals with team members</td>
<td>• share the other team members’ perspectives on the problem and establish a common understanding</td>
<td>• treasure working as a team and take initiative to foster synergy for attaining the team goals</td>
</tr>
<tr>
<td>• follow the rules and instructions set for the team work</td>
<td>• identify and capitalise on the talents and potential of members</td>
<td>• show mutual respect and support when dealing with difficult people and situations</td>
</tr>
<tr>
<td>• participate actively in the team and contribute to achievement of the team goals</td>
<td>• be able to work with different people and accept the adjustments to plans or roles in changing situations</td>
<td>• take initiative to propose plans or make adjustments to the plans and roles in changing situations</td>
</tr>
</tbody>
</table>

\(^{16}\) Adapted from OECD 2015 PISA Framework
### Communication
- comprehend messages with an open mind and ask questions to identify the problem and team goals
- express oneself clearly to team members by verbal and/or non-verbal means
- show courage in sharing new or unconventional ideas
- ask meaningful questions that clarify the vision, goals and viewpoints for better solutions
- respond specifically to queries raised during the problem solving process
- enhance mutual understanding through effective means and with a respectful attitude
- negotiate for consensus and foster a cooperative atmosphere to resolve conflicts
- take the initiative in introducing new resources and exploring further ideas to facilitate the team to progress further

### Problem solving
- propose solutions or strategies to solve a problem
- complete the task assigned to one’s role in the team
- select a problem solving strategy and develop an action plan
- execute actions that comply with the planned distribution of roles and make adjustments when necessary
- select a problem solving strategy and prepare alternative plans
- monitor and evaluate individual and team effectiveness

---

**Example of implementation in Mathematics**

**Key Stage:** 1

**Learning Unit:** Fractions (I)

**Description of Activity:**

Students were required to form groups and to design a vegetable garden in the school backyard. The piece of land has dimensions $6 \text{ m} \times 6 \text{ m}$ and has already been divided into 36 square pieces of dimensions $1 \text{ m} \times 1 \text{ m}$. Students needed to decide which pieces they would plant with which vegetables based on the following requirements: one-fourth of the pieces for tomatoes, at least 6 pieces for lettuce, double as much as lettuce for corn, the remaining for peppers, except for some pieces reserved as one or several paths to reach each kind of vegetable because the land is only accessible from one of the four sides.
The lesson is designed to promote the integrative use of generic skills. The activity particularly facilitates the development of **collaborative problem solving skills**, which involve the application of collaboration, communication and problem solving skills. This learning activity emphasises synergy among learners, team work and communication to enhance creativity and the quality of solutions.

<table>
<thead>
<tr>
<th>Learning Activities</th>
<th>Generic Skills Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The groups are guided to calculate how many pieces of small square land and also the fractions of land for planting each vegetable in the garden under the given requirements.</td>
<td>Problem solving</td>
</tr>
<tr>
<td>2. In each group, learners</td>
<td>Collaboration and communication</td>
</tr>
<tr>
<td>• share their design ideas and explain how the ideas satisfies the requirements;</td>
<td>Problem solving</td>
</tr>
<tr>
<td>• discuss and plan a vegetable garden for the group;</td>
<td></td>
</tr>
<tr>
<td>• discuss ways of enhancing their garden, such as the shape and position of each vegetable area;</td>
<td></td>
</tr>
<tr>
<td>• evaluate the possible ways of enhancing their garden based on some criteria compromised by members;</td>
<td></td>
</tr>
<tr>
<td>• choose a plan, draw the garden for presentation and label each vegetable area.</td>
<td></td>
</tr>
<tr>
<td>3. The groups report their design of the vegetable garden and explain their rationale behind their planning. They could use fractions to represent the quantities in their explanation.</td>
<td>Communication</td>
</tr>
<tr>
<td>4. Based on the ideas generated from the sharing of other groups, each group re-fines their design, such as adjusts the shape of the area for each vegetable.</td>
<td>Collaboration and problem solving</td>
</tr>
</tbody>
</table>
Holistic Thinking Skills

Critical Thinking Skills, Creativity and Problem Solving Skills are conventionally categorised as Higher Order Thinking Skills. These three skills can be combined and employed integratively as Holistic Thinking Skills to deal with complex issues. Holistic Thinking Skills enable learners to deploy Critical Thinking Skills to assess the validity of given information, Creativity to explore other possibilities, and Problem Solving Skills to examine the feasibility of each alternative.

The expected achievements of learners in “holistic thinking skills” are classified according to level of mastery.

Learners will learn to

<table>
<thead>
<tr>
<th></th>
<th>Beginning</th>
<th>Developing</th>
<th>Mastering</th>
</tr>
</thead>
</table>
| **Critical Thinking: enquiring and assessing** | • ask questions to explore matters that attract interest  
• identify main ideas and clarify meaning in information | • pose questions to explore issues related to their immediate contexts  
• comprehend complementary and contradictory information | • pose questions that probe complex and abstract ideas about issues beyond local context and contemporary period  
• synthesise points from complementary and contradictory information |
| **Creativity: generating** | • come up with new ideas by linking imagination and reality  
• create analogies by matching two ideas  
• brainstorm suggestions | • draw parallels between known and new scenarios and use ideas, patterns and trends to consider new possibilities  
• produce alternative or unconventional solutions  
• suspend judgment to consider alternative ideas and actions | • generate a large number of raw ideas  
• combine good ideas to make even better ideas  
• use existing knowledge in a novel way  
• temporarily suspend pragmatic and rational thinking to allow new possibilities to emerge |
| **Critical Thinking and Problem Solving: analysing and comparing** | • realise real world constraints in drafting solutions | • estimate the cost and benefit of possible solutions from multiple | • compare the possible outcomes of each solution against both their own and |
• compare advantages and limitations of various solutions

<table>
<thead>
<tr>
<th>perspectives</th>
<th>rate and select solutions according to criteria, such as feasibility, desirability and ethical considerations</th>
<th>prevailing values</th>
</tr>
</thead>
<tbody>
<tr>
<td>• mediate opposing viewpoints and acknowledge the limitations of one’s view</td>
<td>• synthesise different considerations into a solution</td>
<td></td>
</tr>
</tbody>
</table>

**Creativity and Problem Solving: predicting and fine-tuning**

<table>
<thead>
<tr>
<th>ask “what if” questions</th>
<th>consider ways of tackling possible consequences</th>
<th>make adjustments to avoid possible pitfalls (e.g. ambiguity, stereotyping and misunderstandings) in planning and presentation of solutions</th>
<th>fine tune plans with reference to new developments</th>
</tr>
</thead>
<tbody>
<tr>
<td>• anticipate adverse impacts and suggest precautionary or compensatory measures accordingly</td>
<td>• consider alternative courses of action in changing situations</td>
<td>• consider ways of tackling possible consequences</td>
<td>• be sensitive to stakeholders’ reactions</td>
</tr>
</tbody>
</table>

**Problem Solving: executing and monitoring**

<table>
<thead>
<tr>
<th>choose a solution and devise an implementation plan, using support and advice given</th>
<th>execute the plan, monitor progress and revise the strategies when necessary</th>
<th>monitor the progress with established check points or criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>• turn the plan into workable parts with measures for implementation</td>
<td>• realise the adverse effect of over-reacting and using emotional words</td>
<td>• suggest ways to catch up with delays or optimise the results</td>
</tr>
<tr>
<td>• manage over-reactions and strong emotions</td>
<td></td>
<td>• manage over-reactions and strong emotions</td>
</tr>
</tbody>
</table>

**Problem Solving and Critical Thinking: evaluating and reflecting**

<table>
<thead>
<tr>
<th>reflect on whether the task is accomplished</th>
<th>evaluate the quality of outcomes and the solution process</th>
<th>evaluate the effectiveness of solutions with due regard for positive values</th>
</tr>
</thead>
<tbody>
<tr>
<td>• be open to comments and feedback</td>
<td>• invite and evaluate feedback</td>
<td>• anticipate possible problems arising from the solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• make judicious use of comments and feedback</td>
</tr>
</tbody>
</table>
Example of implementation in Mathematics

Key Stage: 3

Learning Unit: 3-D Figure, Mensuration

Description of Activity:

Students are required to construct right circular cylinders using a piece of 20cm × 20cm paper. They need to formulate possible strategies of making right circular cylinders with the largest surface area from a piece of square-shaped paper.

The teacher can first familiarise students with the way to calculate the surface area of a right circular cylinder, and guide learners to compare the surface areas of the cylinders constructed. The activity facilitates students’ development of holistic thinking skills, which involves the application of critical thinking skills to assess and synthesise given information, creativity to explore possibilities, and problem solving skills to examine the feasibility of each alternative of making the cylinder having the largest possible surface area.

<table>
<thead>
<tr>
<th>Learning Activities</th>
<th>Generic Skills Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Each student brainstorm and explores as many designs as possible the nets of right circular cylinders on the given pieces of square-shaped paper.</td>
<td>Creativity</td>
</tr>
<tr>
<td>2. Each student • calculates and compares the surface areas of cylinders designed; • shares with his/her labour the best design explored, discusses and compares the designs from multiple perspectives, e.g. whether a right circular cylinder can really be built from the net; • makes adjustments if needed to finalise the net designed.</td>
<td>Critical thinking and problem solving</td>
</tr>
<tr>
<td>3. The teacher chooses some students to present their nets designed. Students are required to compare their nets with those of other classmates. Students could also share</td>
<td>Problem solving</td>
</tr>
</tbody>
</table>
with their classmates how they come up with the design, learn from other students and fine-tune their problem-solving strategies.
Appendix 4 : Suggested Websites and Application Software (Apps)
for Learning and Teaching of Mathematics

Below listed some suggested websites and application software for supporting the learning and teaching of Mathematics. Some of them can be accessed using tablet computers. For a more comprehensive list of websites, please refer to the website of Mathematics Education Section (http://www.edb.gov.hk/en/curriculum-development/kla/ma/link/index.html).

<table>
<thead>
<tr>
<th>Suggested website/apps</th>
<th>Primary (P)</th>
<th>Secondary (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(1) Apps of e-geoboard</strong>&lt;br&gt;Geoboard, by The Math Learning Center</td>
<td></td>
<td>P</td>
</tr>
<tr>
<td><strong>(2) Apps for measuring angles</strong>&lt;br&gt;Angle Meter (FREE)</td>
<td></td>
<td>P, S</td>
</tr>
<tr>
<td>iOS <a href="https://itunes.apple.com/hk/app/angle-meter-free/id436775826?mt=8">https://itunes.apple.com/hk/app/angle-meter-free/id436775826?mt=8</a></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(3) GeoGebra</strong> (multi function mathematics application software)&lt;br&gt;Website: <a href="https://www.geogebra.org">https://www.geogebra.org</a></td>
<td></td>
<td>P, S</td>
</tr>
<tr>
<td>Suggested website/apps</td>
<td>Primary (P) Secondary (S)</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------</td>
<td></td>
</tr>
</tbody>
</table>
| (4) **Mathway** (computer algebraic system with graphing tool)  
Website: [https://mathway.com/](https://mathway.com/)  
Apps:  
| (5) **Nrich enriching mathematics** (exploratory activities)  
Website: [http://nrich.maths.org/stemnrich](http://nrich.maths.org/stemnrich) | P, S |
| (6) **Thatquiz** (quiz on different topics)  
Website: [https://www.thatquiz.org/](https://www.thatquiz.org/) | P, S |
| (7) **Websites on STEM Education**  
**STEM Education**  
Website: [http://stem.edb.hkedcity.net/en/home/](http://stem.edb.hkedcity.net/en/home/)  
**Intel Education**  
(search for “Free teaching resources”)  
**STEM Learning**  
Website: [https://www.stem.org.uk](https://www.stem.org.uk)  
(search for “Primary mathematics resource packages”, access other resources by the link “RESOURCES”) | P, S |
| (8) **Center for Algebraic Thinking** (website on learning algebra)  
Website: [http://www.algebraicthinking.org/](http://www.algebraicthinking.org/)  
Apps provided: [http://algebraicthinking.org/tech#Apps](http://algebraicthinking.org/tech#Apps) | S |
| (9) **Desmos** (graphing tool)  
Website: [https://www.desmos.com/](https://www.desmos.com/)  
Apps:  
<table>
<thead>
<tr>
<th></th>
<th>Suggested website/apps</th>
<th>Primary (P) Secondary (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10)</td>
<td><strong>iCrosss Lite</strong> (3-D figure tool)</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>Apps:</td>
<td></td>
</tr>
<tr>
<td>(11)</td>
<td><strong>Minds of Modern Mathematics</strong> (introduce the development of Mathematics)</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>Apps:</td>
<td></td>
</tr>
<tr>
<td>(12)</td>
<td><strong>Sketchometry</strong> (geometric construction tool)</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>Apps:</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix 5: Notes on Interface for Junior Secondary Mathematics


#### Number and Algebra Strand

<table>
<thead>
<tr>
<th>Learning Unit</th>
<th>Notes on Interface</th>
</tr>
</thead>
</table>
| 1. Directed Numbers and the Number Line           | • At primary level, students learn the concepts of whole numbers, fractions and decimals, as well as their interconversions.  
• Recognising negative numbers is not required by the primary Mathematics curriculum.  
• At primary level, the mixed operations handled by students do not involve nested parentheses; while at junior secondary, the calculations of directed numbers may involve nested parentheses with different forms of brackets.  
• Teachers may revise with junior secondary students the function of the bracket notation and introduce multiple levels of brackets. |
| 2. Numerical Estimation                            | • At primary level, students acquire experience in estimating answers of calculations involving whole numbers, fractions, decimals and percentages.                                                                 |
| 3. Approximation and Errors                        | • At primary level, students learn to round off a whole number to a certain place, and a decimal to the nearest tenths or hundredths.                                                                                     |
| 4. Rational and Irrational Numbers                 | • At primary level, students are introduced with different representations of numbers, such as fractions and decimals. They have not recognised the concept of rational numbers.                                             |
| 5. Using Percentages                               | • At primary level, students learn the basic concepts of percentages and the conversion between percentages and decimals or fractions.                                                                                   |
| 6. More about Percentages                          | • They also learn to solve simple daily life problems involving percentages, e.g. problems about discount.                                                                                                         |
| 7. Rate and Ratio                                  | • Students’ experience in handling fractions at primary level could facilitate them to understand the concept of ratio.  
• Students acquired some understanding of rates through the learning of speed at P6.                                                                                                           |
| 8. Formulating Problems with Algebraic Language    | • At primary level, students learn to use letters to represent numbers and form simple expressions. This serves as a foundation for the construction of more complicated expressions using the language of algebra.  
• Regarding patterns of number sequences, the primary curriculum... |

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<table>
<thead>
<tr>
<th>Learning Unit</th>
<th>Notes on Interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Unit</td>
<td>does not require students to formally explore number patterns nor represent number patterns by algebraic language.</td>
</tr>
<tr>
<td></td>
<td>• Students who have learnt the enrichment topic “Number patterns” at primary level have come across some simple number patterns such as square numbers and triangle numbers.</td>
</tr>
<tr>
<td>9. Manipulations of Simple Polynomials</td>
<td>• This learning unit is not directly built on learning units of the primary curriculum.</td>
</tr>
<tr>
<td>10. Laws of Integral Indices</td>
<td>• Some junior secondary students might have learnt the enrichment topic “Squares and square roots” at the primary level, which is related to the learning of integral indices.</td>
</tr>
<tr>
<td>11. Factorisation of Simple Polynomials</td>
<td>• The concept of factors and H.C.F. of integers learnt at the primary level are bases for the learning of factorisation of polynomials in junior secondary. It would be beneficial to students in learning this unit and the handling of algebraic fractions in Learning Unit 15 “Formulas” if they are enriched with the concepts of prime numbers and prime factorisation.</td>
</tr>
<tr>
<td>12. Linear Equations in One Unknown</td>
<td>• At primary level, students learn to solve equations involving at most two steps in the solutions. Equations with the unknown as subtrahend, divisor or denominator are not required.</td>
</tr>
<tr>
<td>13. Linear Equations in Two Unknowns</td>
<td>• This learning unit is not directly built on learning units of the primary curriculum.</td>
</tr>
<tr>
<td>14. Identities</td>
<td>• This learning unit is not directly built on learning units of the primary curriculum.</td>
</tr>
<tr>
<td>15. Formulas</td>
<td>• At primary level, students learn the formulas of perimeters, the areas of some 2-D shapes and volumes of some 3-D shapes. These serve as a foundation for the strengthening of the concept of value substitution and the learning of change of subject.</td>
</tr>
<tr>
<td>16. Linear Inequalities in One Unknown</td>
<td>• At primary level, students learn to compare the values of whole numbers, fractions and decimals.</td>
</tr>
</tbody>
</table>

**Measures, Shape and Space Strand**

<table>
<thead>
<tr>
<th>Learning Unit</th>
<th>Notes on Interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>17. Estimation in Measurement</td>
<td>• At primary level, students acquire basic understanding in measurements involving real objects and simple 2-D shapes, and experience in performing estimation before measurements.</td>
</tr>
</tbody>
</table>
### Learning Unit: Simple Idea of Areas and Volumes
- At primary level, students learn the formula of areas of squares, rectangles, parallelograms, triangles and trapeziums, and volumes of cubes and cuboids.
- Students also learn to find the areas of simple shapes formed by combining and dissecting the aforesaid 2-D shapes.
- At primary level, students have not learnt the formula for finding the area of a circle.

### Learning Unit: More about Areas and Volumes
- At primary level, students learn to find the circumference of a circle by its radius/diameter.

### Learning Unit: Introduction to Geometry
- Students recognise the meaning of right angle, obtuse angle, acute angle, parallel lines, and perpendicular lines at primary level.
- Students in general have not learnt the degree measure of angles at primary level except those have studied the enrichment topic “Angle (degree)”.
- Students also acquire basic understanding on triangles, quadrilaterals, polygons, prisms, pyramids, circular cylinders, circular cones and spheres at primary level.
- At primary level, students recognise the different cross sections of prisms, pyramids, circular cylinders, circular cones and spheres.
- At primary level, students acquire the experience in using basic tools to draw parallel lines, perpendicular lines and polygons. They may also construct polygons using drinking straws, wooden sticks or geoboard.
- Students who have learnt the enrichment topic “Angle (degree)” at their primary level should be able to use protractors to measure and construct angles up to 360°.

### Learning Unit: Transformation and Symmetry
- At primary level, students develop an understanding of symmetrical shapes. They acquire experience in finding the lines of symmetry and in making symmetrical shapes.
- Some students may have learnt the enrichment topic “Rotational symmetry” at primary level.

### Learning Unit: Congruence and Similarity
- At primary level, students construct right angles, parallel lines, perpendicular lines, polygons and circles using tools like rulers, grids, and real objects.

### Learning Unit: Angles related with Lines and Rectilinear Figures
- This learning unit is not directly built on learning units of the primary curriculum.
### Learning Unit Notes on Interface

24. **More about 3-D Figures**
   - At primary level, students develop an understanding of symmetrical 2-D shapes, acquire experience in making symmetrical shapes and learn to find the lines of symmetry.
   - Those students who have studied the enrichment topic “Rotational symmetry” have recognised the concept of rotational symmetry in 2-D shapes and acquired experience in making 2-D rotational symmetrical shapes.
   - At primary level, students learn to make nets of cubes and cuboids.

25. **Simple Introduction to Deductive Geometry**
   - This learning unit is not directly built on learning units of the primary curriculum.

26. **Pythagoras' Theorem**
   - This learning unit is not directly built on learning units of the primary curriculum.

27. **Quadrilaterals**
   - At primary level, students acquire basic understanding of trapezium, parallelogram, rhombus, rectangle and square, such as the number of parallel opposite sides and the possessing of right angles.
   - At primary level, the inclusion relations between the quadrilaterals are not emphasised, e.g. a rectangle is not introduced as a parallelogram having a right angle.

28. **Introduction to Coordinates**
   - At primary level, students learn to use the 8-point compass bearing to describe directions and positions. They do not learn to describe the location of a point by coordinates.

29. **Coordinate Geometry of Straight Lines**
   - This learning unit is not directly built on learning units of the primary curriculum.

30. **Trigonometric Ratios and Using Trigonometry**
   - This learning unit is not directly built on learning units of the primary curriculum.

### Data Handling Strand

<table>
<thead>
<tr>
<th>Learning Unit</th>
<th>Notes on Learning and Teaching</th>
</tr>
</thead>
</table>
| 31. Introduction to Various Stages of Statistics | At primary level, students acquire experience in collecting data about students in the same class through counting directly the frequencies of options.  
At primary level, students also learn various types of statistical graphs and charts (pictograms, block graphs, bar charts and... |
<table>
<thead>
<tr>
<th>Learning Unit</th>
<th>Notes on Learning and Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>broken line graphs) for representing data.</td>
</tr>
<tr>
<td>32. Construction and Interpretation of Simple Diagrams and Graphs</td>
<td>• At primary level, students learn to use pictograms, block graphs, bar charts and broken line graphs to represent data.</td>
</tr>
<tr>
<td>33. Measures of Central Tendency</td>
<td>• At primary level, students learn to find the mean of a set of ungrouped data.</td>
</tr>
<tr>
<td>34. Simple Idea of Probability</td>
<td>• At primary level, students are not required to learn the concept of probability. Students who have learnt the enrichment topic “Chance” at primary level have acquired an elementary experience of the chance of happening of an event and learnt to describe it using phrases like certain, often, seldom, etc.</td>
</tr>
</tbody>
</table>
## Appendix 6: Learning and Teaching Resources List for Mathematics

(A) Learning and teaching materials developed by the EDB

<table>
<thead>
<tr>
<th>No.</th>
<th>Title / Description</th>
<th>CRC call no./Web link</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary (P1 – P6)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1 | Addenda Series for Primary Mathematics (Volume 1)  
| 2 | Addenda Series for Primary Mathematics (Volume 2)  
| 3 | Addenda Series for Primary Mathematics (Volume 3) (Chinese version only)  
This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities. | EDB 372.7044 X53 2004  
| 4 | Addenda Series for Primary Mathematics (Volume 4) (Chinese version only)  
This volume of the series is compiled according to the *Mathematics Curriculum Guide (P1 – P6)* (2000) to provide teachers with examples of learning and teaching activities. | EDB 372.7044 X53 2005  
| 5 | Addenda Series for Primary Mathematics (Volume 5) (Chinese version only)  
| 6 | Addenda Series for Primary Mathematics (Volume 6) (Chinese version only)  
<table>
<thead>
<tr>
<th>No.</th>
<th>Title / Description</th>
<th>CRC call no./Web link</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This volume of the series is compiled according to the <em>Mathematics Curriculum Guide</em> (P1 – P6) (2000) to provide teachers with examples of learning and teaching activities on developing students’ number sense.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This volume of the series is compiled according to the <em>Mathematics Curriculum Guide</em> (P1 – P6) (2000) to provide teachers with examples of learning and teaching activities on nurturing critical thinking skills and creativity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This volume of the series is compiled according to the <em>Mathematics Curriculum Guide</em> (P1 – P6) (2000) to provide teachers with examples of hands-on activities.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This booklet is published for sharing with teachers the ways to promote student learning through diversified modes of assessment.</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Remedial Teaching in Primary Mathematics (Chinese version only)</td>
<td>[<a href="http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Primary">http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Primary</a> Remedial/MPR index.htm](<a href="http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Primary">http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Primary</a> Remedial/MPR index.htm)</td>
</tr>
<tr>
<td></td>
<td>This booklet is published to assist schools to recognise students’ learning difficulties and to provide follow-up actions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This material aims at allowing students to learn the topics in the Sharp and Space Strand effectively through diversified activities and enjoy the learning process.</td>
<td>EDB 372.7044 K66 2000</td>
</tr>
<tr>
<td></td>
<td>This material is compiled for Key Stage 2, which covers the use of calculators for basic operations and conversions between fractions, decimals and percentages.</td>
<td>EDB 372.7044 Y86 1999</td>
</tr>
<tr>
<td>No.</td>
<td>Title / Description</td>
<td>CRC call no./Web link</td>
</tr>
<tr>
<td>-----</td>
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</tr>
<tr>
<td>14</td>
<td>Areas of Triangles (Chinese version only)</td>
<td>EDB 372.7044 S26 2002 <a href="http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/triangle_c/triangle_c/index.htm">http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/triangle_c/triangle_c/index.htm</a></td>
</tr>
<tr>
<td>15</td>
<td>Plane Figure can be fun (Chinese version only)</td>
<td>EDB 372.7044 P56 2002 <a href="http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Plane/Plane/index.htm">http://cd1.edb.hkedcity.net/cd/maths/tc/ref_res/Plane/Plane/index.htm</a></td>
</tr>
<tr>
<td>No.</td>
<td>Title / Description</td>
<td>CRC call no./Web link</td>
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</tr>
<tr>
<td></td>
<td>This learning and teaching package contains learning targets, learning objectives, notes on teaching and examples for the learning and teaching of the Measures, Shape and Space Strand in Key Stage 3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This learning and teaching package contains learning targets, learning objectives, notes on teaching and examples for the learning and teaching of the Data Handling Strand in Key Stage 3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This e-learning tool provides interactives activities of different scenarios, which facilitate students to learn the applications of percentages, coordinates, coordinate geometry of straight lines, approximation &amp; errors and laws of indices.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The package consists of learning and teaching aids to help students recognise the reflectional and rotational symmetries in cubes and regular tetrahedral.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Senior Secondary (S4 – S6)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The explanatory notes aim at further explicating:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• the requirements of the Learning Objectives of the Compulsory Part, Module 1 and Module 2;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• the strategies suggested for the teaching of the Compulsory Part, Module 1 and Module 2;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• the connections and structures among different Learning Units of the Compulsory Part, Module 1 and Module 2;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• the context of development from different key stages, such as Key Stage 3, to the Compulsory Part; and</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Title / Description</td>
<td>CRC call no./Web link</td>
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</tr>
<tr>
<td></td>
<td>the curriculum articulation between the Compulsory Part and the Extended Part.</td>
<td></td>
</tr>
</tbody>
</table>

### Cross Levels

<table>
<thead>
<tr>
<th>No.</th>
<th>Title / Description</th>
<th>CRC call no./Web link</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This e-learning tool aims to help teachers explain the concepts of angles in 3-dimensional figures with the use of animated examples.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The aim of this booklet is to introduce to teachers how open-ended questions can be used to assess learning.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This booklet aims at assisting schools in promoting remedial teaching in secondary Mathematics by introducing the ways of implementation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. NSSMC Compulsory Part</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. NSSMC Module 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. NSSMC Module 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Mathematics Gems</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. Applications of Mathematics : Image Processing - Revolution of Matrix</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6. Applications of Mathematics : Investment Portfolios and Market Efficiency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7. Applications of Mathematics : Analysis of Genes and Protein</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8. Probability Kaleidoscope</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9. Portrait of a Middle-aged Mathematics Teacher</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11. Looking into the Conceptions of Mathematics and Mathematics Teaching from “Introduction of Calculus”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14. Mathematics teachers are no longer intimidated by students’ questions - the mathematics knowledge required for primary and secondary teachers</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Title / Description</td>
<td>CRC call no./Web link</td>
</tr>
<tr>
<td>-----</td>
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<td>-----------------------</td>
</tr>
<tr>
<td>17.</td>
<td>Paper folding and Mathematics</td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>Universal Scaling Laws</td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td>Three Crises in Mathematics</td>
<td></td>
</tr>
</tbody>
</table>

The booklets of this series aim at providing reference materials on the learning and teaching of Mathematics to teachers.


This booklet aims at providing an English-Chinese glossary of commonly used mathematical terms in primary and secondary Mathematics for teachers’ reference.

(B) Other useful resources / web links

<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Web link / Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Inquiry in Mathematics Education</td>
<td>EDB 510.71 I57 2010 / Booklet</td>
</tr>
<tr>
<td>6</td>
<td>ETV</td>
<td><a href="http://resources.hkedcity.net/etv/">http://resources.hkedcity.net/etv/</a></td>
</tr>
</tbody>
</table>
(C) Community Resources

<table>
<thead>
<tr>
<th>No.</th>
<th>Organisation</th>
<th>Web link</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Census and Statistics Department</td>
<td><a href="http://www.info.gov.hk/censtatd">http://www.info.gov.hk/censtatd</a></td>
</tr>
<tr>
<td>2</td>
<td>Hong Kong Association for Mathematics Education</td>
<td><a href="http://www.hkame.org.hk">http://www.hkame.org.hk</a></td>
</tr>
<tr>
<td>3</td>
<td>Hong Kong Association for Science and Mathematics Education</td>
<td><a href="http://www.hkasme.org">http://www.hkasme.org</a></td>
</tr>
<tr>
<td>4</td>
<td>Hong Kong Mathematical Society</td>
<td><a href="http://www.hkms.org.hk">http://www.hkms.org.hk</a></td>
</tr>
<tr>
<td>5</td>
<td>Hong Kong Statistical Society</td>
<td><a href="http://www.hkss.org.hk">http://www.hkss.org.hk</a></td>
</tr>
<tr>
<td>6</td>
<td>International Mathematical Olympiad Hong Kong Committee</td>
<td><a href="http://www.imohkc.org.hk">http://www.imohkc.org.hk</a></td>
</tr>
</tbody>
</table>
### Appendix 7: List of Collaborative Research and Development (“Seed”) Projects for Mathematics

<table>
<thead>
<tr>
<th>Year</th>
<th>&quot;Seed&quot; Project Title</th>
<th>Project Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001/2004</td>
<td>Developing Thinking Abilities through Primary Mathematics Curriculum</td>
<td>MA0101</td>
</tr>
<tr>
<td>2003/2004</td>
<td>Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Primary Level</td>
<td>MA0103</td>
</tr>
<tr>
<td>2004/2005</td>
<td>Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics</td>
<td>MA0104</td>
</tr>
<tr>
<td>2005/2006</td>
<td>Developing Students' Number Sense through Primary Mathematics Curriculum</td>
<td>MA0105</td>
</tr>
<tr>
<td></td>
<td>Strengthening Teachers Mathematics Pedagogical Content Knowledge through Peer Lesson Observation</td>
<td>MA0205</td>
</tr>
<tr>
<td></td>
<td>Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics at Primary Level</td>
<td>MA0405</td>
</tr>
<tr>
<td>2006/2007</td>
<td>Developing Students' Number Sense through Primary Mathematics Curriculum</td>
<td>MA0106</td>
</tr>
<tr>
<td></td>
<td>Assessment for Learning on Primary Mathematics</td>
<td>MA0206</td>
</tr>
<tr>
<td>2007/2008</td>
<td>Assessment for Learning on Primary Mathematics</td>
<td>MA0307</td>
</tr>
<tr>
<td></td>
<td>Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space</td>
<td>MA0107</td>
</tr>
<tr>
<td>2008/2009</td>
<td>Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space</td>
<td>MA0108</td>
</tr>
<tr>
<td></td>
<td>Alternative Assessment for Learning in Mathematics : Performance Task Assessment</td>
<td>MA0408</td>
</tr>
<tr>
<td>2009/2010</td>
<td>Fostering Students' Critical Thinking and Creativity through Primary Mathematics Curriculum in the Dimensions of Measures, Shape and Space</td>
<td>MA0109</td>
</tr>
<tr>
<td></td>
<td>Alternative Assessment for Learning in Mathematics : Performance Task Assessment</td>
<td>MA0409</td>
</tr>
<tr>
<td>2010/2011</td>
<td>Fostering Students' Critical Thinking and Creativity through the Primary Mathematics Curriculum in the Dimensions of Measures, and Shape and Space</td>
<td>MA0110</td>
</tr>
<tr>
<td>2011/2012</td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape &amp; Space</td>
<td>MA0111</td>
</tr>
<tr>
<td>2012/2013</td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape &amp; Space</td>
<td>MA0112</td>
</tr>
<tr>
<td>2013/2014</td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Shape &amp; Space</td>
<td>MA0113</td>
</tr>
<tr>
<td>2014/2015</td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Dimension of Measures</td>
<td>MA0114</td>
</tr>
<tr>
<td>2015/2016</td>
<td>Exploration and Development of Learning and Teaching Strategies in the Dimension of Measures</td>
<td>MA0115</td>
</tr>
<tr>
<td>2016/2017</td>
<td>Exploration and Development of Learning and Teaching Strategies in the Dimension of Measures</td>
<td>MA0116</td>
</tr>
<tr>
<td></td>
<td>Exploration and Development of Effective Strategies for Promoting and Implementing STEM Education in Primary Mathematics</td>
<td>MA0316</td>
</tr>
<tr>
<td>Year</td>
<td>“Seed” Project Title</td>
<td>Project Code</td>
</tr>
<tr>
<td>------------</td>
<td>---------------------------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>2002/2003</td>
<td>Open-ended Assessment in the Learning and Teaching of Secondary Mathematics</td>
<td>MA0202</td>
</tr>
<tr>
<td></td>
<td>Learning and Teaching of Problematic Topics in the Revised Secondary Mathematics Curriculum (1999)</td>
<td>MA0303</td>
</tr>
<tr>
<td>2004/2005</td>
<td>Developing and Using a Learning Outcomes Framework to Enhance Learning and Teaching of Mathematics</td>
<td>MA0104</td>
</tr>
<tr>
<td></td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Mathematics Curriculum at KS4</td>
<td>MA0204</td>
</tr>
<tr>
<td></td>
<td>Exploration and Development of Effective Learning and Teaching Strategies in the Mathematics Curriculum at KS4</td>
<td>MA0305</td>
</tr>
<tr>
<td></td>
<td>School-based Assessments of Secondary Mathematics</td>
<td>MA0605</td>
</tr>
<tr>
<td>2006/2007</td>
<td>Diversified Modes of Assessments in Mathematics</td>
<td>MA0306</td>
</tr>
<tr>
<td>2007/2008</td>
<td>Diversified Modes of Assessments in Mathematics</td>
<td>MA0407</td>
</tr>
<tr>
<td></td>
<td>Incorporation of History of Mathematics into Learning and Teaching</td>
<td>MA0207</td>
</tr>
<tr>
<td>2008/2009</td>
<td>Investigation and Inquiry: Learning and Teaching of Mathematics through Modelling and Experiments</td>
<td>MA0208</td>
</tr>
<tr>
<td></td>
<td>Interface on Learning and Teaching between Primary and Secondary Mathematics</td>
<td>MA0308</td>
</tr>
<tr>
<td>2009/2010</td>
<td>Interface on Learning and Teaching between Primary and Secondary Mathematics</td>
<td>MA0309</td>
</tr>
<tr>
<td></td>
<td>Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3</td>
<td>MA0209</td>
</tr>
<tr>
<td></td>
<td>Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices</td>
<td>MA0509</td>
</tr>
<tr>
<td>2010/2011</td>
<td>Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3</td>
<td>MA0210</td>
</tr>
<tr>
<td></td>
<td>Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices</td>
<td>MA0310</td>
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<td>Exploration and Development of Effective Learning, Teaching and Assessment Strategies for Geometry at KS3</td>
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<td>Investigation and Development of Effective Strategies that Turn the Rationale behind the Internal School-based Assessment in the Senior Secondary Mathematics Curriculum into Classroom Practices</td>
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<td>Exploration and Development of Effective Self-directed Learning Strategies in Junior Secondary Mathematics</td>
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<td>Exploration and Development of Effective Strategies for Promoting and Implementing STEM Education in Secondary Mathematics</td>
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Membership List
Membership of the Curriculum Development Council Committee on Mathematics Education (from September 2015 to August 2017)

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NTHYK Yuen Long District Sec. School
(Member from 1.9.2015, Chairperson from 1.9.2016)

Mr SUM Sing-wah
Buddhist Kok Kwong Secondary School
(from 1.9.2015 to 31.8.2016)

Vice-Chairperson: Mr WAI Kwok-keung
Mathematics Education Section, Curriculum Development Institute, Education Bureau
(from 16.2.2016)

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Ebenezer New Hope School
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The Chinese University of Hong Kong
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<td>School Heads</td>
<td>Mr MOK Sui-kei</td>
<td>Po Leung Kuk C. W. Chu College</td>
<td>6.10.2016</td>
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<td>Mr SHUM Yiu-kwong</td>
<td>F.D.B.W.A. Chow Chin Yau School</td>
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<td>School Teachers</td>
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<td>STFA Cheng Yu Tung Secondary School</td>
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<td>Mr SIU Kwok-leong</td>
<td>Ho Ngai College (Sponsored by Sik Sik Yuen)</td>
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<td>Mr TANG Hok-shu</td>
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<td>Ex-officio Members</td>
<td>Mr NG Siu-kai</td>
<td>Quality Assurance Sections, Quality Assurance &amp; School-based Support Division, Education Bureau</td>
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<td>Hong Kong Examinations and Assessment Authority</td>
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<td>Secretary</td>
<td>Dr NG Yui-kin</td>
<td>Mathematics Education Section, Curriculum Development Institute, Education Bureau</td>
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